

Business Analytics for Management Decision
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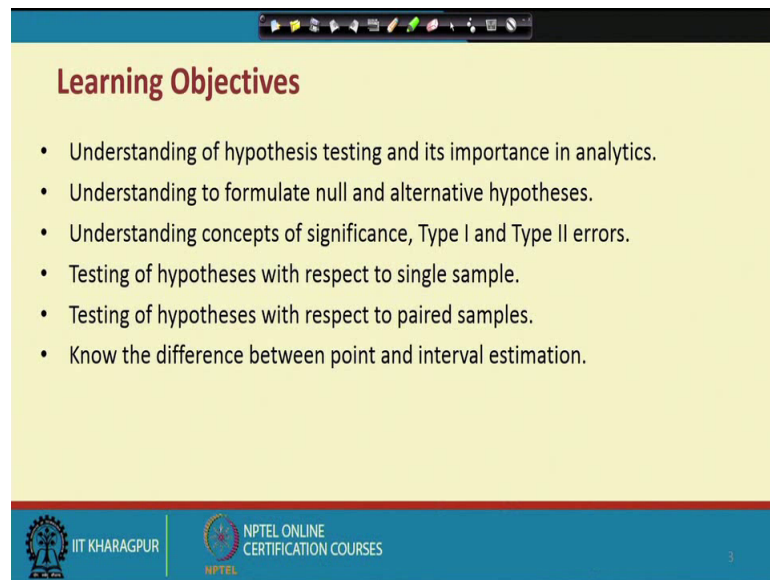
Lecture - 16
Inferential Analytics

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| Course Contents | |
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| Weeks | Lecture Names |
| Week 1 : | Introduction to Business Analytics |
| Week 2 : | Exploring Data and Analytics on Spreadsheets |
| Week 3 : | Descriptive Analytics |
| Week 4 : | Inferential Analytics 1 |
| Week 5 : | Inferential Analytics 2 |
| Week 6 : | Predictive Analytics 1 |
| Week 7 : | Predictive Analytics 2 |
| Week 8 : | Predictive Analytics 3 |
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Hello everybody, this is Rudra Pradhan here. First of all wish you all very happy New Year; and we are here to start our unit four lectures that is on inferential analytics, and this is what the detailed structure. And in the last couple of you know weeks we have already covered introduction to business analytics exploring data and analytics and spreadsheets; and in the last week, we have discussed descriptive analytics. So, now in the case of you know unit 4, we have inferential analytics and that too we have two parts inferential analytics one and inferential analytics two. So, this week we will cover in inferential analytics ones, and let me highlight what are the coverage under the inferential analytics.

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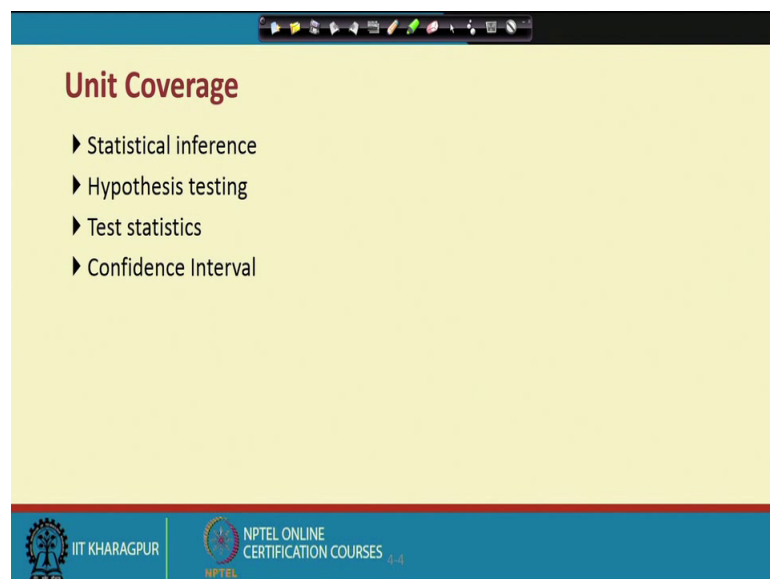
Learning Objectives

- Understanding of hypothesis testing and its importance in analytics.
- Understanding to formulate null and alternative hypotheses.
- Understanding concepts of significance, Type I and Type II errors.
- Testing of hypotheses with respect to single sample.
- Testing of hypotheses with respect to paired samples.
- Know the difference between point and interval estimation.

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And before that let us start with you know details about the objective behind this inferential analytics. So, here in this unit we like to understand the hypothesis testing, and its importance in analytics. Understanding to formulate null and alternative hypothesis, understanding concepts of significance type 1 and type 2 errors, testing of hypothesis with respect to a single samples. Testing of hypothesis with respect to paired samples, and know the difference between the point and interval estimation. So, this is these are the details the objectives.

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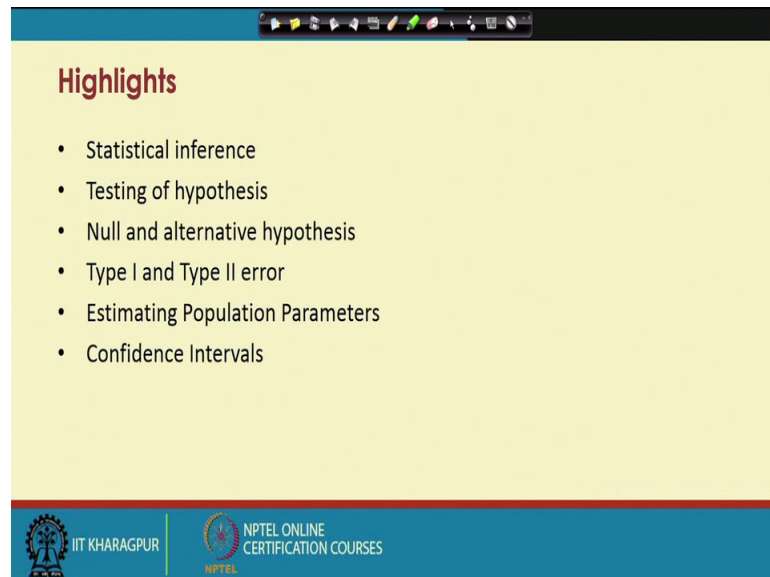
Unit Coverage

- ▶ Statistical inference
- ▶ Hypothesis testing
- ▶ Test statistics
- ▶ Confidence Interval

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And corresponding to these objectives and our unit coverage will be like this. So, the first one is a statistical inference then hypothesis testing, test statistics, then confidence intervals. So, now today in this lecture series, so we will start with first statistical inference.

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Highlights

- Statistical inference
- Testing of hypothesis
- Null and alternative hypothesis
- Type I and Type II error
- Estimating Population Parameters
- Confidence Intervals

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And accordingly the highlights of the lecture will be statistical inference, testing of hypothesis, and knowing null and alternative hypothesis, type 1 and type 2 error, and estimating population parameters and finally, we will touch upon confidence intervals.

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Statistical Inference

- **Statistic:** A function of random variables (usually a sample random variables in an estimation) which does not contain any unknown parameters.
 \bar{X}, S^2, etc
- **Estimator** of an unknown parameter $\theta: \hat{\theta}$, a statistic used for estimating θ .
 $\hat{\theta}: \text{estimator} = U(X_1, X_2, \dots, X_n)$
 $\bar{X}: \text{Estimator}$
 \downarrow
 $\bar{x}: \text{Estimate: A particular value of an estimator}$
An observed value

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So, then the structure will be like this. So, here the discussions will be on the basis of statistical inference. So, in the case of you know descriptive analytics, we started the discussion on the kind of you know problems; and the kind of you know requirements. And in order to know the kind of problems and the basic kind of investigations, we have gone through various descriptive analytics structures, the data visualization, data analysis and the kind of you know overview of the problems.

So, descriptive analytics will give you some kind of you know inference to go for some kind of you know in depth analysis, but here with the basis of you know descriptive hint and the kind of you know problem structures, we will go some kind of you know in depth investigation process. So, as a result we have to know certain you know things, so that you know we can do the investigations and come to a conclusions as per the management requirement or the kind of you know management decision.

So, in the kind of you know business analytics, we have a couple of things and we try to you know investigate the problem under this particular you know umbrella. So, the first structures which you like to connect or you know like to know is sample versus populations and then this statistical inference, statistics versus population you know parameters. So, now the idea is that you know we have a problems, then we have to investigate the problems, and there are various steps through which you have to do the kind of you know investigation so that may be transfer into modeling. So, it is this set of

variable descriptions, and the kind of you know functional relationship. And then with the help of you know data and with the help of you know the kind of you know structures, so we have to investigate.

So, since it is a question of you know data analysis or you know the kind of you know investigation with respect to data, so certain rules and regulation you have to follow. And on the basis of these rules and regulations, you have to come to a particular conclusions that means, we will get some kind of you know inference and on the basis of these inference, so we have to go for some kind of you know decision. So, this is how the structure.

So, here so we start with the concept called as a statistic. So, a function of random variables which does not contain any unknown parameters. So, that means so when we will go for some kind of you know investigations so we have a variables and then the variables can be connected with your particular functional form and in these functional forms so we have some kind of you know variables and then followed by some kind of you know parameters. So, we sometimes the parameters are you know unknown; and with the help of you know data and the kind of you know business analytics. So, we like to know the values of these unknown parameters.

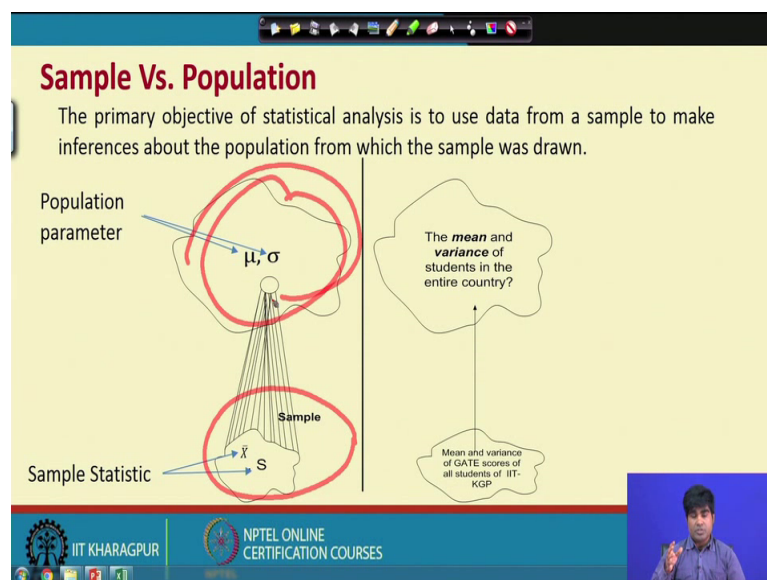
So, the business analytics will help you or to obtain all these you know unknown parameters corresponding to a particular model and corresponding to a particular problem. And then once we get the parameters value, on the basis of parameters value and the kind of you know variable linkage, so we like to go for some kind of you know predictive structures and then we will go for some kind of you know prescriptive structures.

So, here one particular structure is called as you know the kind of you know process. So, \bar{X} , S^2 these are all called as you know sample statistics so that means, we have a variables and corresponding to the variables, we have a data, the data may be with respect to cross sectional units or time series units. So, we have a number of you know data points corresponding to a particular variables, and corresponding to data points with that particular variables we have to calculate the sample statistics. So, the sample statistic can be represented in the form of mean \bar{X} or you know variance S^2 . So, you know we have already discussed all these you know statistic in the last kind of you know

structure, but here we like to against get these parameters or in the statistic values and then on the basis of these values, you have to go for some kind of you know further investigation.

So, when you look for a kind of you know parameters value that some something called as you know estimator of an unknown parameter, then it is nothing but you know function of you know variables. And then with the help of you know data we have to estimate all these you know values of these parameters. So, the kind of you know examples are you know mean, standard deviations, variance all these things right we with the help of these we have to calculate the; or you have to investigate the problems.

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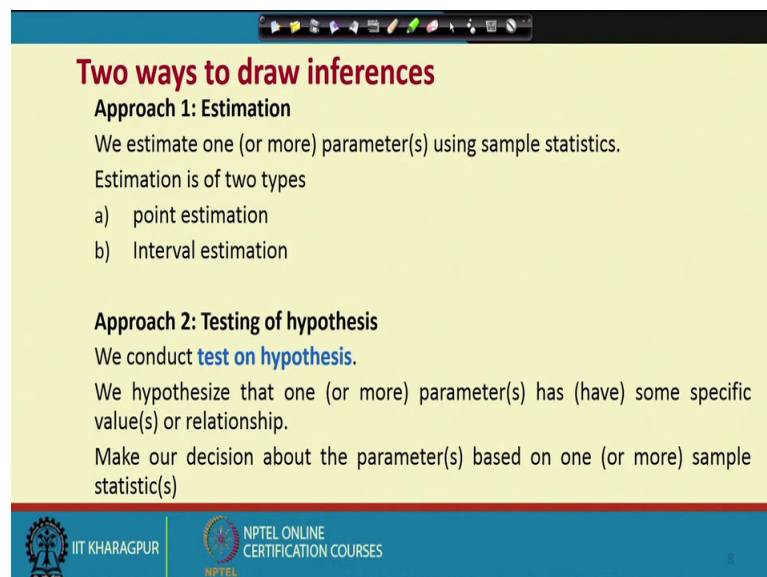
So, now this is what the particular you know structure investigation structures. So, what we have already discussed, it is always the game between population and samples. Most of the instances, we have to draw sample from the population and then investigate the process. For instance, so here in this first case it is the kind of you know population structure and the kind of you know a sample which you have drawn from the population is a called as you know sample statistics, and means the process which you investigate then we will get some kind of you know sample statistics.

So, that means, technically this is the population umbrella and as few items if you a not take and then create a kind of you know structure that is called as you know sample. And sample is a part of the population and when we investigate the population then we have a

population parameters. And usually any kind of you know investigation process, the population parameters represented as you know mean μ and σ you know standard deviation σ . And then corresponding to a particular sample, the sample statistic will be a say x is a sample then corresponding to X , so \bar{x} is this sample mean; and S is the sample variance. This is you know then I mean σ^2 is you know kind of you know parameters through the samples you know the sample statistically represent.

So, every times our approach is to check you know how sample statistics are you know different to population parameters. So, the validations you know or the problem may be authentic or you know reliable when the sample statistic will be close to population statistics. If not then there will be difference and if this difference will be there, so you have to find out against why these difference either the pickup you know samples from the population may be wrong or the populations parameters which are already available is a kind of a you know question mark. So, the idea behind this investigation or inference is just to find out the reality and then do the adjustment accordingly as per the requirement, and then accordingly we have to take some kind of you know management decision corresponding to a particular problems.

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Two ways to draw inferences

Approach 1: Estimation
We estimate one (or more) parameter(s) using sample statistics.
Estimation is of two types

- a) point estimation
- b) Interval estimation

Approach 2: Testing of hypothesis
We conduct **test on hypothesis**.
We hypothesize that one (or more) parameter(s) has (have) some specific value(s) or relationship.
Make our decision about the parameter(s) based on one (or more) sample statistic(s)

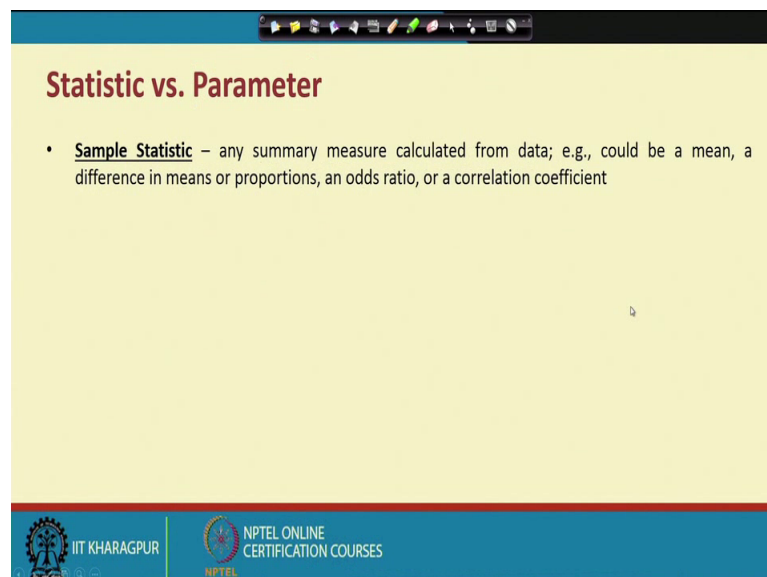
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So, there are two ways to draw inference from this cut from this particular you know process of you know investigation. So, the first approach is called as you know estimation, and the second approach is called as you know testing of hypothesis. In the

estimations, our idea is to estimate the parameters using a sample statistics. And the process which you follow we called as you know estimations. And it is of two types and first one is the point estimation; and second one is the interval estimations which we have already discussed in the previous lectures. And like you know interval what is this test statistic or interval estimation and what is the test statistic for you know point estimation.

And in the second approach, we have to follow the testing of hypothesis and which is actually connected with the first approach that is the estimation. And here we like to you know assume that one or you know few parameters has some kind of you know specific values or some kind of you know relationship. And then on the basis of you know population parameters, so we have to take our decisions that means we have to here compared with the population statistic with you know sample statistics. And whether there is a difference; if there is a difference, so what is the kind of you know inference and if there is a difference you know or if not difference then what is the kind of you know inference so that means, so whatever may be the kind of you know structures. So, we have to conclude accordingly. So, here the process will be like this.

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The slide is titled "Statistic vs. Parameter" in a dark red font. It contains a bullet point defining "Sample Statistic" as any summary measure calculated from data, with examples like mean, difference in means, proportions, odds ratio, and correlation coefficient. The slide has a yellow background and a blue footer with logos for IIT Kharagpur and NPTEL.

Statistic vs. Parameter

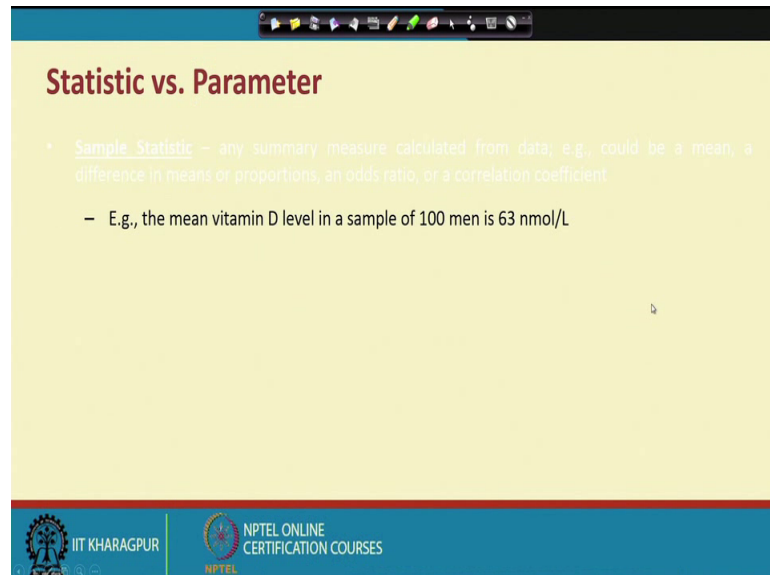
- **Sample Statistic** – any summary measure calculated from data; e.g., could be a mean, a difference in means or proportions, an odds ratio, or a correlation coefficient

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So, statistic work you know versus parameter. So, the first one is the sample statistics. So, it is a kind of a; you know summary sheet measure or like you know what we can say

that you know mean a difference in means or proportions or ratio correlation coefficient. So, these are all represented as a sample statistics.

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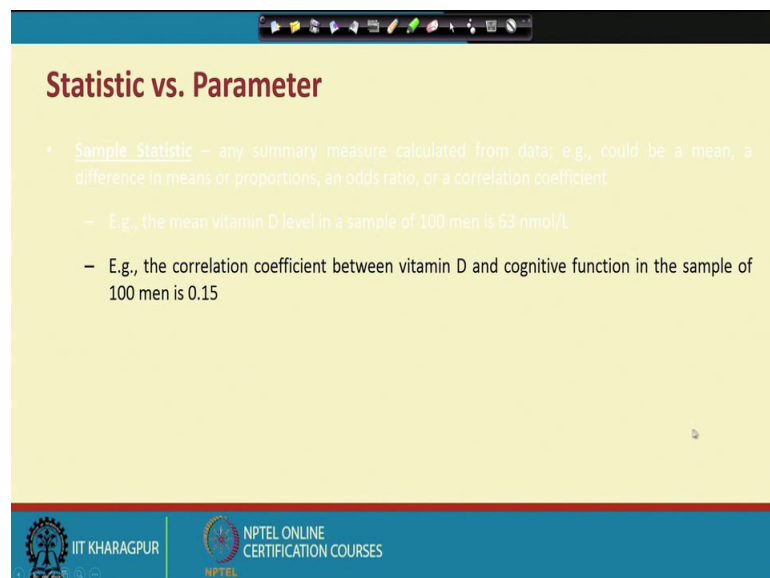
Statistic vs. Parameter

- Sample Statistic – any summary measure calculated from data; e.g., could be a mean, a difference in means or proportions, an odds ratio, or a correlation coefficient
 - E.g., the mean vitamin D level in a sample of 100 men is 63 nmol/L

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So, this is a slight examples behind how to calculate the sample statistics.

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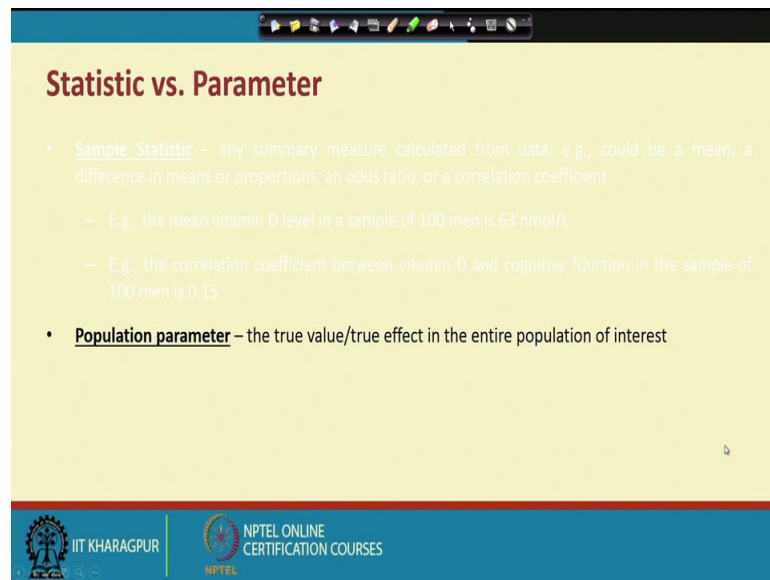
Statistic vs. Parameter

- Sample Statistic – any summary measure calculated from data; e.g., could be a mean, a difference in means or proportions, an odds ratio, or a correlation coefficient
 - E.g., the mean vitamin D level in a sample of 100 men is 63 nmol/L
 - E.g., the correlation coefficient between vitamin D and cognitive function in the sample of 100 men is 0.15

This slide is identical to the previous one but includes an additional example in the list: 'E.g., the correlation coefficient between vitamin D and cognitive function in the sample of 100 men is 0.15'.

And then this is actually correlation coefficient between vitamin D and the cognitive function in the sample of you know 100 men.

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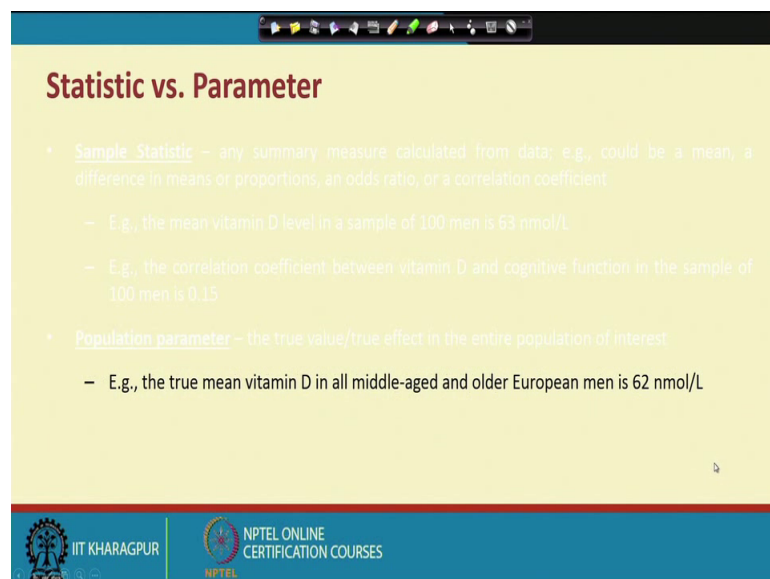
Statistic vs. Parameter

- Sample Statistic – any summary measure calculated from data; e.g., could be a mean, a difference in means or proportions, an odds ratio, or a correlation coefficient
 - E.g., the mean vitamin D level in a sample of 100 men is 63 nmol/L
 - E.g., the correlation coefficient between vitamin D and cognitive function in the sample of 100 men is 0.15
- Population parameter – the true value/true effect in the entire population of interest

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So, it is the kind of you know you know sample statistic which is represented through a particular you know data structures; and then accordingly we have to calculate and then compared with the population parameter.

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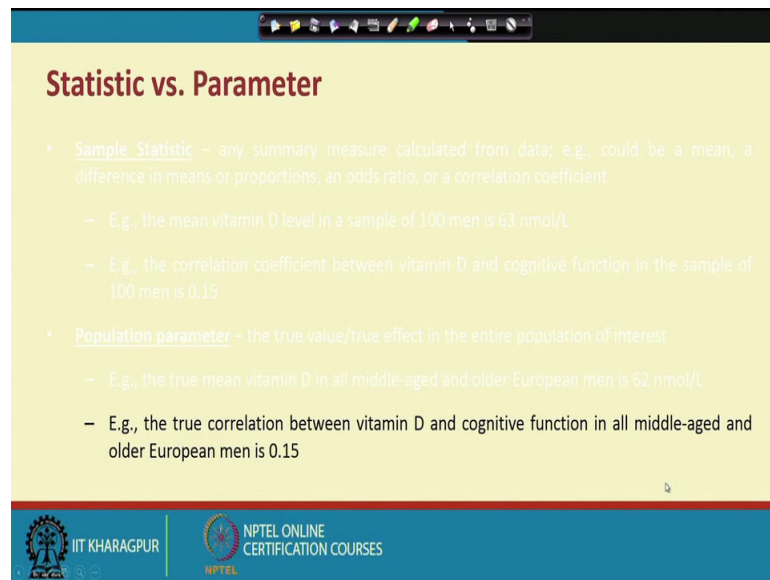
Statistic vs. Parameter

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 - E.g., the mean vitamin D level in a sample of 100 men is 63 nmol/L
 - E.g., the correlation coefficient between vitamin D and cognitive function in the sample of 100 men is 0.15
- Population parameter – the true value/true effect in the entire population of interest
 - E.g., the true mean vitamin D in all middle-aged and older European men is 62 nmol/L

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So, the population parameter is the true value or true effect in the entire population of interest. So, the classic example is the true mean vitamin D in all the middle-age you know older people in European countries means the men side is you know 62, let us say.

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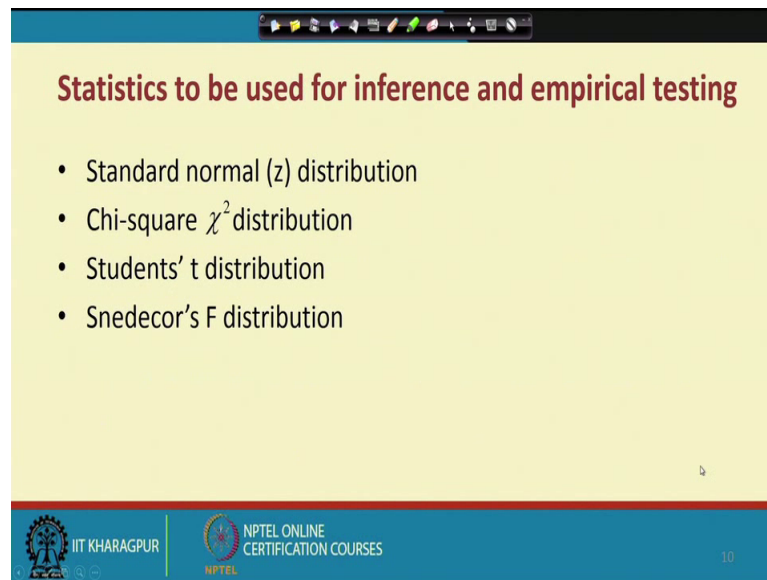
Statistic vs. Parameter

- **Sample Statistic** – any summary measure calculated from data; e.g., could be a mean, a difference in means or proportions, an odds ratio, or a correlation coefficient
 - E.g., the mean vitamin D level in a sample of 100 men is 63 nmol/L
 - E.g., the correlation coefficient between vitamin D and cognitive function in the sample of 100 men is 0.15
- **Population parameter** – the true value/true effect in the entire population of interest
 - E.g., the true mean vitamin D in all middle-aged and older European men is 62 nmol/L
 - E.g., the true correlation between vitamin D and cognitive function in all middle-aged and older European men is 0.15

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Then corresponding to these figures so we have to calculate the sample statistic then you have to compare the kind of you know comparisons. Now, similarly the correlation coefficients between vitamin D and cognitive functions of this particular intelligence, so that means, here the idea is actually, so you have a kind of you know population figure and you have a kind of you know sample figures. So, you have to check you know how sample will be representative to your; you know population. Sometimes you know sample may be good sometimes sample may not be good. So, we like to get some kind of you know inference, you have to check and get the inference and then on the basis of that you have to take the management decisions.

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Statistics to be used for inference and empirical testing

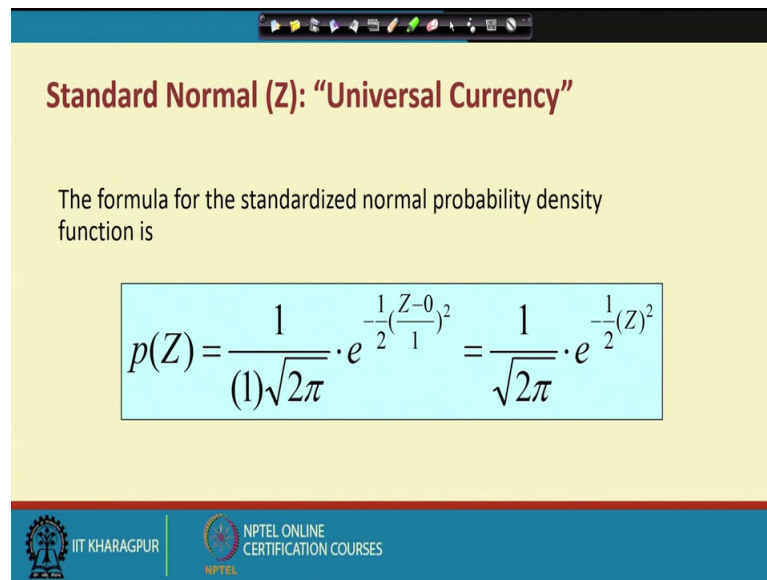
- Standard normal (z) distribution
- Chi-square χ^2 distribution
- Students' t distribution
- Snedecor's F distribution

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So, now one of the procedure through which you have to go for some kind of you know inference of course, we have already something you know the structure about the population sample, then some kind of you know sample statistics then population parameters. So, these are all they are actually, but you know a so far as the testing is concerned. So, you have to follow a particular you know test structures. So, here some of the statistic, we like to use to get some inference and on the basis of you know the empirical testing.

So, there are you know first standard test statistic through which you have to go for this empirical investigations and then to get the inference. So, the first one is called as a standard normal distributions that is usually called as a z statistics, then chi square distributions, then third one is student t distributions then finally, the last one is the f distributions. So, we have to here investigate the kind of you know problems on the basis of z statistic, chi square statistic, t statistic and F statistics.

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Standard Normal (Z): "Universal Currency"

The formula for the standardized normal probability density function is

$$p(Z) = \frac{1}{(1)\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{Z-0}{1}\right)^2} = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}(Z)^2}$$

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So, now we will discuss in details about all these you know test statistics first then we will connect with your problem. And we like to know how this test statistic will help you to get some kind of you know inference on the basis of your problem investigation. The standard and normal is operation standard normal z statistic is concerned. So, the particular first structure is followed by a normal distributions, and this is what the normal distribution functions. And when the basis of you know normal distribution functions, we have to calculate the z statistic.

And it is the particular formula here actually the particular variable will be followed by a normal distribution with mean zero and standard deviation one. So, if that is the case then you know it will give you some kind of you know normality or some kind of you know pattern of the data or you know so far as a distribution is concerned. So, then on the basis of this particular structures, you have to calculate the test statistic and then compare with the; you know population kind of you know structures.

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Standard Normal Distribution (Z)

All normal distributions can be converted into the standard normal curve by subtracting the mean and dividing by the standard deviation:

$$Z = \frac{X - \mu}{\sigma}$$

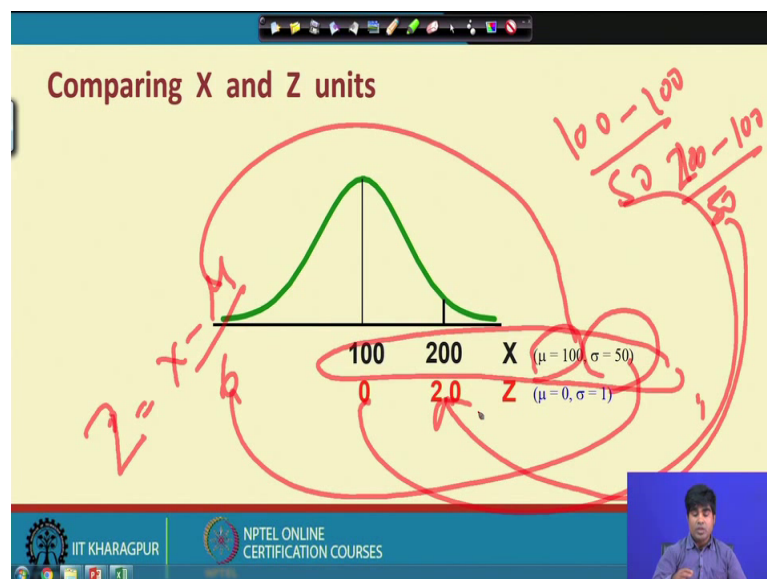
Somebody calculated all the integrals for the standard normal and put them in a table! So we never have to integrate!

Even better, computers now do all the integration.

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So, here then you like to know here how to calculate actually z. So, z is actually standard normal distributions, and usually the procedure to calculate the z is a x minus mu by standard deviations. And mu is nothing but called as a population parameter and standard deviation sigma is nothing but actually population standard deviations. And we like to check, we like to calculate the z and that is what the standard normal distributions. And then accordingly, we have to see the; a kind of the structure through which you we are in a position to take some kind of you know decisions.

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So, we have already discussed the particular structure in the previous lectures. So, here, so the; what is the idea exactly though you have to do some kind of you know transformations. Since, it is the kind of you know interval game, so we need to find out you know integrals and you know the kind of you know probability between you know two different interval or two different range. So, since by calculation it is a difficult one. So, we have to follow the kind of you know z route and then on the basis of you know z route we can get these same structures with a kind of you know simple structure.

So, the idea is here. So, the first one is the x information that is the sample information. So, now, it is a population mean is actually 100 and you know population standard deviation is 50 and then x values you know having 100 and 200 and that will convert into z structure with you know 0 and it to you know 2.0. So, that means, the typical transformation will be z equal to x minus mu by standard deviations so that means, so this is actually the structure is z equal to x minus mu by standard deviation. So, mu is here actually mu is here 100 and standard deviation is actually 50 so that means, so the structure will be 100 a minus 100 divided by 50, so that that will give you the zero structure. And then again 200 for x 200 minus 100 divided by 50, it will give you actually the structure you know 2.0, so that means, the original value job x will be convert into the z and then we will go for some kind of you know investigations.

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Example


- For example: What's the probability of getting a math SAT score of 575 or less, $\mu=500$ and $\sigma=50$?

$$Z = \frac{575 - 500}{50} = 1.5$$


• i.e., A score of 575 is 1.5 standard deviations above the mean

$$\therefore P(200 \leq X \leq 575) = \int_{200}^{575} \frac{1}{(50)\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-500}{50}\right)^2} dx \longrightarrow \int_{-\infty}^{1.5} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}z^2} dz$$





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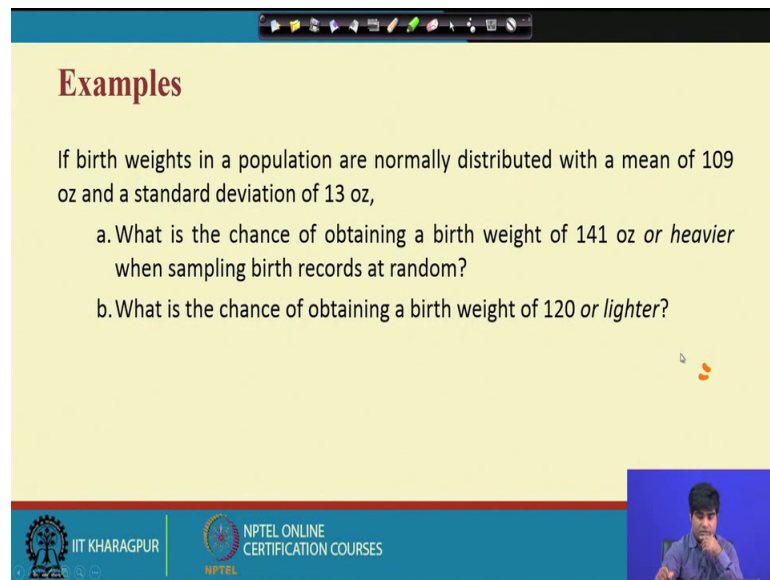


So, this is how the particular you know structure about the kind of you know z distributions and this is what the standard examples. So, what is the probability of getting a math sat score of you know 575 or less where you know μ population figure is μ equal to 500, and population standard deviation is 50. So, now the following transformation we like to follow z equal to x minus μ by standard deviation. So, here so the x is actually x is here 575 and then μ equal to 500, and 50 equal to population standard deviations.

So, now 575 or less, so you can first calculate at 575 then accordingly, you have to find out the particular structure and find out the probability of you know that particular z only. And it is very easy to calculate because we know that total probability is always equal to 1. So, you find out a specific value case then the remaining structure will be calculated by 1 minus that specific probability value. So, by default it will give you the kind of you know inference through which you have to take a decision.

Similarly, there is another example. So, you know let me first highlight you the same examples how you to actually calculate. So, it is actually through you know definite integrals and in fact, actually we need not require to calculate the definite integrals. So, you have to go for you know z transformation. So, once you get the z value then we have actually the simulated z tables; on the basis of simulated z tables you can get the particular probability that is what the level of prediction we will get. And on the basis of that predictions or you know, the probability you will get some kind of you know inference.

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Examples

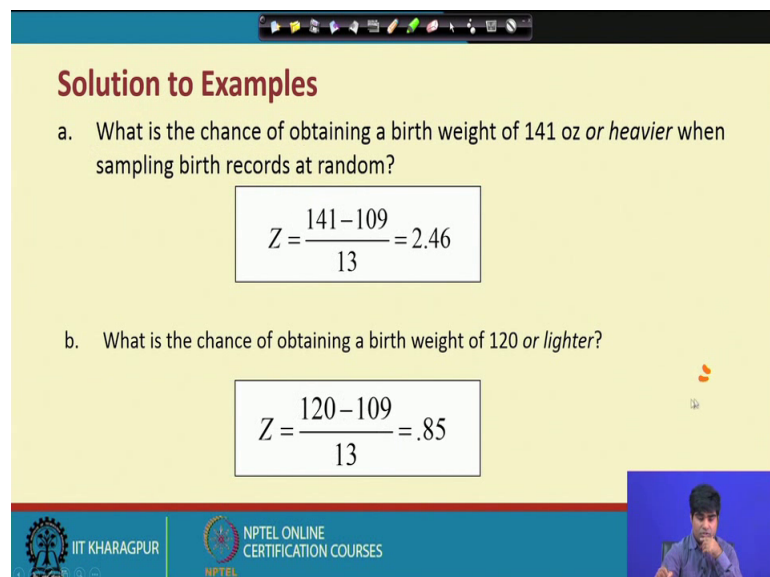
If birth weights in a population are normally distributed with a mean of 109 oz and a standard deviation of 13 oz,

- What is the chance of obtaining a birth weight of 141 oz *or heavier* when sampling birth records at random?
- What is the chance of obtaining a birth weight of 120 oz *or lighter*?

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So, now, I will give you the kind of you know structure here. So, this is another examples. And similarly, you can calculate.

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Solution to Examples

- What is the chance of obtaining a birth weight of 141 oz *or heavier* when sampling birth records at random?
$$Z = \frac{141 - 109}{13} = 2.46$$
- What is the chance of obtaining a birth weight of 120 oz *or lighter*?
$$Z = \frac{120 - 109}{13} = .85$$

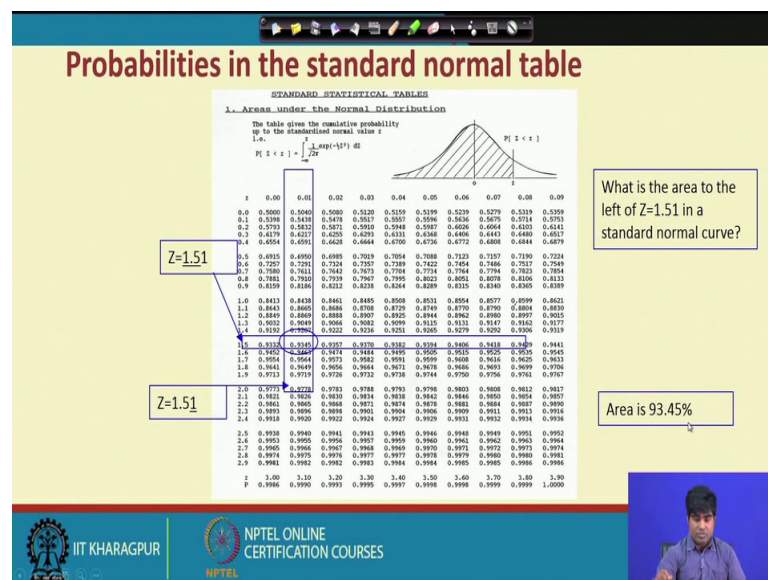
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And this is what you know structures. So, the corresponding to the; you know weight of x value 141 let us say here. So, this is 141 actually x, and then this is actually population mean 109, and population standard deviation 13. And as a result z statistic will be 2.46. Similarly, in the second example, so here x will be 120. And against we will follow the same population parameter mu equal to 109 and standard deviation 3. So, it will give you

0.85. So, now, we will check what is the; you know corresponding to this z. So, what is the probability?

So, again there is no requirement to find out the definite integral. So, you have to just go to the z tables and then you have to report the probability, so that is how the inference you can draw from this particular structure.

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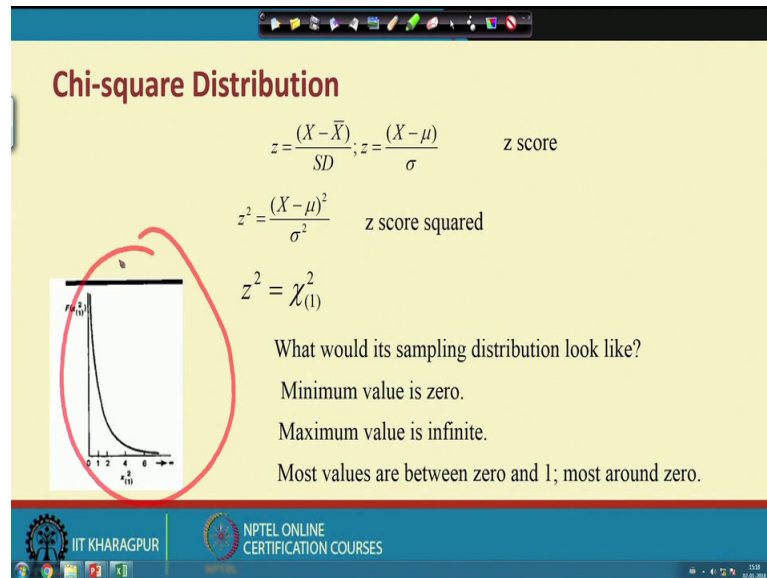


So, this is what the standard you know you know z table. So, usually so the particular structure is like this. So, this is what actually the kind of you know z structure. This is what actually z structure. And then and so for instance let us say one point actually a 401 so 1.401, this is 1.4 and then this is 01, so that means, 0.98 to 9807 that means, to 92 percent. So, likewise when the probability actually said to you know z value is equal to 2.5 then the probability value will be 0.9938, so that is how the particular you know structure. So, it will start with you know 0.0, so that means, the entire tables you will find the values of you know probabilities only.

So, the usual procedure is having the population figures and the kind of you know x specifications. So, you have to first calculate the z value. So, once you get the z value, corresponding to the z value, you can know the probability that is the kind of you know percentage through which we have to get the inference and then you will conclude as per your you know management requirement. So, next so this is actually these two classic

examples, which I have already highlighted. So, this is 1.5, 1.5, 1.5, 1.51, then this will be coming 0.9345.

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So, likewise you can calculate for other values of the z. And the second distribution is called as you know chi square distributions. And here z we have already reported that you know x minus x bar by standard deviations or in other words z equal to x minus mu by standard deviations. So, now so what will you do, you are just squaring the z statistics then you will get x minus mu squares by sigma square. And the z square is nothing but actually chi square. So, what means the question is what would its sampling distributions look like.

So, this is actually the kind of you know example. And this is the usual structure of you know kind of chi square distributions. And your idea is to minimize where minimum value is 0 and maximum value is a infinite so that means, most of the values are between 0 and 1. And most around actually 0; so that means actually a corresponding to this z, so the idea is actually z and chi square actually very close to each other, but just you have to check the kind of you know structures. And then accordingly you have to calculate and then you go for some kind of you know inference.

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Chi-square Distribution

$$z = \frac{(X - \bar{X})}{SD}; z = \frac{(X - \mu)}{\sigma} \quad \text{z score}$$
$$\chi^2_{(k)} = \sum_{i=1}^k \frac{(X_i - \mu)^2}{\sigma^2} \quad \text{z score squared, chi-sqr with k degrees of freedom}$$

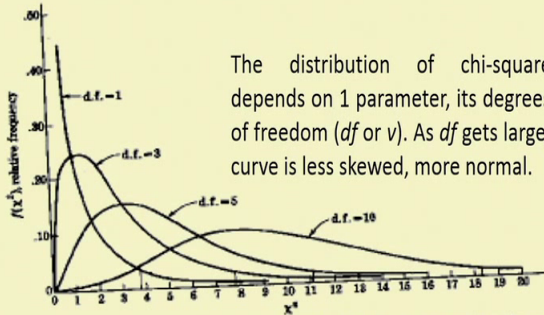
Chi-square is the distribution of a sum of squares. Each squared deviation is taken from the unit normal: $N(0,1)$. The shape of the chi-square distribution depends on the number of squared deviates that are added together.

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So, this is what the usual procedure of you know for individual kind of you know problems you have to calculate the chi square value.

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Chi-square Distribution



The distribution of chi-square depends on 1 parameter, its degrees of freedom (df or ν). As df gets large, curve is less skewed, more normal.

FIG. 13.1 Chi-square distribution and 5 per cent critical regions for various degrees of freedom. (From Francis G. Cornell, *The essentials of educational statistics*, John Wiley & Sons, Inc., New York, 1966.)

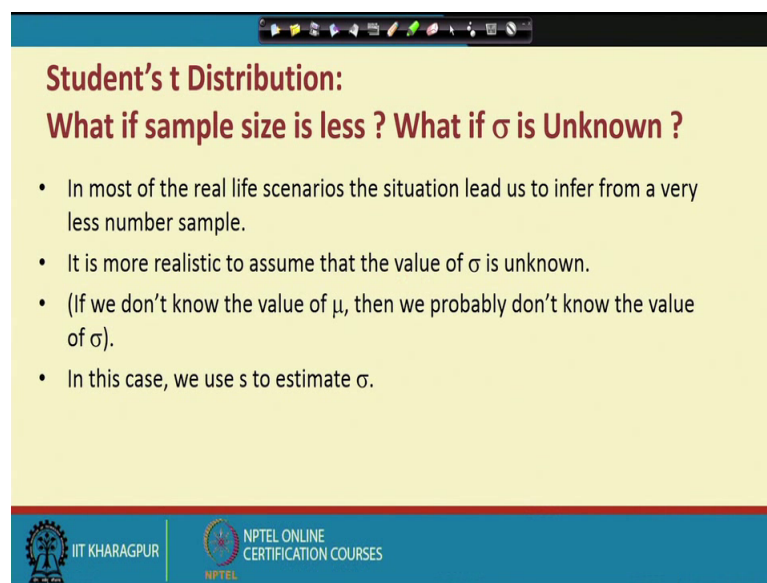
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And again like you know you know z tables, you have actually chi square tables and once you get the chi square value. So, you can go for some kind of you know probability and that is the a structure through which you will get some kind of you know inference. And one of the linkage or you know understanding you may hear that you know when your sample size will be actually large that is you know that will be followed by a

concept called as a degree of freedom, then your distribution will be you know skewed to normal. So, that is what we have already discussed through central limit theorem.

So, every time when will go for some kind of you know investigation that is the problem investigations, so to get inference or to get some kind of you know in depth kind of you know findings or some kind of you know output, so your sample size should be strong enough. It is not actually statistically a kind of you know true or statistical requirement, so it will give you enough exposure to go or to go in depth about this particular you know problems and then you will get solid inference; and on the basis of that your management decision will be very perfect. So, that is how every times the suggestion is that you know you must have actually sufficient sample points or some sufficient data, so that you can investigate the problem in a kind of you know more attractive way.

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Student's t Distribution:
What if sample size is less ? What if σ is Unknown ?

- In most of the real life scenarios the situation lead us to infer from a very less number sample.
- It is more realistic to assume that the value of σ is unknown.
- (If we don't know the value of μ , then we probably don't know the value of σ).
- In this case, we use s to estimate σ .

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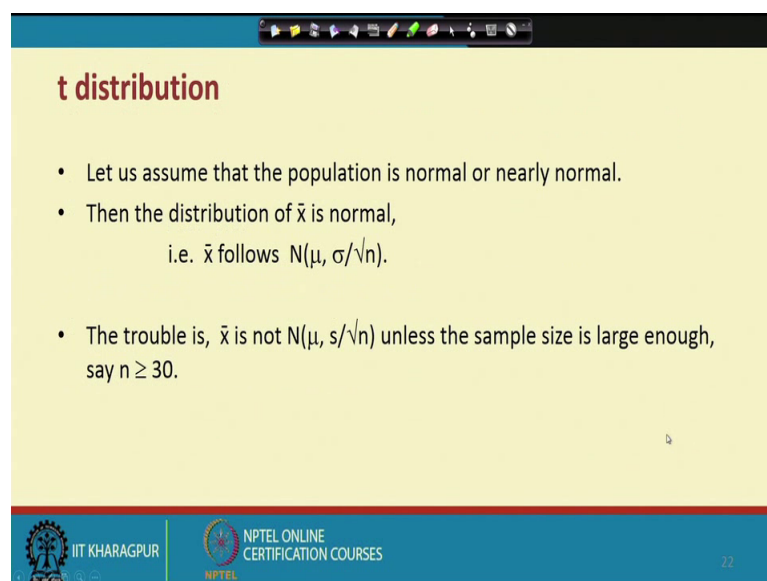
And then another statistics is called as you know student's t distributions. So, here the question is standard question is what if sample size is a less. So, we have actually two different sample structure, one is called as a small samples and the second one is called as a large sample. So, usually the cutoff rule is you know 30. So, any sum any problems having greater than 30 means it is usually called as you know large sampling case and when your sampling is less than 30 is called as you know small samples. But what you know the question is not actually whether it is the structure less than 30 or greater than 30, it depends upon you know how many variables in here in you know system and that

we will discuss in details later stage you know depending upon a particular variable structure. So, what should be the optimum sample size.

But in any case so the usual structure is that you know if you have a large sample then your accuracy will be very high; it is not you know statistical accuracy it is also problem accuracy. Usually if you have more data points or in more information then; obviously, the inference will be more attractive and inference will be more efficient, so that is how every time you look for you know more data points and that is what we called as you know more sample size. So, if your sample sizes will be big enough for you know large one then obviously, statistically it is very true or statistically you are in right track. And accordingly you your investigation process will be very effective to get some kind of you know better management decision.

And sometimes you know you will be find two different situations. In one case, sometimes here you know population parameters are known, and sometimes population parameters are not known. So, when population parameters are known, then you know you can use the z distributions; if population parameters are not known then the particular statistic will be followed by t distributions. Here you have to find out first the parameters value and then on the basis of the parameters value you will get the t statistics. And again you we will be go for as usual you know drawing the inference. So, this is actually. So, one of the particular structure is called as you know t distributions.

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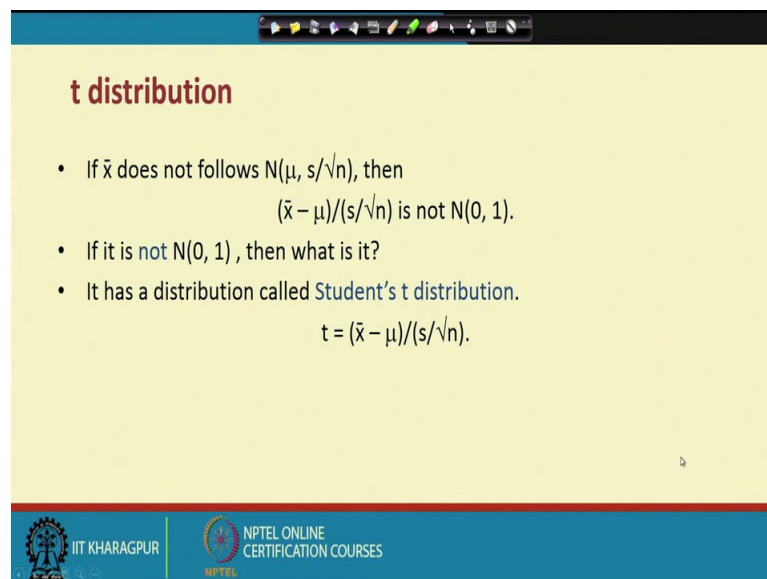
t distribution

- Let us assume that the population is normal or nearly normal.
- Then the distribution of \bar{x} is normal,
i.e. \bar{x} follows $N(\mu, \sigma/\sqrt{n})$.
- The trouble is, \bar{x} is not $N(\mu, s/\sqrt{n})$ unless the sample size is large enough, say $n \geq 30$.

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And in the case of t distributions, your whole idea is to get the actual parameters value first and then you will connect with the inference you know kind of you know requirement. And sometimes what happened you know your sample statistic may not be close to normal distribution and or it may be nearly close to normal distributions, but again the suggestion is that you know you increase the sample size, so that you know it will be close to the normal distributions.

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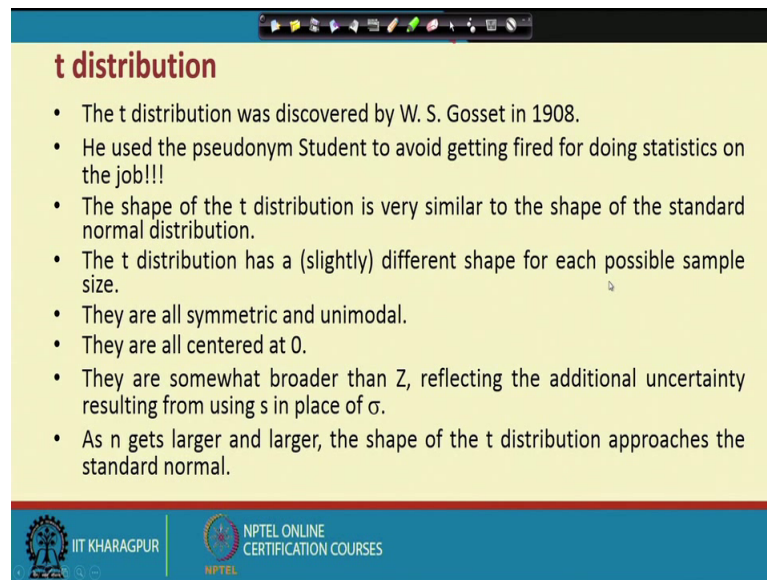
t distribution

- If \bar{x} does not follow $N(\mu, s/\sqrt{n})$, then
 $(\bar{x} - \mu)/(s/\sqrt{n})$ is not $N(0, 1)$.
- If it is not $N(0, 1)$, then what is it?
- It has a distribution called Student's t distribution.
$$t = (\bar{x} - \mu)/(s/\sqrt{n})$$

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And this is how when the distribution is not actually normal then you know you have to calculate the t distributions, and the particular you know t statistic formula is here \bar{x} minus μ by s by root n . So, s is actually a sample variance so that means sample statistic will be represented as here you know the parameters value. So, it is actually in the case of you know z , it is you know x minus μ by standard deviation that is actually the standard deviation is population parameter and it is known to it is readily available or it is known. But here in this case so the standard deviations is not known. So, it will be calculated through you know sample information only so that is how the kind of you know σ is nothing, but actually s by root n . So, that is the adjustment vector which you have to do here in the case of you know t distribution to get the particular you know inference as per your; you know problem investigation.

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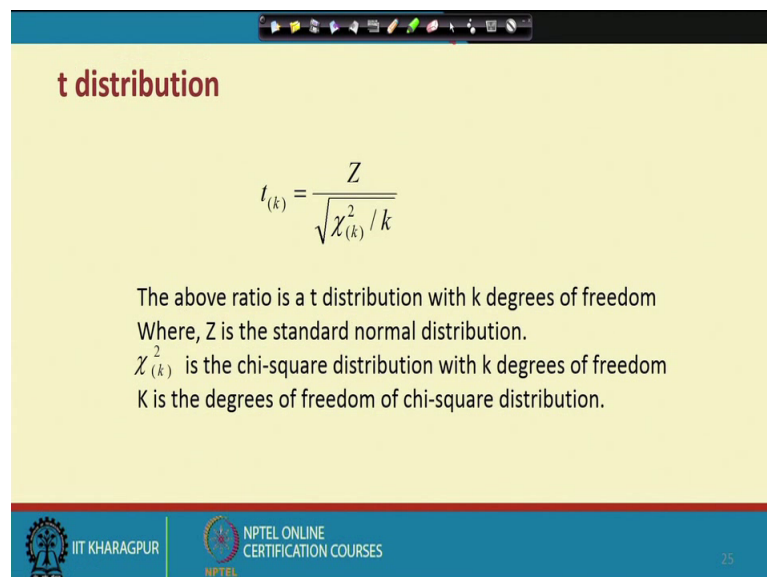
t distribution

- The t distribution was discovered by W. S. Gosset in 1908.
- He used the pseudonym Student to avoid getting fired for doing statistics on the job!!!
- The shape of the t distribution is very similar to the shape of the standard normal distribution.
- The t distribution has a (slightly) different shape for each possible sample size.
- They are all symmetric and unimodal.
- They are all centered at 0.
- They are somewhat broader than Z, reflecting the additional uncertainty resulting from using s in place of σ .
- As n gets larger and larger, the shape of the t distribution approaches the standard normal.

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So, the details about the t distribution is like this. So, let me take you to this particular you know structures how you have to calculate.

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t distribution

$$t_{(k)} = \frac{Z}{\sqrt{\chi_{(k)}^2 / k}}$$

The above ratio is a t distribution with k degrees of freedom
Where, Z is the standard normal distribution.
 $\chi_{(k)}^2$ is the chi-square distribution with k degrees of freedom
K is the degrees of freedom of chi-square distribution.

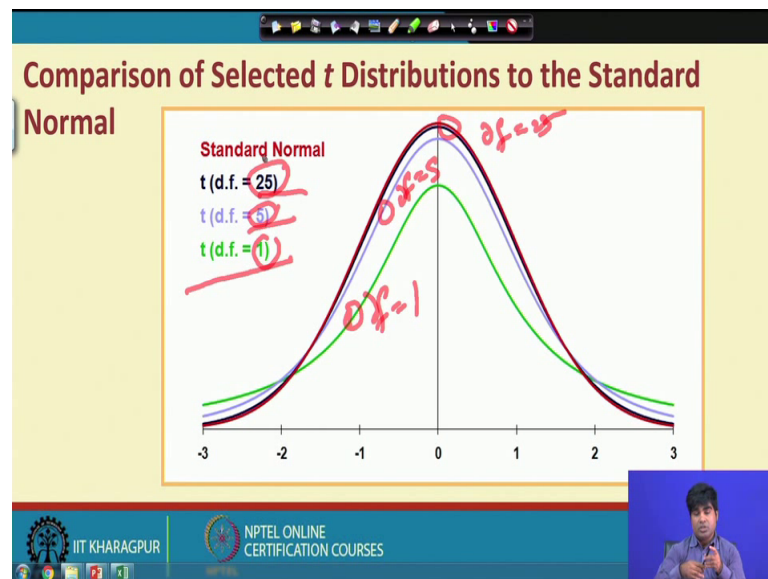
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So, I have already given you the idea you know $x - \mu$ is actually z statistics. So, t distribution has well connected with the z distribution and chi square distribution. So, this is what actually the typical linkage. So, this is what the typical linkage. So, the t distribution with k degrees of freedoms, where z is the standard normal distributions, and chi square is the a distribution with the k degrees of freedom, and k is the degree of

freedom for chi square distributions, so that means, there is a kind of you know proper linkage. So, once you get a particular distributions, so then obviously, you can connect with you know other distribution and get the inference as per here you know a problem requirement. So, now, I will take you to the kind of you know structures to know how these distributions are well connected with you know sampling structure.

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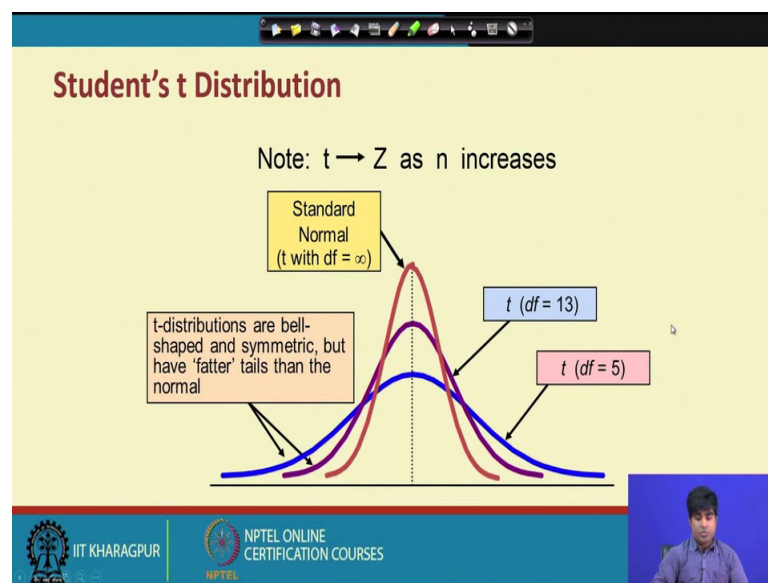
Every time when you are going to investigate any kind of you know problem, your requirement is that you know the distribution should follow actually normal because it is the best distribution so far as the investigation is concerned. And what we have already discussed that you know or the typical suggestion is that you know every time you must have you know high sample size or high degree of freedom, which is the difference between sample size and the number of variables involvement or number of parameters involvement. And here the typical example is you see here there are three shape of the distributions, and typical shape is you represented by three you know three different colors altogether. So, the green colors, then the blue colors and the black colors.

So, here so degree of freedom is 1, degree of freedom is 5, and degree of freedom is 25, so that means, if you put in other words so sample size increasing from one point to another point then when sample size is increasing then by default degree of freedom will be increasing. So, when sample size will increase your degree of freedom will increase then what is the typical structure of the particular you know distribution. Now, see here,

so this is the green where you know say your degree of freedom is equal to 1 and here is degree of freedom is equal to 5, and here the degree of freedom is actually degree of freedom equal to 25. Now, there is a red color graph and that is actually standard normal, so that means, actually 1, 5, 25 maybe it is actually a degree of freedom 50 or something like that. So, that means, you know when sample size will be more and more and more, then your distribution will be every distribution will be close to the normal distribution.

So, since normal distribution is the symmetrical distribution and best distributions so far as a problem investigation is concerned or to draw the statistical inference, so every times the idea is that you should have a enough sample size to represent the particular fact or you know to investigate the process and to get the best inference or the best decision as per the problem requirement. So, this is what the typical structure.

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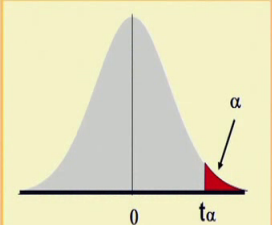
So, the t distributions another if you know make another look, so this is actually t equal to degree of freedom 5, and when degree of freedom is a 13, and here actually when actually it is turns towards you know infinite. So, that means, it will give you what do we call as you know normal distribution and that is what we know called as you know bell shaped kind of you know structure. Or in other words in statistical angles, we called as you know completely symmetrical distributions, here mean median and mode will be coincide.

In other case, when your sample size will be less and less and less, then you know the asymmetrical distribution will be moved to our you know some kind of you know skewed distributions, where you know mean median mode will not be a kind of you know same. So, it will not coincide if you know sample size that means, there is a high chance that you know they will not follow symmetrical distribution if your sample size will be very less. But the same structures can be remove toward you know normal distribution or symmetrical distribution where you know sample size will be is substantially very high and you know indefinitely increasing, so that is how the structure you have to follow to get some kind of you know.

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Table of Critical Values of t

| df | $t_{0.100}$ | $t_{0.050}$ | $t_{0.025}$ | $t_{0.010}$ | $t_{0.005}$ |
|-----------|-------------|--------------|-------------|-------------|-------------|
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.656 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 |
| 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 |
| 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 |
| 120 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 |
| ∞ | 1.282 | 1.645 | 1.960 | 2.327 | 2.576 |

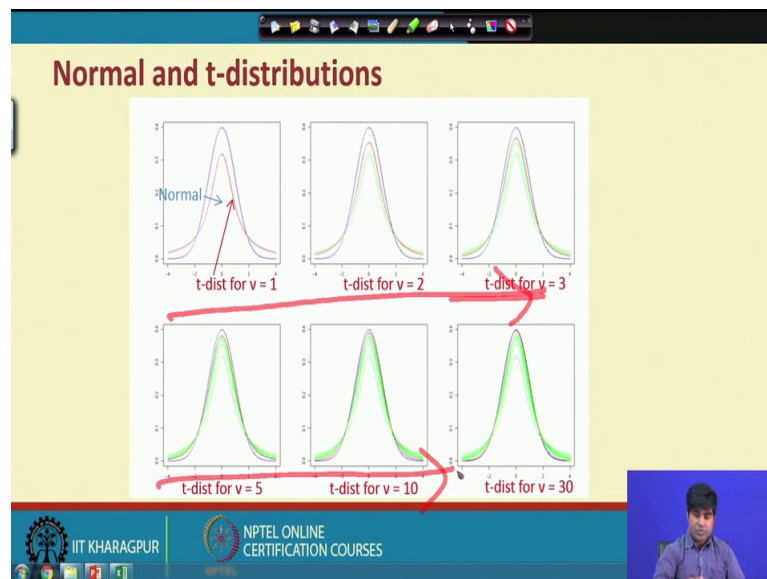


With $df = 24$ and $\alpha = 0.05$,
 $t_\alpha = 1.711$.

So, this is the standard structure of you know t tables. So, we have already you know connected with his z tables, and this is the kind of you know structure about the t tables. In the t tables, the right hand part is nothing but called as you know probability kind of you know structure. And I will let you know the details about these particular you know structures when will you go for you know hypothesis testing a in details. And here in this case it is the kind of you know degree of freedom. So, a degree of freedom actually if you start you know increasing or when sample size will be increasing by default degree of freedom will be increasing, provided a particular variable is constant or you know set of variables will be constant. And then once your degree of freedom will be increasing, then your some you know the t values will be start you know declining over there.

So, this will give you some kind of set up through which you can you know understand that you know when your sample size will be increasing, your degree of freedom will be increasing. So, it will be give you better kind of you know structure to get you know some kind of you know better inference, and then you have to take a decision as per your problem requirement. So, this is what actually.

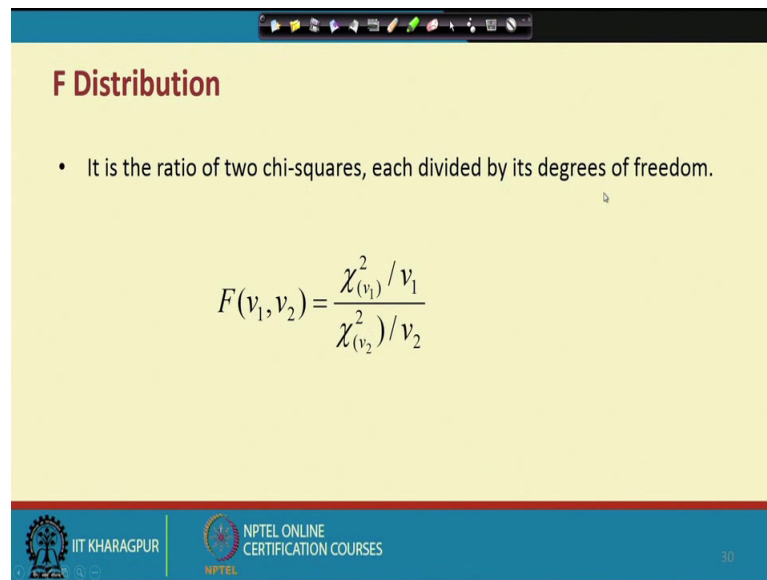
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And in the normal and t distributions, so this is actually the typical you know structures with different kind of structures you are starting with you know degree of freedom small to large then your distribution will be follow a kind of you know symmetrical. So, you will be see here the kind of you know movement. If you will follow this particular movement, so this is the kind of you know movement you will be follow you will it will give you some kind of you know towards you know normal distribution.

This gives you know better picture and this clearly highlights that you know and your sample size will be more and more and more, so it will be follow the normal distributions and the distribution which can give you know better inference for a particular you know problems.

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F Distribution

- It is the ratio of two chi-squares, each divided by its degrees of freedom.

$$F(v_1, v_2) = \frac{\chi^2_{(v_1)} / v_1}{\chi^2_{(v_2)} / v_2}$$

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And the last one is actually called as you know f distributions. And it is the ratio of you know two chi-squares, you know values and that is what the particular structures it is the chi square of you know the first case and the chi square of you know second case what a corresponding to you know t and chi t and chi square. So, f value has a two degree of freedom.

So, the first degree of freedom with the upper one chi-squares that is the; this part and the second one is degree of freedom is with respect to the second chi square. So, as a result if you go to the f tables, you will find two different degree of freedoms and then corresponding to a particular probability you will you will get a kind of you know critical value. So, let me show you how is this particular you know structure altogether.


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F Distribution


- F depends on two parameters: v_1 and v_2 (df_1 and df_2). The shape of F changes with these. Range is 0 to infinity. Shaped a bit like chi-square.
- F tables show critical values for df in the numerator and df in the denominator.



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So, F distributions you know this is the details about the F distribution.

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F table – critical values

| | Numerator df: df_N | | | | |
|--------|----------------------|------|------|------|------|
| df_D | 1 | 2 | 3 | 4 | 5 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 |
| 10 | 16.3 | 13.3 | 12.1 | 11.4 | 11.0 |
| 50 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 |
| 100 | 10.0 | 7.56 | 6.55 | 5.99 | 5.64 |
| 500 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 |
| 1000 | 9.33 | 6.94 | 5.95 | 5.41 | 5.06 |
| 5000 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 |
| 10000 | 8.86 | 6.51 | 5.56 | 5.04 | 4.70 |


e.g. critical value of F at $\alpha=.05$ with 3 & 12 $df=3.49$



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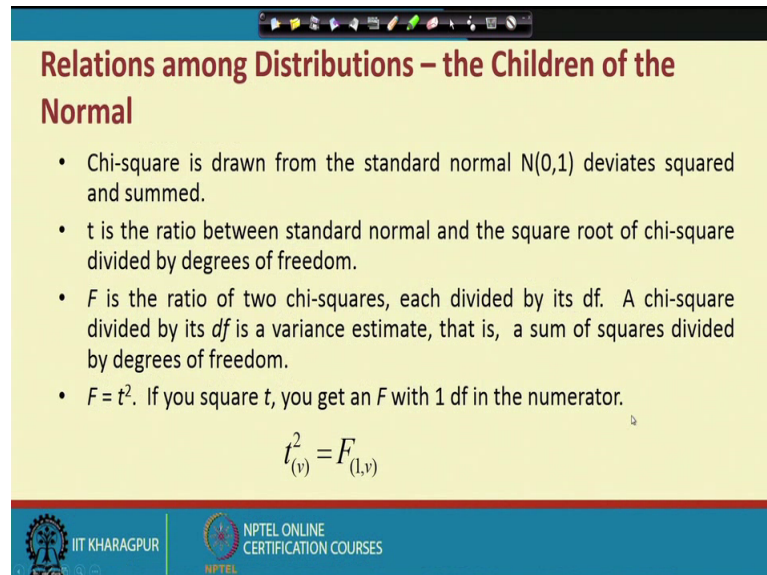
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So, the look of the f distribution is like this. So, this will be follow the kind of you know degree of freedom of the; a first case and the degree of freedom of the second case. So, now in the hypothesis testing structures, you have to first you know fix up the probability levels then corresponding to the degree of freedoms. So, f critical will be calculated. And on the basis of you know f calculated and this kind of in a sample kind of you know structures, you can get to know what is the exact structures or what is the exact inference

through which you have to draw and a conclude as per your you know problem requirement.

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Relations among Distributions – the Children of the Normal

- Chi-square is drawn from the standard normal $N(0,1)$ deviates squared and summed.
- t is the ratio between standard normal and the square root of chi-square divided by degrees of freedom.
- F is the ratio of two chi-squares, each divided by its df . A chi-square divided by its df is a variance estimate, that is, a sum of squares divided by degrees of freedom.
- $F = t^2$. If you square t , you get an F with 1 df in the numerator.

$$t_{(v)}^2 = F_{(1,v)}$$

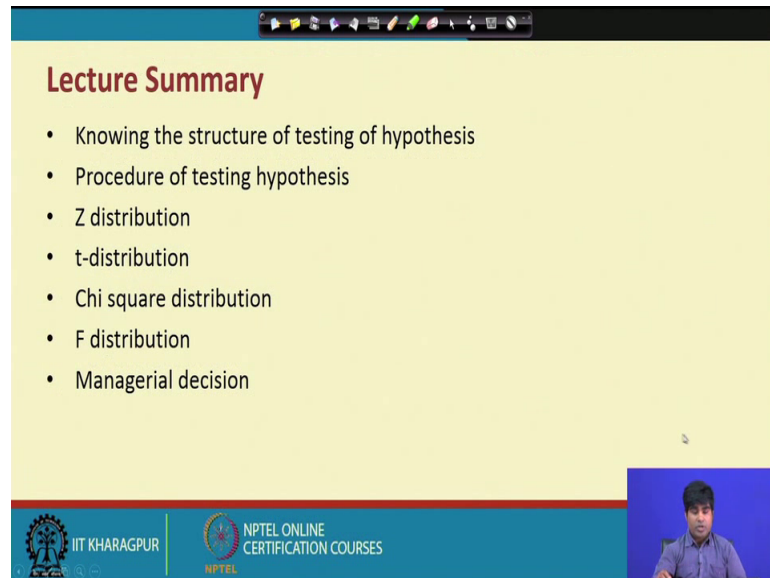
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With this I will let you know that you know in any kind of you know investigation process, so you have actually couple of test statistic through which you have to investigate the problem and get some kind of you know better inference; and on the basis of this inference, you will take some kind of you know management decision. So, I strongly you know suggest that there is a very high linkage with you know all these distributions, but once you acquainted with one particular distribution then by default you will acquainted with you know other distributions. And sometimes you know in a particular problems, you have to choose a particular distribution or you know test statistic through which you can get, you can investigate the problem and to get the inference.

And some test statistics are you know you know it is a kind of you know problem specific. For instance, f distributions if you have now it means the requirement of f distribution that you know you must have at least two variables in the systems, but that is not the case in the case of you know lets a z you know or you know chi square or you know in the kind of you know t . But in the case of you know f , your requirement must be with it to two variables then you like to know what is the actually within and between

difference with respect to their; you know sample point and the corresponding the variable specifications.

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The image shows a presentation slide titled "Lecture Summary" in red text. The slide has a yellow background and a list of topics in black text. At the bottom, there is a blue banner with logos for IIT KHARAGPUR and NPTEL ONLINE CERTIFICATION COURSES. A small video inset of a speaker is visible in the bottom right corner.

Lecture Summary

- Knowing the structure of testing of hypothesis
- Procedure of testing hypothesis
- Z distribution
- t-distribution
- Chi square distribution
- F distribution
- Managerial decision

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Likewise you will find you know plenty of you know structure through which you can you know investigate the problems, and then you will get some kind of you know inference so that means, in the inferential you know analytics we have a typical structures. So, it is the discussion you know is like this you know you have to understand the sample versus populous on them sample statistic then the population statistics, then every times you have to check the comparison between sample statistic and population statistic.

And then so far as you know inference and you know drawing conclusion is concerned then you have to connect with your particular you know test statistic that may be with respect to z statistic, t statistics, chi square statistic and x statistics depending upon the kind of you know problems. And then, the kind of you know sample structure. And on the basis of that you have to get you have to go to you know investigation process and accordingly so you will get some kind of you know inference. And on the basis of that inference so; obviously, the problem you know the management can give you some kind of you know structure through which you can you know take a decisions. And with this we will stop here.

And thank you very much, have a nice time.