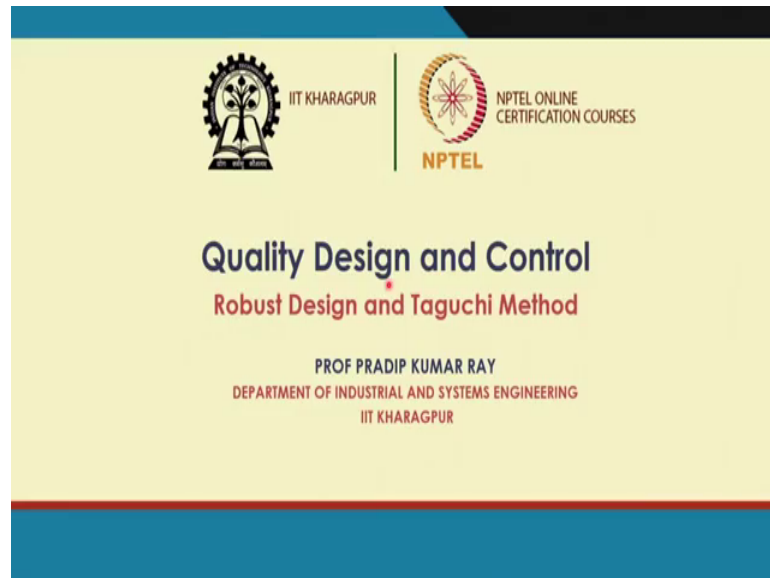


Quality Design and Control
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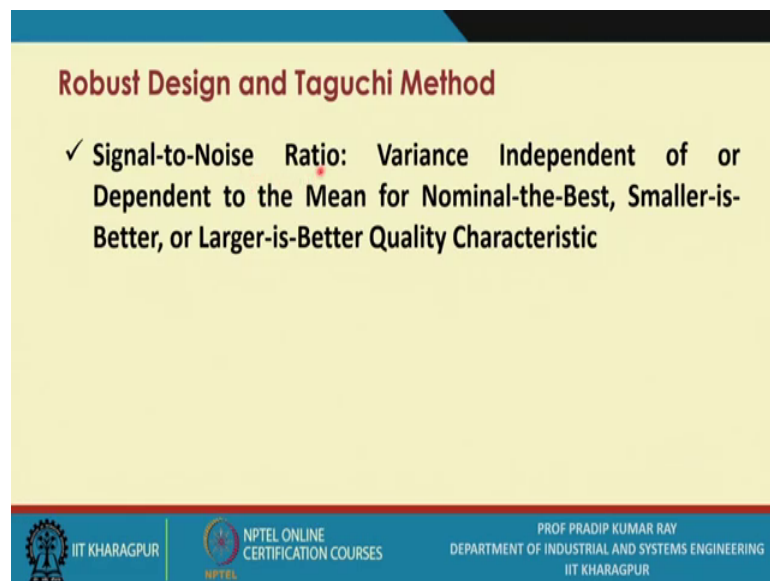
Lecture – 60
Robust Design and Taguchi Method (Contd.)

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So, during this session under robust design and Taguchi method ah.

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I will be referring to an important topic called signal to noise ratio and under different the conditions under different situations, you need to compute this ratio as a performance measure against a particular product and so, all these conditions which you come across all the situations that also, we are going to discuss and like say two conditions like variance may be independent of the mean for the given quality characteristics or the variance may be dependent to the mean for the given quality characteristics.

And as we have already been mentioning that the three kinds of quality characteristics will be dealing with right.

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Tolerance Design and Tolerancing

- It is important for the designer to **establish the optimal values of a product's parameters and their tolerances besides a product's performance, appearance, and reliability.**
- **Examine the effect of tolerance design on the quality evaluation of a product and present methods for economic design of product tolerancing and tolerance.**
- The **nominal values and functional limits generally define the characteristics of a product.** The tolerance of a product characteristic have a significant affect on the total quality loss. Therefore, optimal tolerances are those that minimize the total quality loss.
- The **three sources of variation of product characteristics from their nominal values are:**
 - **Environmental factors**
 - **Deterioration factors**
 - **Imperfections in manufacturing processes.**

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Now, before I discuss signal to noise ratio the; let me they refer to the tolerance design and the tolerancing if you remember in the last lecture sessions, we were referring through the customer tolerance as well as the manufacturing tolerance.

Now, whenever we talk about the Taguchi method as you are aware that there are you know that there are three steps in the Taguchi method the first step is the system design the next step is the parameter design and the last step is the tolerance design. So, we will just highlight a certain important aspects related to tolerance design and tolerancing, it is important for the designer to establish the optimal values of a products parameters and their tolerances besides a besides a products performance appearance and reliability in simple and plain terms, it means that suppose you identify a parameter called say you know the spindle speed related to machining operation.

Now, while you specify the spindle speed you say given that particular component and its quality characteristics and it is its materials the property and all I say that that the target value of the spindle speed should be 500 rpm, but you know this is just a point estimate. So, it is always better that you also specify the tolerances against the nominal value. So, in this case instead of saying say 500 rpm why do not you say that I have determined the tolerance range for the rpm and it should be between say you know the four hundred to 600, is it clear.

Now, how to determine this range? So, that is an important issue that we are going to discuss examine the effect of tolerance design on the quality evaluation of a product; that means, what you need to do if the value of rpm is four hundred what is the performance of the system or the machining or if suppose it is 600; that means, the other side; that means, other extreme value that is 600 rpm what is the performance of are the machining and whether you are going to accept it or not and for which; that means, the effect is to be known and for which you must be aware of the methods for the economic design of product tolerancing and the tolerances ok, the nominal values and the functional limits generally define the characteristics of a product the tolerance of a product characteristics have a significant effect on the total quality loss therefore, optimal tolerances are those that minimize the total quality loss.

So, ok; so, here your basis is in determining this these tolerance for a given quality characteristics is that I will set the tolerance in such a way that even if the value is at the extreme; that means, upper specification limit or the lower specification limit or upper tolerance limit or lower tolerance limit, I am I am incurring I will be incurring a loss, and this loss is acceptable to you the 3 sources of variation of product characteristics from their nominal values are; obviously, you know you cannot expect that all the values of the quality characteristics will be exactly at m, ok.

So, what do you what are the sources of variations? So, first one is the environmental factors the second one is the deterioration factors and the third one is the imperfections in manufacturing processes. So, this just you remember the three important you know the sources of variations now.

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Tolerance Design and Tolerancing

- Product designer addresses the first two causes of variation by considering the **parameter design and the tolerance design**. The production and manufacturing people address on the third cause of variation, that is, variations due to manufacturing process imperfections.
- There are **three approaches for obtaining the optimal tolerances**. They are
 - **The nominal-the-best**
 - **The larger-the-better**
 - **The smaller-the-better.**

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So, there are product designers who address the first two causes of variation by considering the parameter design and the tolerance design ok. So, I have already mentioned the production and the manufacturing people address the third cause of variation that is the variations due to manufacturing process imperfections, is it ok.

So, ah; so, the responsibility is assigned against as a three kinds of the sources of variations. Now, there are three approaches for obtaining the optimum tolerances against all these three types of quality characteristics.

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Tolerance Design and Tolerancing

- **The Nominal-the-Best**
In this type of tolerance, the nominal measurement of product characteristic such as dimensions and viscosity are considered and the variation is reduced to a minimum.
- **The Larger-the-Better**
In larger-the-better is applicable to product's characteristic such as the hardness of materials, fuel efficiency, and thermal properties of materials. The product designer and the manufacturer aim for the larger value of the characteristic.
- **The Smaller-the-Better**
In this case, the product designer and the manufacturer aim for the smaller value of the characteristic of a product such as wear and tear, deterioration and noise level, etc.

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The first one the nominal is the best in this type of tolerance the nominal measurement of product characteristics such as dimensions and viscosity these are examples. In fact, are considered and the variation is reduced to a minimum in the larger the better the larger, the better is applicable to products characteristics such as the hardness of materials fuel efficiency and the thermal properties of materials these are three examples.

The product designer and the manufacturer aim for the larger value of the characteristics as high as possible as large as possible and similarly, the third one that is the smaller the better case the product designer and the manufacturer aim for the smaller values of the characteristics of the product such as wear and tear deterioration and the noise level etcetera.

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Multiple Components

- The objective here is to confirm the output characteristic of the product consisting of k components to the functional limit. The tolerance design of each of these k components is required since the tolerances of the components have a direct affect on the quality characteristics of the product or process.
- The desired characteristic of the product, y with functional limit Δ can be written as

$$y = m_0 + C(x - m)$$

where

- m_0 = the nominal value of y (the target value)
- C = a constant
- x = characteristic
- m = the nominal value

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So, these are the examples. In fact, when you deal with the multiple components this is a very special case. So, here the objective is to confirm the output characteristics of the product consisting of key components to the functional limit as I have already told you during our discussions on reliability like say most of the product has got n number of components and while you design a product you first determine means the number of components the types of components and as well as the configuration, is it ok.

So, so supposing, there are k number of components the tolerance designed for each of these k components is required, since the tolerances of the components have a direct effect on the quality characteristics of the product or the processes, is it ok. So, the

desired characteristics of the product y with functional limit Δ can be written as $y = m_0 \pm \Delta$. So, m_0 is the nominal value plus Δ into x minus m , is it ok. So, m is the nominal value x is the characteristics and C is a constant ok.

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Multiple Components

- The functional limits are $m_0 \pm \Delta$. Loss function is given by



$$L(y) = \frac{A}{\Delta^2} (y - m_0)^2$$

$$L(y) = \frac{A}{\Delta^2} [C(x - m)]^2$$
- Price of the component with the characteristic x is equal to $L(y)$. Therefore

$$L(y) = A_0 = \frac{A}{\Delta^2} [C(x - m)]^2 \quad \bullet \quad x = m \pm \sqrt{\frac{A_0}{A}} \left(\frac{\Delta}{C} \right)$$

which is the optimal tolerance specification for the component characteristic x . For k components, the tolerances for each component are given by

$$\Delta_k = \sqrt{\frac{A_0}{A}} \left(\frac{\Delta}{C_k} \right)$$

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So, this is the equation. Now, the functional limits are $m_0 \pm \Delta$. So, the loss function, you have computed like this, is it and the price of the component with characteristics x is equal to $L(y)$, is it ok. So, this is basically the loss means which price you are going to pay, therefore, you know this loss is equals to A_0 that is the proportionality constant into C into x minus m square and ultimately you get an expression for x like this, is it ok. So, you can derive these expressions; no problem which is optimal tolerance specifications for the component characteristics x for k components the tolerances for each components are given by Δ_k equals to this one is it root over A_0 i by capital A into capital Δ by C into k .

So, this is A_0 , you know generalized formula for capital Δ_k .

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Multiple Components

when $x_i = m_i + \Delta_i$
 Assuming $C_i \Delta_i$ represents the contribution to the displacement y caused by quality characteristic x_i . Overall tolerance δ can be computed as follows

$$\delta^2 = \sum_{i=1}^k (C_i \Delta_i)^2 \quad i = 1, 2, k$$

$$= \left(\sqrt{\frac{A_{01}}{A}} \Delta \right)^2 + \left(\sqrt{\frac{A_{02}}{A}} \Delta \right)^2 + \dots + \left(\sqrt{\frac{A_{0k}}{A}} \Delta \right)^2$$

$$= \left(\frac{A_{01} + A_{02} + \dots + A_{0k}}{A} \right) \Delta^2$$

$$\delta = \sqrt{\frac{\sum_{i=1}^k A_{0i}}{A}} \Delta$$


So, when x_i equals to m_i plus Δ_i assuming $C_i \Delta_i$ represents the contribution to the displacement y caused by the quality characteristics x_i over all tolerance δ can be computed as follows, is it ok. So, you just follow these steps the delta square expressions is known to you and you just substitute all these values and ultimately you simplify this expression and ultimately you know the expressions for small delta is this one; that means, root over sigma I equals to one to k A_{0i} divided by capital A into capital Δ , is it ok.

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Multiple Components

if $\sum_{i=1}^k A_{0i} \ll A$, then $\delta \ll \Delta$.

and $\sum_{i=1}^k A_{0i} \gg A$, then $\delta \gg \Delta$

if $\sum_{i=1}^k A_{0i} \approx A$, then $\delta = \Delta$

$x = m \pm \sqrt{\frac{A_0}{A}} \left(\frac{\Delta}{C} \right)$ can be used to determine the tolerance of each component.



So, if you have the multiple components. So, if this is this one; that means, sigma i equals to 1 to k A 0 i less than equals to a then delta; obviously, there could be different you know the conditions. So, small delta is less than equals to capital delta. So, this is basically you know when you refer to the manufacturing tolerance. So, you make sure the small delta is less than significantly less than the capital delta and if this condition holds; that means, this is this one the sigma A 0 i is greater than significantly greater than A.

So, that these values you must compute and then so small delta is significantly greater than capital delta, then this one is approximately A, is it ok, there is hardly any difference, then small delta equal to capital delta. So, what is the value of x x is m plus minus this one into capital delta into small c. Now these expression you can use to determine the tolerance of the each component.

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Tolerances are Not Equal

- In this case, the tolerances are in general set at unequal distances from the target value m . That is, they are set at $m_{-\Delta_1}^{+\Delta_2}$, where Δ_1 and Δ_2 are the lower and the upper limits of the tolerance, respectively.
- The loss due to the deviation of y from m is given by Fig.

$$L(y) = \begin{cases} \frac{A_1}{\Delta_1^2} (y-m)^2 & \text{if } y \leq m \\ \frac{A_2}{\Delta_2^2} (y-m)^2 & \text{if } y > m \end{cases}$$

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So, when the tolerances are not equal. So, you have this expression the loss due to the deviation of y from m is given by this one; that means, likes asymmetric case. So, y is less than equals to m and y is greater than equals to m already you have computed.

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Tolerances are Not Equal

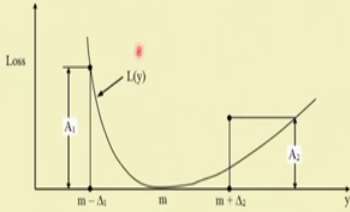


Fig. Loss due to deviations from m.

Where, A_1 is the loss caused by y being below the lower limit of tolerance, and A_2 is the loss caused by y being the upper limit of tolerance. When n observations are taken, the expected loss L is obtained as

$$L = \frac{1}{n} \left[\frac{A_1}{\Delta_1^2} \sum (y-m)^2 + \frac{A_2}{\Delta_2^2} \sum (y-m)^2 \right]$$

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And this is the case loss tolerances are not equal several examples we have put forward.

So, these values are greater than m , is it ok. So, you will find it is it is a loss is there, but the loss is less whereas, if the actual value is less than m ; that means, the loss is very is very high significantly high. So, this we have explained. So, when n observations are taken the expected loss L is obtained as this one is it so; that means, the entire loss is grouped under you know the two categories ; that means, a loss where when the value is greater than m and the loss you incurve when the value is less than m .

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Signal-to-Noise Ratio

- The quality of a product or a process is greatly affected by two classes of input parameters:
 - design factors
 - disturbance factors
- The designer can specify the design factors or parameters rather freely while the disturbance factors are the parameters that are uncontrollable or not practical to control.
- The input and output parameter variability is classified into four categories. As mentioned earlier, the term signal, or the average value of the quality characteristic, represents the desirable component, which will preferably close to a specified target value.
- Noise represents the undesirable component and it is a measure of the variability of the output characteristic, which should preferably be as small as possible.

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So, this is the final expressions you have. So, go through; now, let us talk about a particular ratio called signal to noise ratio as you know that when you determined the parameter or the parameters of say the optimal. So, optimally parameters of a process and the process is used a to produce a component what is important is that you said the parameter values in such a way that the performance of the product is it is held at the maximum level.

Now, the question is how do you specify how do you determine the performance. Now what the Genichi Taguchi has pointed out that these the performance of a product or a process must be where you know measured with a ratio called signal to noise ratio; what actually he says that you know the performance when you have the performance we will have two components one component is the desirable component and the second one is the undesirable component the desirable component is and both will be a both will be occurring.

So, the desirable component is measured with the signal and the undesirable component is measured with the noise, you cannot come across a situation where you know you get only the desirable components or it is least likely that you will be getting only the undesirable components, is it ok. So, ah; so, the signal is the desirable one and the noise is undesirable one. So, what you suggest that the performance must be measured with this ratio called signal to noise ratio.

So, this is the basic idea behind the proposing signal to noise ratio as are the performance measure. So, the quality of a product or a process is greatly affected by two classes of input parameters one is definitely the design factors ok. So, that is why when we talk about offline quality control we essentially we are dealing with the design factors and then these another factor called disturbance factors; that means, there are many external the factors all uncontrollable noise factors and you have to you have to get the performance of the product definitely you try to achieve the best possible performance, but this the product you are running in a given environment and the environment; that means, we will have we have many types of factors on which you do not have any control.

So, directly or indirectly these factors are affecting the performance of the performance of the product. So, the design factors you need to consider as well as you need to

consider the disturbance factors the designer can specify the design factors or the parameters that is the job of a designer. In fact, in fact design process when you prescribe essentially given a product is it you need to determine those design factors and so, any the designer does it and while the disturbance factors are the parameters that are uncontrollable or not practical to control like say in a given situations are the temperature or the humidity or the dusts ok, you cannot avoid, but are you are you are you willing to you know they remove those even if you try to your best will not be able to do that; is it ok.

So, the input and output parameter variability is classified into four categories now there are input parameters, therefore, output parameters ok. So, input variables output variables output variables are also known as the response variables as mentioned earlier the terms signal or the average value of the quality characteristics; that means, that is those values are acceptable right represents the desirable component average which will preferably close to a specified target value the target value is m that is the quality characteristics.

So, noise represents the undesirable component and it is a measure of the variability of the output characteristics which should preferably be as small as possible; that means, there are two components one component is you will be getting several values of y now out of all these values what do you compute you compute the average. So, that is basically the signal and that is basically the desirable component whereas, you will have a standard deviation ok. So, the standard deviation; that means, the variability actually represents the undesirable component, is it ok?

So, and many a time as you know that when you refer to a particular distribution there is a variability we say that this variability is due to you know the system related or say uncontrollable the noise factors, is it ok.

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Signal-to-Noise Ratio

- Four categories of the variability of the input and output parameters are:
Input Variability
 1. **Tolerances**

Tolerances are defined as the limit at which some economically measurable action is taken. Tolerances are the normal variability in design factors. The customer tolerance corresponds to the point at which a significant number of customers take economic action because of off-target performance.
 2. **Inner noise**

Inner noise is a result of variation due to the deterioration of parts and material. Examples include wear out of parts due to friction, and increase in resistance of resistors with age. Thus, inner noise is a long term change in product characteristics over time due to deterioration and wear.

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So, like say these are due to the common causes. So, what you need to do in order to improve the performance of a product or the process you need to maximize the value of signal and you need to simultaneously you need to minimize the value of noise, is it ok.

So, the four categories of the variability of the input and output parameters are like say when you talk about the input variability; that means, in order to improve the performance of the product what do you need to do; that means, you need to consider input parameters as well as the input variables and are the sources of variation of the input variables you must know. So, the first issue is the tolerances. Tolerances are defined as the limit at which some economically measurable action is taken tolerances are the normal variability in design factors, is it either there could be a loose tolerance or there could be tight tolerance, but the tolerance a do exist the customer tolerance corresponds to the point at which a significant number of customers take economic action because of off target performance already we have refer too.

Now, what is the inner noise inner noise is a result of variation due to deterioration of parts and materials; that means, this is happening within the manufacturing system examples include wear out of parts due to friction is it and increase in resistance of registers with age; that means, here is here is a system or here is a product. So, there are

you know the components and the functioning of these components are very very important and those are the internal components.

So, what might happen; that means, there could be wear and tear of the sub the mating parts as well as increase in resistance of registers we these are the examples. In fact, so, the inner noise is a long term change in product characteristics over time due to deterioration and wear like say your product could be a machine tool. So, this happens as we as you start working with the machine tool.

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Output Variability

3. Variational noise
Variational noise is the short term unit-to-unit variation due to the manufacturing process.

4. Outer noise
Outer noise corresponds to the variability of the disturbance factors that contribute to output variability. Environmentally related noise factors, which affect variation within a product (temperature, humidity, operators, etc.). The variables external to a product that affect the product performance are known as external noise factors. Examples include variations in temperature, humidity, and dust.

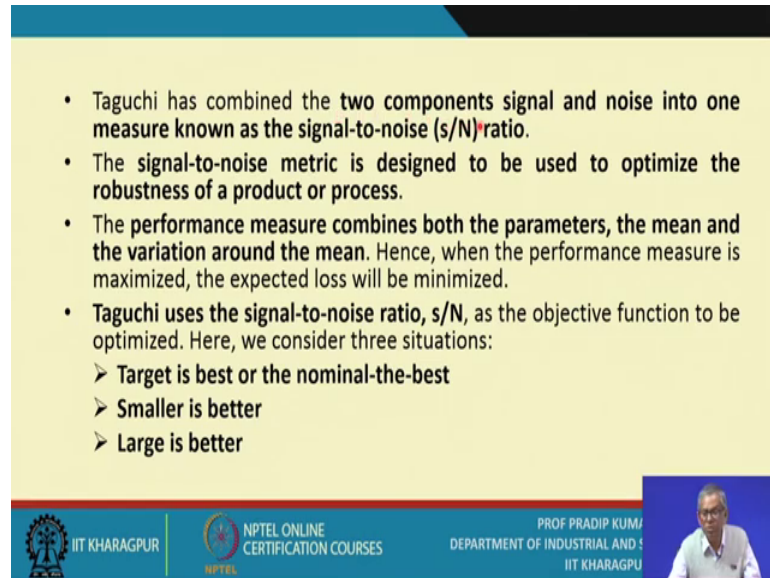
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Now, as far as the output variability concerned first one is the variational noise; that means, the variational noise what is the variational noise is the short term unit to unit variation due to the manufacturing process, I have already pointed out that what do you mean by the unit to unit variation both the both the units are produced with the same set of specifications, but can you just say that both the units are identical you cannot be, is it ok. So, there will be even if they are closely identical they cannot be hundred percent identical.

So, this is referred to as the unit to unit variation. So, this is one and the second one is the outer noise outer noise corresponds to the variability of the disturbance factors that contribute to the output variability. So, you need to identify that what are the sources of this outer noise. So, environmentally related noise factors which affect variation within a product like, I have already mentioned like say the temperature humidity operators, is it

ok, etcetera, the variables external to a product that affects the product performance are known as external noise factors examples include variations in temperature humidity and dust.

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The slide contains a list of bullet points on a yellow background. The first bullet point states that Taguchi has combined signal and noise into one measure, the signal-to-noise (s/N) ratio. The second bullet point explains that this metric is used to optimize the robustness of a product or process. The third bullet point notes that the performance measure combines both the mean and the variation around the mean, and that maximizing it minimizes expected loss. The fourth bullet point states that Taguchi uses the signal-to-noise ratio, s/N, as the objective function to be optimized, and lists three situations: Target is best or the nominal-the-best, Smaller is better, and Large is better. The slide footer includes logos for IIT Kharagpur and NPTEL, and identifies the speaker as Prof. Pradip Kumar, Department of Industrial and Manufacturing Engineering, IIT Kharagpur.

- Taguchi has combined the **two components signal and noise into one measure known as the signal-to-noise (s/N) ratio.**
- The **signal-to-noise metric is designed to be used to optimize the robustness of a product or process.**
- The **performance measure combines both the parameters, the mean and the variation around the mean.** Hence, when the performance measure is maximized, the expected loss will be minimized.
- **Taguchi uses the signal-to-noise ratio, s/N, as the objective function to be optimized.** Here, we consider three situations:
 - **Target is best or the nominal-the-best**
 - **Smaller is better**
 - **Large is better**

Taguchi has combined these two components signal to noise signal and noise into one measure known as the signal to noise ratio as I have already pointed out the signal to noise metric is designed to be used to optimize the robustness of a product the process; that means, make sure that even if there is existence of external noise factors, the performance of the product as measured by signal to noise ratio is not significantly affected.

The performance measure combines both the parameters the mean and the variation around the mean that is essentially the standard deviation or the variance and hence when the performance measure is maximized the expected loss will be minimized Taguchi uses the signal to noise ratio s/N ok, this is that is the notation we use s by n as the objective function to be optimized here, we consider three situations again there are three quality characteristics what you need to do you need to give an expression. So, provide an expressions for signal to noise ratio for each of these three cases.

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Target is Best or the Nominal-the-Best

The expected loss associated with the output characteristic is given by

$$E[L(y)] = \frac{A}{\Delta^2} [\text{variance}[y] + (\mu - m)^2] = k[\text{MSD}]$$
$$= \frac{A}{\Delta^2} [\text{variance of } y + \text{squared bias of } y]$$

where

- A = the associated loss
- Δ = the tolerance
- m = the target value of y
- y = the value of the quality characteristic
- $k = \frac{A}{\Delta^2}$ = quality loss coefficient
- MSD = the mean square deviation
- μ = the mean value of y

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So, when the target is the best nominal is the best you know that this is the expected loss that is k into MSD; already we have derived these expressions ok. So, this is the squared bias of y and this is the variance of y and this is the expressions for the proportionality constant.

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Target is Best or the Nominal-the-Best

- It is desirable for the output characteristic to be on target while at the same time the variance to be as small as possible.
- The mean square deviation [MSD] for the target is best or nominal the best can be written as
- $$\text{MSD} = [s^2 + (\bar{y} - m)^2]$$
- There are two different nominal-the-best cases that require modifying the MSD in different ways to obtain the form most likely to be independent of the adjustment.

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So, it is desirable for the output characteristics to be on target while at the same time the variance to be as small as possible, is it ok, the mean square deviation MSD or the target is best or nominal is the best can be written as this one, ok, there are two different

nominal is the best cases that require modifying the MSDs in different ways to obtain the form most likely to be independent of the adjustment.

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Variance Independent of the Mean

- Response values are continuous and can take on either positive or negative values and the target can be zero. Typical examples of this type are temperature controller errors around the nominal value, and the offset voltage for a power supply.
- In this situation, the performance measure, which reduces the variance of the output characteristic, is given by $\xi = -\log(\text{variance } y)$
- An estimate of which the performance statistic is given by $z = -\log(s^2)$ where s^2 is the sample variance of y , which is given by

$$s^2 = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n - 1}$$

n = sample size and \bar{y} = the sample mean of y_i

- We observe that the larger the performance statistic Z , the smaller the variability

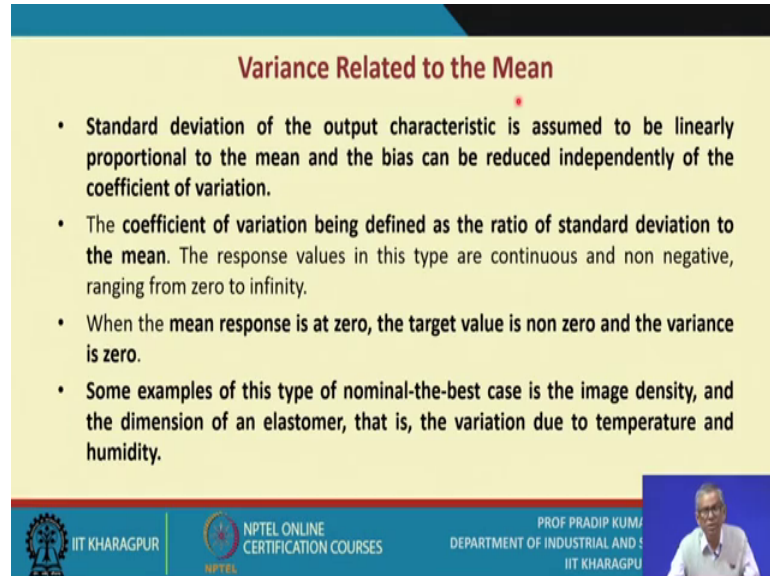
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So, there are the two different forms two different assumptions. So, the first assumption is we assume that the variance is independent of the mean. So, this is the first case; that means, you have collected the a set of data related to a particular quality characteristic and then you compute its mean and you compute the variance and then you assume that the variance is independent of the mean response values are continuous and can take on either positive or negative values and the target can be 0, typical examples or types are temperature controller errors around the nominal value and the offset voltage for a power supply.

So, you please go through all these examples there is several examples, I have provided. Now, in this situation the performance measure which reduces the variance of the output is given by ξ equals to minus log variance of y you y why did use the log scale because in that case you know very low value as well as very large value the variation is not known. So, all those values can be accommodated in the same scale an estimate of who is the performance statistics that is you have this, an estimate this is the general expression estimates ξ is z minus log s^2 ; that means, this is the variance of y . So, you get a sample and this is the sample variance.

So, you compute the sample variance like this we observe the larger the performance statistic z the smaller the variability ok.

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Variance Related to the Mean

- Standard deviation of the output characteristic is assumed to be linearly proportional to the mean and the bias can be reduced independently of the coefficient of variation.
- The coefficient of variation being defined as the ratio of standard deviation to the mean. The response values in this type are continuous and non negative, ranging from zero to infinity.
- When the mean response is at zero, the target value is non zero and the variance is zero.
- Some examples of this type of nominal-the-best case is the image density, and the dimension of an elastomer, that is, the variation due to temperature and humidity.

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When the variance is related to the main and this is this might happen in majority of the cases at least 60 to 75, 70 percent of cases, you find that this assumption holds. So, the standard deviation of the output characteristics is assumed to be linearly proportional to the mean and the bias can be reduced independently of the coefficient of variation, is it ok.

So, you know; what is coefficient of variation the coefficient of variation being defined as the ratio of the standard deviation to the mean, I presume that you know what is coefficient of variation already you have studied coefficient of variation response values in this type are continuous and non negative ranging from 0 to infinity when the mean response is at 0 the target value is nonzero and the variance is 0 some examples of this type of nominal the best case is the image density and the dimension of an elastomer that is the variation due to temperature and humidity.

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Variance Related to the Mean



- The method involves identifying the adjustment parameters and their ranges for which the s/N ratio is constant. **The S/N ratio defined by Taguchi is given by**

$$\zeta = \frac{\mu^2}{\sigma^2}$$

where

- μ = the mean of the output characteristic y
- σ = standard deviation of the output characteristic y
- σ^2 = variance of the output characteristic y
- y = output characteristic

- The performance statistic for this type is given by

$$z = 10 \log \left(\frac{\bar{y}^2}{s^2} \right)$$



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So, here how do you calculate the s N ratio that is mu square by sigma square is it and when ah? So, corresponding performance statistics; that means, when you deal with a sample; that means, this is z and 10 times log of y bar square divided by the sample you know the standard deviation square is it the sample variance sample variance.

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Smaller is Better



- The smaller-the-better s/N ratio is based on the smaller-the-better loss function. The loss function is given by

$$L(y) = \frac{A}{\Delta^2} [s^2 + \bar{y}^2] = k[s^2 + \bar{y}^2]$$

- The expected loss or average loss is given by

$$E(L(y)) = \frac{A}{\Delta^2} [\text{MSD}] = \frac{A}{\Delta^2} \left(\frac{\sum_{i=1}^n y_i^2}{n} \right)$$

- The response values or quality characteristics for this case are continuous and non negative and the desired value of the response is zero. The objective here is to minimize the mean and variance simultaneously.

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So, in the smaller the better case you have these expressions and correspondings expected value is also this one we have already derived so ok.

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Smaller is Better

- The performance characteristic is given by

$$z = -10 \log \left(\frac{\sum_{i=1}^n y_i^2}{n} \right)$$

or

$$z = -10 \log[s^2 + \bar{y}^2]$$

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So, for the smaller is better case the performance characteristics is given by that is z statistic that is minus 10 log like say sigma y i square divided by n is it ok. So, this is a simplified expression what small z.

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Larger is Better

- The larger-the-better case is the inverse of the smaller-the-better case. The response values are continuous non negative numbers ranging from 0 to infinity and the desired value of the response is infinity or the largest number possible.
- The loss function for this case is given by $L(y) = k \left(\frac{1}{y^2} \right)$
- The expected loss is given by $E[L(y)] = A\Delta^2 \left[\frac{1}{n} \sum_{i=1}^n \left(\frac{1}{y_i^2} \right) \right]$

or

$$= A\Delta^2[\text{MSD}]$$

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And for the larger is better case this is the expression for the expected loss this is the loss function this is the expected loss when you deal with n number of data points and this is the expressions ok, already we have derived and this is the expression for the performance characteristics given by z equals to minus 10 log this one, is it ok.

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Larger is Better

- The performance characteristic is given by
$$z = -10 \log \left(\frac{\sum_{i=1}^n \left(\frac{1}{y_i^2} \right)}{n} \right)$$
- The objective here is to find the parameter settings that will maximize the performance statistic.
- Two of such design factors of importance are (1) control factors – which affect mainly the S/N ratio, but not the mean, and (2) signal factors – which affect mainly the mean response of the performance characteristic.
- The control factors are first used to minimize the output variability and then the signal factors are used to shift the mean to the desired target value.
- Taguchi method advocates the economical maximization of the output performance characteristics while simultaneously minimizing the effect of output variability.

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So, objective here is to find the parameter settings that will maximize the performance statistics ok. So, two of such design factors of importance are one is the control factors which affect mainly the s N ratio, but not the mean that is most important. In fact, and the second one is the signal factors which affect mainly the mean response of the performance characteristics is it ok. So, this point is to be noted.

So, the control factors are first used to minimize the output variability. So, that is your first you know the choice and there could be some you know the engineering solution for this one and then the signal factors are used to shift the mean to the desired target value as we have been pointing out even in control charting we say that the first you concentrate on the variability.

So, if you can control the variability that is you know the necessary condition is satisfied next what you do and you try to you know the shift or the value of mean is it and always will find that the controlling variability is difficult in many cases, you must be aware of or say the changing this design you need to change the design altogether whereas, the shifting the mean may not be that problematic in majority of the cases.

Taguchi method advocates the economical maximization of the output performance characteristics while simultaneously minimizing the effect of output variability.

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Example

- In order to determine which injection molding machine has the best performance with respect to part shrinkage, a full factorial experiment based on four different machines and three noise factors (three different types of plastic materials) is designed and carried out. The control factors are the machines 1, 2, 3, and 4. The data in Table gives the noise in inches of the four machines under each of the three noise conditions. The customer tolerance is given as 0.03 in and the customer loss is set at \$35. Determine which machine gives the best performance.

Machine	Noise (inches)		
	Noise 1	Noise 2	Noise 3
1	0.009	0.012	0.014
2	0.005	0.008	0.006
3	0.008	0.018	0.014
4	0.020	0.018	0.008

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So, that is very important. So, here just I will give one example and the solution is also given. So, my suggestion is I will just read out this example and you follow the steps that how to how you are determining the performance the measure in order to that determine which injection molding machine has the best performance with respect to part shrinkage a full factorial experiment based on four different machines and three noise factors already you have gone through in the in the previous week, we have already discussed that the experimental design and one particular experimental design is a full factorial experiment.

So, we are referring to that full factorial experiment. So, we have four different machines and three noise factors. So, three different types of plastic materials is designed and carried out the control factors are the machines one two three and four the data in table gives the noise in inches of the four machines under each of the three noise conditions the three customer tolerance is given as 103 inch and the customer tolerance is given as 0.03 inch and the customer loss is set at 35 dollars determine which machine gives the best performance.

So, you have four machines corresponding noise values are given, is it and this table information.

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Solution

The most desirable condition is that the shrinkage should be zero. This example illustrates the smaller-the-better case and the use of the quality loss function and the s/N ratio to quantify the performance of several alternatives.

For smaller-the-better case, the s/N ratio is given by Eq

$$S/N = -10 \log[s^2 + \bar{y}^2]$$



where

$$\bar{y} = \sum_{i=1}^n \frac{1}{n} y_i$$

and

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (y_i - \bar{y})^2}$$

The average loss is given by Eq.

$$E[L(y)] = \frac{A}{\Delta^2} \left[\frac{1}{n} \sum_{i=1}^n y_i^2 \right]$$



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So, how do you get the solution? So, for the smaller is the better case the s/N ratio you compute with this formula, is it ok. So, what do you do with the given data you calculate y bar, is it and similarly you calculate the sample standard deviation by using this formula and ultimately the average loss you compute, is it ok.

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Solution

Machine	Noise 1	Noise 2	Noise 3	Mean	s	Loss	s/N
1	0.009	0.012	0.014	0.01166	3.56×10^{-6}	0.3150	38.6611
2	0.005	0.008	0.006	0.00633	8.89×10^{-6}	0.0972	43.9673
3	0.008	0.018	0.014	0.01333	1.42×10^{-6}	0.2489	37.5012
4	0.020	0.018	0.020	0.01933	2.20×10^{-7}	1.5555	34.2739

We observe from Table that machine 2 gives the lowest shrinkage and the best performance. The value of s/N ratio increases as the mean increases. Similarly, s/N ratio increases as the variability decreases.

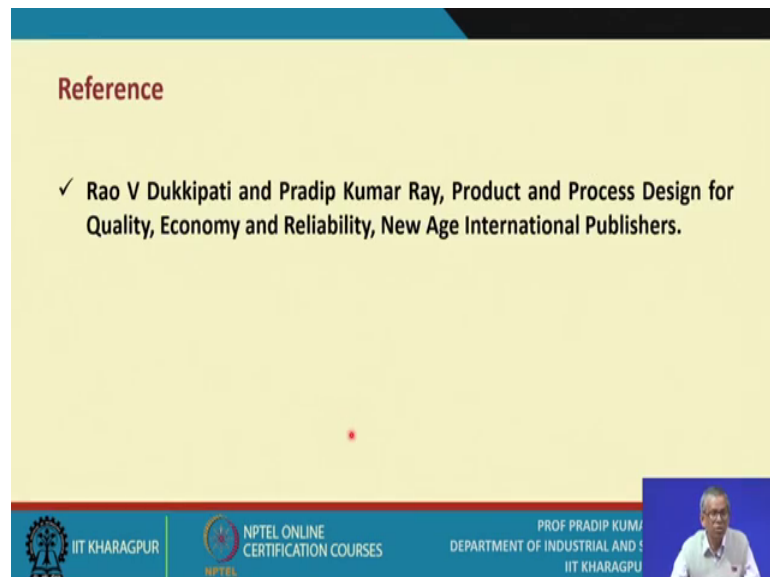
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So, this formula is already we have explained and ultimately you have against machine was noise one noise two noise three mean you have computed the standard deviation you

computed the loss by using the loss formula you have computed the loss and you have also computed the s N ratio, is it ok.

So, this process this computational steps you repeat for all other three machines machine two machine three and machine four and if this create a table we observed from the table that machine two gives the lowest shrinkage and the best performance, if you look at all these values its giving the best performance the value of s N ratio increases as the mean increases, this is your observation and similarly signal to noise ratio increases as the variability decreases.

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So, my suggestion is you refer to this particular textbook there are many examples in subsequent you know the assignments which will be providing; o, a number of the numerical examples will be referring to as a part of assignments, is it ok. So, here we conclude our discussions on robot design and Taguchi method we have covered almost all the important aspects and I suggest that you go through or the numerical examples and if you go through the numerical examples your understanding will be very very high, is it ok.

Thank you.