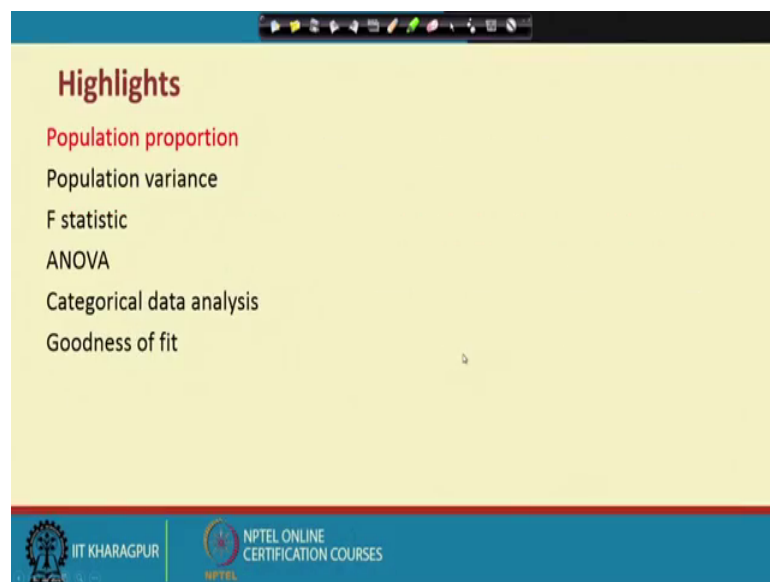


**Business Analytics for Management Decision**  
**Prof. Rudra P Pradhan**  
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**Indian Institute of Technology, Kharagpur**

**Lecture – 23**  
**Inferential Analytics Part – 2 (Contd.)**

Hello everybody this is Rudra Pradhan here and welcome you all to BMD lecture series today our discussion is on inferential analytics and that we will continue for business problem relating to multiple sample case and in fact, we have already discussed this concept with respect to t statistics and Z statistics and then we will continue with respect to population proportions and chi square distributions and F distributions.

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Let us have the first case for you know the kind of you know population proportion and some of the highlights of this lecture are like this the a this population proportions, population variance, F statistics, then analysis of variance ANOVA and a categorical data and finally, goodness of fit.

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**Large-Sample Test for Difference between Two Population Proportions**

- Hypothesized difference is zero
  - ✓ I: Difference between two population proportions is 0
    - $p_1 = p_2$ 
      - »  $H_0: p_1 - p_2 = 0$
      - »  $H_1: p_1 - p_2 \neq 0$
- ✓ II: Difference between two population proportions is less than 0
  - $p_1 \leq p_2$ 
    - »  $H_0: p_1 - p_2 \leq 0$
    - »  $H_1: p_1 - p_2 > 0$

*Handwritten notes: Case 1 (next to I), Case 2 (next to II), Case 3 (next to III)*

- Hypothesized difference is other than zero:
- ✓ III: Difference between two population proportions is less than D
  - $p_1 \leq p_2 + D$ 
    - »  $H_0: p_1 - p_2 \leq D$
    - »  $H_1: p_1 - p_2 > D$

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So, the discussion is with respect to 2 population proportions so; that means, here the idea is a we have a 2 different samples and we like to check whether there is a significant difference between 2 sample mean and here the sample observation with respect to population proportion.

So, as a results we have altogether 3 different situations, in the first situations. So, both the samples means since there are 2 samples let us say  $p_1$  and  $p_2$ . So, this  $p_1$  is the proportion of first one and  $p_2$  is the proportion of the second one. So, then accordingly, either they are equals; that means,  $p_1$  equal to  $p_2$  and the second case is if not  $p_1$  less than to  $p_2$  or  $p_1$  greater than to  $p_2$ .

So, accordingly we are going to address this problem with 3 different situation, the first situation on the hypothesis will be  $p_1$  equal to  $p_2$  and as a result the difference between  $p_1$  minus  $p_2$  equal to 0 and the corresponding alternative hypothesis will be corresponding alternative hypothesis will be  $p_1$  minus  $p_2$  not equal to 0 and this is the case 1 and the case 2 will be  $p_1$  less than equal to  $p_2$ . So, this is the case 2 case 2 option and in this case. So,  $p_1$  minus  $p_2$  less than equal to 0 then by default the alternative hypothesis will be  $p_1$  minus  $p_2$  greater than equal to 0.

So; that means, say a in the accordingly the third case will be the difference between the 2 will be having some kind of you know a particular difference which is less than to D. So, that is what actually  $p_1$  less than equal to  $p_2$  plus D. So, accordingly the difference

between  $p_1$  minus  $p_2$  will be followed by the difference you know the D component. So, accordingly the corresponding alternative hypothesis will be  $p_1$  minus  $p_2$  greater than to greater than to D so; that means, this is case 3, we have altogether 3 different cases that we have to test with the case of you know 2 population proportions so; that means, there are 2 populations then we have to draw the samples and we like to check whether there is a significant difference between the 2 and if there is a significant difference which one is the greater and which one is the lessers and how much this difference and which one you know whether the particular differences are statistically significant. So, this is the kind of you know things we are we are going to check and address.

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**Comparisons of Two Population Proportions When the Hypothesized Difference Is Zero: Test Statistic**

When the population proportions are hypothesized to be equal, then a pooled estimator of the proportion ( $\hat{p}$ ) may be used in calculating the test statistic.

A large-sample test statistic for the difference between two population proportions, when the hypothesized difference is zero:

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where  $\hat{p}_1 = \frac{x_1}{n_1}$  is the sample proportion in sample 1 and  $\hat{p}_2 = \frac{x_2}{n_2}$  is the sample proportion in sample 2. The symbol  $\hat{p}$  stands for the combined sample proportion in both samples, considered as a single sample. That is:

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

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So, accordingly, the structure which we like to follow is the Z statistics and here the idea is that you know we have to select a means we have to test for the last samples. So, like the earlier case. So, we can use Z statistics provided sample size will be greater than to 30 and then you know there is availability of you know population variance accordingly will it test the particular you know kind of you know requirement.

So, the corresponding test statistics will be. So, like you know  $z$  equal to  $p_1$  minus  $p_2$  that is actually sample mean and this is also sample mean and their difference need to be tested whether it is a statistically significant or not then accordingly the population proportions will be  $p_1$  equal to  $p_2$  then the as a result the difference will be equal to 0.

Then in the case of you know  $p_1$  here the sample a sample case will be  $x_1$  by  $n_1$  and in the case of  $p_2$ , it will be  $x_2$  by  $n_2$  and accordingly the total probability for this particular you know selection will be  $x_1$  plus  $x_2$  by  $n_1$  plus  $n_2$ , this is  $x_2$ ,  $x_1$  plus  $x_2$  by  $n_1$  plus  $n_2$ .

So, as results we will get you know the probability of happenings then the counterpart will be you know  $1 - q$  that is not happening. So, accordingly this  $1 - p$  is nothing, but actually a  $q$ . So, this or the standard deviation of this particular component will give you the kind of you know standard errors and which will add the value for these testings that to the Z testings.

So, accordingly, we can test a particular you know situation and check whether the difference will be statistically significant or not. So, the procedure of testing you know more or less same, we will fix the probability you know that is the alpha component and then we will allow the particular case whether it is you know one tail or to tail then you have to specify the critical points that is the confidence interval and then the rejected zone.

So, we like to check whether the  $z$  calculated will be falling under the accepted zones or you know rejected zones if it is under the rejected zones then you are rejecting the true null hypothesis that in that case  $p_1$  equal to  $p_2$  and if not then you have to accept the null hypothesis. So, the business inference will be drawn accordingly and as a result the management decision can be taken care. So, in order to justify the particular you know structures let us you know go by you know a little bit elaboration.

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**Sampling Distribution of Differences in Sample Proportions**

For large samples

1.  $n_1 \cdot \hat{p}_1 > 5$ ,
2.  $n_1 \cdot \hat{q}_1 > 5$ ,
3.  $n_2 \cdot \hat{p}_2 > 5$ , and
4.  $n_2 \cdot \hat{q}_2 > 5$  where  $\hat{q} = 1 - \hat{p}$

the difference in sample proportions is normally distributed with

$$\mu_{\hat{p}_1 - \hat{p}_2} = P_1 - P_2 \quad \text{and}$$
$$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{P_1 \cdot Q_1}{n_1} + \frac{P_2 \cdot Q_2}{n_2}}$$

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So, here in this case, we are transferring the population proportion to single ones that is actually  $p$ . So, which is nothing, but  $x_1$  plus  $x_2$  by  $n_1$  plus  $n_2$  then corresponding  $q$  equal to  $1 - p$ , but now when they are you know having actually difference then as a result the sample variance will be considered like this. So, corresponding to the previous ones it is only single one. So, that is  $p$  into  $q$  or subject to the adjustment of you know sample observations. So, here  $P_1 Q_1$  is the first part and  $P_2 Q_2$  is the second part that is for the in a second samples then the combined standard deviation will be square root of  $P_1$  into  $Q_1$  by  $n_1$  plus  $P_2$  into  $Q_2$  by  $n_2$ . So, accordingly, the structures of you know testing will be in like this.

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**Z Formula for the Difference in Two Population Proportions**

$$Z = \frac{(\hat{P}_1 - \hat{P}_2) - (P_1 - P_2)}{\sqrt{\frac{P_1 \cdot Q_1}{n_1} + \frac{P_2 \cdot Q_2}{n_2}}}$$

$\hat{P}_1$  = proportion from sample 1  
 $\hat{P}_2$  = proportion from sample 2  
 $n_1$  = size of sample 1  
 $n_2$  = size of sample 2  
 $P_1$  = proportion from population 1  
 $P_2$  = proportion from population 2  
 $Q_1 = 1 - P_1$   
 $Q_2 = 1 - P_2$

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So, then in this case it is just same structures, but now here in this case you know with the help of you know kind of you know null hypothesis. So, we have to see the difference will be exactly equal to 0.

So, then correspond corresponding to this particular structure Z will be finally, you know the difference between 2 proportion sample proportions and that will be adjusted with the standard deviations then finally, we will get the kind of you know the kind of Z calculated and accordingly. So, this is actually first proposition sample 1 then proportion from the sample 2 and this is n 1 the sample size for the first ones and this is n 2 the sample size for the second one and this is population proportion P 1 and this is population proportion P 2. So, Q 1 by default will be 1 minus P 1 and Q 2 it will be 1 minus P 2.

So, now, having the information from the problem we have to actually put all these figures in the Z you know Z statistic then we like to calculate the you know Z value. So, then on the basis of you know Z calculated value. So, we have to compare with Z critical and then we will take the particularly you know decisions.

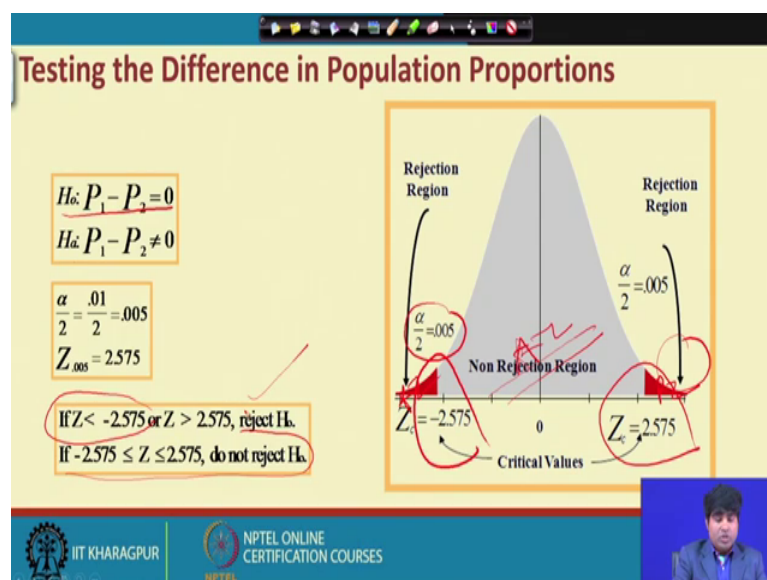
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### Z Formula to Test the Difference in Population Proportions

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (P_1 - P_2)}{\sqrt{(\bar{P} \cdot \bar{Q}) \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}$$
$$\bar{P} = \frac{X_1 + X_2}{n_1 + n_2}$$
$$= \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2}$$
$$\bar{Q} = 1 - \bar{P}$$


So, here is the same structures in this case you know we have actually single component instead of you know  $p_1$   $Q_1$   $p_2$   $Q_2$ . So, we are putting actually  $p$  into  $Q$ , where  $p$  equal to  $X_1$  plus  $X_2$  by  $n_1$  plus  $n_2$ . So, as a result the total observations will be  $n_1 p_1$  and then  $n_2 p_2$  and accordingly the adjustment will be  $Q$  equal to  $1$  minus  $p$ .

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So, now these are the typical structure through which we have to follow and here is the testing procedure and in this case the null hypothesis will be a  $P_1$  minus  $P_2$  and that is equal to  $0$  because we are assuming that you know both are you know same proportion

and the alternative hypothesis  $Z$  not equal to. So, as a result they are difference equal to not equal to 0 and we are fixing here alpha and that to 1 percent.

So, as a result if we allow 2 sides then the alpha by 2 will be 0.005 this side and 0.005 this side and this is what actually a rejected zones and this you know this is accepted zones and this is what the rejected zone and this is what the rejected zone and accordingly the  $Z$  criticals is 2.575 and also this sides 2.575 so; that means, technically if the  $Z$  calculated is a less than 2 minus 275 we are going to reject the true null hypothesis; that means, in that case population of proportion  $P_1$  equal to  $P_2$  and if not then and then we have to worry a you know if we are not in a position to reject the null hypothesis and then we have to accept the alternative hypothesis; that means, a  $P_1$  minus  $P_2$  not equal to 0.

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**Testing the Difference in Population Proportions**

$n_1 = 100$   
 $X_1 = 24$   
 $\hat{p}_1 = \frac{24}{100} = 0.24$

$n_2 = 95$   
 $X_2 = 39$   
 $\hat{p}_2 = \frac{39}{95} = 0.41$

$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{24 + 39}{100 + 95} = \frac{63}{195} = 0.323$

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (P_1 - P_2)}{\sqrt{\bar{p} \cdot \bar{q} \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$= \frac{(0.24 - 0.41) - (0)}{\sqrt{(0.323)(0.677) \left( \frac{1}{100} + \frac{1}{95} \right)}}$$

$$= \frac{-0.17}{0.067} = -2.54$$

Since  $-2.575 < Z = -2.54 < 2.575$ , do not reject  $H_0$ .

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So, accordingly you will get some kind of you know inference and on the basis of this inference we can get some kind of you know you have to take a kind of you know decision and to verify the particular structures we are taking actually 2 different options and in the first case is actually sample 1 and this is what the sample 2 and  $n_1$  is 100 and  $X_1$  is 24. So, by default  $P_1$  you know estimated will be 24 by 100.

So, that is copying 0.24 and similarly in the second case we have a sample size 95 and we are assuming you know  $X_2$  equal to 39 and as a result the proportion sample proportion will be 39 by 95. So, it is coming actually 0.41. So, now you have  $P_1$



estimate that is 0.24 and we have  $P$  to estimate that is 0.41 and as a result you can also equally calculate  $P$  combined and then that is nothing, but actually  $X_1$  plus  $X_2$  by  $n_1$  plus  $n_2$ . So, that is nothing, but actually 24 plus 39 divided by 100 plus 95. So, this 100 and this 95 this is actually coming under the  $n_1$  plus  $n_2$  and then  $X_1$  24 and  $X_2$  39 it will be coming here the upper part. So, as a result your  $P$  combined  $P$  estimate will be 0.323.

So, now on the basis of that you know we can come to this you know  $Z$  statistics here,  $P_1$  minus  $P_2$  that is the difference. So, as a result that is nothing, but 0.24 and minus 0.41 and since we are assuming that you know  $P_1$  and  $P_2$  are equal. So, their difference will be coming equal to 0.

So, this is what we are inputting 0 here and then,  $P$  component and you know  $P$  is coming 0.323 as a result  $Q$  will be 1 minus 0.323. So, it is coming 0.677 and this is the adjustment factors  $1$  by  $n_1$  plus  $1$  by  $n_2$  and after simplifications we are getting  $Z$  value equal to minus 2.54. So, now, the critical value which we have already highlighted in the previous slide is nothing, but minus 2.575 and this value is coming actually minus 2.54 which is actually a you know lessers.

So, as a result you are not in a position to reject the true null hypothesis so; that means, technically there is a you know equality between 2 sample proportions. So,  $P_1$  equal to  $P_2$ . So, that is what the inference we have to draw with this particular you know case and then in order to get you know more details we will take another examples.

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**Comparisons of Two Population Proportions When the Hypothesized Difference Is Zero: Example**

Carry out a two-tailed test of the equality of banks' share of the car loan market in 1980 and 1995.

Population 1: 1980	Population 2: 1995
$n_1 = 100$	$n_2 = 100$
$x_1 = 53$	$x_2 = 43$
$\hat{p}_1 = 0.53$	$\hat{p}_2 = 0.43$

$H_0: p_1 - p_2 = 0$   
 $H_1: p_1 - p_2 \neq 0$

$(p_1 - p_2) + 0 = 0.53 - 0.43$

$$z = \frac{\hat{p}_1 - \hat{p}_2 - (p_1 - p_2)}{\sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.10}{\sqrt{0.004992}} = \frac{0.10}{0.07065} = 1.415$$

$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{53 + 43}{100 + 100} = 0.48$

Critical point:  $z_{0.05} = 1.645$

$H_0$  may not be rejected even at a 10% level of significance.

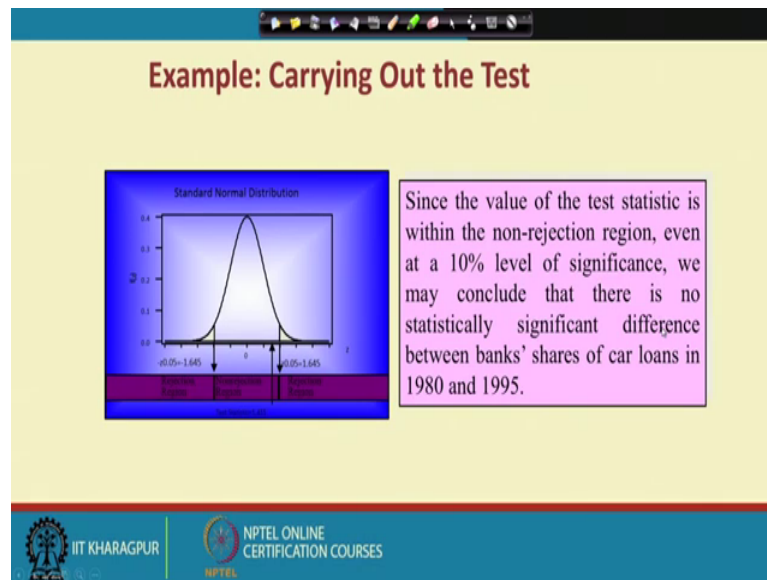
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In this case, the populations this is actually sample information 1 and this is what the sample information 2 and then we will get you know combine samples. So, as a result again,  $p_1$  minus  $p_1$  estimate minus  $p_2$  estimate, this minus you know this which is nothing, but actually this much and since we are assuming  $p_1$  equal to  $p_2$  population proportion. So, that is where here and as a result there difference equal to 0, so putting 0 here. So, the final value will be 0.53 minus 0.43 and the alternative hypotheses by default  $p_1$  minus  $p_2$  are not equal to 0. So, we have to take a decision whether to reject this one or whether you have to accept this once right.

So, then accordingly you have to calculate Z and after simplifications the Z value is coming 1.415 now putting actually 5 percent probability labels. So, critical value of Z degree coming actually 1.645 and this is the positive difference and as a result we can go to the right side of the testing. So, here we are not in you know position to again you know reject the true null hypothesis, so accordingly here to accept that you know they are actually you know same.

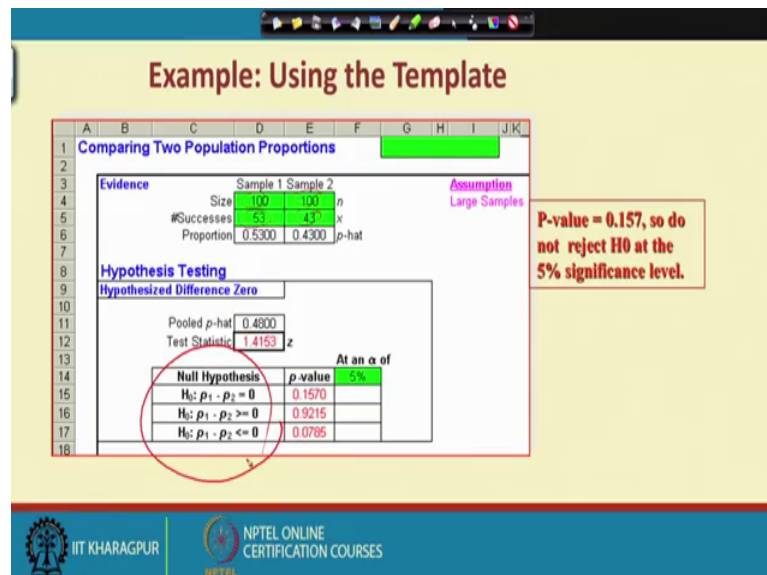
So, let us have this particular view. So, this is what the kind of you know structure, here.

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So, this is actually Z criticals and this is what the Z criticals, now, placing the Z value. So, it is coming actually the accepted zone. So, as a result, you cannot actually in a position to you know reject the true null hypothesis so; that means, we have to accept this particular you know case and  $P_1$  equal to  $P_2$ .

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So, then again if you will take into the excel sheet then the particular structure will be like this. So, this is 2 sample case and we have actually drawn you know this 100 and 100. So, 53 43, it is again between actually  $p_1$   $q_1$   $p_2$   $q_2$  and then you know we have

the particularly in a case like this and we have to test and whatever test we have actually obtained the kind of you know difference between a critical and you know the tabulated. So, then accordingly we have to conclude and in this case we against come to the conclusion that you know we cannot reject the true null hypothesis so; that means, both are actually you know same, there is a no difference.

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**Comparisons of Two Population Proportions When the Hypothesized Difference Is Not Zero: Example**

Carry out a one-tailed test to determine whether the population proportion of traveler's check buyers who buy at least \$2500 in checks when sweepstakes prizes are offered as at least 10% higher than the proportion of such buyers when no sweepstakes are on.

**Population 1: With Sweepstakes**  
 $n_1 = 300$   
 $x_1 = 120$   
 $\hat{p}_1 = 0.40$

**Population 2: No Sweepstakes**  
 $n_2 = 700$   
 $x_2 = 140$   
 $\hat{p}_2 = 0.20$

$H_0: p_1 - p_2 \leq 0.10$   
 $H_1: p_1 - p_2 > 0.10$

$(\hat{p}_1 - \hat{p}_2) - D$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - D}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$$

$$= \frac{(0.40 - 0.20) - 0.10}{\sqrt{\frac{(0.40)(0.60)}{300} + \frac{(0.20)(0.80)}{700}}} = \frac{0.10}{0.03207} = 3.118$$

Critical point:  $z_{0.001} = 3.09$

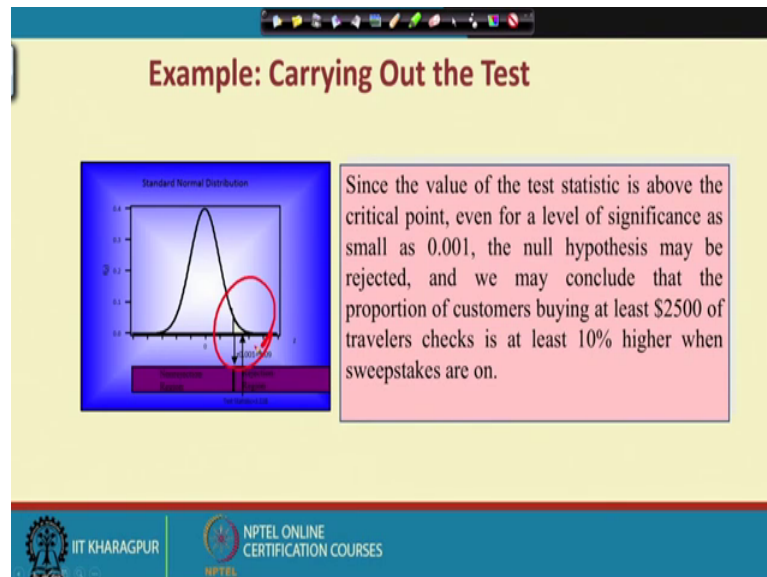
$H_0$  may be rejected at any common level of significance.

So, this is another you know example similar kind of in examples, but it is the case of you know third in the earlier case we are you know assuming that you know  $p_1$  equal to  $p_2$  and as a result they are difference equal to 0. Now, in certain you know business problems. So, it can be tested with the condition that you know their difference may not be equal to 0. So, their difference if not 0 either you know it will be less than to 0 or it will be actually you know something greater than 0.

So, now, what will we see in this case let us assume that you know, their difference is subject to actually reflected by the d component, as a result, the sample or the difference between sample means minus then we will put that you know the difference component that is the d component right. So, in this case, earlier we are putting here actually 0 and now in this case, this is the value and assuming that you know this value is actually 0.10 then after again the same procedure you 2 follow and in after simplifications the value is coming 3.118 and then at you know 1 percent priority levels.

So, Z critical is coming 3.09 and since actually the calculated value is 3.118, as a result, this particular structure is coming in the rejected regions and here we are in a position to reject the true null hypothesis compared to previous 2 situations in this case you can in a position to reject the true null hypothesis.

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So, accordingly we have to conclude and this is what the typical case and we are here actually to draw the conclusion. So, since it is coming under the rejected zone. So, you can actually reject the 2 null hypothesis and then he will conclude as per the particular you know problem structure.

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**Confidence Interval to Estimate  $P_1 - P_2$**

$$(\hat{p}_1 - \hat{p}_2) \pm z \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} \leq P_1 - P_2 \leq (\hat{p}_1 - \hat{p}_2) \pm z \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$

$(\hat{p}_1 - \hat{p}_2) \pm z$

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So, now we have actually discussed the kind of you know sample difference between 2 d you know 2 different structure and you know 3 different instances we have solved the problems in the first instance the population sample proportions are same and in the second and third case sample proportions are not same.

So, this is their difference will be reflected by some amount that is the difference between 2 population proportions and accordingly we have a tested and then we give the signal that you know and the sample difference are you know same or you know they are different, but now like earlier case we have a single sample case we have actually calculate the mean then you know comment about the population parameters and we are checking the difference between sample mean 2 population mean and checking whether there is a significant difference or not and again with the help of you know sample mean population mean and you know the sample size we can actually in a position to find out the confidence interval for population mean right.

So, in this case we can also do the kind of you know structures you know same structures. So, since we are going through checking the difference between 2 proportions. So, here for this difference; that means, the 2 different population proportion. So, we can clear the confidence interval. So, the same structure we can follow and then we are in a position to find out the confidence interval for the difference

between 2 population proportions and accordingly the structure of this difference should be like this.

So, this is actually the difference between the population proportion that is  $p_1$  minus  $p_2$  and their confidence interval typically depends upon the difference between 2 sample proportion that is  $\hat{p}_1$  minus  $\hat{p}_2$ . So, as a result, the actually, it is the structure  $p_1$  minus  $p_2$  or worse you know. So, that is the actually the difference sample difference and this is plus minus  $Z$  of once the particular you know this particular component right. So, as a result, in the left hand side, this will be  $p_1$  minus  $p_2$  minus this much and in the right hand side that is the upper bound. So,  $p_1$  minus  $p_2$  plus  $Z$  upon this much, then accordingly you can create a confidence interval for the difference between 2 population proportions.

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**Example Problem: When do men shop for groceries?**

$n_1 = 400$ $X_1 = 48$ $\hat{p}_1 = \frac{48}{400} = 12$ $\hat{q}_1 = 1 - \hat{p}_1 = 88$	$n_2 = 480$ $X_2 = 187$ $\hat{p}_2 = \frac{187}{480} = 39$ $\hat{q}_2 = 1 - \hat{p}_2 = 61$
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For a 98% level of confidence,  $Z = 2.33$ .

$$(\hat{p}_1 - \hat{p}_2) \pm Z \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} \leq P_1 - P_2 \leq (\hat{p}_1 - \hat{p}_2) \pm Z \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$

$$(12 - 39) \pm 2.33 \sqrt{\frac{(12)(88)}{400} + \frac{(39)(61)}{480}} \leq P_1 - P_2 \leq (12 - 39) \pm 2.33 \sqrt{\frac{(12)(88)}{400} + \frac{(39)(61)}{480}}$$

$$-27.064 \leq P_1 - P_2 \leq -27.064$$

$$-334 \leq P_1 - P_2 \leq -206$$

$$\text{Prob}[-334 \leq P_1 - P_2 \leq -206] = 0.98$$

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In order to justify these particular structures let us take an example and the corresponding example is here and this is the sample case and this is the sample 1 case and this is the sample 2 case and here the idea is we are not checking the difference whether statistically significant or not. So, here the objective is using the sample information we have to create a confidence interval for the population proportion. So, as a result the same structures, the difference between 2 sample mean will be  $p_1$  minus  $p_2$  and then we have to put actually  $Z$  confidence interval first, at 98 percent  $Z$  is coming to 2.33.

So, as a result this value  $Z$  will be reflected here and then the sample adjustment is through actually  $p_1 q_1$ . So, which is actually available here and then the sample observation and this is for the sample 1 and this is for the sample 2, now square root of this will give you the kind of you know structure and after you know once you simplify then finally, you will get actually the range minus 0.33 for 2 minus 0.206. So, now, in order to you know present in a kind of you know structure with respect to a 98 confidence interval. So, the probability between these 2 range that is population proportion between minus 0.334 to minus 0.206 is nothing but 0.98.

So, in other words, in this particular situation your population means with respect to this sample information 1 and sample information 2. So, the difference between the confidence interval for the difference between 2 population proportion will be minus 0.334 and minus 0.206, that is the lower limit and that is the upper limit. So, likewise with the help of you know sample informations, you can create the confidence intervals. So, whether it is a single case or in a single sample case or it is kind of you know multiple sample case that is the case yet in the case of you know 2 sample case.

So, you can create a confidence interval for population parameters. So, in this case that is the difference between a  $p_1$  and  $p_2$ . So, the population proportion 1 and population proportion 2.

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**Example: Using the Template**

**Evidence**

	Sample 1	Sample 2
n	120	140
#Successes	40	28
Population	0.4000	0.2000

**Hypothesis Testing**

Hypothesized Difference Nonzero

Test Statistic: 3.1188 z

At an  $\alpha$  of

Null Hypothesis	p-value	Decision
$H_0: p_1 - p_2 = 0$	0.0009	Reject
$H_0: p_1 - p_2 \geq 0$	0.9991	Reject
$H_0: p_1 - p_2 \leq 0$	0.0009	Reject

**Confidence Interval**

1 - $\alpha$	Confidence Interval
95%	(0.1371, 0.3629)

P-value = 0.0009, so reject  $H_0$  at the 5% significance level.

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So, some things actually you can take it to you know excel sheet and then you know with the help of you know and that analysis. So, you can come to the conclusion that you know the sample profile sample proportions written a between you know sample 1 and sample 2 are you know statistically different and again with the help of you know sample informations you can create a confidence intervals.

So, now accordingly, you know whatever we have actually information we have in the sample 1 and sample 2 after getting the actually the difference and the kind of you know Z value that is you know Z confidence intervals. So, we can actually create a confidence interval. So, at the 95 percent, the range of this population variance you know population proportion is nothing, but actually 0.200 plus minus 0.0629. So, that this particularly this particular item is the nothing, but you know difference between 2 population proportion that is  $p_1$  had minus  $p_2$  heads and then this is the adjustment factors Z upon the is a standard error and then with the help of you know the kind of you know structures we can find out their lower bound and you can find out the upper bound so; that means, with the given sample informations.

So, the difference between 2 population proportions will be lying between 0.137 to 0.263, likewise you can solve some of the problems connecting to connecting to this you know in the kind of you know or testing, that is the case of you know 2 sample case .

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**Confidence Intervals for the Difference between Two Population Proportions**

A  $(1-\alpha)$  100% large-sample confidence interval for the difference between two population proportions:

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

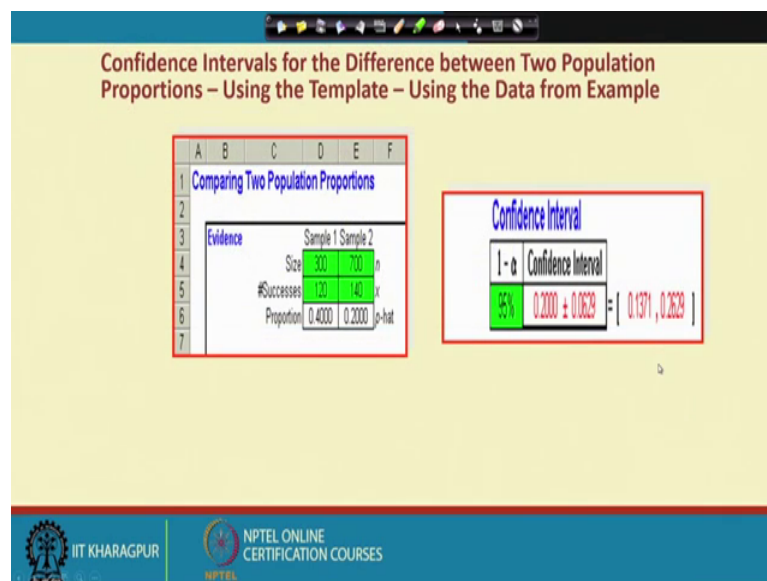
$$(\hat{p}_1 - \hat{p}_2) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} = (0.4 - 0.2) \pm 1.96 \sqrt{\frac{(0.4)(0.6)}{300} + \frac{(0.2)(0.8)}{700}}$$

$$= 0.2 \pm (1.96)(0.0321) = 0.2 \pm 0.063 = [0.137, 0.263]$$

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So, this is typically actually again confidence interval corresponding to the previous problem we can actually create a confidence interval here. So, these are the integer the information about the sample 1 and this is the sample 1 information and this is the sample 2 information and this is this difference between 2 sample proportions and the confidence interval by default will be coming 0.137 and 0.263 corresponding to information available for sample 1 and sample 2 and accordingly and we can actually solve some of the problems.

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So, same things we are actually solving through excel sheet and the other part of this particular discussion is the F statistics.

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The slide is titled "F Test for Two Population Variances". It contains the following formulas:

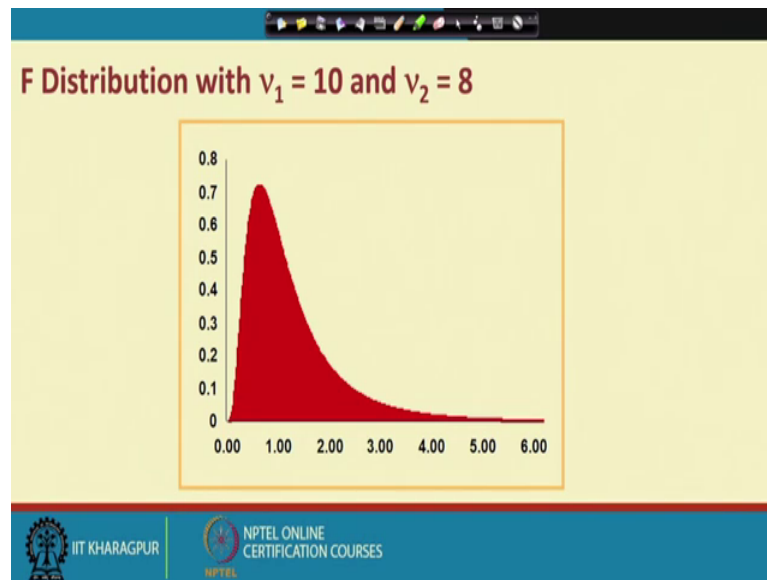
$$F = \frac{S_1^2}{S_2^2}$$
$$df_{\text{numerator}} = \nu_1 = n_1 - 1$$
$$df_{\text{denominator}} = \nu_2 = n_2 - 1$$

The slide also features the IIT Kharagpur logo and the NPTEL Online Certification Courses logo at the bottom left, and a small video feed of a presenter at the bottom right.

Now, means whatever we have discussed you know it is kind of in a sample proportion and the kind of you know population proportions and that is with respect to 2 sample case and we have also discussed the similar problem in 1 sample case. So, now, here we are going to discuss a situation where you know it is a kind of you know multiple sample case again and that to through F test and typically F test is nothing, but you know the ratio between 2 you know vary 2 different situations that is actually  $S_1^2$  by  $S_2^2$  square.

So, it is actually a ratio between 2 chi square statistic and then we have to check whether you know this ratio is a coming statistically significant or not corresponding to Z test and t test. So, here is actually you need 2 degree of freedoms corresponding to 2 different chi square observations. So, that is with respect to  $n_1 - 1$  and  $n_2 - 1$ . So, your critical value of this F depends upon 2 degree of freedom and corresponding to sample 1 and sample 2 we can get the kind of you know F calculated. So, once you get the F calculated then with the help of you know confidence intervals that is F confidence intervals. So, you can it can be in a position to report whether you are you know able to reject the kind of you know difference or you have to accept the kind of you know difference.

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So, let us see here the kind of case, this is typically you know F distribution or in you know F structures you know out of you know 4 test statistic which we have already discussed. So, chi square and F is typically positive square distributions and here the requirement of epi g you know you must have a 2 degree of freedom corresponding to a sample 1 and corresponding to sample 2.

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**A Portion of the F Distribution Table for  $\alpha = 0.025$**

	1	2	3	4	5	6	7	8	9
1	647.79	799.48	864.15	899.60	921.83	937.11	948.20	956.64	963.28
2	39.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44

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So, this is the typical you know F table structure and here is the kind of you know structure the degree of freedom part 1 and this is the kind of degree of freedom 2 and

depending upon the 2 degree of freedom then you have to specify the alpha component like this in this case 0.025, as a result you are getting this critical value.

So, now, depending upon the problems you have to calculate the F abstract F calculated and then you have to compare with you have tabulated and then you have to put your comment or get the inference corresponding to the you null hypothesis we have actually it means we are going to test. Now on the basis of you know the first sample and the second samples you can find out the d f 1 and d f 2 and once you fix df 1 df 2 then you can actually get the critical value subject to the alpha fixations that is the alpha here 0.025 and corresponding to point 0.025. So, you are you know F critical will be coming 3.59.

So, this is here actually the 11th case so; that means your sample information for n 1 equal to 11 and in this case it is coming 9. So, n 2 is equal to say 10 so; that means, it is the n1 equal to 11 and n 2 equal to 10 and alpha equal to 0.5 at 210. So, this is how the typical case to get the critical value of F and once you get the critical value. So, then you can compare with the calculated F and accordingly you can get the kind of inference.

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**Example: Hypothesis Test for Equality of Two Population Variances (Part 1)**

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_a: \sigma_1^2 \neq \sigma_2^2$$

$\alpha = 0.05$   
 $n_1 = 10$   
 $n_2 = 12$

$F_{0.025, 9, 11} = 3.59$

$$F = \frac{S_1^2}{S_2^2}$$

$$df_{\text{numerator}} = \nu_1 = n_1 - 1$$

$$df_{\text{denominator}} = \nu_2 = n_2 - 1$$

$$F_{975, 119} = \frac{1}{F_{0.025, 9, 11}}$$

$$= \frac{1}{3.59}$$

$$= 0.28$$

If  $F < 0.28$  or  $F > 3.59$ , reject  $H_0$ .

If  $0.28 \leq F \leq 3.59$ , do not reject  $H_0$ .

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So, now, here the hypothesis will be since it is a variance test and we are assuming that you know 2 variants are same and here the variance is nothing, but actually sigma 1 is equal to sigma 2 squares and the alternative hypothesis will be sigma 1 squares not equal to sigma 2 squares.

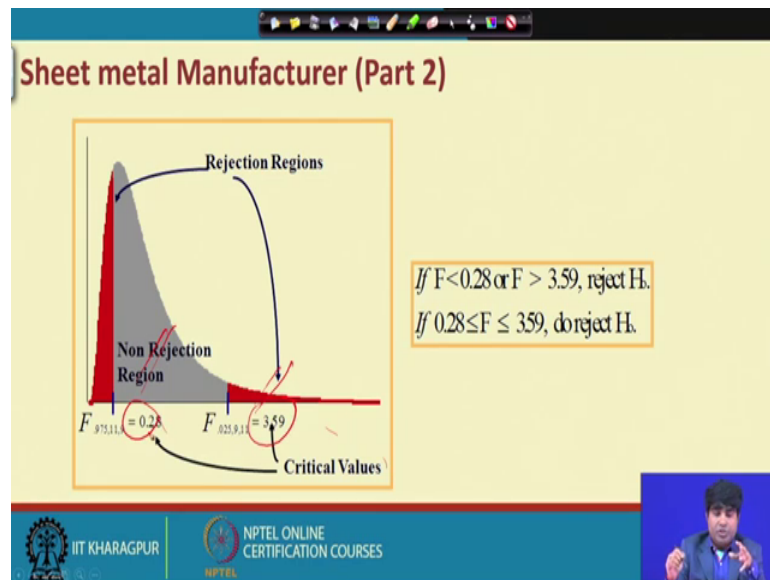
So; that means, the difference between the two,  $\sigma_1^2$  minus  $\sigma_2^2$  equal to 0 and the counterpart alternative hypothesis with the difference not equal to 0. So, now, corresponding to sample actually which you have already you know discuss let say  $n_1$  equal to 10 here and  $n_2$  equal to 12 and as a result your degree of freedom will be 9 and 11 and whatever we have already highlighted in the previous slide. So, your F critical is coming 3.59 which you have derived from the table.

And then with the sample informations, we have to calculate actually  $S_1^2$  and then  $S_2^2$  in this case  $S_1^2$  is coming 1 and  $S_2^2$  is coming 3.59 and as a result, the F calculated will be 0.28 and since actually it is the lower limit, so the typical case will be if F is a 0.28 and in this case F greater than to 3.59. So, reject  $H_0$  and 0.8 to F it between actually 3.59 and you know you have to actually reject  $H_0$ .

So, now the typical structure is actually, you have to calculate the sample variance and then the ratio between 2 sample variance will give you the F calculated and with the help of yesterdays sample observation 1 and sample observation sample of generation 2 that is the df 1 and df 2 and alpha.

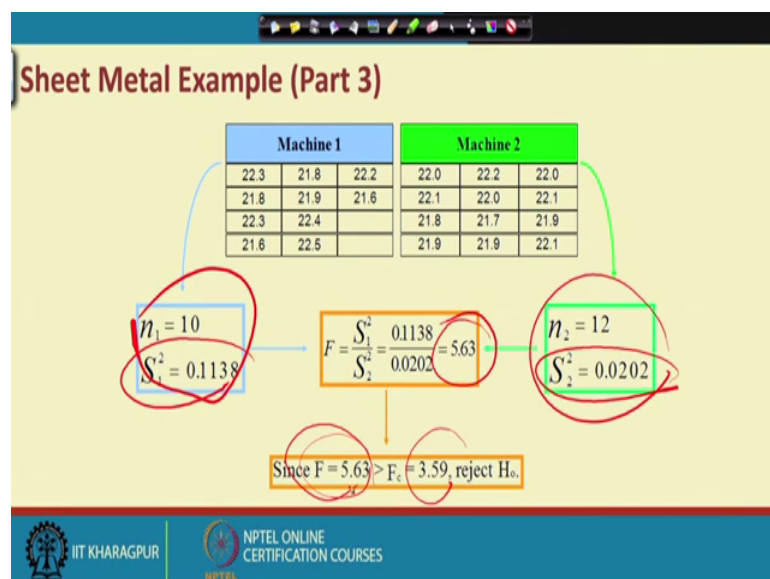
So, you have to find out critical value and if the calculated will overtake the criticals then you are going to reject the true null hypothesis; that means, there is a significant difference between 2 sample and if not then you have to accept the alternative hypothesis this is the similar kind of in a case, but here the structure is a little bit different compared to Z statistic and t statistics.

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This is what the kind of you know structures and whatever we have discussed and here the same structures we have actually classified in this particular you know structure. So, this is non rejected zone and this is what the rejected zone and this is what you know F criticals and then this is actually lower limit of F criticals and accordingly here to see what is the positions of you know F calculated, then accordingly you have to take the decision.

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So, let us take an example and since there are 2 samples machine 1 and machines 2 and the sample observations  $n_1$  is nothing, but actually a 10 and then in this case  $n_2$  is equal to actually 12 and you using actually excel spreadsheet you can calculate  $S_1$  square and then you can also calculate  $S_2$  squares.

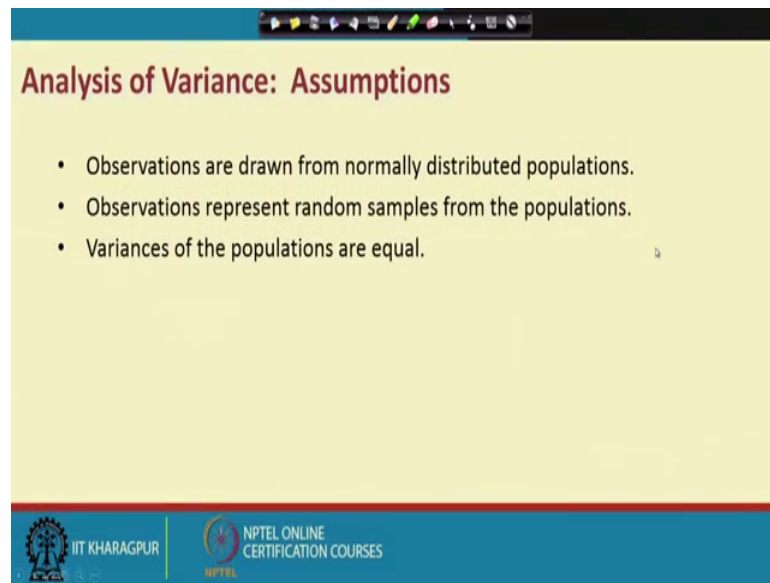
Now the ratio between  $S_1$  square by  $S_2$  will give you the F statistic that is F calculated and with the help of you know previous kind of you know structures we have got the F criticals instead of calculating F critical. Now F calculated will be compared with you know F criticals and since the F calculated is coming 5.63 and which is actually substantially higher than 2 F critical that is 3.59. So, you have to reject this hypothesis; that means, there is a significant difference between the variance of these 2 samples.

Now, so, whatever problems we have discussed till now. So, it is the question of you know whether there is a difference between 2 sample mean or 2 variance of this 2 sample means and whether this difference is statistically significant or not so; that means, with the help of you know t statistic and Z statistics we have calculated the mean difference and then we have checked the mean difference whether statistically significant or not and again with the help of you know F and chi squares we have to check whether there are sample variance means whether the sample variance are you know different within 2 sample variance are statically significant or not.

So, this is the case for you know F statistics, having 2 different samples within and some sample size. So, we have seen here that you know that there is a significant difference between there you know variance statistic so; that means, these 2 samples are not. So, consistent and that their variance is becoming actually a statistically significant; that means, it is actually interesting thing that you need to take decision and accordingly have to check where the variance is high and where the variance is low and accordingly management decision need to be taken into consideration and.



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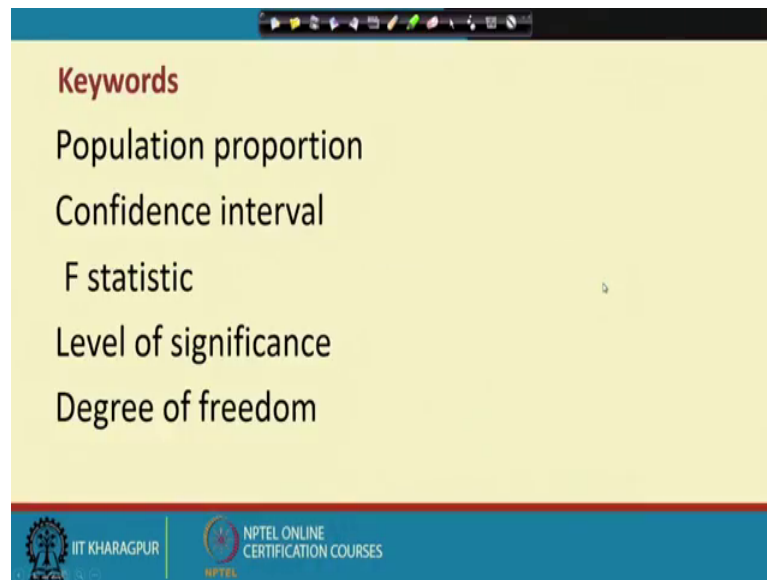
**Analysis of Variance: Assumptions**

- Observations are drawn from normally distributed populations.
- Observations represent random samples from the populations.
- Variances of the populations are equal.

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So, far analysis of variance is concern some of the assumptions are you know observations are drawn from randomly that is typically actually a random sampling and random samples drawn from the populations and variance of these populations are you know equals. So, that is the null hypothesis we have set and then accordingly we have to take 2 different samples and taken their you know sample variance and we are in a position to check whether their sample variance are you know equal with you know population variance and accordingly we have to come to a conclusion that you know there is a significant difference and corresponding to this particular you know problem. So, management needs to be in decision to be taken care accordingly.

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**Keywords**

- Population proportion
- Confidence interval
- F statistic
- Level of significance
- Degree of freedom

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So, these are the typical you know discussion we have you know one through the population proportion, the confidence interval and then F statistic that is the analysis of variance and the level of significance and degree of freedom. So, this is all about the structure about the population proportions and the kind of F statistic.

And in the next class we will discuss more about this F statistic and we will go for multiple sample case and then we see how is the particular you know structure and what kind of solution or inference we need when we have actually multiple samples parallelly and some situation you can actually a clubbing all these things and get some kind of you know inference and then after you know getting the kind of you know significant difference with the check the difference between and which particular pair should be coming actually significant and we will discuss all these in the next class with this we will stop here.

Thank you very much very nice time.