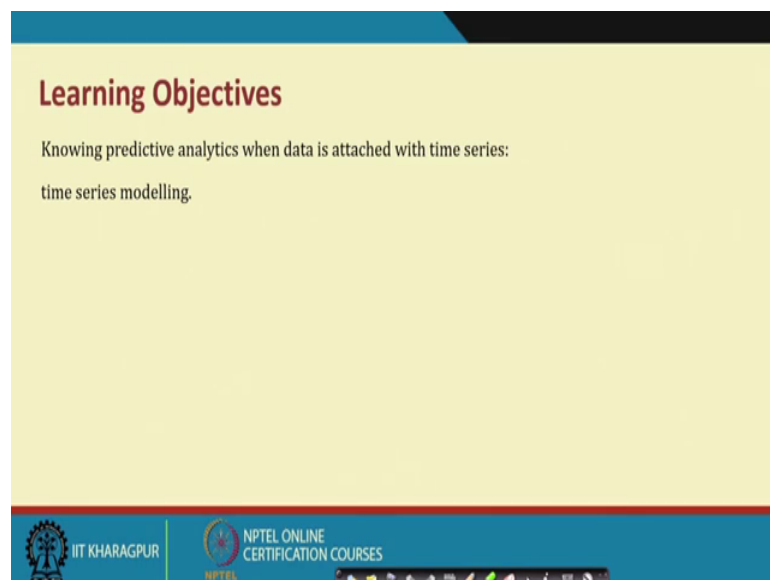


Business Analytics for Management Decision
Prof. Rudra P Pradhan
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Lecture – 36
Predictive Analytics: Time Series Forecasting (Contd.)

Hello everybody. This is Rudra Pradhan here. Welcome to BMD lecture series. Today, we will continue with predictive analytics and that too coverage on time series forecasting.

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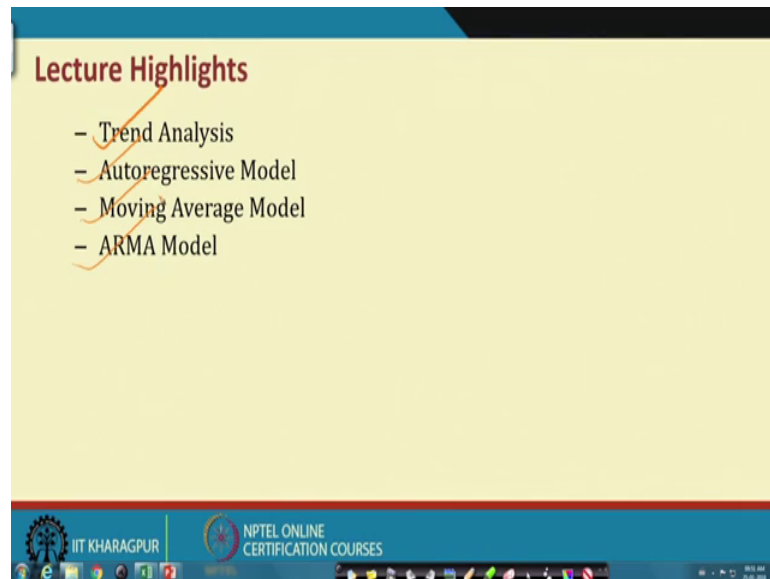
Learning Objectives

Knowing predictive analytics when data is attached with time series:
time series modelling.

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In fact, we have already discussed this particular component.

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And in this particular lectures we specifically highlights for different models, time series models and that too trend analysis , this is trend analysis, autoregressive model, moving average model and ARMA model in our last lectures, we have discussed lots of time series forecasting ; that means, we have discussed couple of you know models through time series data and the idea behind this particular discussion is we have historical data of a particular variables and that too we like to develop a models whether it is a kind of you know naïve forecasting model or moving average models or smoothing models.

So, in all the cases we like to use the historical data and develop a kind of you know forecasting model and then we will go for the kind of you know prediction and forecasting for the future requirement as per the particular you know problem or as per the particular management you know requirement. So, in the similar lines we will discuss you know couple of you know other time series models through which we can do the similar kind of you know predictions and similar kind of you know forecasting by using the historical data.

So, we in the same way we have to develop a model and then we will get the estimated model on the basis of historical data and before we go for you know prediction and forecastings, we have the check the kind of you know accuracy and the similar kind of you know accuracy indicators we can use here's to validate first the models and then we will go for the kind of you know prediction and forecasting.

So, let us start with the first the trend analysis then we will proceed for autoregressive model moving average model and autoregressive moving average model.

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Measures of Forecast Error

(Forecast Error = $Y_t - F_t$)

- Bias - Mean Error
- MAD - Mean Absolute Deviation
- MAPE - Mean Absolute Percentage Error
- Mean Square Error (MSE)
- Root Mean Square Error (RMSE)
- Bias, MAD, and MAPE - typically used for time series

$$\text{Bias} = \frac{\sum_{t=1}^T (\text{forecast error})}{T}$$

$$= \frac{\sum_{t=1}^T (Y_t - F_t)}{T}$$

$$\text{MSE} = \frac{\sum_{t=1}^T (\text{forecast error})^2}{T}$$

$$= \frac{\sum_{t=1}^T (Y_t - F_t)^2}{T}$$

$$\text{MAD} = \frac{\sum_{t=1}^T |\text{forecast error}|}{T} = \frac{\sum_{t=1}^T |Y_t - F_t|}{T}$$

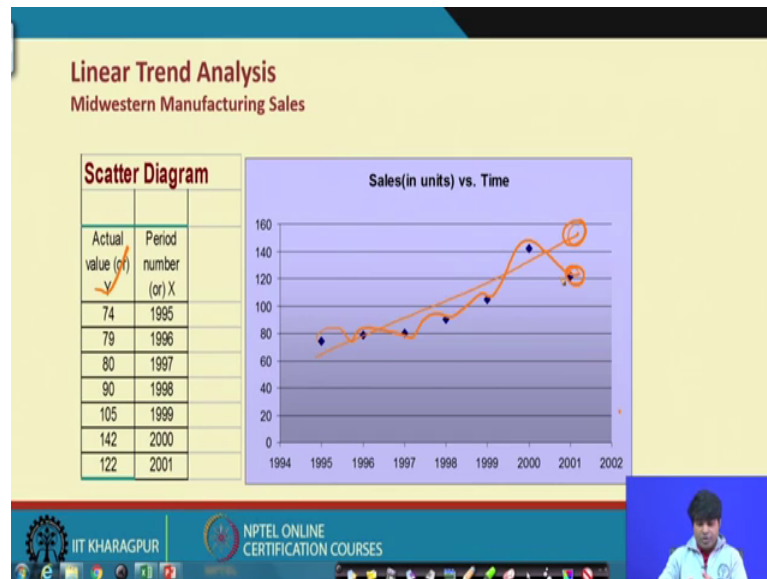
$$\text{MAPE} = 100 \sum_{t=1}^T \frac{|Y_t - F_t|}{Y_t} / T$$

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So, in the trend analysis so, the basic structure is like this formation and the forecasted informations that is the difference between in this case here Y_t minus F_t and Y_t is the actual information of a particular variables and F_t is the predicted informations of the of this particular variable through the forecasted model and once we have the forecasted model and the forecasted information. So, the difference between the actual information and forecasted information will give you the error component and these error components are you know need to be evaluated first before you go for the prediction and you know forecasting.

In fact, the particular structure we have already discussed in our previous lectures. So, here the similar kind of you know indicators like Mean Error, Mean Absolute Deviation, Mean Absolute Percentage Error, Mean Square Error, Root Means Square Error. So, all these indicators are you know finally, used to validate the model before we go for the kind of you know prediction and forecasting.

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So, what will you do actually? So, we start with the first you know trend analysis and then we will follow by other models like you know autoregressive model, moving average model and ARMA model in the trend analysis the particular you know it can be a linear trend structure it can be non-linear trend structure, but generally a we have a particular variable and that variables information is with respect to time, it may be annually, it may be weekly, it may be quarterly, but using these informations, we like to predict a kind of you know structures not exactly like you know moving average and naïve forecasting it is through a kind of you know well structures that is similar kind of you know structure which we have discussed in the case of you know regression analysis.

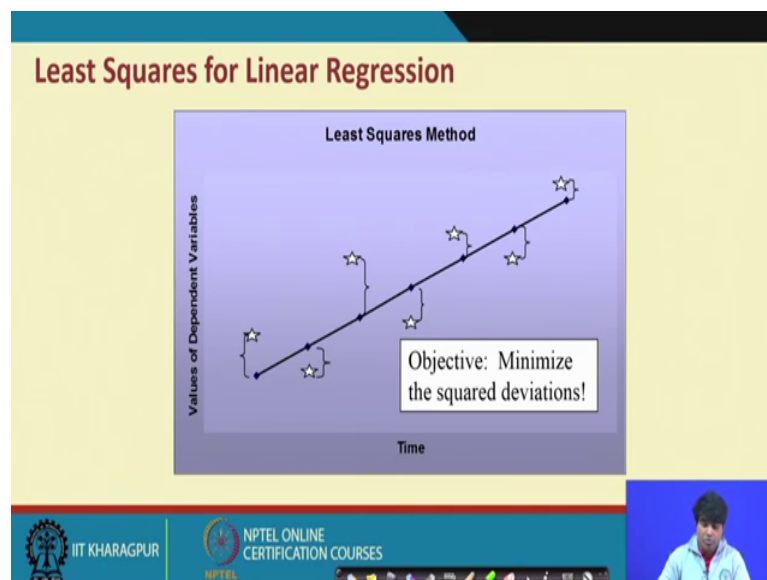
So, what will you do technically here is, in this case so, we first you know it is in yearly informations we like to plot these information ah. So, these are all actual values. So, this is the actual value and then these are the actual value and with the actual value we have a time period. So, what we can do first. So, we plot these actual values with respect to time. So, then if you join all these points so, this will give a kind of you know actual behaviour of this variables here in this case we are actually going to predict the sale structure of a particular company and once you join all these points then it will give you the actual movement of this you know sales for this particular you know company.

So, what we need to you know do here. So, we will like to develop forecasting model. So, which can give you the you know better forecasting or you know better structure through which you can actually analyse this particular you know sales figures or we can

you know discuss the particular you know you know business performance. So, what will do here is. So, means the technically how will you do this, this seems to be a predicted line and this is actually the actual behaviour of sales.

So, then the difference will give you the kind of you know error component once you get the error component that need to be validate before you go for actually further sales forecasting or you know sales prediction. So, what will you do technically? So, having this particular you know information. So, we will go to this particular you know trend analysis and then we will get the trend line.

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So, the focus of the trend line will be like this. So, these are all kind of you know actual points, these are all actual points and in between the line straight lines is called as you know trend line. So, we start with the first linear one and similar kind of you know various package will be used here to get this particular you know trend line and once you get the trend lines. So, it will give you the forecasted figures and the actual figure and forecasted figure will give you the error component.

So, now, once you get the error component and you know the kind of you know structure you know indicators which you have. So, we will validate the error component before with the before you go the kind of you know forecastings. So, let us see you know how is this particular you know structure. So, the kind of you know structure will be like this.

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Linear Trend Data & Error Analysis

Midwestern Manufacturing Company

Forecasting Linear trend analysis

Enter the actual values in cells shaded YELLOW. Enter new time period at the bottom to forecast

Period	Actual value (or) Y	Period number (or) X
Year 1	74	1
Year 2	79	2
Year 3	80	3
Year 4	90	4
Year 5	105	5
Year 6	142	6
Year 7	222	7
Intercept	56.714	
Slope	10.536	
Next period	141,000	8

Forecast	Error	Absolute error	Squared error	Absolute % error
67.250	6.750	6.750	45.563	9.12%
77.786	1.214	1.214	1.474	1.54%
88.321	-8.321	8.321	69.246	10.40%
98.857	-8.857	8.857	78.449	9.84%
109.393	-4.393	4.393	19.297	4.18%
119.929	22.071	22.071	487.148	15.54%
130.464	-8.464	8.464	71.644	6.94%
Average		8.582	110.403	8.22%
		MAD	MSE	MAPE

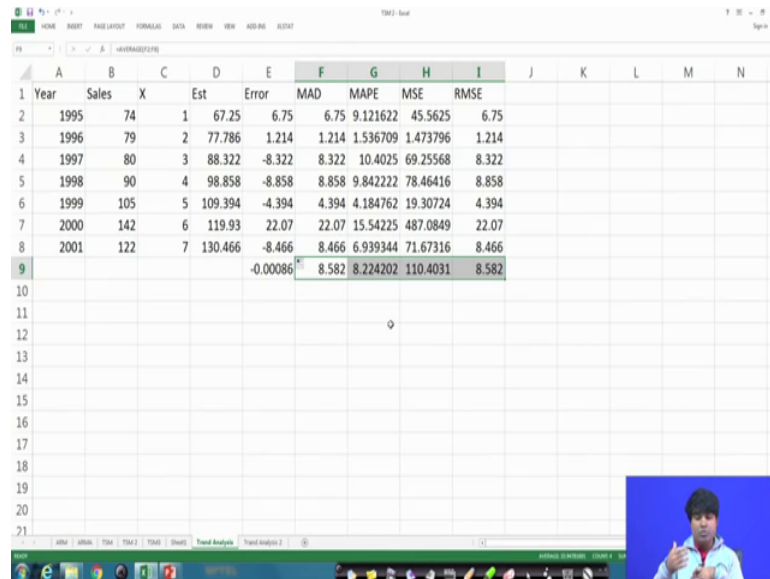
$y = a + bt$

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So, this is what the kind of you know time periods and this is the actual sales and now in the trend analysis. So, the time component can be used as you know X variables and then we like to the regress Y upon X where Y is the actual sales and X is the kind of you know time periods. So, accordingly so, we will apply the simple you know linear equations and then we estimate the linear equation through various mechanisms and the various mechanism structures will be like this.

So, we start with a simple equation like this Y equal to a plus b and t represents the time period and a is the parameter and b is the parameter initially they are unknown and with the help of this data and the kind of you know various structures. So, we can get to know a value and b value. So, accordingly, so, a value is here is this much and b value is this much and then we will go for the kind of you know forecasting for instance you know in order to know the kind of you know structure. So, what will you do? So, we can technically go to the excel sheet and get to know.

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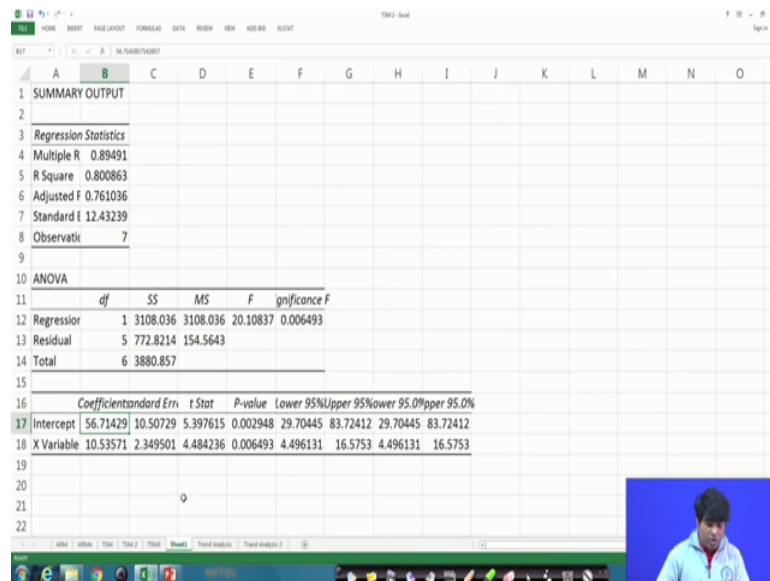
	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Year	Sales	X	Est	Error	MAD	MAPE	MSE	RMSE					
2	1995	74	1	67.25	6.75	6.75	9.121622	45.5625	6.75					
3	1996	79	2	77.786	1.214	1.214	1.536709	1.473796	1.214					
4	1997	80	3	88.322	-8.322	8.322	10.4025	69.25568	8.322					
5	1998	90	4	98.858	-8.858	8.858	9.842222	78.46416	8.858					
6	1999	105	5	109.394	-4.394	4.394	4.184762	19.30724	4.394					
7	2000	142	6	119.93	22.07	22.07	15.54225	487.0849	22.07					
8	2001	122	7	130.466	-8.466	8.466	6.939344	71.67316	8.466					
9					-0.00086	8.582	8.224202	110.4031	8.582					

How it is actually happening see here. So, this is what actually the kind of you know, the kind of you know figures this is what the sales figure yearly sales figures and what will you do. So, we create a new X with respect to the time period and the first year we represent 1, the second year represent 2 likewise we have actually 1995 to 2001.

So, the first step of this particular trend analysis to transfer this 1995 to 2001 to 1 to 7 so, accordingly we have now sales data which can be represented as a Y and here X here which is represented by 1 to 7. So, as a result so, the first step of the process you go to the you know data analysis and as usual in the data analysis. So, we choose the simple regression and once you know go to regressions. So, you can indicate with the dependent structure independent structure. So, accordingly so, sales can be indicated as a dependent variable here. So, we can you can ensure the kind of you know items then we go to the X you know series.

So, where you know X informations will be available from 1 to 7. So, the moment we put you know.

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SUMMARY OUTPUT						
Regression Statistics						
Multiple R	0.89491					
R Square	0.800863					
Adjusted R Square	0.761036					
Standard Error	12.43239					
Observations	7					
ANOVA						
	df	SS	MS	F	Significance F	
Regression	1	3108.036	3108.036	20.10837	0.006493	
Residual	5	772.8214	154.5643			
Total	6	3880.857				
Coefficients						
	Coefficient	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	56.71429	10.50729	5.397615	0.002948	29.70445	83.72412
X Variable 1	10.53571	2.349501	4.484236	0.006493	4.496131	16.5753

So this will give you the a linear regression results and that too here this is a kind of you know linear trend you know analysis so; that means, we need actually A coefficient and B coefficient. So, the A is here 56.71, B is 10.53. So, accordingly we can go to the particular you know structure. So, here so, what it will do. So, we just you know estimate the particular you know sales figure and on the basis of this parameter A equal to 56.714 and B equal to 10.536 and then once you actually set the particular equation and you scroll it then; obviously, it will give you the estimated figures.

So, once you get the estimated figures and we can you can obtain the error component which is the difference between the actual sales this is the B column and the you know you know forecasted sales that is the D column. So, the difference will give you the error component which is available in E column and then against corresponding to each you know actual sales and forecasted sales. So, we will find a error terms.

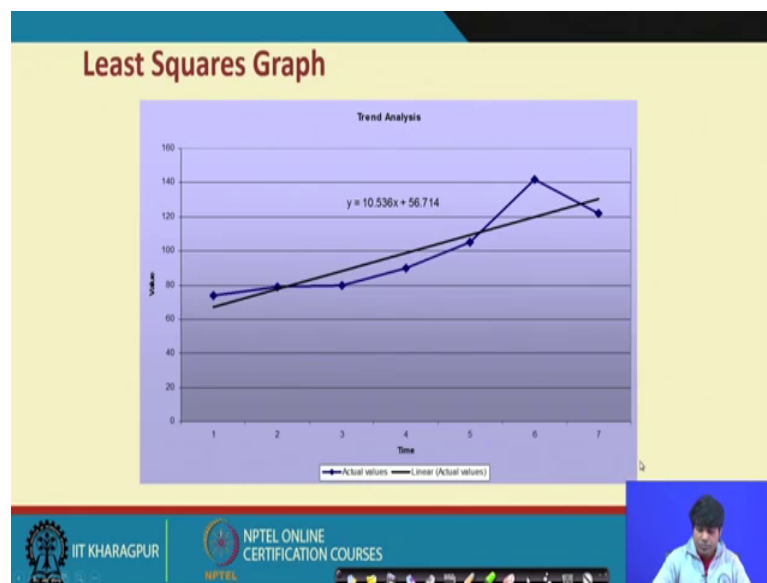
So, once you get the error terms so, the first check is to check the bias that is the difference between actual sale and forecasted sales. So, this sum of the bias should be equal to 0 and accordingly we can have you can check the particular you know error kind of error validations through mean absolute deviation, mean absolute percentage error, mean square error, root mean square error.

So, we have already calculated all these things and then we just check it you know as usual the previous discussions. So, then you know you can validate so. In fact, you know for any kind of you know models. So, you know once the forecasted model is ready. So,

you can get the error component and following the error component you can get mean error, mean absolute deviation, mean absolute percentage error, mean square error, root mean square error, but now the question is whether the model is actually feasible one for the prediction and forecasting for that we can have a comparative kind of you know structure and the model which can be used for you know finally, for the prediction and forecasting that should have a minimum error minimum you know mean absolute deviation, minimum, mean absolute percentage error, minimum mean square error and minimum root mean square error.

So, accordingly we can decide which model should we finally, use for the prediction and forecasting accordingly. So, the model is available here in our case. So, this is what the forecasted figure and the error component and then finally, the particular you know validation is you know validated through mean absolute deviation mean square error and mean absolute percentage error.

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So, likewise so, we can actually plot this one. So, this is the actual sales and this is the actual sales and this is the predicted sales and this particular structure is called as you know trend analysis. So, the trend analysis can give you now this is available for a particular you know pre outs and for any kind of you know future requirement you just enter the, you know particular date. So, automatically it will give you the forecasted figures.

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Least square Method (Linear Model)

$$Y_c = a + bx$$

Where,

- Y = Trend value to be computed
- X = Unit of time (Independent Variable)
- a = Constant to be Calculated
- b = Constant to be calculated

Example:-

Draw a straight line trend and estimate trend value for 2017:

Year	2012	2013	2014	2015	2016
Production	8	9	8	9	16

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So, the in this way so, we can actually develop a trend analysis and go for the kind of you know predictions. So, these are all you know mathematical structure through which you can apply least square mechanism to find out the trend line. So, what we have already discussed for a; you know another example which we can site here to check the particular you know validation.

So, this is the early information 2012 to 2016 and we like to estimate the trend value for 2017 and against. So, we can start with a simple structure linear model Y_c equal to a plus bx and the idea is to get the, a value and b value, once you get the, a value and b value. So, now, you know for X we can start with 2012 1, 2013 2, 2014 3, 2015 you know 4 and 2016 5. So; obviously, for 2017 the series will be 6. So, accordingly once you get a value and b value. So, then put X 6 then by default we will get the forecasted figure of the sales for 2017.

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Model Output

$$\sum Y = Na + b \sum X$$

$$\sum XY = a \sum X + b \sum X^2$$

$$50 = 5a + 15(b)$$

$$166 = 15a + 55(b)$$

$$Y_c = a + bx;$$

$$Y = 5.2 + 1.6X$$

Now we calculate the trend line for 2017:-

$$Y_{2017} = 5.2 + 1.6(9) = 14.8$$

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So, likewise so, these are the mathematical structure well structure through which you get the particular you know models. So, this is the estimated model 5.2 you know plus 1.6X and then accordingly. So, the trend structures so, this is the estimated model. So, now, for 2017 so, this is the kind of you know figure need to be adjusted here. So, then accordingly we can get the forecasted figure. So, this is how this linear structure through which we can do the predictions and get the kind of you know forecasted figure for you know next year.

So, these are the process through which we can actually you know do the kind of you know trend analysis and then we will get the kind of you know structure you know prediction structure and forecasting structure.

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Least square Method (Non-Linear Model)

Model Structure:

$$Y_c = a + bx + cx^2$$

There are three parameters 'a', 'b' and 'c'.

$$\sum Y = Na + b \sum X + c \sum X^2$$

$$\sum XY = a \sum X + b \sum X^2 + c \sum X^3$$

$$\sum X^2 Y = a \sum X^2 + b \sum X^3 + c \sum X^4$$

So, it is not necessarily that we can every retine go for you know linear trend. So, we can also follow the non-linear trend. So, what is the usual structure we have to just you know plot the data and check the movement of a particular you know variables. So, corresponding to the you know data visualizations that is through graphically. So, we can know the particular functional form and if the functional forms as you know second degree equation then again various mechanisms can be applied accordingly and then you can again re estimate and get the parameters.

In this case you know this can be like this so, it is a you know secondary equation. So, here we need to know what is a value, b value and c value. So, for that you know for the, for you know various structures. So, we have 3 standard equations, structural equation through which you can solve the particular problem. So, similarly we have a Y that is the actual sales and x is the time informations then x is represented as a t and then. So, we can get x square and other components through which you can actually you know solve these 3 equations and that too get the values of you know a, b, c. So, once you get values of a, b, c. So, this will be the forecasted figure and then we have actual figure and the forecasted figures by default we will get the error component again we will follow the similar mechanism to test the model first before you go the go for the kind of you know prediction and forecastings.

So; that means, technically so, you know the trend analysis will help you to get the kind of you know prediction of a particular variables and as per the kind of you know baseness requirement or you know management requirement.

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Forecasting by Autoregressive Model

An autoregressive model of order p , an $AR(p)$ can be expressed as

$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + u_t$$
$$y_t = \mu + \sum_{i=1}^p \phi_i y_{t-i} + u_t$$

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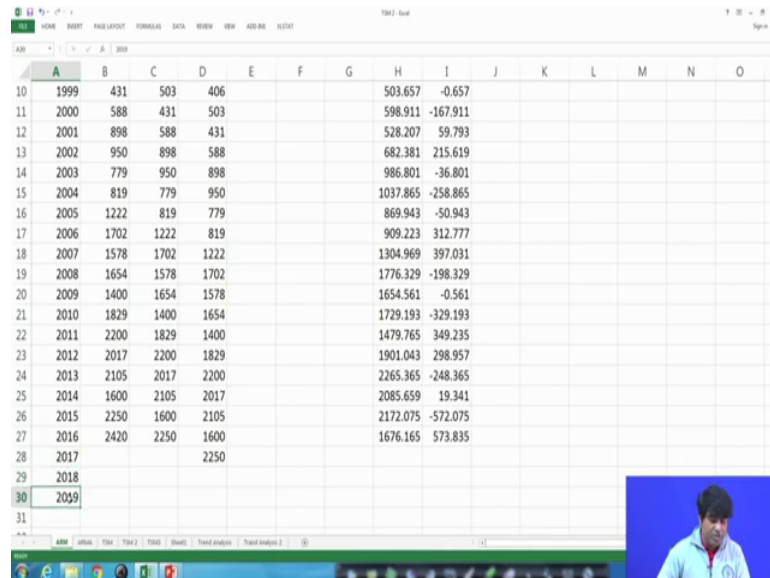
So, likewise the second type you know in the time series, second type of you know structure which we will like to discuss here is the autoregressive scheme and we have a 3 models all together here. Autoregressive model, moving average model and autoregressive moving average models; that means, the combination of autoregressive and moving average. In the autoregressive models the typical structure is you know the actual sales need to be predicted with you know previous sales, that is actually you know technically we can you know club with a or connect with a log variables.

So, Y_t represents sales you know at a current you know time frame and then followed by the previous you know sales that is represented by Y_{t-1} and Y_{t-2} Y_t you know minus 3 and so on. So, many log variables you can create. So, once you get the log variables. So, the actual you know actual sales at a particular you know time can be connected with a previous time periods and then we will get a again the forecasted model and the like the previous trend analysis. So, we have actual sales and the predicted sales then we will find the error component and again the error component need to be tested or you know validate before you go for the prediction and forecastings.

So, here the measure is in how many log lines you can create for this particular you know autoregressive models. So, let me first you know give you the structure how the log variables can be created, then we will go for the kind of you know prediction. So,

what will you do here? So, this is what the particular you know model. So, you go to this particular you know auto regressive models see here.

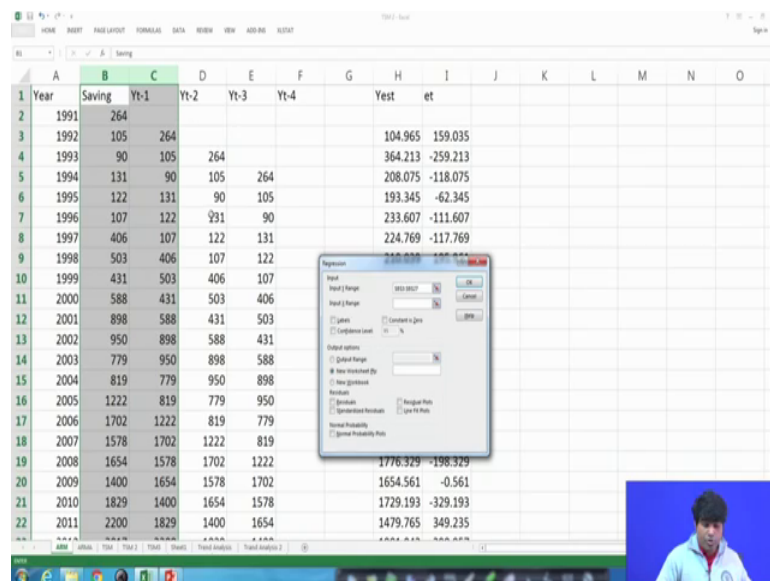
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Year	Saving	Yt-1	Yt-2	Yt-3	Yt-4	Yest	et
1999	431	503	406			503.657	-0.657
2000	588	431	503			598.911	-167.911
2001	898	588	431			528.207	59.793
2002	950	898	588			682.381	215.619
2003	779	950	898			986.801	-36.801
2004	819	779	950			1037.865	-258.865
2005	1222	819	779			869.943	-50.943
2006	1702	1222	819			909.223	312.777
2007	1578	1702	1222			1304.969	397.031
2008	1654	1578	1702			1776.329	-198.329
2009	1400	1654	1578			1654.561	-0.561
2010	1829	1400	1654			1729.193	-329.193
2011	2200	1829	1400			1479.765	349.235
2012	2017	2200	1829			1901.043	298.957
2013	2105	2017	2200			2265.365	-248.365
2014	1600	2105	2017			2085.659	19.341
2015	2250	1600	2105			2172.075	-572.075
2016	2420	2250	1600			1676.165	573.835
2017			2250				
2018							
2019							

So, this is the actual you know data set and let us say this is a saving of a particular economics and that is available from 1991 to 2016 and we need to predict a this saving rate for 2017, 2018 and 2019 and that too through auto regressive model.

(Refer Slide Time: 18:41)



Year	Saving	Yt-1	Yt-2	Yt-3	Yt-4	Yest	et
1991	264						
1992	105	264				104.965	159.035
1993	90	105	264			364.213	-259.213
1994	131	90	105	264		208.075	-118.075
1995	122	131	90	105		193.345	-62.345
1996	107	122	90	105		233.607	-111.607
1997	406	107	122	131		224.769	-117.769
1998	503	406	107	122			
1999	431	503	406	107			
2000	588	431	503	406			
2001	898	588	431	503			
2002	950	898	588	431			
2003	779	950	898	588			
2004	819	779	950	898			
2005	1222	819	779	950			
2006	1702	1222	819	779			
2007	1578	1702	1222	819			
2008	1654	1578	1702	1222		1776.329	-198.329
2009	1400	1654	1578	1702		1654.561	-0.561
2010	1829	1400	1654	1578		1729.193	-329.193
2011	2200	1829	1400	1654		1479.765	349.235

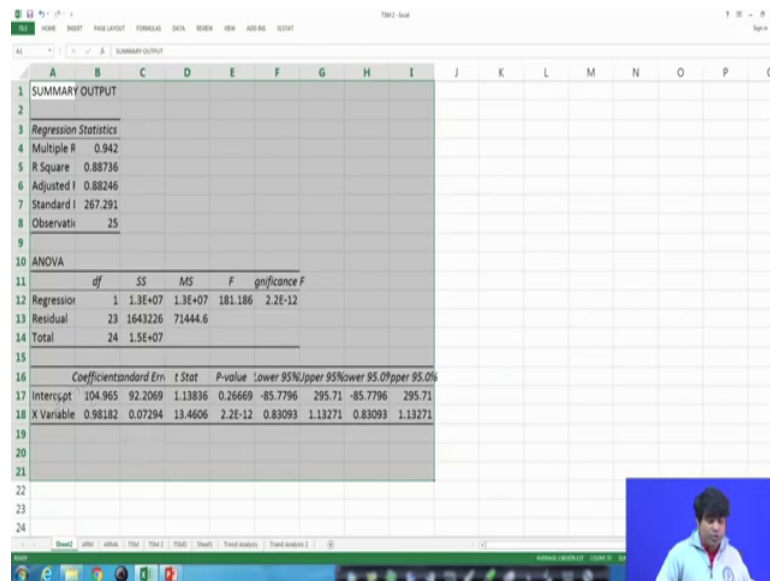
So, by default this saving data you know for this time period 1991 to 2016. So, it can be represented as a Y_t and then. So, the way you know you can create the log variable is like this. So, just copy the particular variable and then you can paste one log behind.

So, by default so, like this we have already created here. So, Y_{t-1} , Y_{t-2} , then for Y_{t-3} we can paste it here. So, likewise you know one log behind by default now this is Y_t and then this Y_t can be connected with Y_{t-1} and against Y_t it can be connected with the Y_{t-2} , Y_t can be connected with Y_{t-3} . So; that means, we can have here one model with Y_t , Y_{t-1} , again we can connect with Y_t , Y_{t-1} , Y_{t-2} ; that means, 3 variables simultaneously where Y_{t-1} and Y_{t-2} can be treated as independent variables and Y_t is the dependent variable. So, likewise again we can create a third log variables Y_{t-3} then again Y_t can be connected with Y_{t-1} , Y_{t-2} and Y_{t-3} so; that means, since we have actually more data points. So, we can create actually n number of log variables.

So; obviously, the first issue is a how many log variables can be connected at a particular point of time. So, that ; that means, you know it is a very interesting, kind of you know structure. So, what is the challenge that you know first you fix the log length, then you know then you go for the kind of you know estimation. So, one there are you know different indicators through which you can fix the log length, but you know manually you can you know test yourself and you know check the particular you know structure. What we can start means what we can do, we can start with the first Y_t and Y_{t-1} like you know simply go to the data analysis and then what will you do?

So, you can connect with a Y_t same regressions and then we can connect with the particular you know indications. So, this is ok. So, we can start here ok. So, same data point so, we can start here from this data to this data ok. So, this is X indications and then accordingly. So, your you know independent variable will be Y_{t-1} . So, that will start from the second you know second unit and as a result you just put ok.

(Refer Slide Time: 21:13)



SUMMARY OUTPUT									
Regression Statistics									
Multiple R	0.942								
R Square	0.88736								
Adjusted R Square	0.88246								
Standard Error	267.291								
Observations	25								
ANOVA									
	df	SS	MS	F	Significance F				
Regression	1	1.3E+07	1.3E+07	181.186	2.2E-12				
Residual	23	1643226	71444.6						
Total	24	1.5E+07							
Coefficients									
	Coefficient	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%	
Intercept	104.965	92.2069	1.13836	0.26669	-85.7796	295.71	-85.7796	295.71	
X Variable	0.98182	0.07294	13.4606	2.2E-12	0.83093	1.13271	0.83093	1.13271	

So, this will give you the kind of you know moving average models where Y_t as a function of Y_{t-1} . So, likewise if you add Y_{t-2} then again so, we have actually interested. So, then V_1 coefficient, V_2 coefficient so, the model will be extended accordingly. So, technically so, we have actually a particular variable, but we can create a number of variables through which you can actually predict the particular you know saving rate. So, obviously, the first is you know how many lag lines you can check.

So, what will you do? So, you connect with Y_t Y_{t-1} this is the first model and you can connect with Y_t Y_{t-1} and Y_{t-2} this is second model then Y_t Y_{t-1} Y_{t-2} third model. So, likewise you know we can have a plenty of you know models every time you estimate and get the error component and against you can find out the model indicators mean absolute deviation, mean square error, root mean square error.

Then now for you know Y_t Y_{t-1} then Y_t Y_{t-1} , Y_{t-2} . So, every case you have to find 2 error component and all the model indicators like mean square error, mean absolute deviation, mean absolute percentage error. Then finally, check where the particular you know you know model is very reliable or you know very perfect with respect to minimum of all these indicators like you know mean error, mean square error, mean absolute deviation, mean percentage error.

So, once you confirm that this particular model is you know perfectly fit compared to other models then you will go for the kind of you know prediction and the kind of you know management requirement. So; obviously, the first you know structure is here the choice of the log length and we have a. So, you know some standard indicators through which you can check the log length. So, there is A I C statistic and S I C statistic through which you can find out the kind of you know optimum log length through which you can actually indicate the particular you know structure before you go for the estimation.

Otherwise manually you have to you know develop a model every time and you know check them finally, you can do the comparative analysis and fix which model is perfectly for this particular you know prediction and forecasting. So, this is what the kind of you know auto regressive structure and accordingly you can have another model called as you know moving average model.

(Refer Slide Time: 23:31)

Forecasting by Moving Average Model

$$y_t = \mu + \theta_0 u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \dots + \theta_q u_{t-q}$$

It is a q^{th} order moving average model $MA(q)$.

Handwritten notes:

- $y_t = a + b_1 y_{t-1} + b_2 y_{t-2} + \dots + b_q y_{t-q}$
- $e_t = y_t - f_t$
- $y_t = f(y_{t-1})$
- $y_t = f(y_{t-1}, y_{t-2})$

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So, in this context so, in the it is actually similar lines like, you know it is the similar line you know similar structures here is like you know Y_t can be forecasted with you know error component. So; that means, technically we start with a simply let us say Y_t equal to function of Y_t minus 1 like the moving average model and in we can have also model like Y_t as a function of Y_t minus 1 and Y_t minus 2, so; that means, we have a n number of ways, we can develop model for you know for this particular you know structure in the you know moving average structures you can start you know connecting with first Y_t

and Y_{t-1} and then according to this particular structure your model will be simply $Y_t = a + b Y_{t-1} + \text{error terms}$ so, the first requirement is to find out a and b .

So, then once you get a and b by simple regression, then you can get the error component which is nothing, but you know equal to Y_t minus forecasted figures f_t and once you get the error component. So, this will be treated as you know e_t . So, it is a variable now. So, like you know Y_t we can create Y_{t-1} , Y_{t-2} , Y_{t-3} etcetera. So, we can also create e_{t-1} , e_{t-2} and so, on. So, for this what we can do? So, you can go to the particular you know excel sheet and then you check here. So, what will you do here?

So, in this case so, what will do? So, you go just you know you know same data sets saving rate. So, we have actually Y_t this 1s, then this is Y_{t-1} , Y_{t-2} , which I have already you know mentioned how to create Y_{t-1} , Y_{t-2} , Y_{t-3} , Y_{t-4} and for e_t so, what will you do, first you know go to the data analysis and then you just connect with the you know Y_t and Y_{t-1} for this we have already indicated here the particular you know requirement.

So, the first requirement is to indicate Y_t information and that too that too from B 3 to B 27 then against. So, you can give the indication about the independent variables, in this case independent variable will be Y_{t-1} and against from C 3 to you know C 27 and then you just estimate. So, what you know in this case, you will get the A coefficient and B coefficient, once you get the A coefficient and B coefficient then you come to again excel sheet and then you know develop the estimated equations by putting A equal to 100.965 and B 0.982. So, you can actually just connect with you know Y_{t-1} then you get the error component.

So, now once you generate the error component first you check the all you know biasness mean square error etcetera ah, but in the requirement of you know moving average we need to create you know error component. So, that is the difference between you know actual sales that is Y_t and the forecasted sales and as a result you will get the error component like this. So, now, once you get this error component. So, this error component need to be actually pasted one point behind every times like this you know like this.

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Year	Saving	Yt-1	Yt-2	Yt-3	Yt-4	Yest	et	et-1	et-2
1991	264								
1992	105						104.965	159.035	
1993	90	105	264				364.213	-259.213	523.213
1994	131	90	105	264			208.075	-118.075	223.075
1995	122	131	90	105			193.345	-62.345	152.345
1996	107	122	131	90			233.607	-111.607	242.607
1997	406	107	122	131			224.769	-117.769	239.769
1998	503	406	107	122			210.039	-195.961	-88.961
1999	431	503	406	107			503.657	-0.657	406.657
2000	588	431	503	406			598.911	-167.911	670.911
2001	898	588	431	503			528.207	-59.793	371.207
2002	950	898	588	431			682.381	-215.619	372.381
2003	779	950	898	588			986.801	-36.801	934.801
2004	819	779	950	898			1037.865	-258.865	1208.865
2005	1222	819	779	950			869.943	-50.943	829.943
2006	1702	1222	819	779			909.223	312.777	506.223
2007	1578	1702	1222	819			1304.969	397.031	824.969
2008	1654	1578	1702	1222			1776.329	-198.329	1900.329
2009	1400	1654	1578	1702			1654.561	-0.561	1578.561
2010	1829	1400	1654	1578			1729.193	-329.193	1983.193
2011	2200	1829	1400	1654			1479.765	349.235	1050.765

So, you can actually connect continuously like this. So, that you know you will get actually this is e_t and again this will be e_{t-1} and similarly you can create e_{t-2} , e_{t-3} , e_{t-4} . So, like this you know you can create a number of you know error terms. So, now, in the moving average structures. So, the final model will be so, Y_t which can be connected with the error terms. So, now, instead of you know Y_t , Y_{t-1} , Y_{t-2} , Y_{t-3} , Y_{t-4} .

So, now, the model will be Y_t upon you know error terms e_t and then you go for the estimations, similarly Y_t can be connected with the e_t and e_{t-1} , then Y_t can be connected with the e_{t-1} , e_{t-2} . So, against we have a plenty of you know different models ah. So, every times you have to estimate them get the error component and check the validation of you know error component then finally, fix which model is perfectly fit for this particular you know prediction and again you go for the you know the kind of you know management requirement that is with respect to forecasting and the kind of you know the saving prediction of a you know upon a economic right.

So, what will you do here, like the previous case moving average you know autoregressive models in the case of you know moving average. So, same problems will be with respect to log length and; obviously, you go by you know different kind of you know models and check all these error components and finally, if you pick up a particular model which is having actually minimum error or minimum root mean square

error, minimum mean absolute percentage error then finally, that model will be used for you know forecasting.

So, against so you know the choice of the log length is very important because what is happening if you connect with the wide with you know more number of you know error you know log variables or more number of you know log you know dependent variable then; obviously, so, what is happening, you know you have a less degree of freedom to validate the particular model so; obviously, the first and requirement of this kind of you know model is to fix the optimum log length.

Then you can go for the kind of; you know estimations once you go you know fix the optimum log length. So, you will get the particular structure and then the estimation process will give you the kind of; you know forecasted models. So, once you get the optimum forecastings. So, the error will be finally, checked and validate before you go for again you know future forecasting like the kind of you know future predictions.

So, the third model corresponding to you know you know auto regressive model and moving average.

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Forecasting with ARMA Model

$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \dots + \theta_q u_{t-q} + u_t$$

$$E(u_t) = 0; E(u_t^2) = \sigma^2; E(u_t u_s) = 0, t \neq s$$

AR: $y_t = f(y_{t-1})$ MA: $y_t = f(e_t)$ ARMA: $y_t = f(y_{t-1}, e_{t-1})$

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The third model is actually a clubbing of you know both auto regressive and moving average that too it is called as a ARMA models. So, in the ARMA models so, the Y t can be connected with the kind of you know this is Y t connected with the log of the Y you

know Y variables and the log of you know error component so; that means, in the first steps you connect with Y_t and Y_{t-1} and then get the you know parameters value. So, on the basis of the parameter value you can get the estimated equation.

So, the actual information and the estimated information will give you the error term and once you get the error terms you can create you know log error variable and then finally, in the excel sheet we have actually dependent variable Y_t followed by independent variables the log variables Y_{t-1} , Y_{t-2} , Y_{t-3} and so, on and against the error log variables u_{t-1} , u_{t-2} , u_{t-p} and accordingly. So, you can fix up a particular you know structure and then go for the kind of you know prediction.

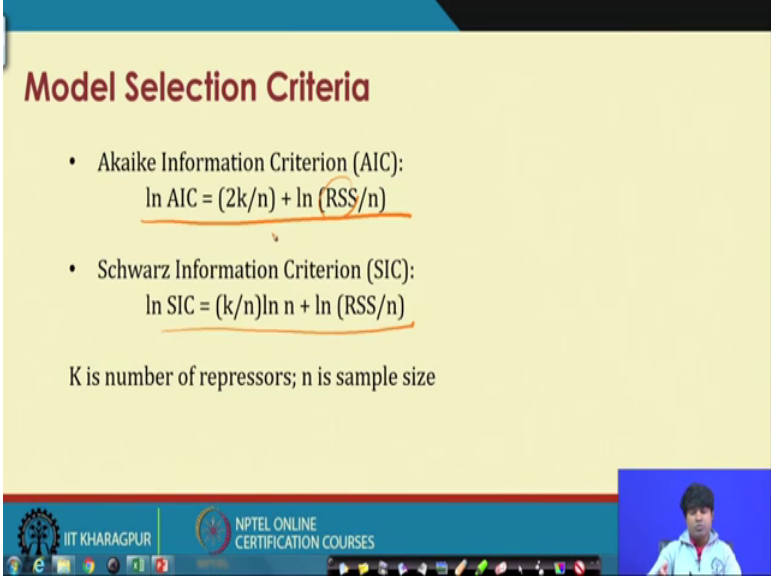
For instance corresponding to let us say this is actually auto regressive structure and this is you know moving average structure and this is ARMA structure. So, in the case of you know auto regressive structures. So, let us say we start with a first one. So, Y_t equal to function of Y_{t-1} and here in the moving average Y_t as a function of e_{t-1} or e_t simply, where in the case ARMA. So, Y_t is a function of both Y_{t-1} s and e_{t-1} s. So, like so, this is actually similar kind of you know structure.

So, in one case you are connecting the actual variable with log variables, in another case you are connecting with actual variable with a error log variables and finally, in the third case that is auto regressive moving average. We are just clubbing the auto regressive scheme and moving average schemes then we you know have a different kind of you know models where the particular variable will be connected with the log of that particular variables and the log of the error variable which is derived from the particular variable Y_t .

So; obviously, the same problem is here is like you know fixing optimum log length. So, once you fix the optimum log length then you know you find out the estimated equations, then once you get the estimated equations the error can be error component can be generated accordingly and once you get the error series the forecast actual sales and the forecasted sales then finally, you will get the error component again and that error component need to be validate before you go for you know future forecastings and future predictions. So, likewise you know this is this is a kind of you know time series structure through which you know you will have a interesting structure through which

you can get the particular you know prediction kind of you know environment through which you can do the proper predictions as per the management requirement.

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Model Selection Criteria

- Akaike Information Criterion (AIC):
$$\ln AIC = (2k/n) + \ln (RSS/n)$$
- Schwarz Information Criterion (SIC):
$$\ln SIC = (k/n) \ln n + \ln (RSS/n)$$

K is number of regressors; n is sample size

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So, what we have actually you know discussed in the; you know last slide. So, with respect to the auto regressive model, moving average models and auto regressive moving average model. So, every time what we have actually seen. So, we have actually plenty of scopes to develop you know simple bivariate structure to you know a multivariate structure so; that means, we have a kind of you know flexibility here to develop you know very interesting you know multivariate framework through which you can do the kind of you know prediction as per the management requirement.

But you know mathematically it is very easy to develop you know or connect the models and extend the model from bivariate to multivariate, but finally, so, the obvious choice is actually log length because you cannot have a multivariate you know kind of you know infinite structure you start with the Y_t and if you have actually more data points you can create you know n number of you know large variable with respect to both you know the dependent variable and the kind of you know error variable. So, technically we have actually plenty of you know models through which you can actually do the kind of you know forecasting. So, ah; obviously, the first hand choice is the to check you know the optimum log length and that is one of the you know important step in the case of you

know auto regressive model, moving average model and auto regressive moving average models.

So, once you actually fix the optimum log length then you know it will give you the kind of you know structure through which you can proceed for the prediction and forecasting. So, what I have actually mentioned earlier. So, the choice of log length is very important for both these you know for both these models that is auto regressive and moving average and that too again ARMA model and so, the choice of log length you know to fixing the kind of you know optimum structure.

So, we have actually Akaike Information Criteria that is A I C statistics and then S I C statistics. So, the particular formula is here available to you know report the akaike you know information criteria that is A I C statistics and this is S I C statistics and here R S S is a Residual Sum of Squares that is actually derived from the the estimated equations and then k and n depends upon you know number of you know regressions and the kind of you know sample size and once you get actually a the particular you know estimated equations.

So, by default anova analysis are variance stable so, they will provide you explained sum of squares and total sum of square, but residual sum of square is the input to the A I C statistic and S I C statistic and once you get all these things. So, calculate the A I C statistics and S I C statistic like the kind of you know model choice with respect to mean square error, mean absolute deviation, mean root, mean square error here a final models with respect to log length particular log length depends upon the minimum value of you know A I C and S I C statistics.

So, it is a kind of you know iterative and continuous process with respects to fixing the optimum log length and fixing a particular you know models whether with go ahead with you know auto regressive schemes or moving average schemes or the kind of you know ARMA schemes. So, depending upon a particular structure and the kind of you know best predictions we have to first pick you know pick up a particular model as per the requirement and then we will go for the prediction and forecasting which is as per the management requirement or the then in that context you can go for you know having good management decision as per the particular you know business requirement.

So, likewise you have actually discussed couple of you know time series techniques and in all these time series techniques you have a historical data and you have to develop a forecasting model and with the help of forecasting models you will go for you know future prediction and future forecastings as per the kind of you know business requirement or management requirement with this we will stop here.

Thank you very much have a nice time.