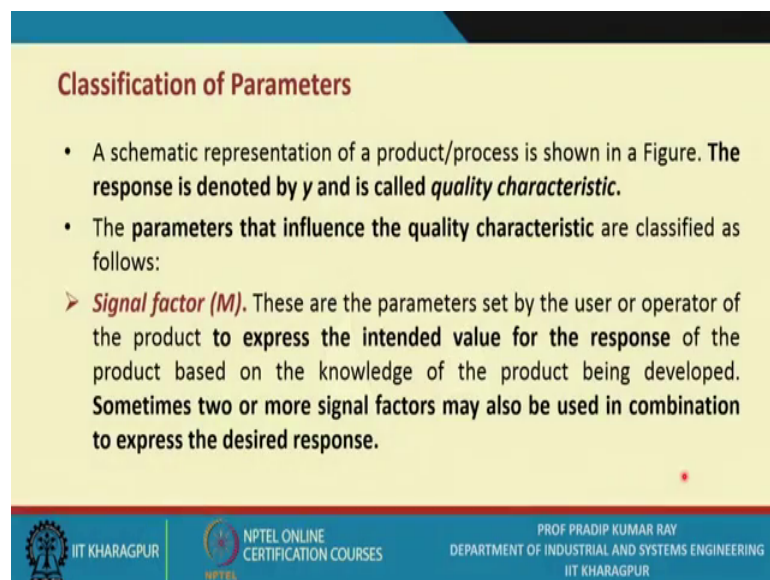


**Quality Design and Control**  
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**Indian Institute of Technology, Kharagpur**

**Lecture – 58**  
**Robust Design and Taguchi Method (Contd.)**

During this session again under robust design and Taguchi method, I will be referring to 1 particular aspect very important aspect that is classification of the parameters and the quality evaluations ok. So, these are the topics we are going to discuss in this session.

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**Classification of Parameters**

- A schematic representation of a product/process is shown in a Figure. **The response is denoted by  $y$  and is called *quality characteristic*.**
- The parameters that influence the quality characteristic are classified as follows:
  - **Signal factor ( $M$ ).** These are the parameters set by the user or operator of the product to express the intended value for the response of the product based on the knowledge of the product being developed. Sometimes two or more signal factors may also be used in combination to express the desired response.

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So, the question remains that you have probably understood by this by this time, that if you want to apply the Taguchi method to create a robust product. Now you should be knowing the performance of the product with respect to various possible conditions and when you start producing the product with respect to or in a process are using a number of processes.

Now, what are the factors what are the parameters affecting the output quality characteristic of the product you must be able to identify first and then you must suggest the settings of this all important or all the relevant parameters and the factors of the process. So, and there could be in a in a particular case there could be many types of parameters. So, the first thing you need to do that means, you need to identify the process parameters a relevant process parameters and you must be able to classify them.

So, how do you classify them a schematic representation of a product or a process is shown in a figure? So, you I will show you the figure and what you find in this figure, the response is denoted by  $y$  and is called the quality characteristic this is always we use the term quality characteristics.

The parameters that influence the quality characteristic are classified as follows and these parameters are sometimes they refer to the process parameters. So, the first the term we use that is the signal factor notation is  $m$  ok, the Taguchi has used the term called signal and he has proposed a performance measure called signal to noise. So, we will we will be elaborating on that later on in another lecture sessions.

So, you also must know what is this signal factor, these are the parameters said by the user or operator of the product to express the intended value for the response, of the product based on the knowledge of the product being developed. So, that is the signal factor; that means, you know this is the intended value of the response; that means, you have you could have you can identify several output quality characteristics, there also referred to as response variable or the responses simply responses output responses and the signal factor means you prescribed values for all those output quality characteristics and plain and simple terms.

Sometimes two or more signal factors may also be used in combination to express the desired response is it ok? So, depending on what type of product you are design.

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**Classification of Parameters**

- **Noise factors (x).** Certain parameters are not controllable by the designer and are known as *noise factors*. Parameters whose settings (also called *levels*) are difficult to control in the field or whose levels are expensive to control are also considered *noise factors*. The levels of the noise factors change from one unit to another, from one environment to another, and from time to time. The noise factors cause the response  $y$  to deviate from the target specified by the signal factor  $M$  and lead to quality loss.
- **Control factors (z).** These are parameters that can be specified by the designer. Control factor can take multiple values, called *levels*.

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Next one is the noise factors that this term we have been using and these noise factors can be of 2 types internal noise factor as usual external noise factors, we have already cited the several examples of these noise factors notation is  $x$ , certain parameters are not controllable by the designer and are known as noise factors; that means, these are sometimes referred to as uncontrollable noise factors.

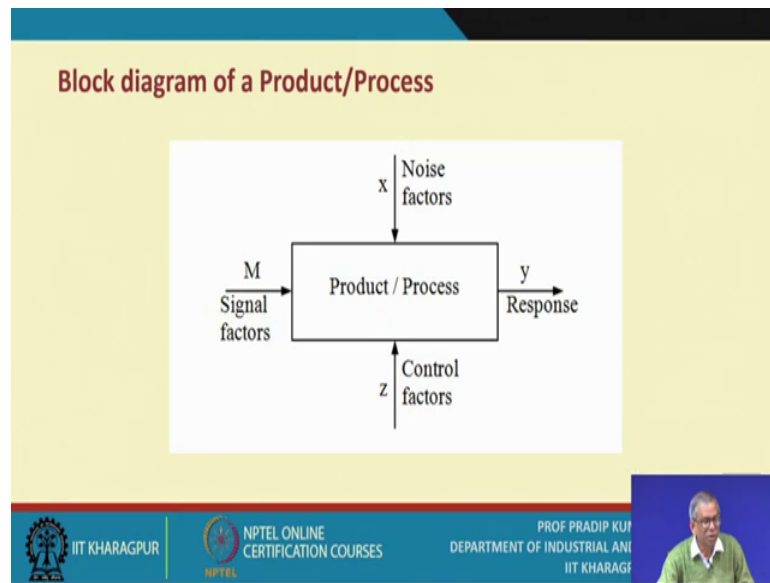
Parameters who settings also called the levels is it ok, there is another term we used when we discussed the design of experiments. Obviously, now we use the term called levels, so we are already aware of what is level. So, parameters who settings or levels are difficult to control in the field or whose levels are expensive to control are also considered noise factors like say you know ambient temperatures or say the humidity in a given context suppose they are difficult to control. So, what do you do you say that these are I am considering them as a noise factors.

The levels of the noise factors change from 1 unit to another from 1 environment to another and from time to time is it, so this is known there are several examples the noise factors caused the response  $y$  to deviate from the target specified by the single factor  $m$ ; that means,  $m$  value of  $m$  will be affected and lead to quality loss it is clear.

So, against each quality characteristics there is a signal you have to specify you as a designer you must be able to specify and then you check that; what are the noise factors that may be affecting the value of  $m$  or the value of the signal? So, at any point in time you know that these are the set of noise factors, like say you know like humidity the temperature the dust fumes etc, so there are many examples.

What are the control factors the notation is  $z$ , these are the parameters that can be specified by the designer like say in a typical say you know the machining operation, the turning operation is it. What do you find that the spindle speed is very important sign it you can control it, like say spindle speed  $y$  rpm. So, you say that I am a settings of rpm that is between 500 to 700; that means, any value between 500 and 700 including 500 including 700 any value is acceptable to you. So, these are basically the control factors or the feed rate for metal cutting operations right.

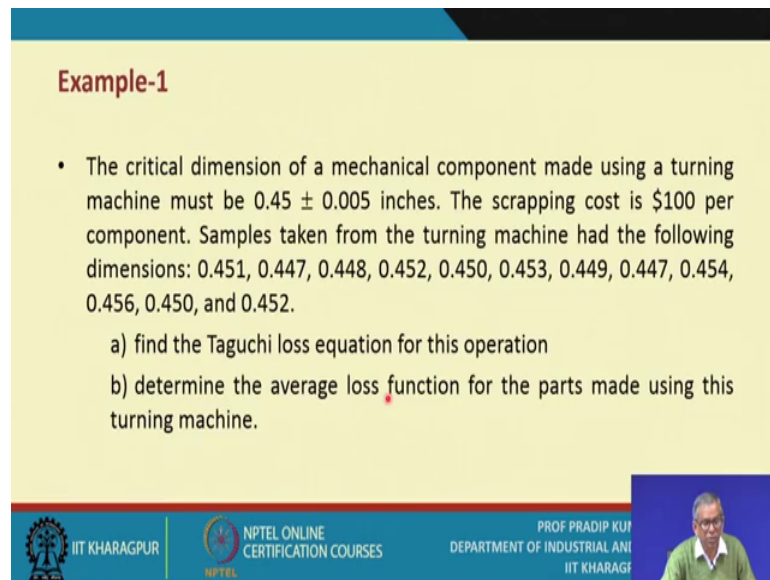
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So, there could be many control factors and control factors can take multiple values called levels is it right. So, this is the block diagram of a product of the process. So, you must have in your mind this particular block diagram. So, what is the block diagram; that means, there is a product or the process whenever you talk about the process the process can be of 1 step or process can be of 100 steps.

So, from this product or the process you expect the response the notation is  $Y$  this process or the product performance is affected by several kinds of noise factors, both internal as well as the external type. There are the signal factors signal factors is like this that is the  $M$  and these are the control factors that is  $Z$  is it clear. So, this is the block diagram of product or the process.

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**Example-1**

- The critical dimension of a mechanical component made using a turning machine must be  $0.45 \pm 0.005$  inches. The scrapping cost is \$100 per component. Samples taken from the turning machine had the following dimensions: 0.451, 0.447, 0.448, 0.452, 0.450, 0.453, 0.449, 0.447, 0.454, 0.456, 0.450, and 0.452.
  - find the Taguchi loss equation for this operation
  - determine the average loss function for the parts made using this turning machine.

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Now, let us take one numerical examples, so that you understand the whole concept more clearly. The critical dimension of a mechanical component made using a turning machine must be 0.45 plus minus 0.005 inches ok. So, this is just one example. So, this is the tolerance the scrapping cost is 100 dollars per component, samples taken from the turning machine had the following dimensions.

As I have been telling you that for assessing the loss or for measuring the loss what you need to do you have to collect data on  $y$  and so these data points; that means, the possible this value of  $y$  will vary it is expected to vary from 1 unit to another. So, these values you consider you have collected these data points 0.451 0.447 0.448 0.452 and so on is it ok. So, how many data points you have considered 1 2 3 4 5 6 7 8 9 10 11 12. So, 12 is a sample size so this data set you have collected.

Find the Taguchi loss equation for this operation is it that is the first question and the second question is determine the average loss function for the parts made using these turning machine.

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**Solution**

a)  $L(y) = k(y-m)^2 = \frac{A}{\Delta^2}(y-m)^2 = \frac{\$100}{(0.005)^2}(y-m)^2 = 4 \times 10^6 (y-m)^2$

b)  $Q = k[(\mu - m)^2 + \sigma^2]$




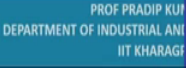

$$\mu = \bar{x} = \frac{0.451 + 0.447 + 0.448 + 0.452 + 0.450 + 0.453 + 0.449 + 0.447 + 0.454 + 0.456 + 0.450 + 0.452}{12} = \frac{5.409}{12} = 0.451$$

$$\sigma = \sqrt{\frac{1}{n-1} \sum (y_i - \mu)^2} = \sqrt{\frac{1}{12-1} (0.000090)} = \sqrt{\frac{0.000090}{11}} = 0.00286$$

$$Q = 4 \times 10^6 [(0.451 - 0.450)^2 + (0.00286)^2] = 4 \times 10^6 [(0.001)^2 + (0.00286)^2]$$

$$= 4 \times 10^6 [1 \times 10^{-6} + 8.180 \times 10^{-6}] = 36.72$$

Average quality loss = \$ 36.72.

So, how do you get the solution the first question is L y you need to calculate. So, it is a case I mean you are dealing with equality characteristics and this quality characteristic is nominal is the best type or the target is the best type.

So, you use this formula that is L y equals to k into y minus n square we have already referred to this particular formula and we know the expressions for small k that is the proportionality constant that is capital A divided by capital delta square is it ok. So, what is capital delta capital delta that is you know the limit of the deviation; that means, the deviation from m and the corresponding loss is capital A.

So, capital A the value of capital A is hundred already is value is given they have already estimated. So, this is an estimated value and this deviation is 0.005 is it because the tolerance is 0.45 plus minus 0.005, so this is an extreme point. So, if the value touches this one that means the deviation is plus delta or deviation is minus delta. So, so the actual value is 0.005 square y minus m; that means, this value is 4 into 10 to the power minus 6 into y minus square.

Now, how do you calculate this y minus m square; that means, you have considered all these twelve values is it and you consider there you know the sample mean. So, the population mean is approximated with the sample mean that is x bar. So, so the twelve values you have taken.

So, all these values you add and divided by 12 you get a value of 0.5 0.451 and then you calculate the standard deviation the notation is sigma is it ok, it is basically a sample standard deviation. So, it is sigma hat so root over 1 upon 1 minus 1 sigma y I minus mu whole square I equals to 1 to 12 is it ok. So, this is the total value you have computed the squared terms and ultimately you get a value of 0.00286.

So, you are required to compute these values independently it is very simple, in fact and then you get the value of the quality loss the average quality loss; that means, this is the value of say this proportionality constant and then you what you do; that means, 0.45 1 minus 0.450 that is for the deviation and the square plus this 1.00286 that is a square deviation.

So, you apply this formula ultimately it becomes 36.72 average quality loss is equal to 36.72 is it ok. So, the first part is understood and the second part is also you just follow the steps and you will get the value of the average quality law. So, this is a typical numerical problem at the very first stage.

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**Quality Evaluations**

- According to the **traditional concept of loss function** that as long as the **product's quality characteristic is within the specification limits or tolerances, there are no losses incurred.** The loss takes the form of a step function with a constant value A outside of the specification limits or tolerances.

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Now, quality when you try to evaluate the quality; obviously, you know right at this stage you know you must be aware of that this evaluation must be based on the quality loss, according to the traditional concept of loss function that as long as the products quality characteristic is within the specification limits.

Now these aspects do we understand very clearly or tolerances there are no losses incurred; that means, what we are saying that we are specifying the tolerance range and as for the traditional concept what we believe in that any value you get within this tolerance range is acceptable to you and as it is acceptable to you, as far the tolerance range specified there is no quality loss and any value within this range will have the same impact or the same loss incurred and you assume that these losses held at very minimum.

Now, this loss takes the form of a step function, with a constant value outside of the specification limit or the tolerances is it now these are going to elaborate with a figure later on.

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**Quality Evaluations**

- **Loss function  $L(y)$  makes no distinction between products**
  - a) whose quality characteristic is exactly on target at  $m$ , or
  - b) whose quality characteristic is just above lower specification limit (LSL), or
  - c) whose quality characteristic is just below the upper specification limit (USL).

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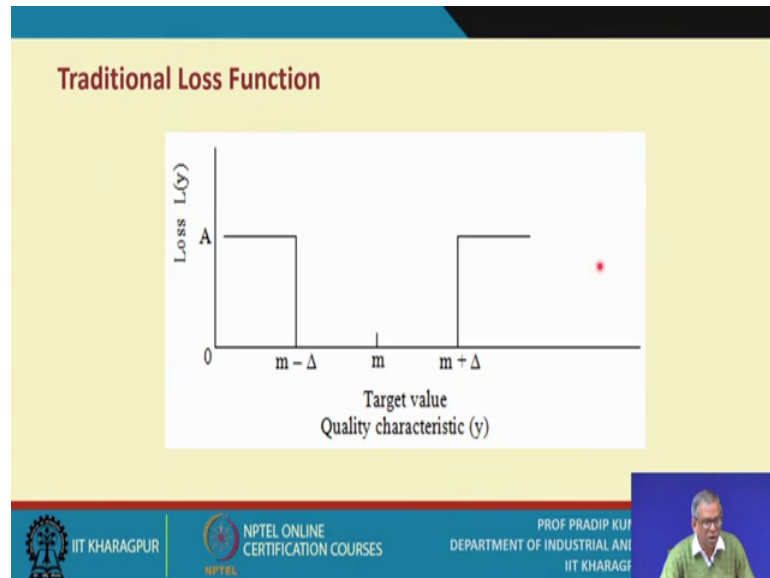
So, loss function  $L(y)$  makes no distinction between products whose quality characteristics is exactly on target a mean or whose quality characteristic is just above your specification limit or whose quality characteristics just below the upper specification limit is it ok. So, just 3 cases we have specified; that means, a value you get it is absolutely around or is very near to  $m$ .

Another value you get that is it is less than LSL, but is almost touching LSL lower specification limit on lower tolerance limit and the third value you get that is definitely it has not crossed USL, but it is very near to USL. Now as per the traditional concept all these 3 points are considered the same identical is it ok, but actually they are not, now



these aspect; that means, this traditional whenever you believe in the concept of say Taguchi loss function so; obviously, the traditional concept is not applicable.

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So, the traditional loss functions; that means, this is your target value this is the quality characteristics of  $y$ , whether the value you get over here the value of  $y$  or here. So, long or here very near to  $m$ . So, long there within there within this range we will assume that there is no loss or even if there is a loss; that means, this will be just minimum level. But as soon as it crosses this value or in sometimes know if it touches this one.

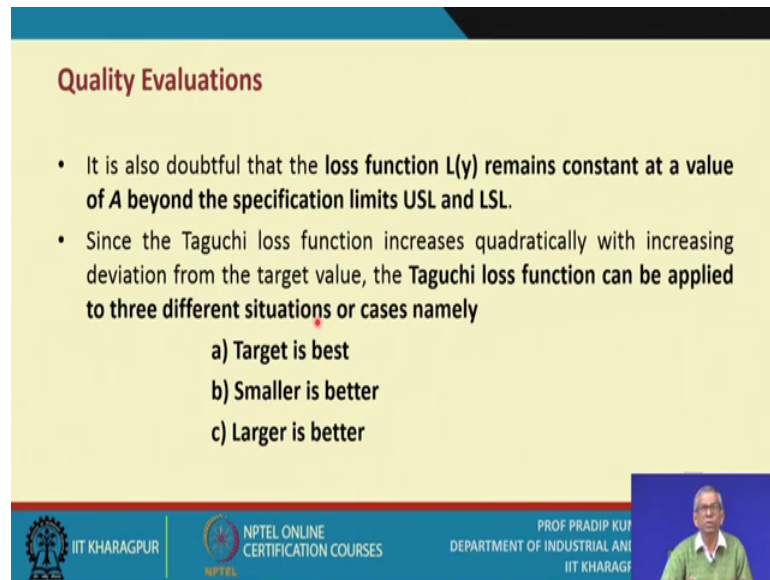
That means, this is the value almost is on say  $m$  plus delta the on the limiting value, you will find that that we say we say that there is a loss and this loss will continue and these loss amount is  $a$ . So, whether the deviation so whether the point is here or here we will assume that the loss remains same [laughter] ok. So, this is the traditional loss function now is it except to you as far Taguchi philosophy Taguchi says it is not acceptable.

Our goal is that you need to produce as far as possible or in as many units as possible, you need to produce the values around  $m$ . It is best if you can produce at  $m$ , but because of some reason supposing cannot reach the target all the time, at least the value you get it is deviation from the  $m$  should be as many as possible.

So, if you have and then you have the quality loss function as you deviate more from say the target value; obviously, we will we will be incurring more loss it is not that it remains

same till you reach, oh say you know the limiting value that is the upper specification limit or the lower specification and if we if you cross this one immediately the loss up to A; that means, is a loss is a just a function you assume ok.

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**Quality Evaluations**

- It is also doubtful that the loss function  $L(y)$  remains constant at a value of  $A$  beyond the specification limits USL and LSL.
- Since the Taguchi loss function increases quadratically with increasing deviation from the target value, the Taguchi loss function can be applied to three different situations or cases namely
  - a) Target is best
  - b) Smaller is better
  - c) Larger is better

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So, in many of the cases this sort of you know the step function approach is not valid.

So, how do you evaluate quality it is also doubtful that the loss function  $L(y)$  remains constant at a value of  $A$  beyond the specification limits USL and LSL. Since the Taguchi loss function increases quadratically with increasing deviation from the target value the Taguchi function can be applied to 3 different situations or cases namely target is based smaller is better larger is better is it.

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**Target is Best or The Nominal-the-Best Case**

- In the Target is the best or the nominal-the-best case the measured response,  $y$ , always has a specific target value  $m$ .
- The quality loss is undesirable on either side of the target.
- The quality loss increases in a quadratic manner as the quality characteristic deviates from the target value.
- Examples of quality characteristics of the target is best or the nominal-the-best category include product's component measurements such as diameter, thickness, and length; and a product's quality characteristic such as the properties of a fluid etc. The nominal-the-best loss function can be applied to just one-part or product or to the average loss associated with more than one unit.

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In the target is the best or the nominal is the best case the measured response  $y$  always has a specific target value  $m$ , then the quality loss is undesirable on either side of the target the quality loss increases in a quadratic manner, as the quality characteristic deviates from the target value that is why it is referred to as a quadratic loss function examples of quality characteristics.

Now, we are referring to the examples of quality characteristics of the target is the best or the nominal is the best category, product component measurements such as diameter thickness length, a products quality characteristics such as the properties of the fluid etc. A nominal the best loss function can be applied to just one part or a product or to the average loss associated with more than 1 units.

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**Target is Best or The Nominal-the-Best Case**

Taguchi's loss function is given by

$$L(y) = k(y-m)^2 \quad (1)$$

where  $k$  = a proportionality constant called *the quality loss coefficient*  
 $m$  = the target value of  $y$   
 $y$  = the value of the quality characteristic  
 $L(y)$  = loss in dollars when the quality characteristic is equal to  $y$

**Note that  $L(y) = 0$  when  $y = m$ .**

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So, this is equation so we have already gone through way revising this one. So, a proportionality constant called the quality loss coefficient that is  $k$ ,  $m$  is the target value  $y$  the value of the quality characteristics  $L$   $y$  is the loss in dollars when the quality characteristics is equal to  $y$  note that  $L$   $y$  equals to 0 when  $y$  equals to  $m$ .

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**Target is Best or The Nominal-the-Best Case**

- The functional tolerance range of the quality characteristic is  $(m-\Delta)$  and  $(m+\Delta)$  which represents the maximum permissible variation. If the consumer's average loss is  $A$  when the quality characteristic is at the limit of the functional tolerance, then  $A$  represents the customer cost for repair or replacement of the product.
- The proportionality constant  $k$  is obtained from

$$A = k\Delta^2 \quad (2)$$

or,

$$k = \frac{A}{\Delta^2} \quad (3)$$

- Then  $L(y)$  in Eq. (1) can be written as

$$L(y) = \frac{A}{\Delta^2} (y-m)^2 \quad (4)$$

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So, this is the way you calculate, so functional tolerance range of the quality characteristics is  $m$  minus  $\Delta$  and  $m$  plus  $\Delta$  and which represents the maximum permissible variation. If the consumers average loss is  $A$  when the quality characteristic is

at the at the limit of the functional tolerances ok, then a represents the customer cost for repair or replacement of the product. So, the proportionality constant is obtained by this is already we have computed and then  $L_y$  is this one ok. So,  $L_y$  is a divided by delta square into  $y$  minus  $m$  square.

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**Target is Best or The Nominal-the-Best Case**

- The **expected loss** is the mean loss over many instances of the product. The **expectation** is taken with respect to the distribution of the quality characteristic  $y$ . The **average quality loss** is obtained by considering an average of the measure of off-target performance  $(y-m)^2$  of the loss function.
- The deviation referred to is from the target value  $m$ .
- The **average loss** can be written as

$$\bar{L}(y) = k (\text{MSD}) \quad (5)$$

where MSD represents the mean square deviation of  $y$  where MSD is estimated

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So, so expected loss is the mean loss over many instances of the product, the expectation is taken from taken with respect to the distribution of the quality characteristics already I have mentioned, the average quality loss is obtained by considering an average of the measure of the off target performance  $y$  minus  $n$  square of the loss function. The deviation referred to is from the target value  $m$ . So, average loss function  $L_y$   $\bar{L}_y$  equals to  $k$  into MSD in case always remember this particular the formula ultimate, we have derived this formula what is a MSD, that means as mean square deviation.



So, we have expressions for our mean squared deviation for the 2 cases 1 is the nominal is the best or the target is the best and the smaller is better ok.

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**Target is Best or The Nominal-the-Best Case**

- $MSD = \frac{1}{n}[(y_1 - m)^2 + (y_2 - m)^2 + (y_3 - m)^2 + \dots + (y_n - m)^2] = \frac{1}{n} \sum_{i=1}^n (y_i - m)^2 = \frac{1}{n} \sum_{i=1}^n (y_i^2 - 2y_i m + m^2)$

Since the sample mean is  $\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$


$$MSD = \frac{1}{n} \sum_{i=1}^n (y_i^2 - 2\bar{y}m + m^2) = \frac{1}{n} \sum_{i=1}^n (y_i^2 - \bar{y}^2 + \bar{y}^2 - 2\bar{y}m + m^2) = \frac{1}{n} \sum_{i=1}^n (y_i - \bar{y})^2 + (\bar{y} - m)^2$$
$$= \frac{1}{n} \sum_{i=1}^n (y_i - \bar{y})^2 + \frac{1}{n} \sum_{i=1}^n (\bar{y} - m)^2$$


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So, this is the expressions we have so you have n number of the data points or the given quality characteristics. So, that each 1 is compared with in and the deviation is squared and all these squared deviations are added, so you get these expressions in fact.

So, sample mean is this already you have computed  $\frac{1}{n} \sum_{i=1}^n y_i$ , i equals to 1 to n number of data points. So, are you compute the MSDs. So, you apply this formula, so mean squared deviation is given by this expression is it. So, this is  $\bar{y} - m$ ; that means, this is a sample mean and how the sample mean is deviating is different from the target, this is the one part and the first part is individual values how the individual values are different from say are say the average value or the mean value that is why  $\bar{y}$  is it.

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**Target is Best or The Nominal-the-Best Case**

$$\text{MSD} = \frac{1}{n} \sum_{i=1}^n (y_i - \bar{y})^2 + \frac{1}{n} \sum_{i=1}^n (\bar{y} - m)^2$$

= (Average square deviation from the mean) + (Square of the deviation of mean from the target value)


= (Population standard deviation) + (Squared bias of  $y$ )

= (Variance of  $y$ ) + (Squared bias of  $y$ )


=  $\sigma^2 + (\bar{y} - m)^2$

Also for  $n > 30$ , the sample variance  $s^2 =$  population variance  $\sigma^2$

Hence, the loss function for  $n$ -units is


$$L(y) = k [s^2 + (\bar{y} - m)^2] \quad (6)$$


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So, if the target is the best of the nominal is the best case we consider then MSD is given by this. So, what is it that means the average squared deviation from the mean average squared deviations from the mean and the square of the deviation of the mean from the target value that is this one. So, this is population standard deviation and the squared bias of  $y$ . So, this is referred as the squared bias  $y$ , so this is variance of  $y$  and this is the squared bias of  $y$  ok. So, you have this expression; that means the sigma square plus  $\bar{y}$  minus  $m$  square is it ok.

Also for equals  $n$  is greater than 30 that means, for the last sample size the sample variance  $S$  square is approximately population variance sigma square. So, that is the many time we assume this, but if the sample size is a small sample size then you need to in many cases you need to use some other you know the techniques, whether it is parametric or nonparametric that you need to consider, but maybe there could be there will be a always some variations or you may go for some empirical study.

Hence the loss function for  $n$  units is this is it right. So, this is  $\mu$  minus  $m$  square, so  $k$  into  $S$  square plus  $\mu$  minus  $m$  square right so you refer to equation 6.

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**Target is Best or The Nominal-the-Best Case**

- To minimize the loss, the MSD must be minimized. This can be achieved by either minimizing  $s^2$  that is, reducing the variability that is causing the deviation of the data from  $m$  or by adjusting the average response to coincide with the target value  $m$ , that is

$$(\bar{y} - m)^2 = 0$$

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To minimize the loss MSD must be minimized that is your main target that is your main objective, this can be achieved by either minimizing  $S^2$  that is the sample variance, that is reducing the variability that is causing the deviation of the data from all by adjusting the average response to coincide with the target value  $m$ , that is  $y$  minus  $m$  square should be equals to 0 ok.

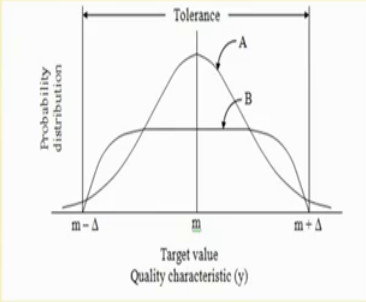
So; that means, if you achieve this condition that is the best conditions you have achieved it is clear. So, if you can do that, that means, what you are saying here; that means, all the averages values average values are falling on say  $m$ . So obviously, that is the best possible situation right and but prior to that you need to considered is it ok, the variance also; that means, the variability should be under control that is the first exercise you do and then when the variability is under control then you try to control the value of mean. So, exactly this thing we do when we use the control charting, if you refer to you know the or the variables control charting like when you refer to say  $\bar{x}$  chart as well as  $x$  bar chart.



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**Target is Best or The Nominal-the-Best Case**

- It should be noted here that the form of the distribution of the quality characteristics also has an influence on the expected loss. Two typical probability distributions of a quality characteristics are shown in Figure, where distribution A is a normal distribution and distribution B is a uniform distribution.



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So, first you try to control the variability of the process with the help of the  $r$  chart and when you are successful in reducing the variability of the process and then you try to control the process mean is it ok. Now here is 1 particular case where you will find that your performance is like this suppose 10000 units you are produced at your plant and you will find that the distribution is like this is your plant. So, suppose A plant B and plant A there it is also using the same another specification limits, but it is performance is like this plant A, now the question is which 1 is better from the quality loss point of view, so it is the question raised.

Now, how to say respond to this particular question, now it should be noted here that the form of the distribution of the quality characteristic also has an influence on the expected loss is it ok; that means, what you need to do if the distribution is quality characteristic distribution is like this, what is this you need to compute what is the expected loss and if suppose with respect to the same specification limits for the same quality characteristics, you get this sort of you know the behaviors or the performance.

Then what is quality loss 2 typical probability distributions of a quality characteristic are shown in figure, where distribution A is a normal distribution is a normal distribution and distribution B is a uniform distribution approximately uniform distribution.

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**Target is Best or The Nominal-the-Best Case**

- The expected loss for the product from distribution A will be less than that for distribution B, since most of the product is tightly clustered around the target value for distribution A compared to that of distribution B. The standard deviation of distribution is about 1/6 of the tolerance equals to 10.
- The expected quality losses of products with similar functions can be evaluated collectively irrespective of their sizes or specifications using the above approach given by Eq. (6). Suppose that components of n different types are supplied to an assembly operation.

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So, expected loss for the product from distribution a will be less than that for distribution B, since most of the product is tightly clustered around the target value or distribution A compared to that of the distribution B is it ok; that means, the majority maybe 10 plus minus say 20 percent of the products is it or maybe say maximum 30 percent of the products are their values will be lying within plus minus ten percent of the target value in a case B.

Whereas you know in the case a you may find that almost a 70 to 80 percent of the values, you will be getting within plus minus 10 percent deviation from the target value. So obviously, you know for say the situation A is much better than situation B.

So, the standard deviation of distribution is about 1 by 6th of the tolerance equals to 10, the expected quality losses of products with similar functions can be evaluated collectively irrespective of their sizes or specifications using the above approach given by equation 6. Suppose that components of n different types of supplied to an assembly operation, so expected quality loss will be like this is it ok.

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**Expected Quality Losses**

$$L(y) = k_1 [\bar{y}_1 - m_1]^2 + k_2 [\bar{y}_2 - m_2]^2 + \dots + k_n [\bar{y}_n - m_n]^2$$

where

$k_i = \frac{A_i}{\Delta_i^2}$  ( $i = 1, 2, n$ ) = quality loss coefficient of component  $i$

$2 \Delta_i$  ( $i = 1, 2, n$ ) = tolerance limit of component  $i$

$A_i$  ( $i = 1, 2, \dots, n$ ) = price (loss) per unit of component  $i$

$[\bar{y}_i - m_i]$  ( $i = 1, 2, n$ ) = deviation of the component  $i$  from the target value  $m_i$

$[\bar{y}_i - m_i]^2$  ( $i = 1, 2, n$ ) = the mean squared deviation of component  $i$  from the target value  $m_i$

$m_i$  ( $i = 1, 2, n$ ) = the target value of component  $i$

$n$  = number of measurable functional quality characteristics of the product

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So, please go through these expressions, in fact this is just another derivation when you deal with a number of components and these components one particular component is specified as  $i$ .

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**Expected Quality Losses**

**Losses due to deviations**

$$L(y) = \sum_{i=1}^n \frac{A_i}{\Delta_i^2} (\bar{y}_i - m_i)^2$$

$$L(y) = \sum_{i=1}^n k_i (\bar{y}_i - m_i)^2 \quad (7)$$

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So, losses due to deviations will be like this, so 1 case is this  $I$  equals to  $1 a_i \Delta_i^2$  and  $L y$  will be  $k_i$  into this right. So,  $k_i$  expressions will be this  $1$ , so you need to compute item wise actually you are doing it item wise right.

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**Expected Quality Losses**

- Eq. (7) gives an indication of the total quality of the product, since it considers all the measurable functional quality characteristics  $n$  of the product.
- For many products, parts, elements and components when a nominal size or characteristics is preferred, we require to use the nominal-the-best or target is best type.
- Bilateral tolerances (tolerances related to a basic dimensions given in two directions-plus and minus) are generally employed in this type of application.

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So, equation 7 gives an indication of the total quality of the product and for many products parts elements and components when a nominal size preferred we require to use, the nominal the best target or the best type bilateral tolerances. That means, tolerances related to the basic dimensions given in the two directions plus and minus as generally employed in this type of application.

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**Example-2**

- Assume the cost of the repair of a failed DVD player in the factory is \$50 per unit. Compare the losses caused by variations from the target value for two DVD players, one produced in factory A, which follows the normal distribution A and the other produced in a factory B, which follows the uniform distribution B as described earlier in Figure. The tolerance interval ranges from  $m - \Delta$  to  $m + \Delta$ , where  $\Delta = 20$ .

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So, this example is another example we are given ok, so please go through these examples.

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**Solution**

The constant  $k$  is computed

$$k = \frac{A}{\Delta^2} = \frac{50}{(20)^2} = 0.125$$

The expected losses caused by deviations in the production of the DVD player in case A is obtained

$$L(y) = \left(\frac{A}{\Delta^2}\right)(y - m)^2 = k(y - m)^2$$


=  $k$ (mean squared deviation from target value  $m$ )

$$= k \left(\frac{40}{6}\right)^2 = \$5.56/\text{unit}$$


The loss caused by deviation in the production of the DVD player in case B is

$$L = k(y - m)^2 = k \left(\frac{40}{\sqrt{12}}\right)^2 = 0.125 \left(\frac{40}{\sqrt{12}}\right)^2 = \$16.67/\text{unit}$$

The above results indicate that the losses caused by deviation for DVD players produced in case B are three times those for the same type of DVD players produced in case A.




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Again we will refer to these examples right say the cost of repair; it is another examples of how to compute the quality loss function. So, I suggest that we just go through these examples or the details are details steps are given over here is it ok. So, the basics we have understood other the issues related to the Taguchi method we will be discussing in the subsequent lecture sessions.