

Business Analytics for Management Decision
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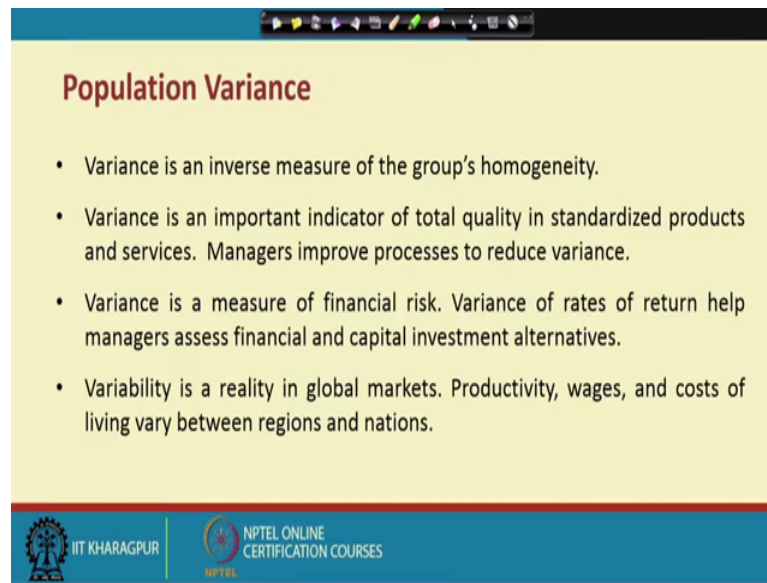
Lecture - 20
Inferential Analytics (Confidential Interval)

Hello everybody this is Rudra Pradhan here. And welcome to BMD lecture series. Today we will again continue the content called as a inferential analytics. And in the last couple of lectures we have discussed this concept regarding the inference analytics. So, here the idea is a two types. So, we have to comment on population with respect to sample information and then in the second instance with the help of you know sample information, we have to create a confidence interval for the population parameters.

So, we have a couple of lectures discuss means couple of lectures we have actually discussed to know how to validate the population parameter by using the sample statistics. And again we have also discussed you know using sample statistic how we have to create a confidence interval for the population parameter. And in this lecture, we again try to find out confidence interval for the population parameter and that to by using different test statistic that is the variance test statistics. And so for as a variant test statistic is concerns we have a two different distributions one distribution is called as a chi square distribution, and then we have a f distributions.

But today's discussion is on you know chi square distribution and that to using chi square distributions we like to know how confidence interval can be built for population parameters. So, since it is a variance in the you know indications, so the population variance need to be a predicted here and that to population variance confidence intervals we have to find out. And there are two parameters population parameters μ and standard deviations that is the variance. And in the last lectures, we have discussed the confidence interval about the population mean subject to using z statistic and t statistics, and then here is we have to create you know confidence interval for population standard deviations and by using a chi square distributions.

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Population Variance

- Variance is an inverse measure of the group's homogeneity.
- Variance is an important indicator of total quality in standardized products and services. Managers improve processes to reduce variance.
- Variance is a measure of financial risk. Variance of rates of return help managers assess financial and capital investment alternatives.
- Variability is a reality in global markets. Productivity, wages, and costs of living vary between regions and nations.

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So, the game is like this. Let us first you know give some kind of you know hint about the variance. So, in any kind of you know empirical testing procedures, so two standard indicators are mean and variance and we have already discussed so many problems by using the mean of the particular you know structures and variance of this particular structure. So, now some of the test statistic and some of the kind of you know problems can be actually investigated by the mean statistics; and some of the problems can be again you know predicted through the variance of this particular you know series.

So, now here is the idea is that you know how variance can be you know use or can be you know it can be helpful for predicting the problems and to get some kind of you know inference about the population parameter. First of all, we should know what is exactly the variance that is the population variance? Variance is an inverse measure of you know of the groups homogeneity. Variance is an important indicators of total quantity in standardized products and services; you know sometimes it will give you some kind of you know indication about the consistency of a particular you know series.

So, if the variance of a particular series is very high then that series is very you know can be considered as a volatile and inconsistent we are in compared to a series where you know variance is very less. So, we try to actually you know predict the situations and try to minimize the variance, so that you know the accuracy or you knows uncertainty factors can be minimized as per our problem is concerned. And it is sometimes you know

in finance it is actually very good measure to you know predict the risk component and you know variance of any kind of you know assets will be predict the kind of you know situation whether you know the kind of you know asset is you know consistent behaviors or you know inconsistent behavior. So, the market prediction completely depends upon the you know variance indicators.

So, the typical structure or a decision making is like that you know when the variance factor will be very high then this the prediction would be not. So, good actual, but in the case of you know variance you know less. So, where you know the prediction can be slightly you know perfect. So, in any situations, so we like to you know find out what is the volatility of this particular series or variance of this particular series, so that you know we may be in a position to predict as per the problem requirement.

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Estimating the Population Variance

- Population Parameter σ^2
- Estimator of σ^2

$$S^2 = \frac{\sum (X - \bar{X})^2}{n - 1}$$

- χ^2 formula for Single Variance

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

degrees of freedom = $n - 1$

Handwritten notes: $\mu = ?$ and $\sigma^2 = ?$

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So, now after knowing all these things, so you would like to know what is exactly the concept and how we can use for this particular you know confidence interval structure. So, for as a population is concerned, so there are two parameters which I have already mentioned sigma square and mu. And in this particular case, we are actually targeting the a population parameter sigma square. And first of all you should to know how you have to calculate the variance which we have already discussed. And that to, so this is actually for any kind of you know samples. So, your variance of a particular series can be

calculated like this. And the square root of this variance by default is called as actually standard deviation.

And for a you know chi square formula for single variance will be will be you know calculated like this. And in fact, actually some of the software by default will give you this values of the chi squares and you know then that will be helpful for you know some kind of you know prediction and some kind of you know comments about the a population parameter. So, here the standard structure is that you know so first you calculate the X squares that is the estimator of you know sigma square and then with the help of you know S square. So, you have to calculate the chi square. So, that is the with respect to sample adjustment and the population parameters. So, this is actually population parameter and this is what actually sample variance. So, on the basis of that, so we have to calculate the chi squares and that to with the help of you know degree of freedom n minus k.

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Confidence Interval for σ^2

$$\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}} \leq \sigma^2 \leq \frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}}$$

$df = n - 1$

$\alpha = 1 - \text{level of confidence}$

Handwritten notes: y^2 , df , 17.54, 12.19

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So now corresponding to this, so you have to you have to see how is this particular you know confidence interval structure. So, now you see previous so what is our objective that you know using chi square value. So, you know we have to create a confidence interval for this sigma squares so that means, in the earlier case. So, having S square information sigma square information, we are calculating chi square value. And then chi square statistic will give you some kind of you know prediction or some kind of you

know comments about the null hypothesis and alternative hypothesis, but here the structure is just opposite.

So, now, having the chi square value and the sample information, we like to create a confidence interval for the population parameters; like the previous lectures, we have actually given you a kind of you know confidence interval for population parameter μ by using sample information and the kind of test statistic like z and t . So, here having the same information you know since we are targeting the variance of this particular you know population, so chi square is the you know right choice you know to use for this particular you know prediction so for as you know confidence interval of you know population various variance is concerned.

So that means, in this particular form like you know same thing earlier we have discussed first you calculate the z statistic, calculate the t statistic then having the sample mean you have to create a confidence interval. So, here the same thing, so having the chi square structure and then you have to create a confidence interval for the population variance. So, that means, here the idea is actually so the population variance needs to be actually you know predicted. So, the targeted element in this particular you know structure is to create a confidence interval for sigma square. So, like we have already discussed the confidence interval for μ in the earlier case by using t and z . So, here the idea is to create a confidence interval for z square subject to availability of you know chi square information sample information and the sample variance, so that is how the kind of you know structure or you know all together.

So, let us see how we can do this. So, this is what the kind of you know. So, this is this is what you know population variance. And now this is actually sample variance, and this is a chi square a chi square test statistics and this is the sample adjustment factor that is nothing but called as you know degree of freedom, so that means, technically to get chi square value to get a chi square value. So, we need actually you know alpha and degree of freedom right. So, alpha is the type one error so that is actually probability of significance and the rejected zones then you know by putting both the sides options, so we like to create a confidence interval. So, that means, we like to know what is the chi square value at a particular point of time by fixing alpha and the degree of freedom.

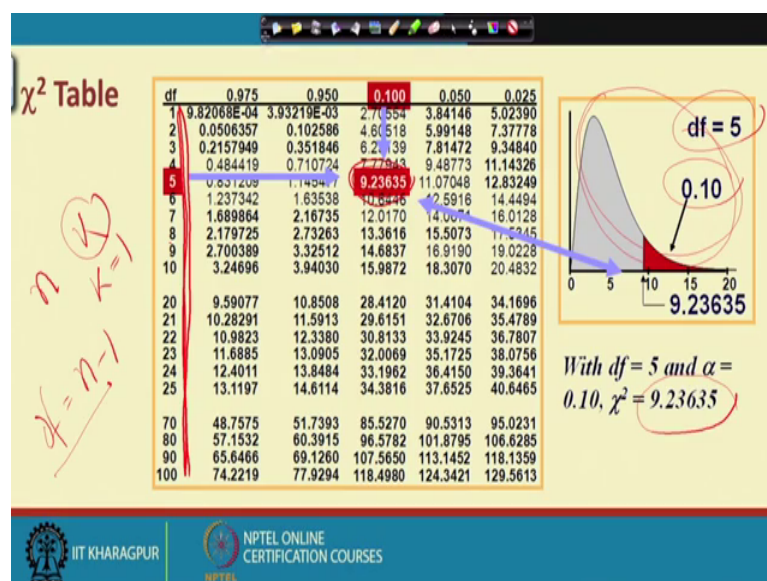
So, from the particular problems, so you can you can easily get the degree of freedom which is nothing but actually $n - 1$, for one sample case. And alpha is in your controls you have to fix a kind of you know structure through which you need actually confidence interval, you can put you know alpha 1 percent, you can put 5 percent, you can put 10 percent, so it can be varies actually. So, now having variation of you know alpha and having the degree of freedom you can get chi square value.

Once you get the chi square value, so then you knows for you know since it is a kind of you know confidence interval lower bound and upper bounds, so by default you will allow alpha to go in the both the sides. So, as a result the choice will go alpha by 2 in the left side and alpha by 2 in the right side. Then remaining 1 minus alpha 50 percent will be in the left zone and 50 percent will be a right zone and that is what the confidence interval and that to for you know population variance sigma square.

So, first you have to actually the first end you know starting point is to fix you know alpha that is what actually the kind of you know confidence interval. So, 1 minus level of confidence interval is nothing but actually alpha, so obviously so confidence interval is nothing but called as you know 1 minus alpha. So, first you fix the alpha and then by default you will create a confidence intervals right. So, this is how the typical structure this is the lower bound structure and this is what the upper bound structures.

So, knowing the S square value and the chi square value you can you may be in a position to know what is the lower range and what is the upper range for the population variance sigma square.

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So, let us take an example. So, how it will actually work? So, let us first you know understand the chi square tables. Then on the basis of chi square values, so you can you may be in a position to create a confidence interval for the population parameter. So, now in the usual you know chi square table will be like this. And here is in the first you know a first column you will find you know ranges of you know degree of freedom starting with you know 1, 2, 3 and so on. So, now, corresponding to a particular you know sample structure, so you can get to know degree of freedom. So, degree of freedom depends upon you know two things. So, your sample observations n and the variables involvement is you know k .

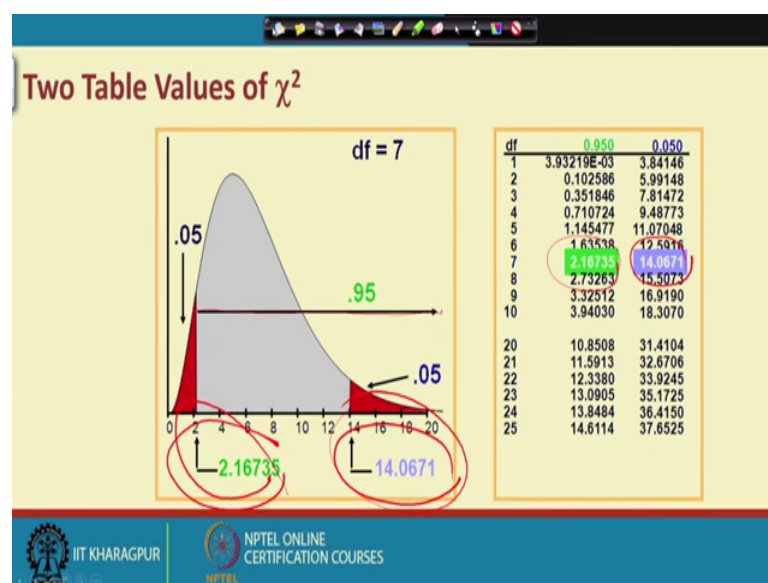
So, since we are actually discussing one sample case, so every times k will be one for one sample case. And this sample size is n so obviously degree of freedom is usually for one sample case is nothing but called as a n minus 1. So, now if your n is equal to say 10, then by default degree of freedom will be 9; if we are n equal to say 20 degree of freedom will be 19; if n equal to say 25 then degree of freedom is a 24. So, like this you know degree of freedom can be calculated and then and you want to specify or you have to specify the kind of you know confidence intervals. So, if you do not fix the kind of you know alpha then you may not in a position to find out the chi square value.

So, one requirement of you know chi square figure is the degree of freedom. So, this is very easy to find out depending upon the number of variables involvement and the

sample size. And then for confidence interval requirement, so it depends upon you know alpha fixation. So, if you put alpha equal to say 10 percent then obviously so corresponding to 10 percent. So, this is actually coming as actually the chi square value. So, it is coming 9.23 you know 6. So, this is what actually the chi square test statistics corresponding to degree of freedom; in this case degree of freedom is coming 5 so that means, sample size is a n and then alpha is actually you know 10 percent. So, as a result your chi square is coming 9.23635.

So, now, we need actually confidence interval for sigma square and subject to you know use of you know chi squares. So, now first of all you need to specify the chi square requirement and then get the chi square critical the chi square test statistics, then using the chi square test statistic you can create a kind of you know confidence intervals. So, you know actually we need actually confidence interval. So, the lower bound and upper bound, but this particular distribution is actually positively skewed distribution. So, as a result so the particular structure will be in the only one particular you know domains and accordingly. So, you have to fix a kind of you know confidence interval. So, that you know it will be positive positively be skewed and then it will be follow us for the particular you know requirement. Let us see here. So, the kind of you know structure. And in the next slides I will give you the kind of you know structure about the confidence intervals.

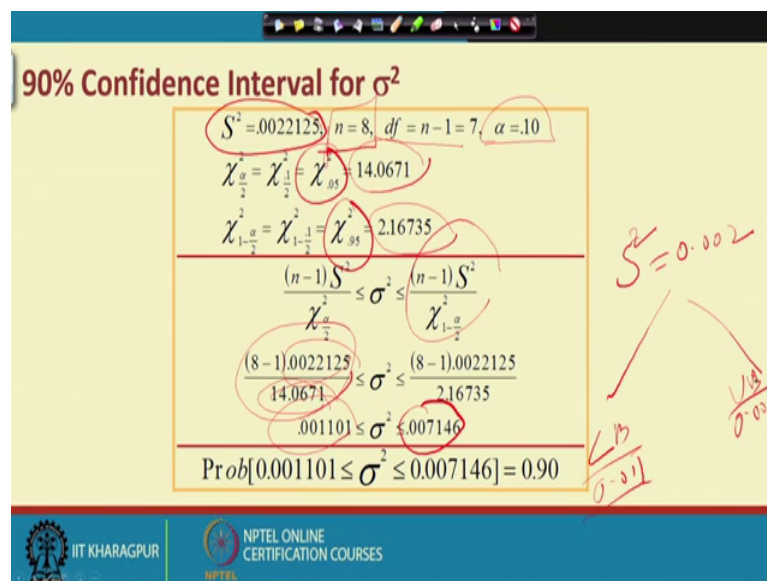
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So, let us start with the fixing of the ranges of the you know lower limit and upper limit. Now, you know for a confidence intervals, you need actually two values and one value one value will be this ones and the you know the upper value will be this one. So, that means, actually corresponding to a degree of freedoms, so you will be find the particular source if you are fixing actually 5 percent here then the 95 percent of you know area will be covering like this.

So, as a result so the 95 percent of you know area coverage and the corresponding to the degree of freedom, the critical value will be coming actually 2.167 that is actually 5 percent you know range then the upper value will be actually 14.07. So, that means, actually we need a lower bound and we need actually upper bound and that to you know to fix the confidence interval for sigma square. So, now in the lower bound, so the chi square value reporting will be this much, and the upper bound the chi square value reporting will be 14.07 and that to at the 5 percent probability levels.

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So, now corresponding to these two figures, so let us see how this particular you know confidence intervals. So, now see here. Let us take a standard examples since in the previous slides, we have actually a fixed at a kind of you know 95 percent of confidence intervals. So, now in this examples we are putting 90 percent of you know confidence interval for actually population variance sigma square, so that means we like to know what is the confidence interval for sigma squares when you are fixing 90 percent of you

know confidence interval. And that to sample size of you know n equal to 8 and sample variance is nothing but actually 0.002.

So, as a result so that means, actually the standard requirement of you know this you know confidence interval first actually the sample statistic, this is that is the sample variance. And then and you are you know this sample size reporting. And as a result you can find out the degree of freedoms, and then you get you fix the confidence interval. So, since we are allowing both the sides then you know chi square alpha by 2 is you know nothing but actually upper side is coming actually 14.07, and then in the other side it is coming actually 2.16735 so that means, that is what actually the difference between these two.

So, chi square 0.95 and chi square actually 0.05. So, the lower range and the kind of you know upper range. So, corresponding to the lower range, so this is what actually the kind of you know critical value, and this is what the kind of you know critical value, where you know 95 percent is concern. So, as a result, so we have to fix the you know lower bound corresponding to this particular you know 0.05 and then corresponding to 95 percent. So, you have to find out the a kind of you know upper bound right. So, as a result, so the sigma square will be a now putting all these values like you know degree of degree of freedom here you know 8 minus 1, then the sample variance, and the chi square value alpha by 2, alpha by 2 for you know 5 percent is coming actually 14.07.

So, after putting all these things, so this will give you the kind of you know value called as you know 0.001. So, this is the lower value or you know lower confidence interval for sigma square. And again you know chi square 95 percent the upper value is coming 2.17. So, now, again with the adjusting degree of freedom and the sample variance, so you will find the upper range. So, the upper range is coming actually here 0.007, so that means, having actually a sigma, you know you know the sample variance 0.0002. So, you are you know lower bound population variance will be a 0.0001 and the upper bound will be 0.007.

So, this is how this is upper bound and this is how the kind of you know prediction about the confidence interval. So, subject to availability of you know sample variance. So, that means, here there are you know two different structure altogether for confidence interval. And one particular structure is to find out population mean and then create a confidence

interval. And for that you can use z statistics, you can use t statistics and that to with you know having information about the population standard deviations and without information of the population standard deviations. And in the case of you know population variance confidence intervals, so we are supposed to use actually chi square distributions and then with the help of you know the kind of you know sample size and you know the alpha choice, we can create a confidence interval for sigma square.

So; that means, technically it is all together actually the kind of you know predictions or inference about the population parameters that is nothing but actually mu and you know standard deviations and that is variance mean mu and you know variance standard deviations. So, now but the choice of test statistic you know will be different as for the particular you know structure and the kind of you know requirement. So, far as you know mean confidence interval is concerned that is population mean confidence interval is concerned, so either z statistic can be applied or t statistic can be applied depending upon the large samples, depending upon the small samples and the kind of you know sigma availability and sigma I mean sigma non and sigma unknown situations.

But in the case of you know population variance confidence intervals, so the standard choice will be always actually chi square you know chi square test statistics. So, having chi square information, you can create a kind of you know confidence interval and that is what the kind of you know structure which you have already highlighted. And in this case, so the standard structure requirement is to find out sample variance first and report the sample size, and report the confidence interval that is the alpha value. And on the basis of you know alpha value you can create a confidence interval for the sample variance means population variance, but it will be with the help of you know only sample variance. Without knowing the sample variance, you are not in a position to create a confidence interval for population variance.

So; that means, the entire inferential analytics we have actually two different you know work plans all together. In one case, so having sample information you are committing to the population you know parameters and whether it is with respect to you know mean or you know variance. But in other case having sample statistics, you can predict the kind of you know structure about the population statistics that is actually population mean mu and population variance you know sigma squares. So, both the cases actually you know kind of you know consist what I called as you know very kind of you know

complimentary and once you know a particular situation you can actually predict the other situations and vice versa is also equally true.

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Solution for Demonstration Problem

$S^2 = 1.2544$, $n = 25$, $df = n - 1 = 24$, $\alpha = .05$
 $\chi^2_{\frac{\alpha}{2}} = \chi^2_{\frac{.05}{2}} = \chi^2_{.025} = 39.3641$
 $\chi^2_{1-\frac{\alpha}{2}} = \chi^2_{1-\frac{.05}{2}} = \chi^2_{.975} = 12.4011$

$$\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}} \leq \sigma^2 \leq \frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}}$$

$$\frac{(25-1)(1.2544)}{39.3641} \leq \sigma^2 \leq \frac{(25-1)(1.2544)}{12.4011}$$

$$0.7648 \leq \sigma^2 \leq 2.4277$$

Handwritten red notes on the slide: $S^2 = 1.254$ and a diagram showing a downward arrow labeled 'UL' (Upper Limit) and an upward arrow labeled 'LL' (Lower Limit).

So, now corresponding to these particular structures. So, we will see here so the kind of you know solutions. So, there is another kind of you know example here. So, this is again ah sample statistic sample variance that is actually 1.254, and then degree of freedom is it degree of freedom is actually 24 corresponding to the sample size 25. And then again alpha you can fix actually 5 percent that is 0.05. So, then against you have to find out you know chi square the lower bound and chi square the upper bound. So, the chi square you know lower bound will be this ones and the chi square upper bound will be this one.

So, this will be actually the kind of you know requirement. So, once you get the kind of you know sample variance, then using the sample variance, so you have to see what is the confidence interval for the population variance, so that means, technically having as you know sample variance 1.254. So, what is actually the upper limit sorry lower limit and you know lower limit and upper limit for you know this particular you know problem.

So, again the same structure, so using sample variance. So, a chi square lower limit and chi square upper limit and that to with the help of you know alpha and degree of freedom. So, you can just you know report these figures then automatically you can

simplify. And you may be in a position to you know predict you know what should be the population variance you know lower limit and population variance the upper limit. So, that is actually standard structure through which you can actually frequently you know this you know issues and then predict the problem or the predict the environment as per the particular you know requirement.

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Determining Sample Size when Estimating μ

- Z formula $Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$
- Error of Estimation (tolerable error) $E = \bar{X} - \mu$
- Estimated Sample Size $n = \frac{Z^2 \sigma^2}{E^2} = \left(\frac{Z \sigma}{E} \right)^2$
- Estimated σ $\sigma \approx \frac{1}{4} \text{range}$

Handwritten note: E = 0

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And this is what actually the standard z structures. And usually the standard normal you know normal distributions is a structure is like this z equal to x bar minus mu so that means, see here. So, this is actually sample statistics and this is what the population parameters. Every times we like to means one of the objective in this particular inferential analytics is to find out the difference between sample statistics to population statistics. So, that means, in the first instance you would like to check whether you know sample mean is converging to the population mean. And if sample mean is converging population means that means, it is completely unbiased kind of you know things and this is very efficient and very effective. And if that is the case and your sample selection will be very appropriate for this particular you know problem or you know the kind of you know decision making is concerned.

If the sample mean is not converging population means so then there is a difference. So, the difference is nothing but called as you know bias. In this case, it is called as you know error of estimations. So, now you know we need that you know this bias should be

equal to 0. So, one way to you know make this equal to zero means you know if your samples identification or you know sample selection will be very perfect as for the problem requirement or else you sometimes you can increase the sample size then the biasness will be by default will be reduced slightly.

And every time our approach you know reduce the bias and make the system you know unbiased so that means, sample statistics should be converged to population test statistic. If not then you know then the you know prediction rules or you know the kind of you know prediction environment will be a you know not so effective. So, it may give some kind of you know wrong decision may kind of you know thing. So, that is how every time we try to find out you know how to you know bring you know less and less bias in the system.

And now here one of the interesting thing is that you know in the process of you know this informational analytics, so the first two cases which you have discussed actually to comment about the population parameter subject to sample statistics. And again second is to predict the population parameters and subject to samples you know statistics information that is what actually the prediction about the a population parameter μ and you know population parameter you know σ^2 and that is what actually the confidence interval.

So, now another things which we can actually explore in an inferential analytics is a nothing but actually the kind of you know what we called the sample size. So, now, since you know this is actually well structure, so having the information about test statistic and the parameters value, so sample size can be also actually estimated here. So, that means, actually in one case, so you are you know targeting the population parameter subject to sample statistic then targeting sample statistic is a population parameter, subject to you know information about the alpha you know degree of freedom or sample size. And now in the same process having all these information, you may be in a position to exactly specify what should be the estimated sample size.

So, now by this formula you know it is very easily you can actually detect the kind of you know estimated sample size. So, that means, you know in the empirical investigation process, so it is very difficult to say whether this sample is very accurate or very efficient. So, that is why we you know we need to specify what should be the optimum sample size. So, that you know the prediction can be very perfect as for the as for the

kind of you know problem is concerned. So, in the overall actually empirical process, resource investigation says that you know estimated standard deviations should be equal to 1 by 4 range. Range is nothing but actually the difference between you know lower limit and the upper limit. So, like that you know I will give you some kind of you know examples through which actually you can actually predict the kind of you know sample size in a kind of you know given situations.

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Sample Size When Estimating μ : Example

$E = 1, \sigma = 4$
 90% confidence $\Rightarrow Z = 1.645$

$$n = \frac{Z^2 \sigma^2}{E^2}$$

$$= \frac{(1.645)^2 (4)^2}{1^2}$$

$= 43.30 \text{ or } 44$

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Like you know earlier case you know having sample information having sample size and the sample statistics and the confidence intervals, you can predict the population parameters. So, here also same things having some kind of you know information about you know population information and sample information, then you can check the kind of you know biasness. And with the help of you know biasness, you can able to detect the size of the samples.

So, you know as per the previous discussions, so these you know to for using z statistics you can actually in a position to report what should be the sample size. For instance, so this E is nothing but actually $\bar{x} - \mu$. So, that is the bias created in this particular investigation process. And having sigma equal to 4, and a 90 percent confidence intervals then z is coming 1.645. And then accordingly n will be by default just putting actually z value here and sigma square 4 and then the biasness, so you can able to predict what should be the actually sample size so that is what actually 44.

So; that means, actually the thing is that you know the problems are you know very complementary kind of things. If you know certain things then you are in a position to predict you know other things as per your you know problem requirement. So, whether you know commenting the population parameter or predicting the predicting the population parameter or you are targeting the size of the samples, so whatever you know the requirement, so the system or an inferential analytics will help you lot to generate all such information as for your problem requirement. And the kind of you know management decision is concerned.

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Solution for Demonstration Problem

$E = 2, \text{ range} = 25$
 95% confidence $\Rightarrow Z = 1.96$
 estimated $\sigma: \frac{1}{4} \text{ range} = \left(\frac{1}{4}\right)(25) = 6.25$

$$n = \frac{Z^2 \sigma^2}{E^2}$$

$$= \frac{(1.96)^2 (6.25)^2}{2^2}$$

$$= 37.52 \text{ or } 38$$

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So, I will give you another examples and which can give you this snaps up about the sample selections. Again let us say E equal to x minus mu that is the bias equal to 2, and the range that is it 25 and the 95 percent you know confidence interval, the z value will be 1.96. So, here actually the sample range will be the z square and you know standard deviations using the standard deviations which is nothing but actually 6.25 and sample size will be coming actually 38.

So, that means, having all kind of you know information you know biasness and you know population variance and the z statistics, so you are in a position to calculate the sample size. Sometimes you know why it is actually required to discuss because sometimes you know the structure is so complicated and you may not be in a position to know what is exactly a population size? So, a population may be actually finite type and

infinite type, but exactly what should be the kind of you know size, it will be very easily actually predicted through this particular structure. Since it all are you know correlated and it is very easy to predict the kind of you know requirement as per your objective and the kind of you know problem requirement is concerned.

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Determining Sample Size when Estimating P

- Z formula $Z = \frac{\hat{p} - P}{\sqrt{\frac{P \cdot Q}{n}}}$
- Error of Estimation (tolerable error) $E = \hat{p} - P$
- Estimated Sample Size $n = \frac{Z^2 P Q}{E^2}$

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So, likewise actually it can be also useful for you know sample proportions. So, you know like you know we have discussed you know \bar{x} minus μ , so here actually sample proportion this is actually is you know sample proportion and this is the population proportion. And with the help of sample proportion and population proportion, so you can also in a position to calculate the size of the sample. So, like this you know so this is actually so; that means, this test means what I like to say that you know the test statistics are very useful component to predict certain things as per the kind of you know problem requirement.

So, once you know a particular test statistic structures, so having the sample information, so you can predict the population information. So, adding sample information and population structures, you can increase the kind of you know sample size. So, now, this is another way to you know having the kind of you know biased calculations. So, same way you have to report this sample size; only thing is you know here actually the kind of you know biasness is the difference between you know sample proportion minus you know population proportions. And then with the help of you know the kind of you know

standard errors and the kind of z values, so you are in a position to report the sample size right. So, the estimated sample size for this particular case will be like you know you know like to be reported by this particular you know formula.

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Solution for Demonstration Problem

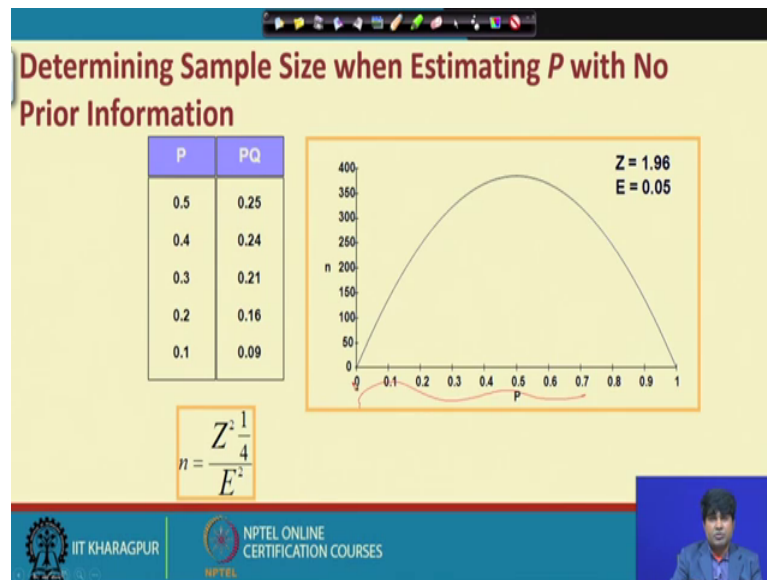
$E = 0.03$
 $98\% \text{ Confidence} \Rightarrow Z = 2.33$
 $\text{estimated } P = 0.40$
 $Q = 1 - P = 0.60$

$$n = \frac{Z^2 P Q}{E^2}$$
$$= \frac{(2.33)^2 (0.40)(0.60)}{(.003)^2}$$
$$= 1,447.7 \text{ or } 1,448$$

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So, let us take an examples, and then will be highlight how is it actually the case. So, let us say you know bias is coming 0.03 and we are fixing 98 percent confidence interval and for that for that intervals z is actually 2.33 and p is actually 0.40 and by default Q equal to 1 minus p and that is coming actually 0.60. And now so having Z figures and having E figures and having the P Q figures so very easily you can predict the actually estimated sample. So, that in this case you know the sample size will be 1488 right. So, this is means this is a typically actually very interesting kind of you know inference we can draw with the help of you know all these you know tests statistic say z statistic, t statistics and chi square test statistic. So, this is very standard kind of you know structure through which actually you have to predict the environment.

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So, now likewise you know in the P can be you know varying subject to you know kind of you know variation. And this is a kind of you know classic kind of you know structure. So, means it gives actually the kind of you know snapshot that you know when actually P is changing one domain to another domain. So, the size of the sample is also actually you know changing. So; that means, actually this graph says that you know at a particular point of time, you can conclude that you know every time you know it is not that you know increasing sample size will give you actually a very consistent results are very effective results. But you know it is not the increased samples it is the concept called as you know optimum samples always give the better decision and you know fair decisions.

Because you know sometimes for example, let us say you are going for you know for stock market predictions, but having stock market predictions you know let us say you know 10 years data or 5 years data it is very you know reliable to do the kind of you know predictions. But you know having a 50 years or you know 100 year data then today's stock market prediction may not to be so effective. So, accordingly so it is clear cut signal that you know it is not the increased sample size always give you better accuracy or you know better kind of you know management as per the requirement. It is the optimum size of the sample will give you better you know kind of you know inference and it will give you better management decision. So, that is why you must know what should be the kind of you know optimum sample for a particular you know

problem. Some problems you know having more samples may give some kind of you know more biasness or you know having less samples it will again give you know you know more and more bias to the systems.

And when your bias is very high in the system in the process of you know investigation then by default it will give you some kind of you know negative kind of you know clue to give you know better kind of you know prediction or to give some kind of you know better management decision. So, that is why at a particular point of time you should know what should be the optimum sample structure so far as you know problem investigation is concern. And this is how the you know the kind of you know structure you know this figure typically you know highlights that you know it is not the increase sample size, it is the optimum sample size will give you the appropriate decisions or you know so far as you know management requirement is concerned.

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Example: Determining n when Estimating P with No Prior Information

$E = 0.05$
90% Confidence $\Rightarrow Z = 1.645$
with no prior estimate of P, use $P = 0.50$
 $Q = 1 - P = 0.50$

$$n = \frac{Z^2 PQ}{E^2}$$
$$= \frac{(1.645)^2 (0.50)(0.50)}{(0.05)^2}$$
$$= 270.6 \text{ or } 271$$

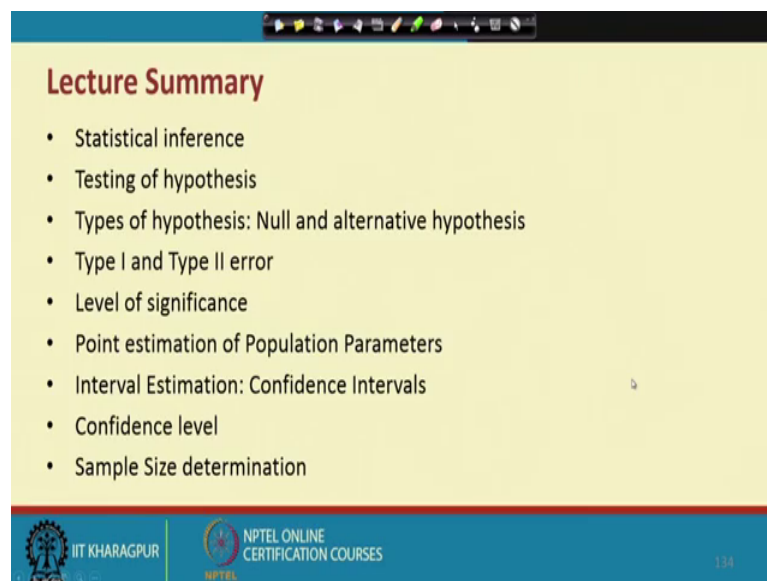
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So, like this is another example through which actually again you can increase the sample size and keeping 90 percent of confidence interval, and biasness 0.05 with the P value 0.50, you can able to calculate again sample size. And in this problem the sample size is again coming actually 271. So, that means, you know there are plenty of you know examples you can cite here. So, far as you know sample information means it is a prediction about the sample size, so that means, three things here we are here doing in this particular you know inferential analytics that to part one. And here actually using

sample information, you are giving comment to the population parameters or population statistics.

And again using sample information, you are you are creating a confidence interval about the population parameters again using actually sample information and the kind of you know population information, you are in a position to predicting the kind of you know sample size. And all these are you know highly relevant so far as you know particular problem is concerned and the management decision is concerned. So; that means, all together we are having actually three ways of you know getting inference for a particular you know problem. So, it will give you better exposure or you know better kind of you know setups through which the your prediction can be perfect. And you may be get you may be in a position to get better inference to solve the problem and to take some kind of you know management decisions and that to as per the problem requirement.

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Lecture Summary

- Statistical inference
- Testing of hypothesis
- Types of hypothesis: Null and alternative hypothesis
- Type I and Type II error
- Level of significance
- Point estimation of Population Parameters
- Interval Estimation: Confidence Intervals
- Confidence level
- Sample Size determination

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So, likewise you know these are the things we are you know supposed to discuss in the case of you know inferential analytics. And so far we have already discussed the kind of you know concept about the statistical inference, hypothesis testing, the kind of you know requirement, the kind of you know structuring, various test statistics the requirement of test statistic, the kind of you know structure of you know test statistics, the objectives behind the test statistics and the kind of you know problem requirement.

And in whatever we have discussed in this particular you know unit it is related to one sample case only.

But you know by the way in real life situations, so many problems will be there which is actually multivariate in natures. So, that means we will be find in many instances so you know there will be a kind of you know problem which can be connected with the multiple samples that means it is a multiple variables environment. And in that case same test statistic can be applied to predict the kind of you know situation and to do some kind of you know means to look for some kind of you know management decisions. But you know the procedure and the kind of you know structuring is slightly different, it is a more or less same, but you know like you know one sample case, but in the case of you know multiple sample case some more test statistic can be applied. But you know same statistic with you know kind of you know different structure.

For instance we have not discussed in this unit one particular distribution is called as you know f distribution which is actually multivariate kind of you know concept. And we will discuss in details in the next unit but by the way so when you have actually one sample case or you know one variable case problem. Then whatever we have discussed in this you know inferential analytics in this particular unit, so I hope you will get enough exposures to predict the kind of you know business requirement and the kind of you know you may be in a position to take some kind of you know good management decision.

Now, in the next unit we will we will discuss details you know when there is a multivariate case, how you will go for some kind of you know prediction that is what is the game plan for you know sampling to population, how to predict the population again with respect to you know creating a confidence interval. Again, how to predict this sample size all these things? So, we will discuss in details in the next lectures in that to any in next unit that is unit five. With this, we will be stop here.

Thank you very much, Have a nice day.