

**Business Analytics for Management Decision**  
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**Lecture – 42**  
**Prescriptive Analytics (Contd.)**

Hello everybody, this is Rudra here, welcome to B M D lecture series. Today, we will continue with prescriptive analytics and that to coverage on linear programming problem.

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**Formulating LP Models**

1. Define the decision variables.
2. Determine the objective functions.
3. Identify the constraints.
4. Determine appropriate values for parameters and determine whether an upper limit, lower limit, or equality is called for.
5. Use this information to build a model.
6. Validate the model.

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And in the last lecture we have already discussed about this particular you know structure and the typical linear programming structure is like this.

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### Characteristics of LP Models

<p><b>Components</b></p> <ol style="list-style-type: none"> <li>1. Objective</li> <li>2. Decision variables</li> <li>3. Constraints</li> <li>4. Parameters</li> <li>5. Nonnegativity</li> </ol>	}	<p>Model structure</p>
<p><b>Assumptions</b></p> <ol style="list-style-type: none"> <li>1. Proportionality</li> <li>2. Additivity</li> <li>3. Divisibility</li> <li>4. Certainty</li> </ol>	}	<p>Model validity</p>

Max  $Z = c^T x$   
 $Ax \leq b$   
 $x \geq 0$

Objective

Decision variables

Constraints

Parameters

Nonnegativity

Model structure

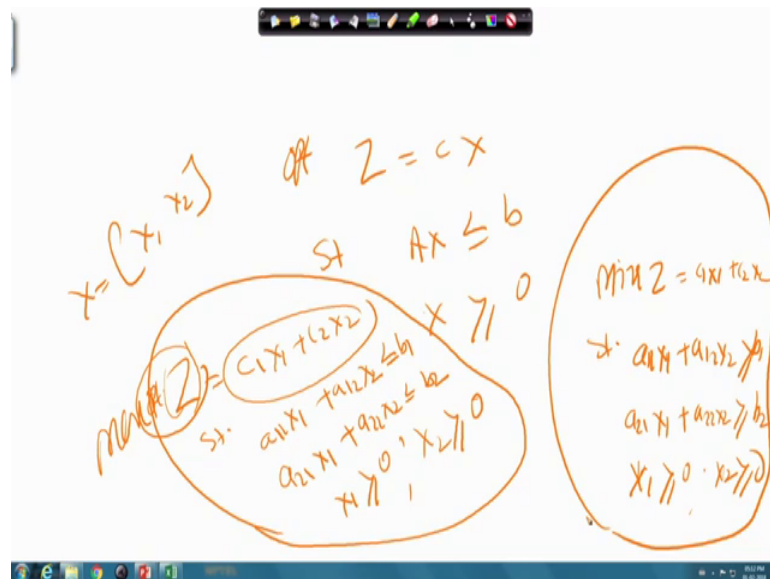
Model validity

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We have a objective function, we have a constraints and we have condition and then we are looking for the optimum solution for a particular you know business problem so; that means, if you summarize then we have the kind of you know structure like you know objective then issue about the decision variables, constraints, parameters, non-negativity. So, that is how the modeling structure altogether. And in fact, let me give you the kind of you know structure here so ok.

So, this is how the particular you know you know issue. So, we have objective functions and we have you know issues about the decision variables, constraints parameters and non-negativity. So now, in order to simplify the particular, you know or we we like to you know understand in a more attractive way.

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So, let us start with like this you know optimize a  $Z$  equal to  $C X$  subject to  $X$  less than equal to  $b$  and  $x$  greater than equal to  $0$ . So, this is how the simple LP structure and let us say  $X$  is the you know you know  $X$  consist of 2 variables  $X_1$  and  $X_2$  that is the you know requirement of you know decision variables.

And accordingly, the problem can be you know analyze like this. So,  $Z$  equal to  $C_1 X_1$  plus  $C_2 X_2$  subject to  $a_{11} X_1$  plus  $a_{12} X_2$  less than equal to  $b_1$ . And  $a_{21} X_1$  plus  $a_{22} X_2$  less than equal to  $b_2$ , and then  $X_1$  greater than equal to  $0$  and  $X_2$  greater than equal to  $0$ . So, this is how the kind of you know LP structure and that too with respect to 2 variables so; that means, we need actually linear combination of you know these 2 variables that too with respect to objective functions of; obviously, this is with respect to optimization  $Z$ .

And the optimization you know structure may be maximization type or the kind of you know minimization type. And the constraints can be of you know 3 types it can be a less than type, it can be greater than type or it may be equality type so; that means, we have a objective functions which can be either you know maximization type or minimization type, then subject to constraints and some of the constraints are you know greater than types some of the constraint less than type and some of the constraints maybe you know equality type. Now knowing the particular you know situations, then you will search for the optimum solutions.

Sometimes you know the problem will be more complicated, you know like you know we have objective function with several constraints, where some of the constraint will be less than type, some of the constraint would be greater than type and some of the constraint will be equality type so; that means, technically we have a true you know 2 kind of you know situations, for instance let us say we start with you know optimizing a particular you know objective functions were  $Z$  equal to  $C_1 X_1 + C_2 X_2$  and then all the constraints the you know in generals or you know as per a kind of you know a simple structure corresponding to objective function you know you know maximization type.

The constraint should be you know less than type and a on the other side if the objective function is the minimization type then the constraints must be greater than type. So now, the usual structure or simplest structure is like this when the objective function is; that means, technically in this context if I put you know maximizing  $Z$  equal to  $C_1 X_1 + C_2 X_2$ , then the first constraint should be less than type second constraint should be less than type again. So, in the kind of you know minimization problems. So, we can write like this minimization of  $Z$  equal to  $C_1 X_1 + C_2 X_2$  subject to same  $a_{11} X_1 + a_{12} X_2$  then it can be greater than equal to  $b_1$ . And then  $a_{21} X_1 + a_{22} X_2$  greater than equal to  $b_2$  and then  $X_1$  greater than equal to 0 and  $X_2$  greater than equal to 0.

So, this is the kind of you know structure called as you know minimization structures. In the kind of you know minimization structure and that to in the simplest way corresponding to objective functions the constraint should be great greater than type. And corresponding to the maximization problem where the objective function is maximization type. Then the constraint should be less than type if that is the case then this is a simple way of you know LP structure and it is very easy to solve this kind of you know problems you know like a maximizing  $Z$  with the less than constraint and minimize minimizing  $Z$  with objective function with respect to greater than kind of you know situation constraints.

So now, in reality you will find the objective function may be maximization type or minimization types where the constraints is like that you know some are less than type some are greater than type and some may be equality types. So, that is a kind of you know complex kind of you know situations and complex kind of you know environment, where the you know problem will be more interesting and then you know it may be more

challenging and then we need a some kind of you know analytic structure to solve the problems as per the particular you know business requirement or the management requirement.

Of course whether it is you know you know the kind situation like you know all are less than type or greater than type corresponding to maximization situation, minimization situation then, we need not require you know extra kind of you know structure through which you can simplify the problem and looking for the values of the decision variable, but in reality if you know objective function is maximization or minimizations and then the constraint will be the kind of you know mixture like you know less than, greater than and equality.

So, we have a kind of you know system through which you know corresponding to a particular objective functions maximization then all the constraints can be transferred into less than type again corresponding to minimization problems. So, the constraints can be transferred into greater than types. So, we have a kind of you know system through which you can and do that first then we look for the kind of you know solution that is the optimum solution so; that means, we are here in the kind of you know business analytics we like to address, how it can be done, how you have to structure restructure through which you can actually easily generalize the problem and then look for the optimality and get the values of the decision variable through which you can address the business problem more effectively.

So; that means, you know knowing all these structures and you know kind of you know you know kind of you know algorithms through which actually you can analyze the problem more effectively and then looking for the kind of you know optimality through which you can actually generalize the problems and then you can go for the kind of you know management decision. So, you know you know to know all these things then you know we we start like this. So, here the structure is like this, what we can say that you know the kind of you know components.

So, 5 components which we have been the kind of you know LP structure that is the objective decision variable constraints and the kind of you know parameters which we have already highlighted here  $C_1 C_2 a_{11} a_{12} a_{21} a_{22}$ . So, these are and  $b_1 b_2$  these are the kind of you know parameters and then the decision variables are you know

$X_1$  and  $X_2$ . And then the constraint is nothing but you know the first constraint and second constraints and the non-negativity. So, that is actually the kind of you know what we can call as you know model structures so; that means, if I like to address this one. So, I can simply put you know like this a a you know let us say maximizing  $Z$  equal to  $C X$  and  $A X$  less than equal to  $b$   $X$  greater than equal to  $0$ .

So; that means, here. So,  $C$   $A$   $b$  are the kind of you know parameters and then you know objective is nothing but the entire you know functions. And so, for as a constraints you know are concerned. So, this is in a vector format. So,  $X$  less than  $b$  equal to is a kind of you know constraint. And then non negativity is nothing but you know  $X$  greater than equal to  $0$  so; that means, you know corresponding to the kind of you know LP structure you have to transfer the problem into this particular you know format depending upon the situation, depending upon the kind of you know requirement, depending upon the kind of you know business problem you know kind of you know issues.

Then finally, we look for the kind of you know optimum solution so; that means, the whole idea is you know understanding the problems you have to first transfer the problem into the particular you know model structure LP structure. And then we think about the kind of you know optimum solutions as per the particularly you know business requirement. And while doing the kind of you know model formulation and the kind of you know solution and that is the looking for the optimality of the kind of you know solution that is, the values of the decision variable. So, we have a kind of you know a you know some kind of you know assumption we like to sight.

And then when the basis of that particular you know structure. So, we like to you know have the optimum solution. And some of the assumptions you know which can be usually you know connected with the LP structure is nothing but you know professionalism, additivity, divisibility and certainty. And these are all called for you know model validations like you know, we have already discussed in that case of you know predictive analytics. So, here also similar kind of you know structures so we have the kind of you know problem develop the model then look for the solution and validate the models as per the particular requirement; that means, the validation is here. You know once you get the values of the decision variables. So, at least you know the decision variable should satisfy the particular you know constraints. If that is not the case, then we cannot quality to know optimality and the values of the decision variables

may not you know address the problem as per the particular you know requirement and the kind of you know the kind of you know effectiveness. So, we like to you know look for the solution which can you know address you know both in the kind of you know objective function and the kind of you know constant.

So; that means, technically we must have a you know situation where the values of the decision variables can be obtained where you know, we can you know optimize the objective function and or you know the constraints can be satisfied equally. And once you set all these things and develop the kind of you know structure or develop the kind of an algorithm by default. So, once you get the values of the decision variables that will be automatically you know or stim line so; that means, the constraints will be satisfied and the objectives can be also fulfilled and then we can actually easily address the business problems as per the particular you know business requirement.

So now, looking for this particular you know structures. So, let us let us move you know how is the LP structures and then the formulation of LP model is like this, and define the decision variables, determine the objective functions, identify the constraints and determine appropriate values for parameters and determine whether an upper limit lower limit or equality is called for and use this information to build a model and then validate the models like you know what we have already discussed here. So, corresponding to this particular you know LP structure.

So, this is how the formulation you know requirement. So, you know decision variables to be you know define properly and then fix the objective whether it is a single objective or multiple objective and then you know and that will be address with you know kind of you know linear combination structures like you know which we have already highlighted with respect to 2 variables. So, the maximization of  $Z$  or minimization of  $Z$  is nothing but you know  $C_1 X_1$  plus  $C_2 X_2$ ; that means that is the kind of you know linear combination.

So, so; that means, we like to know what is the; you know optimum combination or linear combination through which actually you can generalize the problem. And then analyze the problem or you know address the problem in such a way the business objective can be effectively addressed as per the particular you know requirement. So,

likewise you know constraints can be satisfied then you know all the parameters and the kind of you know limit, the kind of you know structures need to be actually fulfilled.

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**Formulating LP Models**

**The Server Problem**  
**General description** A firm that assembles computers and computer equipment is about to start production of two new Web server models. Each type of model will require assembly time, inspection time, and storage space. The amounts of each of these resources that can be devoted to the production of the servers is limited. The manager of the firm would like to determine the quantity of each model to produce in order to maximize the profit generated by sales of these servers.

**Additional information** In order to develop a suitable model of the problem, the manager has met with design and manufacturing personnel. As a result of those meetings, the manager has obtained the following information:

	Type 1 (X <sub>1</sub> )	Type 2 (X <sub>2</sub> )
Profit per unit	\$60	\$50
Assembly time per unit	4 hours	10 hours
Inspection time per unit	2 hours	1 hour
Storage space per unit	3 cubic feet	3 cubic feet

The manager also has acquired information on the availability of company resources. These (daily) amounts are:

Resource	Amount Available
Assembly time	100 hours
Inspection time	22 hours
Storage space	39 cubic feet

The manager also met with the firm's marketing manager and learned that demand for the servers was such that whatever combination of these two models of servers is produced, all of the output can be sold.

$x_1$  = quantity of server model 1 to produce  
 $x_2$  = quantity of server model 2 to produce

maximize  $Z = 60x_1 + 50x_2$   
 Subject to:

Assembly  $4x_1 + 10x_2 \leq 100$  hours  
 Inspection  $2x_1 + 1x_2 \leq 22$  hours  
 Storage  $3x_1 + 3x_2 \leq 39$  cubic feet  
 $x_1, x_2 \geq 0$

Handwritten notes on the slide include:  $Z = Cx$ ,  $Ax = b$ , and  $x \geq 0$ .

So, accordingly a so we can address the problem. So, we start with a simple structure like this. So, let us say this is a simple linear programming problem with it 2 variables, and the problem will be a like this you know a business firm that assembles computers and computer equipment is about to start production of you know 2 new web server models. And each type of model will require you know assembly time and inspection time and storage space and the amounts of each of these resources that can be used to the production of the servers is limited.

And the manager of the business for would like to determine the quantity of each model to produce. So, that we can maximize the profit generated by these cells of these you know servers so; that means, a this is a production problem against with respect to you know 2 variables that to a web server you know that to actually 2 web server models. And the values of the decision variables will be you know represented as you know X 1 and X 2. So, accordingly so you see here. So, this is what you know X 1 the quantity of server model to produce and X 2 is the quantity of you know server model to produce so; that means, we have a 2 models model 1 and model 2 as that is that is now here the kind of you know output of this particular you know business.



That is the component through which actually the entire operation will be you know looking for and a. So, let us assume  $X_1$  is the kind of you know first decision variable and  $X_2$  is the second decision variables and  $X_1$  represents the quantity of server model one to produce and  $X_2$  is the quantity of server model to to produce. So, then we need actually the LP formulations and this is how the beginning where you know  $X_1$  is defined and  $X_2$  is the defined and this is a kind of you know production structures and that too with respect to 2 variable so; obviously, LP structure will be what should be the you know objective function and what are the constraints through which you can you know address the typical problem.

So now, for that so we need actually additional information. So now, the additional information is here in order to develop suitable model of the problem the manager, you know you know with design and manufacturing you know connect with you know design and manufacturing kind of you know department as a result of these you know structures. So, the a you know streamlined of the processes will be like this. So, this is the type 1 structure and type 2 structure; that means, a corresponding to model one and model 2.

And since you know profit is the prime call of this kind of you know business and that to that that to we you know we are actually more interested and. In fact, you know the entire you know prescriptive analytic structure will be like that so; obviously, profit is the prime goal. So, the objective will be the profit function and here the business power manager actually wanted that you know there should be at least you know 60-dollar profit from the type 1 and 50-dollar profit from the type 2. And accordingly, accordingly so this is actually 60 and 50 accordingly there are 3 constraints the a assembly part then a inspection part and the storage space. So now, this is actually type 1 which we represented  $X_1$ s and here type 2 which represented  $X_2$ .

So now, corresponding to the objective function, so now, this is actually objective function parameter coefficient and this is objective function you know parameter coefficient for  $X_2$  so; obviously, So, the  $Z$  can be a  $60 X_1$ s and then plus  $50 X_2$ . So, accordingly so it is formulated here. So, maximize  $Z$  equal to  $60 X_1$  plus  $50 X_2$  and subject to constraints. So, the constraints are here there are 3 constraints. So, first constraint is the assembly part and that to 4 hours required for model 1s and then 10 hours required for model 2.

So; that means, when they will go to the assembly department. So, these product 1s needs 4 hours and product 2 needs actually 10 hours so; obviously, So, the first constraints will be  $4X_1 + 10X_2$ . So now, we look for you know the kind of you know research constraints and how much you know total hours actually available in this particular you know department; that means, again we need additional information. So, this is the part of the additional information that to the right-side of the particular you know right side you know information about the particular you know constraint.

So that means, technically so we have a system called as you know  $Z = CX$  and  $X$  less than equal to  $b$  and  $X$  greater than equal to 0 so; that means, we need the a coefficients and we need the information about the  $C$  coefficient and we need the you know coefficient about you know you know  $b$ . So, accordingly, so, since there are 2 you know 2 products. So,  $C$  becomes  $C_1$  plus  $C_1$  and  $C_2$ . So, accordingly  $C_1 = 60$  and  $C_2 = 50$  and again. So, there are 2 3 constraints so; obviously, so a  $1X_1 + 1X_2$  plus a  $1X_1 + 2X_2$  a  $2X_1 + 1X_2$  a  $2X_1 + 2X_2$  then a  $3X_1$  and a  $3X_2$ .

So, this is are the 3 constraints so; that means, technically we need again the coefficients of these coefficients ah. So accordingly, so the coefficients are here. So, the first you know second constraints that is the inspection. So, it is needs actually 2 hours in the first product and then 1 hour for the second product so; obviously, the second constraints will be a  $2X_1 + X_2$  again we look for the resource availability and then against we go for the second a a second information base here actually inspection time. So, maximum availability of a hour is 22 so; obviously, this will be a 22 and then third constraint will be the storage space and for that you know 3 units for you know type 1 and 3 unit for type 2 accordingly.

So, the left-hand side of the constraint will be  $3X_1 + 3X_2$  and then how much availability. So, storage space availability is actually 39. So, we put actually here you know a less than equal to 39; that means, whatever availability here. So, these are you know maximum availability. So, that is how we need actually a optimizations. So, resource optimization means with a maximum limit and as per the requirement and as per the kind of you know combination. So, we look for the kind of you know you know exact you know structure through which you know everything can be satisfied equally then the business can be it is to you know a particular, you know situation where you know that can be addressed more effectively.

So now corresponding to this problem; so, if you see here so this is what the you know the problem you know a base and the understanding the problem base we transfer the entire problem into simple, you know LP model like this so; that means, we have actually come big information or you know full of you know information's with respect to the problem in details the kind of you know requirement, the kind of you know structure, the kind of you know availability and the kind of you know objectives and the kind of you know constraints.

So, all these things will be simply transferring to a model format. So now, the model is ready to you know operate. So now, once the model is ready; so, then we will look for the kind of you know solution this is how the first-hand size of the particular you know situation. So, let us let us see how is the kind of you know structure through which you can actually move. So, for that we can actually go for the kind of you know solution and uh. So, here the you know since it is actually a bivariate, you know structure that is with respect to 2 variables; that means, here your our objective is a maximization type and that too for 2 variables and according to the problem structure the objective function is the maximize the Z equal to  $60 X_1$  and  $50 X_2$  subject to 3 constraints  $4 X_1$  plus  $10 X_2$  less than equal to 100 and  $2 X_1$  plus  $X_2$  less than equal to 22 and then  $3 X_1$  plus  $3 X_2$  less than equal to 39.

And then  $X_1$  and  $X_2$  greater than equal to 0 because it is the kind of you know production you know kind of you know issue and; obviously, so the values of the decision variable it can be either 0 or it can be positive it can you know no way it can be actually a negative. So, as a result having the kind of you know structuring the kind of you know requirement. So now, we like to you know optimize the process through which we like to know what is the exact values of the decision variable through which, if you know the business firm can reach the optimum profit subject to the available you know constraints or the kind of you know available resource base.

So, accordingly we can move and what we can actually you know address here since it is with respect to 2 variables. So now, you can have actually the solution graphically as well as you know you know algebraically. So now, let us you know see how graphically can be analyzed because you know graphically give you more you know a accurate way to understand those who are you know beginners, they can you know easily can pick up how it is actually the kind of you know structure of the prescriptive analytics that to the

linear programming framework. So, that you know we can actually address and then look for the kind of you know optimality.

So; that means, actually the way we like to highlight here now. So, we like to create a kind of flexible environment; that means, the kind of you know various alternatives because it is the kind of you know best combinations. So, what should be the  $X_1$  pool and what should be the  $X_2$  pool. So, that you know objective function can be at the highest-level subject to you know satisfaction of the kind of you know constraints so; that means, it cannot be actually the kind of you know unique kind of you know structure through which you are looking for the decisions. So, since it is a kind of you know decision making process. So, obviously, we must have some kind of you know alternatives.

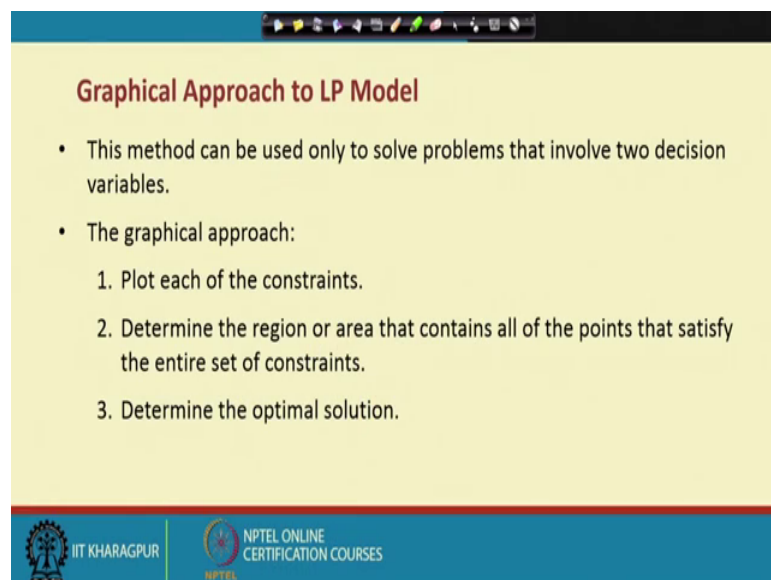
Now here the kind of you know alternative is the kind of you know different combinations and that too with respect to  $X_1$  and  $X_2$ . So, we must have at least you know 2 different you know flexible structure as a result we can think about the you know process and then we take the decision. If it is only one kind of you know situation then there is no need to take a kind of you know you see no need to take kind of you know decision. So, since she actually there are you know more than one kind of you know situation. So, there is need of you know decision which one should be finally, taken into consideration and that too you have to take care the kind of you know objectivity objective issue and the kind of you know constraint issue.

So; obviously, so you know the whole idea is it that you know graphically we see how is the kind of you know structuring; that means, we simply you know streamline the process slowly slowly, then we finally, (Refer Time: 27:08) where you know the kind of you know optimality you know can help us to analyze the problem and then you know address the you know business issue more effectively as per the particular you know requirement. So now, so far as you know graphical you know solution is concerned. So, what will you do actually?

So, we like to it we like to visualize with respect to  $X_1$  and  $X_2$  and then we simply you know put the constraint as per the particular requirement and then, we like to fix a kind of you know boundary and through which actually the resource base can be you know adjusted; that means, it is actually a question of you know optimal location of resources.

Since, it is a kind of you know production process so the simple understanding or theoretical, you know structuring is like this, what should be the optimum production combinations and that too with respect to available resource. you know available resources, right? So, that means, technically so how these resources are you know allocated optimally. So, that you know the particular you know production combination will be more effective or you know more accurate as a result the profit of the business power is at the highest level that is what the actual agenda and the kind of no requirement in this particular you know structure in that too for the prescriptive analytics structure.

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**Graphical Approach to LP Model**

- This method can be used only to solve problems that involve two decision variables.
- The graphical approach:
  1. Plot each of the constraints.
  2. Determine the region or area that contains all of the points that satisfy the entire set of constraints.
  3. Determine the optimal solution.

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So now in a in the first there are 3 steps altogether in the first step. So, plot each of the constraints and then check the kind of you know since it is a kind of you know linear programming problems; obviously, all the constraints are in a kind of you know linear format only and since it is with respect to 2 variables then; obviously, a with a particular you know constraint we can have a kind of you know straight line and then we can check the kind of you know feasibility.

For instance, ah, you know in a kind of you know linear programming problems corresponding to a particular objective. So, the constraint should be less than type or greater than type. So, the first (Refer Time: 29:19) requirement of the graphic structure is a make the constraint into equality. Then you know find out the points you know for you

know what the you know kind of you know structure as per the particular you know equation, then you draw the line and then we allow or you can actually apply the kind of you know a reality. So, whether it is a less than type or you know greater than type for instance you know let us have a kind of you know structure here. So, for instance here the constraint is  $4X_1 + 10X_2 \leq 100$  so; that means, you know first of all, what will you do?

So, we can write here you know simply the kind of you know the kind of you know requirement.

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**Formulating LP Models**

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**Additional information** In order to develop a suitable model of this problem, the manager has met with design and manufacturing personnel. As a result of those meetings, the manager has obtained the following information:

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 Inspection  $2x_1 + 1x_2 \leq 22$  hours  
 Storage  $3x_1 + 3x_2 \leq 39$  cubic feet  
 $x_1, x_2 \geq 0$

Handwritten annotations:  $4x_1 + 10x_2 = 100$ ,  $x_1=0, x_2=10$ ,  $x_1=25, x_2=0$

So,  $4X_1 + 10X_2 = 100$  means. So, it will be like this  $4X_1 + 10X_2 = 100$  so now, this is a straight-line equation. And so, the how you have to draw the straight lines corresponding to the graphical requirement. So, just put  $X_1$  equal to 0 then find out  $X_2$  value and then again put  $X_2$  equal to 0 find out  $X_1$  value so; that means, So, we need actually 2 different point to have a straight line then once you have the straight line then after that you can apply the a actual you know requirement, whether it is a less than type or greater than type for instance; so, here putting  $X_1$  equal to 0  $X_1$  equal to 0  $X_2$  equal to simply 10 so  $X_2$  equal to 100 by 10.

So, it is 10 against putting  $X_2$  equal to 0  $X_2$  equal to 0  $X_1$  equal to 100 by 4 that is 25 so; that means, technically we have 2 different points. So, one point is a 0 10 and another point is a 25 and 0. So now, so the structure will be like this you know, you simply go to

the particular, you know, structure like this and then this is  $X_1$  and this is  $X_2$  and then corresponding to  $X_1$  you know 0. So,  $X_2$  will be 10 so let us assume that somewhere here and then again putting actually a  $X_2$  equal to 0  $X_1$  equal to 25 let us say somewhere here so; that means, it will be in the  $X_1$  line only and corresponding to the first ones it will be on the  $X_2$  line only.

So now we have 2 different you know corner point and just join the particular you know point. So, this is how the line is all about and then. So, the first step of this process is make the constraint into equality and then have 2 different points. So, that you can draw a straight line after drawing the straight lines; so, then you look for the kind of you know actual you know actual fact that is the actual you know reality. So, whether it is less than type or you know greater than type, since it is a less than type by default the resource you know allocation will be only in the left-hand side only.

So, right hand side will not be you know possible because that is the availability of resources only that is the maximum availability so; that means, we may be here. So, we may be here. So, we may be here, so we may be here, so are these are the this line will give you the upper limit and then we have a lots of you know interesting you know uh area here and the solution will be within this particular area somewhere you know maybe a here's may be here like this. So, it depends upon actually you know combination of you know other constraint altogether. So, like for the first constraint the structure will be like this against you go for the second a constraint that is  $2X_1 + X_2 \leq 22$ .

So, the first step of this process again is  $2X_1 + X_2 = 22$ . Then against look for the look for 2 different points draw the lines and then again allow you know the actual fact then that is the less than type. So, you put the less than type then by default you will get the kind of you know you know the actual zone and now the second constraint will be apply a like this. So, for instance here if you put  $X_1$  equal to 0  $X_2$  equal to 22 so; that means, somewhere here and then if you put actually a  $X_2$  equal to 0  $X_1$  equal to 11. So, let us assume somewhere here. So, the line will be like this again since it is less than type. So, the zone will be this side a then come to the third the third kind of you know constraint this is  $3X_1 + 3X_2 \leq 39$ .

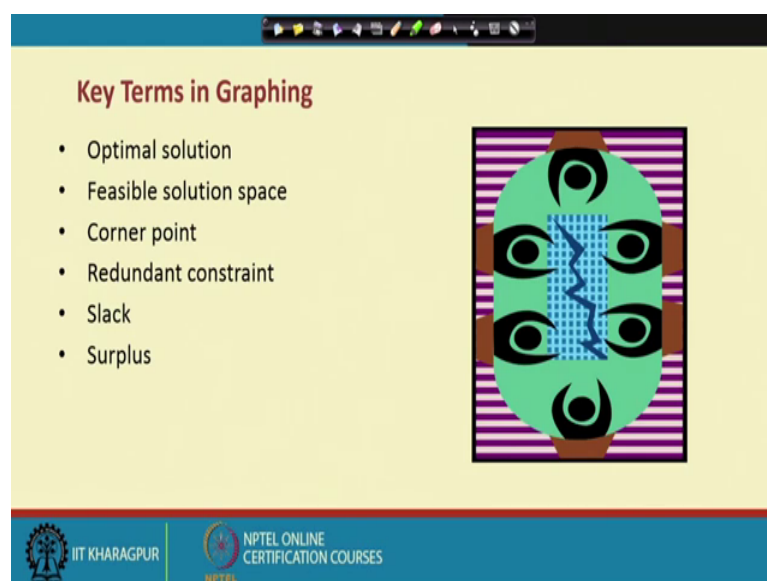
So, put  $X_1$  equal to 0  $X_2$  equal to 13 and that to that to somewhere here you know somewhere here. And then against put you know  $X_2$  equal to 0 then  $X_1$  equal to 13 and

accordingly. So, it will be somewhere here. So, you now join these 2 points ok. So, this is a so it will be like this ok. So now, so we have we have here you know these are all you know zones less than zones so; that means, technically. So, this is the first constraints this is the second constraint and this is the third constraint and then these are the typical zone here's.

So now having this typical zone. So, we have actually you know different you know extreme points and these extreme points can be the possible solutions these are all called as you know technically corner points and so a you know taking care of you know all the constraints and simultaneously, it will give you the you know the kind of you know feasible zones or you know a bounded you know bounded kind of you know structure. And then the boundary we look for the extreme points and these are called as you know corner points.

So now these corner points are the possible solutions you know that is the possible solution means that is the optimal solution for the particular you know business problem. And out of all the corner points one particular you know corner point will give you the optimum solution this is what the basic structure of you know LP and now in order to know the particular you know effect. So, let us go to the kind of you know structures.

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**Key Terms in Graphing**

- Optimal solution
- Feasible solution space
- Corner point
- Redundant constraint
- Slack
- Surplus

The diagram shows a green shaded feasible region bounded by a blue grid. The region is roughly rectangular with a jagged, irregular shape on its right side. It is surrounded by a purple and white striped border. The diagram is set against a light yellow background.

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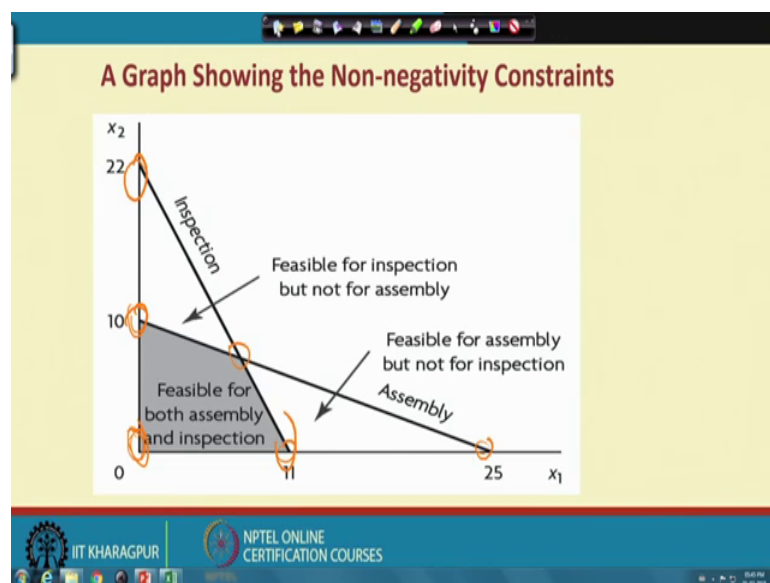
So, this is what the graphic structure. So, we look for you know optimal solutions and we look for you know feasible solution space then corner point solution and then a



redundant constraint so; that means, we have actually couple of constraints. So, while you know operating the particular you know process and looking for the optimality by integrating all the constraints simultaneously depending upon the kind of you know constraint, you know actual structure, some of the constraint may not be operative a in the in the in the kind of you know final you know requirement.

Then we have a kind of you know slack structure and circular structure through which you can understand the particular you know requirement.

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So, also technically so this is what the kind of you know game. So, whatever I have already mentioned. So, this is the; you know first constraints and this is the second constraints. So now, if you omit actually the third constraint still we can get the particular you know solutions for in for this case. So, this is a kind of you know solution corner point and this is a kind of you know corner point and this is a kind of you know corner point and this is a kind of you know corner point so; that means, with respect to 2 constraint. So, we can have a kind of you know feasible structure and that to the boundary will be a like this and we have a 4 different corner points and one of such corner points will be it will give you the optimum solution.

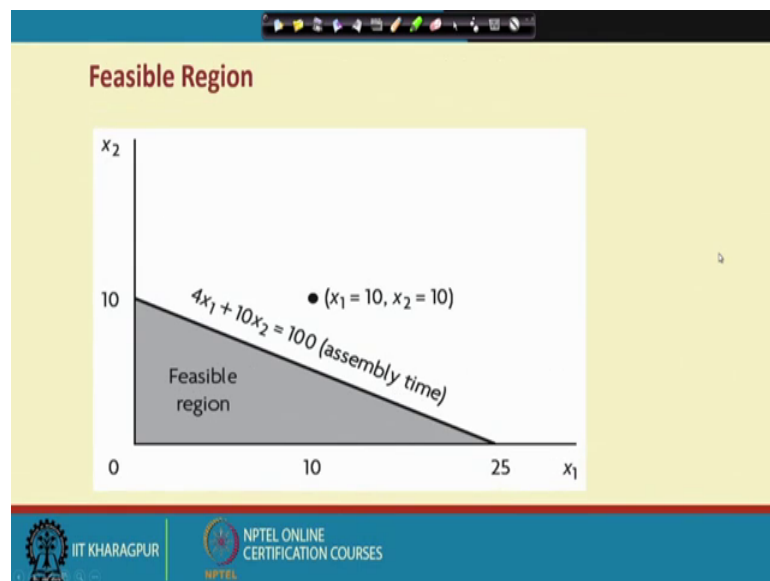
In fact, you know if you have only one constraint still you know. So, let us say this is the only 1 constraint then we have 3 corner point 1 2 and 3. Suppose we will take this particular constraint only then by default we have a 3 corner points again so; that means,

if you take only 1 constraints we have a 3 different flexibility if you take the second constraint only and still we have a 3 different corner points of course, the  $X_1$  position and  $X_2$  position will be different, but still we have a kind of you know flexibility to find out or to take the decision which one should be the final choice, you know to optimize the profit subject to the particular you know available resources

And against when we add another constraint then the (Refer Time: 38:10) of you know the kind of you know feasibility is on will be slightly you know restricted, because you know the entire you know structure is like that you know when will be add one after another constraint then the you know then the kind of you know feasible structure or feasible zone will be restricted you know slowly slowly. So, like you know in the case of you know predictive analytics what we have already discussed when you go one after another variable then; obviously, a the kind of you know the problem complexity will start increasing, and then and there are lots of you know additional problems will be coming up and then finally, we look what the solution after addressing all the you know problem simultaneously. And similarly, this is also a case you know if you add one after another you know structure then it will be you know giving you some kind of you know complexity.

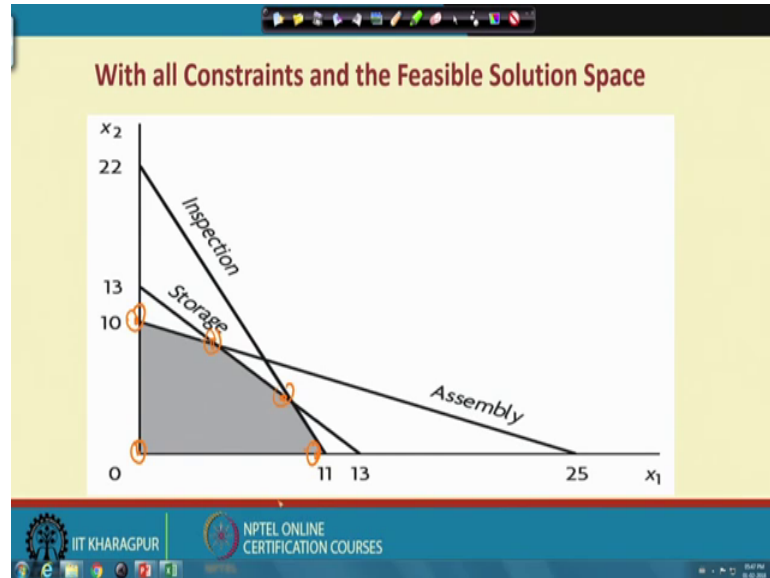
So, let us say you know start with the kind of you know structure here; so, having the kind of you know structure.

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So, let us go to the third constraints. So, this is actually the third constraint and now this is what the ok.

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So, this is what the third constraint. And so, initially we have the kind of you know structure here like this. And so, this is the first constraint and this is the second constraint and then finally, we have the third constraints. So now, obviously so this is the finally, you know zone so; that means, a taking care of you know all the 3 the all the 3 kind of you know you know constraints then finally, you have one corner point here you have another corner point here we have another corner point here we have another corner point here and we have another corner.

That means you know with respect to 1 constraint we have a 3 flexibility with respect to you know means a out of 3 constraint if you go 1 by 1 then every times you have a 3-flexible position. So now, if you add up you know or 2 simultaneously a taking a if you do not consider you know this particular you know constraint then again, we have 1 2 3 you know for different situations similarly if you if you remove a this 1s then still we have a a 1 2 3 4. So, we have different kind of you know combinations now having all the constraints simultaneously. So, altogether we have a 5 different corner points. Then and the optimum solution will be at a particular you know corner points that is how we are actually looking for and searching a searching for this particular you know requirement.

So now what will you do? So, again you know you go to the particular you know structure and check how is the final solution and the particular structure is called as actually the extreme point structures you know you can called as you know extreme points or corner points and through which actually some of you know the particular optimum solution will be lying a; that means, technically out of all these corner points one particular corner point will give you the optimum solutions and that is how what will you do? So, we take all the corner points that is with respect to the combination of  $X_1$  and  $X_2$  and put this value in the objective functions and they check where the object values of the objective function will be at the highest. So, once you get then accordingly we can actually look for the solution and then you know we can fix you know which one is the optimal solution out of you know all possible alternatives.

So, it is it is clearly like you know the kind of you know predictive structures earlier we have discussed you know; that means, say you know if you change a different kind of you know structure we have actually flexible options and then we have to pick up a final one which can actually finally, you know address the problem more effectively and the entire business analytic structure is a like that, you know you we like to choose a particular technique corresponding to a particular problem or the corresponding to the problem, if you know data or you know data availability, where actually the kind of you know, situation is like that you know, we have to pick up a best out of the several alternatives and then uh that particular you know best you know requirement will address the problem more effectively.

So, that is how you know whether you are in the predictive analytics or prescriptive analytics. The whole idea or the you know you know the kind of you know simple message is that; you know, we like to bring a kind of you know flexible structure through which you know, we can pick up a kind of you know requirement which can effectively address the business problems and then we can do the kind of you know management decision as per the particular you know business requirement.

So; that means, that is the beauty of this you know analytics. So, the one of the challenges you know how many alternatives you can bring within a kind of you know particular you know problem with different kind of you know constraints and situations or the kind of you know diagnostics, and the kind of you know sensitivity structure through which you know we can actually analyze the problem more effectively. And then

look for the kind of you know solution as per the particularly you know business requirement. So, with this we will stop here and in the next class we will continue here to get the means we will look for the search of the optimality in a kind of you know effective way as per the particular you know business requirement.

Thank you very much have a nice day.