Business Analytics for Management Decision
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Lecture – 43
Prescriptive Analytics (Contd.)

Hello everybody. This is Rudra here. Welcome to BMD lecture series and today, we will continue with the prescriptive analytics and that too the coverage on linear programming problem. In the last lecture we have we have highlighted you know the kind of you know problem structure, the kind of you know linear programming model and the kind of you know solutions the way we look for the optimality and we will continue you know from this particular you know last lecture and in this last lecture we had the problem like this.

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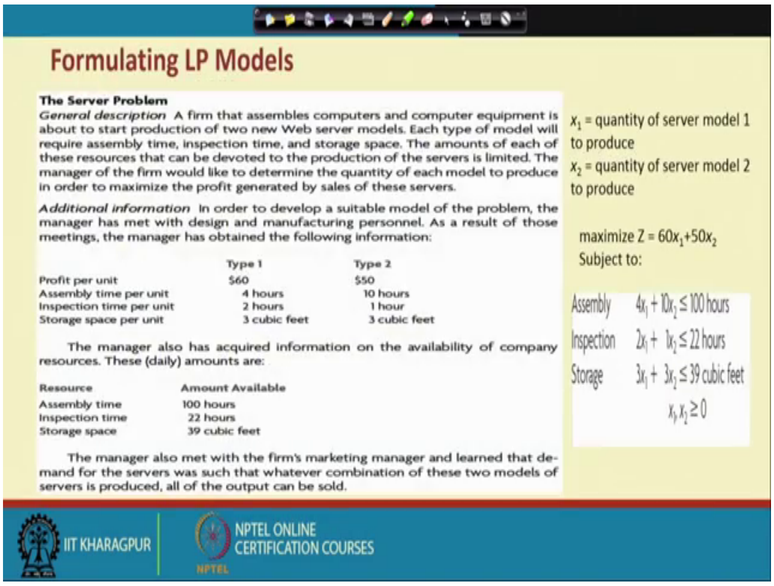

And, then we like to look for you know we like for the optimum solution corresponding to these problems.

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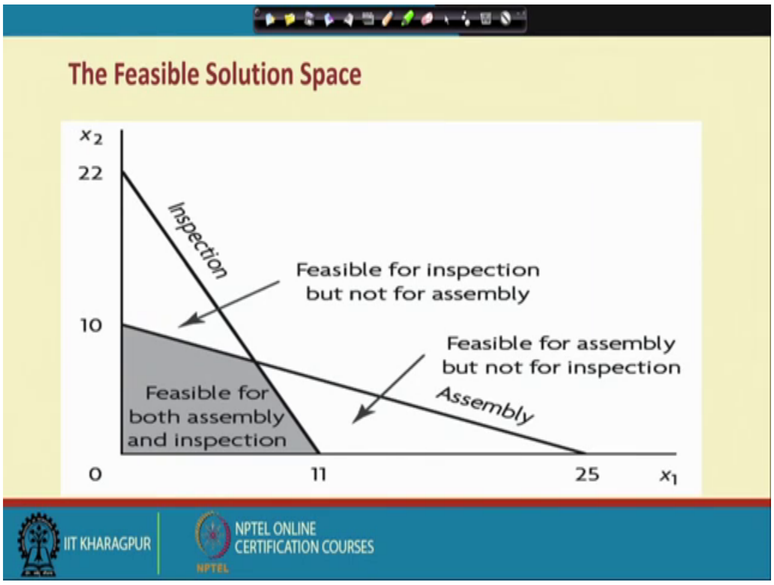

And, then we have a structure like this, that means technically we have three constraints corresponding to the objective functions.

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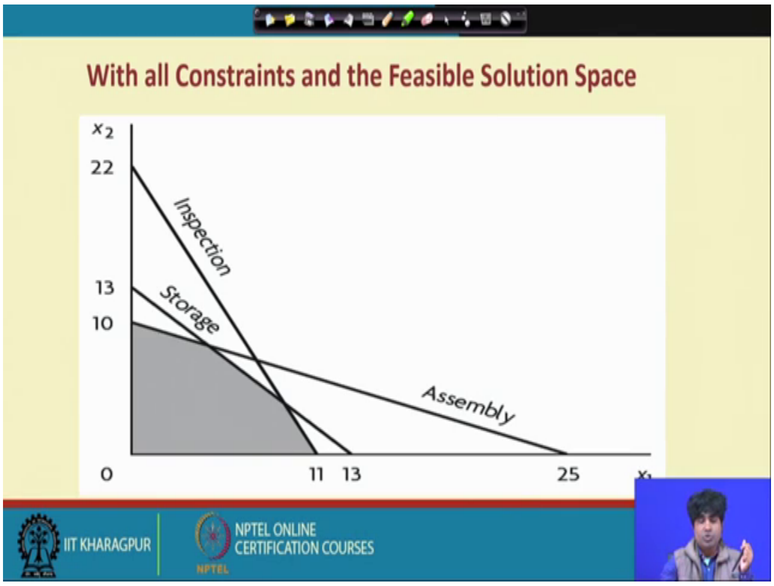

And, then here we are actually having three different constraints and adding all these three constraints we have actually five different you know corner points. And, then we are looking for the optimum solutions, that means, technically we like to know which particular corner point will give you the a efficient solution or optimum solution as per the a particular you know business requirement.

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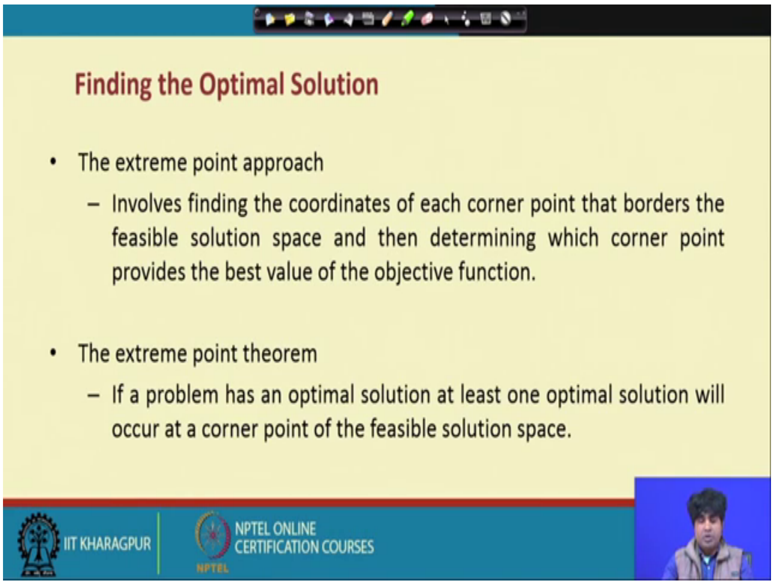

So, now while searching for you know optimum solution, so, we have actually structure called as you know extreme point approach. It involves you know the kind of you know the coordinates of each corner point that borders the feasible solution space and then determining which corner point provides the best value of the objective function. That means, by default it will give you the kind of you know various alternatives and one particular option will be the best as per the particular you know business requirement. If a particular problem has no optimum solutions you know then you know the constraints may not be actually having a means some kind of inconsistent is there.

So, if a problem has an optimum solution then at least one optimal solution will occur at a corner point of the feasible solution space. So, that means, if the all the constraints are you know consistent then obviously, so, we will have actually kind of you know optimum solutions, but if the if you say think that you know there is no optimum solution, that means, simple message is that you know the constraints are not you know consistent which will address in the later stage.

But in the in the mean times in this particular problem all these things are you know very consistent and as a result we have already got the feasible zones and within the feasible zones we have the kind of you know various a you know corner points and then we are looking for the particular you know solutions which can have the optimum value.

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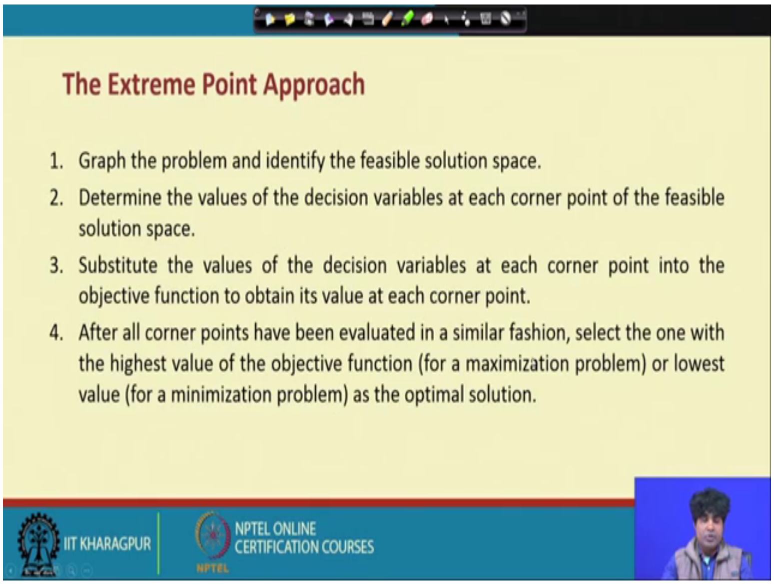

So, in the case of you know extreme point approach, so, graphically actually a you know what will you do so, every corner point will give you the you know quantity values of the decision variables. So, now, we like to put all these values of the decision variable to the objective function then you will check where the values of the objective function will be at the a highest where you know accordingly that will be the kind of you know solution, ok.

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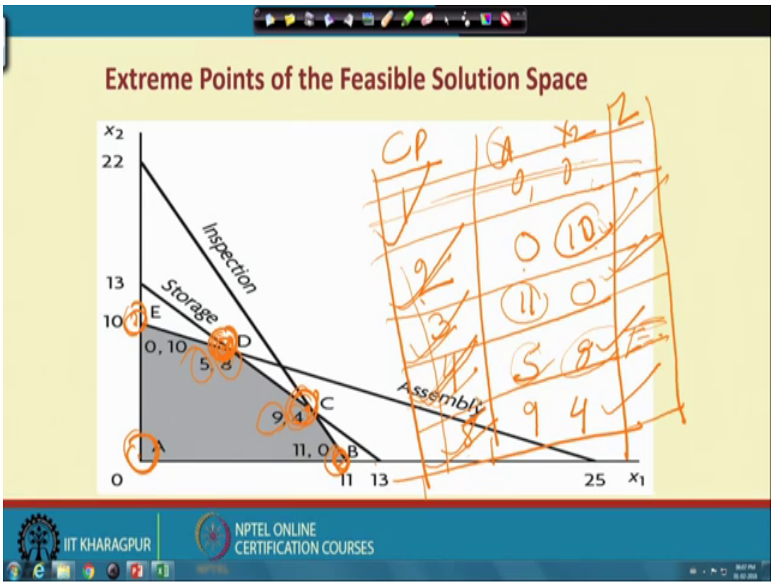

So, now, corresponding to the previous you know you know discussion. So, here you see this is actually the one particular corner point and in this contest the values of the x 1 and x 2 are you know 0, 0 and in this case a this is one corner point where x 1 is having a positive value x 2 equal to 0, this corner point where x 1 is 0 x 2 is having positive point. Now, this is one corner point where actually both x 1 and x 2 will be positive and this is another corner point where both x 1 and x 2 will be positive.

So, that means, what should be the exact point between you know this point and this point that depends upon the intersection of you know two straight line that is two constraint. So, that means, these this is a kind of you know corner point where the storage constraints and the assembly constraints are you know intersecting each other. So, as a result so, these two equations can be simplify and then we can get the values of you know x 1 and x 2. So, after simplification so, x 1 is becoming 5 here, the x 2 is coming 8 here and in this case in this corner point, so this is the intersection of you know storage and the inspection and once you solve these two equations so, we have here x 1 equal to 9 and x 2 equal to 4.

So that means so, , so, we have we have one corner point let us say you know this is the corner point. So, for the first case, so, we have actually 0, 0 and the second case we have x 2 equal to x 1 equal to 0 x 2 equal to 10 and third case we have x 1 equal to 11 and x 2 equal to 0, in the fourth case we have x 1 equal to 5 and x 2 equal to 8 and in the fifth case we have x 1 equal to 9 and we have x 2 equal to 4. So, that means, now this is the you know summary sheet which we can have from this particular you know graphic approach.

So, that means, technically, so, this is one corner point this is another corner point this is another corner point this is another corner point and this is another that means, out of all these five so, one would be the final optimum solution. So, corresponding you know first case we have x 1 equal to 0 so, that means, technically this is x 1 and this is x 2. So, these are the values of the decision variable x 1 and these are the values of the decision variable for x 2 and this is what the combination.

We cannot have the option 0, 10 or 0, 8 like this. Of course, there are you know plenty of option here, but here the particular choice will be the kind of you know combination. So, that means, technically either you choose 1 or you can choose 2 or you can choose 3 or you can choose 4 or you can choose 5. So, that means, technically a you have to choose a particular you know option some. So, if you choose 2 then by default so, x 1 equal to 0 and x 2 equal to 10 you know no way you can take 0, here and you know 0 here or you know 10 here and 11 here. So, with the process we have actually obtained this particular you know you know summer sheet. So, that means, out of you know all these five cases one particular case will be the final optimum solutions.

So, now which one which particular case will be the final choice as per your as per our you know requirement that exclusively depends upon our you know Z value. So, that means, technically so, we like to you know find out Z values corresponding to case 1 where x 1 equal to 0 and x 2 equal to 0 and again you go to the x 2 we find out Z value where, x 1 equal to 0 x 2 equal to 10 and against we go to the case 3 where x 1 equal to 11, x 2 equal to 0 and then we like to have a Z value here and go to case 4 where x 1 equal to 5, x 2 equal to 8 and then we can find out a Z value here for case 4 and then in the case 5 we have 9 here in the case of x 1 and in the case of x 2 here 4, so, we can have the Z value.

So, now, you have to just check all these you know Z values and where the Z value is the highest that will be the case you know which you can declare that you know optimal solution while searching for optimality and we can say that you know the optimum solution can be address in two different ways; one is called as you know economical optimal solution and you know social optimal solutions. So, that means technically or means, you know we like to know what is the actually difference between the economic optimum and social optimum.

In the case of you know economic optimum so, profit is the prime goals we have no you know kind of you know issue whether we like to go for you know only one kind of you know production either x 1 or x 2 and we like to check you know which particular combination having highest profit only. So, that means, let us say in this case either we can go for you know x 2 and then x 1 can be 0 and in this case either you can go for x 1 and x 2 can be 0. So, that means, technically if this 0, 0 cannot be the proper combination because where there is no x 1 no x 2.

So, this cannot be the you know efficient solution or you know optimum solution for any kind of you know business you know problem. So, this can be a possible solution, this can be a possible solution, this can be a possible solution, this can be a possible solution.

That means if you skip the case 1, so, we have four options and out of the four options the first two that is the case 2 and case 3. So, this is called as you know economic optimum provided Z should be a you know highest value compared to other option if you if you choose the case 2 then you know Z should be a Z should have you know high value compared to case 3, case 4 and case 5. Similarly, if you choose 3 then Z value should be a you know high compared to case 2, case 4, case 5.

By default you know case 2, case 3, case 4, case 5, Z value is a high compared to case 1, because in the case of you know case 1 x 1 equal to 0 and x 2 equal to 0 and obviously, in these four cases 2, 3, 4, 5, so, if Z will be you know for the case you know case 2 or you know case 3 so then you know it is called as you know economic optimum where we are actually going for the kind of you know one by one production and keeping the other one.

And, if you know compare with economic you know optimum to social optimum and this you know 4 and 5 can be the right choice, where both x 1 and x 2 is operative and then provided you know means we can choose that particular combination provided Z value should be highest compared to other option. For instance if you choose the case 4 then the Z value of case 4 should be a should be higher than case 2, case 3 and case 5 and against if you choose case 5 then Z value of case 5 should be higher than to case 4, case 3 and case 2. Obviously, it will be higher than to case 1.

So, that means, you know finally, we have no idea which one is the right choice that actually depends upon the Z value, but altogether if the optimal solution will be either 4 or 5 then this is called as you know both economic optimum and social optimum, but if the final choice will be either 2 or 3 then it is called as you know economic optimum not the social optimum because we are actually considering only one production and skipping the other productions.

Obviously, since it is a kind of you know mathematical modeling and through technique or through software we are the we are looking for the solution and the software or algorithms will be operative as per the objective only.

But, you know it is you to you know you know find out the kind of you know structure or you can give some kind of you know compromise so that you know the profit objective can be actually maintained and then both the products of you know production can be operative. Actually, this is this is what the you know kind of you know long run kind of you know business requirement, but ultimately the final solution depends upon you know not only the values of the objective functions you know in the same times it you can also look for the kind of you know business you know requirement.

So, that means, it should not be you know you know strictly economical so you know provided if there is no kind of you know kind of you know strict kind of you know condition conditionalities there otherwise you look for the kind of you know optimality or you know the kind of you know optimum combination where all the values of the variables should be in a kind of you know operative position, but, ultimately it is a kind of you know mechanical process and depending upon you know your you know problem inputs. So, the algorithms or you know the software will give you the output, but now in order to bring the best as per the business requirement it is you to you know adjust the particular new systems.

In fact, you know optimization structures there is lots of you know flexibility through which you can actually change the particular you know requirement and then we can actually find out a situation where we can actually a address the business problem more effectively. Sometimes you know if there is no kind of you know strict a decision about the particular you know objective function then we can have a some kind of you know compromise policy, but ultimately the you know it depends upon you know you know business to business or you know the kind of you know particular you know problems.

So, ultimately means technically once you know connect with these problems and we have a various alternatives then you think about the kind of you know optimum solution which is the economically you know acceptable and socially also acceptable. So, that is depends upon you know means actually that is the a depends upon the long run kind of you know requirement and the kind of you know sustainability kind of you know requirement in the business environment.

So, let us see how is the kind of you know Z values and then finally, how is the kind of you know means what is the kind of you know final corner solutions or the optimum solution for this problem.

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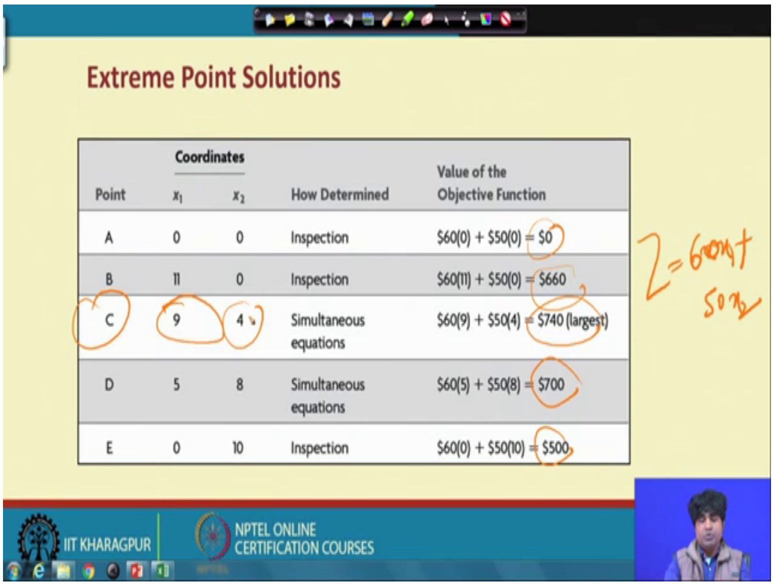

So, now, corresponding to whatever we have already highlighted. So, against it is in the streamlined process here. So, what I have mentioned. So, there are you know 5 different corner points A, B, C, D, E and obviously, 0 0, 11 0, 9 4, 5 8 and 0 10 and how determined actually it is a kind of you know inspection, inspection then this is what it is called as a simultaneous combination, simultaneous equation combination and inspection. That means technically, so that is how the kind of you know you know this is very beautiful way to you know address the kind of you know situation ah.

If you look if you know if you ask for you know kind of you know suggestion or the kind of you know justification I think you know C and B is the more attractive kind of you know position means attractive kind of you know solutions provided Z should be highest and the business will be a kind of you know in a kind of you know more you know a you know accurate position. So, that means, actually our job is not to get the optimum solution you know, but in the same times it should be it should be in a kind of you know effective kind of you know you know structure through which you know the business problem can be addressed more effectively.

So, now, which one will be final choice then that depends upon you know you know a Z values. So, our objective function you know, that means, technically so, these are all possible values of the decision variable, but objective function coefficient is here Z equal to 60 x 1, 60 x 1 plus 50 x 2, so, which was actually already highlighted earlier. So, that means, so, every time you put actually x 1 value and x 2 value then check the you know Z value. So, for instance for corner point a x 1 equal to 0, x 2 equal to 0, so, obviously, sixty into 0 plus 50 into 0.

So, Z value is 0 and against at corner point B x 1 is 11. So, 60 into 11 and x 2 equal to 0 means 50 into 0. So, obviously, so, say 660 and then again, so, the option C x 1 9, x 2 4, so, 60 into 9, 50 into 4, so, 740 and similarly for D 60 into 5 and 50 into 8, 700 and then against for E, x 1 equal to 0, x 2 equal to 10 so, obviously, 50 into 10, 500. So, now, if you compare all these you know possible values of the Z then C is the right choice and in fact, my personal suggestion is that you know C is the best because it is not economically feasible, it is also socially acceptable as both the combination are you know operative.

And, I think this is a you know best combination or you know best management decision which you can make. Since you know we are getting directly you know from the objective function and the kind of you know constraints so, that say we have no you know additional kind of you know third process you know you can apply here. But, in some instants you know a since it is a kind of you know mechanical way of you know getting the optimum solutions there is a high chance that you know when the optimum value of optimum value of Z is at the highest level where only x 1 will be their x 2 not will not there and vice versa can also true, where x 2 will be their x 1 will not there, but in that case it is a kind of you know challenge actually.

So, it is not that you know what mechanical process will give that you have to just accept, but in the mean times you have to think actually the kind of you know reality and sometimes we do little bit you know compromise a as per the particular you know requirement and the kind of you know need as per the as per the particular you know business is concerned. So, obviously,. So, this is how this is what the a kind of you know extreme point solution.

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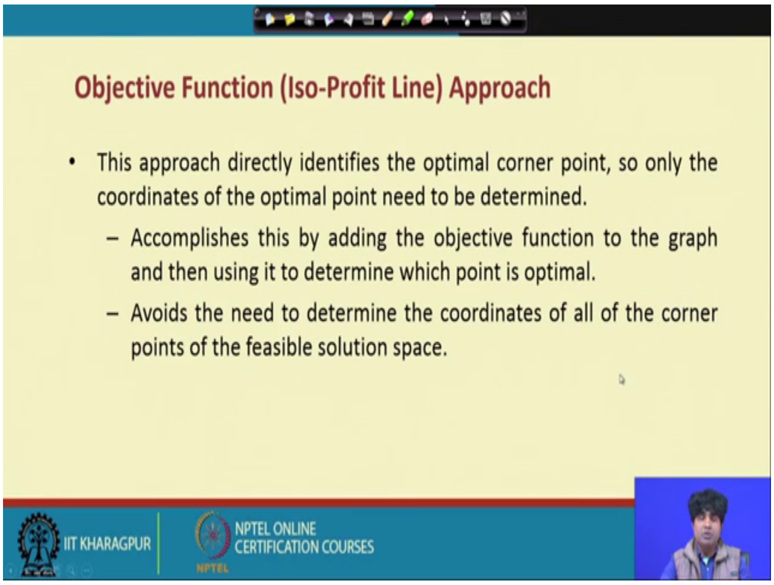

And then and so, what is actually called as you know isotropic line approach. So, technically the objective function is called as a isotropic line and the constraints are called as an isoquants and we are looking for the optimum solution which can satisfy the isoquants and the kind of you know business in it. So, this approach directly identifies the optimal corner point. So, only the coordinates of the optimal point need to be determines.

So, that means, technically just after getting the you know values of the decision variables then you know you just you know combined since it is a kind of you know linear combination of these decision variables. So, you know once since the parameters are you know availables and the x 1 and x 2 are reported then accordingly we can just you know add up and combine then we will think it means we will get the kind you know values of the Z and then we think about the kind of you know solutions.

So; that means, actually the parameters corresponding to objective function and you know the constant is a kind of you know you know big deal or you know big issue through which you can get the values of the decision variable. Actually, the kind of you know model is that to objective function and constraints. So, we are you know putting the objective function like you know Z equal to C 1 x 1, C 2 x 2 subject to constraint x less than equal to B or x greater than equal to B and x greater than 0 a in the prescriptive kind of you know prescriptive analytics structures. So, the parameters values are you know available where you are looking for the values of the decision variables.

So, that means, technically you know means in reality the particular parameters are usually obtained from the you know descriptive analytics sorry a kind of you know operative analytic structures of, obviously, descriptive structure and predictive analytic structure will give you the these parameters value and that too it is obtained through the kind of you know data structure and that means, information you know availability.

Using the information and the kind of you know model predictive kind of you know model then we like to get the kind of you know parameters value and once you get these parameters value and set the objective and set the constraints as per the resource availability as per the business requirement then we are looking for the kind of you know values of the decision variable which can be very optimum as per the business requirement and that too with respect to constraints and you know conditions, that means, it should be consistent with both constraints and condition and then a the particular problem can be addressed effectively a as per the business requirement.

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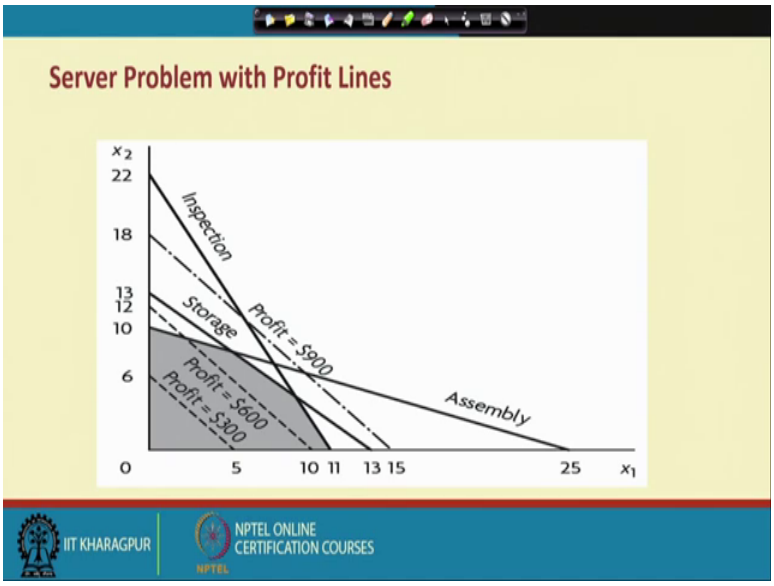

So, now, a to more about the particular you know structure here the particular constraint will be profit structures and then with respect to objective function just you check and then finally, you come with a kind of you know solutions. Right?

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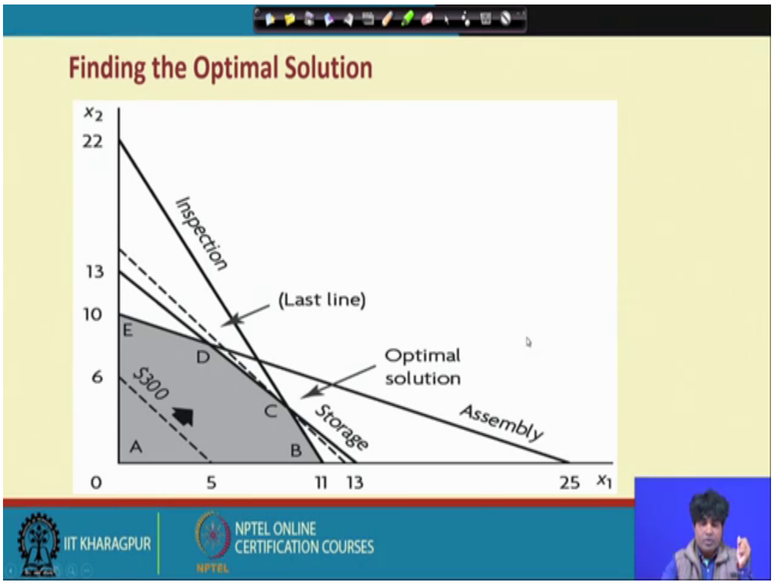

So, this is another way of you know visualizations technically again you know like you know predictive analytics, here the problem is also having alternatives and against we always look for you know more and more alternatives by you know changing the structure and you know setups. And, then finally, you freeze a kind of you know situation where you know the particular you know problem will be more a you know effectively address and the management decision can be taken into considerations.

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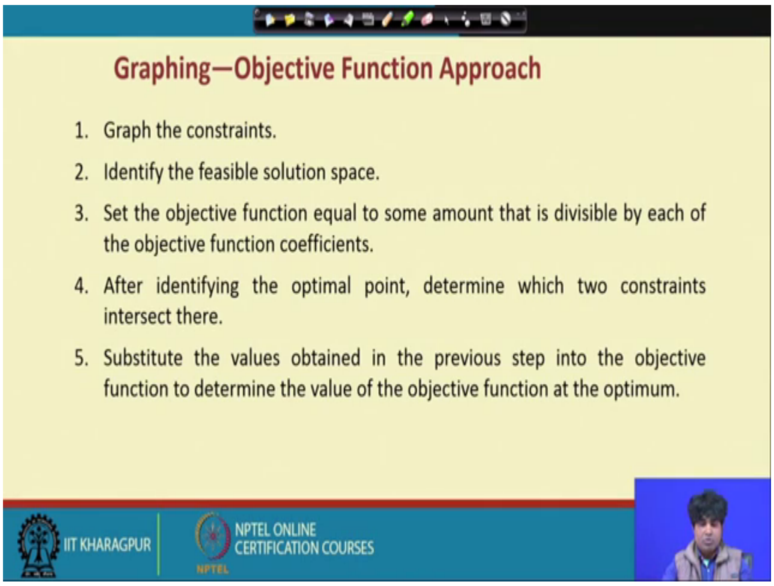

So, likewise a you know now to summarize you know the kind of you know graphic structure about the objective function approach. So, see first you know step is you know graph the constraints that too you know the inequality constraint you transfer into equality constraint then you know draw the straight line and then apply the particular you know constraint to know the you know the feasible zones whether it is in the left side and the right side to know the kind of you know means that is with respect to resource availability.

And, that is why it is called as you know identify the feasible solution space and set the objective function equal to some amount that is divisible by each of the objective function coefficients and after identifying the optimal point determine which two constraints you know intersect there and substitute the values obtained in the previous steps into the objective function to determine the value objective value of the objective function at the optimum.

So, that means, it is a means a it is the kind of you know some you know summary through which actually we start the process or initiate the process and then finally, freeze the a optimality and that is how the kind of you know structures and, to know more about the particular structure.

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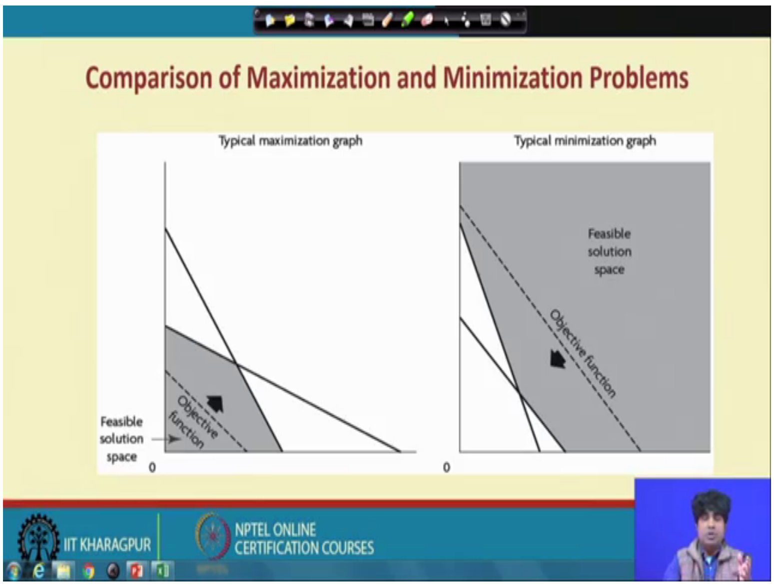

So, you know we like to you know compare the kind of you know optimization that to minimization structure and maximization structure and in fact, you know the problem which you have address you know in a kind of you know structures how to analyze means how to start with the you know issue how to you know freeze the kind of you know requirement and obviously, to as you know maximization problem and now the structure is like you know minimization to maximization and in the case of you know minimization the look is like this you see here and this is what the objective function and this is a maximization type where you know most of the instances or you know the usual or normal framework is the constraint should be less than type.

So, that you know very easily can get the a kind of you know optimum value, but in the case of you know minimization it is actually the feasible zone will be in the right side of the game that is actually usually greater than type and then we are looking for the minimum point in the corner solution like this.

So, means technically a so, it is a two different you know you know ballgame altogether and in the one side means objective function which maximizes maximization type we like to have the system is where the feasible zone should be bounded in nature so that you know we can easily get the optimum solution. But, in the other case in the minimization type it is you know easy to have the upper bound structure and then we look for the minimum point. As a result we can actually get the optimum value very easily to address the business as per the particular you know requirement.

So, that means, typically so, this is a kind of you know comparative kind of you know structure how is the look of you know maximization structure and how is the look of you know minimization structures and ultimately whether it is a maximization structure or minimization structure it is not you know arbitrary choice or it is not a kind of you know artificial kind of you know requirement and it all together depends upon the particular you know business problems and the business objectives. Corresponding to the business problem business objective we have to we have to actually prepare or you know design the objective functions and then you know optimize as per the particular requirement whether it is a maximization type and the kind of minimization type.

For instance within a particular you know business problems some of the you know variables or values of the decision variables will be like you know you know with respect to profit or the kind of know cost and the kind of you know revenue. So, if the target is the with respect to cost then obviously, we assume that you know the objective function needs to be actually minimized. For instance you know in another instance if it is a kind of you know appropriate then; obviously, we like to you know have a structure that you know the objective function stood objective function to be you know maximize.

Similarly, in the case of you know revenue then the kind of you know structures need to be actually optimize that to again maximization type and the constants ultimately depends upon the a the particular problem and the kind of you know business environment that the resources are limited or you know kind of you know unlimited what is the minimum requirement or what is the maximum capacity or maximum availability like this.

So, these are you know the kind of you know you know kind of you know structure through which actually linear programming linear programming structure can be addressed, can be analyzed and then use this particular you know model for the a solution of you know particular you know business problem and ah.

In this contest prescriptive analytics you know has a kind of you know rule to find out the situations as per the particular you know business requirement and the kind of you know management requirement.

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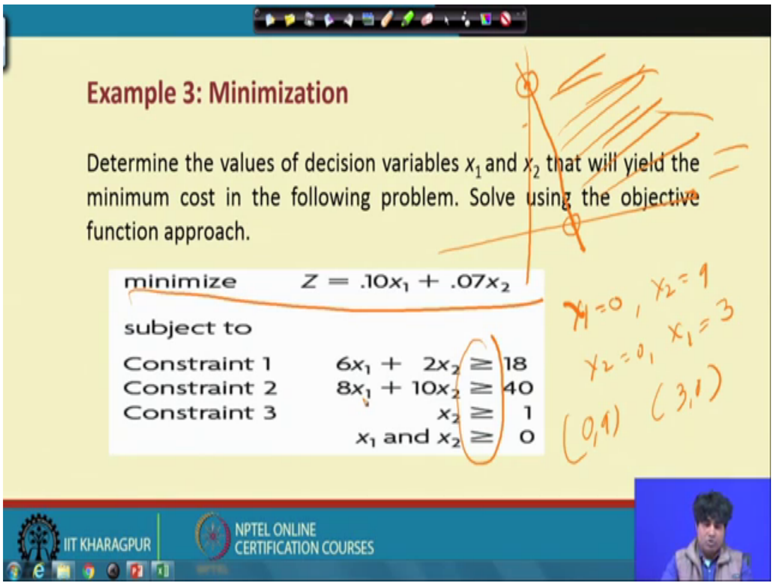

So, likewise you know, so, this is a another kind of you know similar problems. Corresponding to previous problem that is the maximization type, so, this is a simple problem called as you know minimization type and in this contest. So, the objective function is minimization type and what I have mentioned earlier if your objective function is minimization type and your constraint will be or greater than type then it is very easy to get the solution and it is actually the as usual general format of you know minimizations. And, in the counterpart the maximization type the constraint should be less than type and a corresponding to the previous problem this is actually a normal situation since the objective function is minimization and we have all greater than constraints. So, we can actually you know easily get the solution and the structuring is very easy heres.

And, see you know like the previous case first hand requirement is you make the constant into equality and draw the straight lines, then apply the inequality, that means, 6 for instance 6 x 6 x 1 plus 2 x 2 equal to 18. So, obviously, a put x 1 equal to 0, x 1 equal to 0. So, we only have x 2 equal to 9 and similarly, put x 2 equal to 0 then, x on[e] x 1 equal to x x x 1 equal to 3. So, obviously, we have a 0, 9 in one side and another side 3, 0. So, now, you know you just you know draw the lines corresponding to 0, 9 and 3, 0 and then after that you know you can actually apply the particular you know constraint.

For instance let us say this is how the case. So, let us say this is actually 0, 9 means x 1 equal to 0, x 2 equal to 9 somewhere here and a x 1 equal to 3 x 2 equal to 0 means somewhere here. So, now, you draw the line here like this and since it is greater than type. So, see so, the solution will be in the right side only so, that means, the optimality will be in the right side only, but to with respect to the first constraints again you apply the second constraint then you check the kind of you know feasible zone and again apply the constraint 3 and a find out the feasible zones and then finally, you club all together you know constraint 1 constraint 2 and constraint 3 and, agains after you know applying all these constraints.

So, it will give you the kind of you know corner points possible corner points where the a typical solution will be there and then take these corner points to the kind of you know objective functions, that means, every corner point you have x 1 value and x 2 value maybe x 1 0, x 2 a you know x 2 is positive or x 2 0 x 1 is the positive or x 1 0, x 2 0 or in case or in you know x 1 positive and x 2 positive by the way x 1 and x 2 cannot be negative. So, we start with either you know 0, 0, but in the case of you know minimization problems so, the constraint is like that. So, that you know x 1 and x 2 is not possible. So, obviously, there are 3 different cases all together, either x 1 0, x 2 is not 0; x 2 0, x 1 not 0 or x 1, x 2 both are you know positive.

So, that is the possible you know solution in this particular problem that too in the kind of you know minimization type, that means, we have lots of you know flexible kind of you know situation through which you can address the problem and then look for the kind of you know solution and as likewise actually you can you can actually just visualize and then think about the kind of you know solutions.

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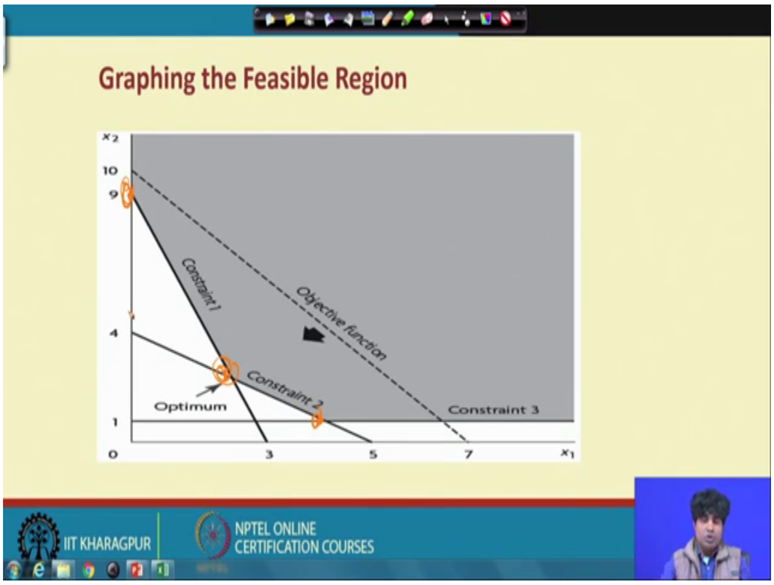

So, looking for the solutions, so, this is why the constraint 1, constraints 2 and then this is this is how the feasible zone. So, now, here the possible you know corner points will be corresponding all these you know constraints. So, you know this is what the constraints and then you know the corner points possible corner points you know like this you know this could be one corner point, this could be another corner point, this could be another corner point, that means, there are three corner points here and here it is actually x 1 is positive and x x 2 is 0 and x 2 is positive here 9 and x 1 equal to 0.

Now, here both x 1 and x 2 will be positive and that too solve these two constraints and then get to know what is x 1 value and x 2 value. So, then take typically we have three corner point and in all the three corner points you have x 1 you know possible x 1 values and the possible x 2 values then which one is the finally, the optimal choice or you know optimal requirement that depends upon you know the values of the objective function put these values and get the a you know Z value and then and that will give you the kind of you know optimality.

So, now compared to maximization problem here you know the choice will be where Z is the minimum one. So, because it is a kind of you know minimization problem and accordingly we will look for the kind of you know optimality and corresponding to the kind of you know three corner point, so, the possible structure will be like this.

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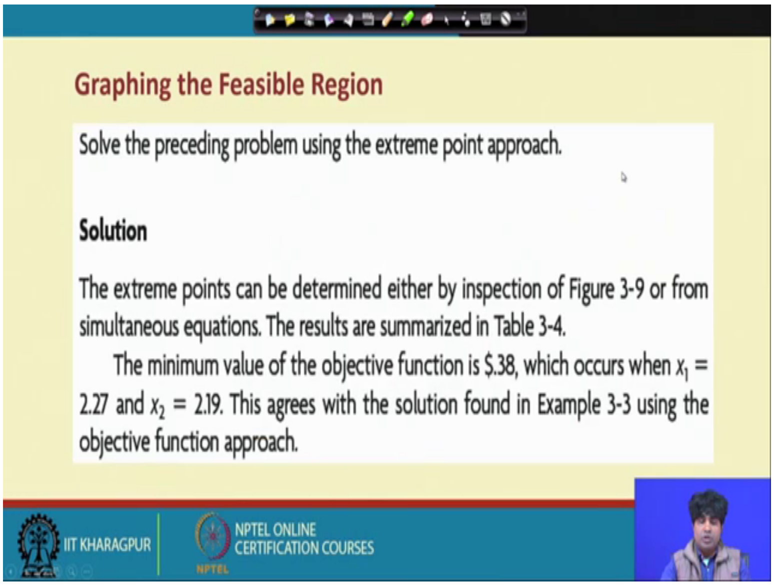

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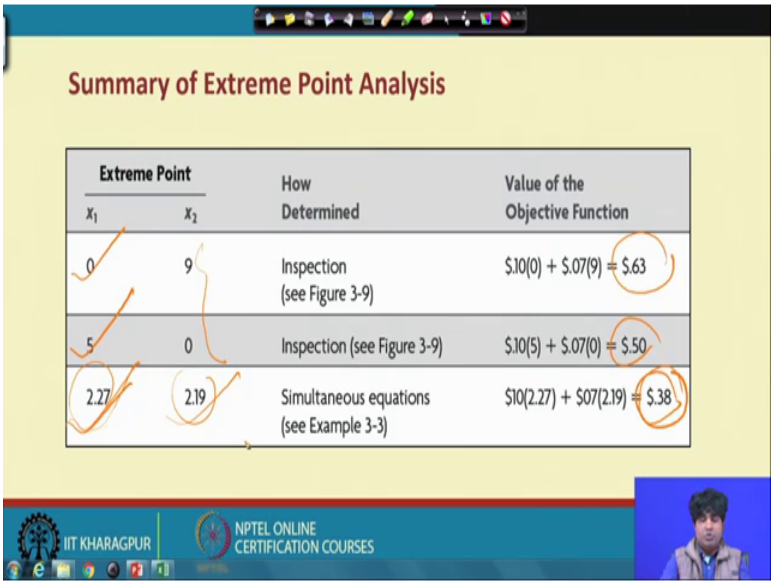

And, this is what the graphical structures you see heres. So, 0 9, 5 0 and this is what the combinations and then finally, so, this is case 1, this is case 2 and this is case 3 and obviously, like you know previous discussion these are all economically feasible not be necessarily feasible. So, in this case both combination 1 and combination 2 are effective, but ultimately depends upon you know where the a Z value is the minimum corresponding to the objective function. Now, putting all this value we find here as you know 0.63, then this is actually 0.50 and this is point 0.38.

So, that means, technically compared to if you compare you know Z values in all these three cases this is the minimum; that means, you know this will be the right choice and here x 1 equal to the values of the decision variable for x 1 is a 2.27 and x 2 is 2.19 this is economically feasible socially desirable and then you know our problem is very effectively address as per the particular you know requirement. So, that means, we are lucky that you know compared to you know maximization type and minimization type.

So, we have you know a kind of you know optimal solution where both the values of the a decision variable are positive and we have the you know for you know Z value you know having you know high weightage or you know high choice as a result this problem can be addressed more effectively as per the particular you know vision business requirement and then accordingly we can you know or take the management decision as per the particular you know requirement.

With this we will be stop here and thank you very much. Have a nice day.