Quality Design and Control
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Lecture – 57
Robust Design and Taguchi Method (Contd.)

During this session on robust design and Taguchi method, I am going to discuss the topic called a quality loss function, as well as the variations of quadratic loss function and average quality loss.

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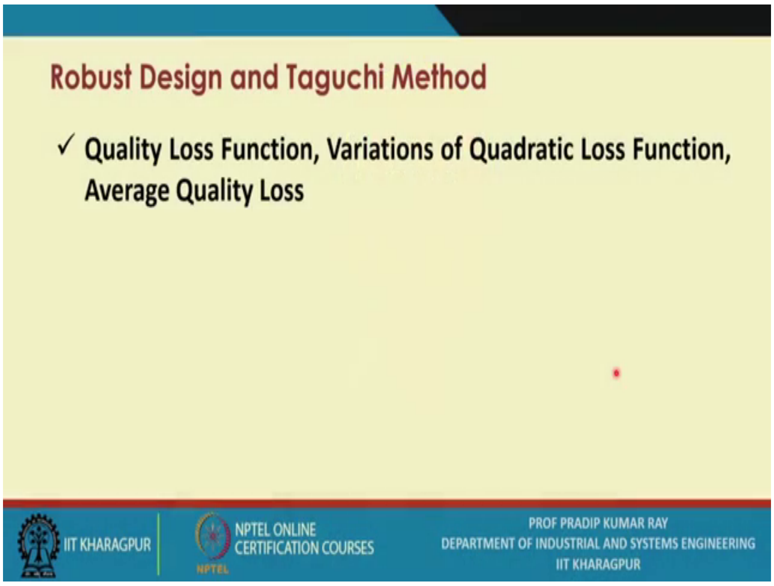

So, we will be discussing the 3 important topics related to robust design and Taguchi method.

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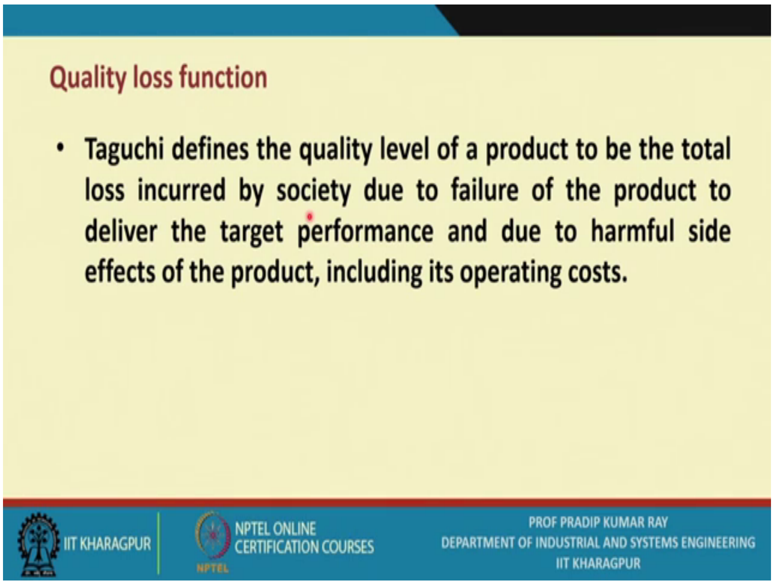

So, now we are already aware of that what is the definition of quality as given by Taguchi, so Taguchi defines the quality level of a product to be the total loss incurred by society societal loss, due to failure of the product to deliver the target performance and due to harmful side effects of the product including it is operating costs; so, that means operating cost means, that means operating processes; that means, there cannot be any product which when produced does not produce any harmful side effects ok.

So, what we need to do that is it is unavoidable, so you design the product as well as the process in such a way that this harmful side effect comes to a minimum level, that is the minimum as well as you know the failure of the product failure occurrences come down to a minimum level and if it does not it does not fail or even if it fails it fails really then what do you expect that the performance product is excellent and if the performance product is excellent as well as there is a minimum harmful side effect we say the quality is excellent; that means, what do you conclude that under these conditions the societal loss is minimum. So, what is the relationship between the quality of the product and the societal loss?

That means, if the societal loss that you compute if it is found to be very very high, you say that the quality of the product is very very poor and similarly if you compute or if the value of the societal loss there could be different formulations there could be under different assumptions and then you conclude that the societal loss is very very minimum is that and so it is a bare minimum. So, immediately you conclude that the quality of the product that you have designed is excellent is it clear. So, you need to do certain calculations and this calculation is related to a particular term called quality loss function is it. So, how do you define the quality loss function under Taguchi philosophy?

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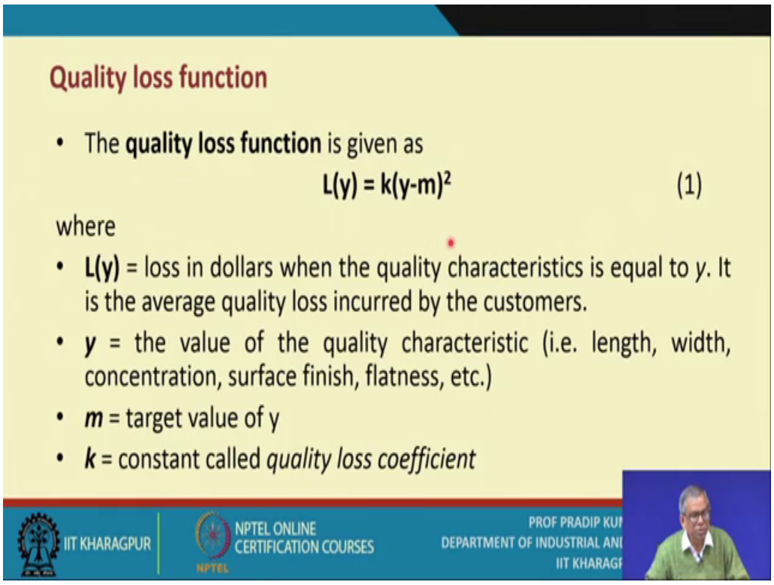

So, here we have 1 formulation let me explain this formulation, the quality loss function is given as Ly is equals to k times y minus m whole square is it ok, but let me first explain these terms which use what is Ly Ly is a loss in dollars or in monetary terms. When the quality characteristics is equal to y it is the average quality loss incurred by the customers; that means, in a continuing running say continuously running production system given a particular product or given a particular component on a particular process what do you find that you start producing a particular component in large numbers. So, there will be many units and obviously, against a particular unit you measure the value of the quality characteristic.

So, that value is why and; obviously, is this is y is a random variable, now what is so that is why we say that y is the value of the quality characteristic which is a random variable. For example, this could be the length this could be the width concentration surface finish flatness etc etc; that means, these are the variables and we have already defined what is a variable quality characteristics; that means, they are measurable and the exact the value will be known. Now what is m m is the target value of y and what is k? k is the constant called quality loss coefficient it is fine, now how do you explain it that means, what we say there is the loss is proportional is proportional too.

The square of the deviation of y from the target value, so this is the square of the deviation so obviously it is proportional, so the when you write so when you consider the proportionality constant, the equation becomes Ly equals to k times that is the proportionality constant into y minus m square. So, this is the Taguchi assumption and it is a quadratic loss function it is a quadratic loss function. What actually is has assumed; that means, he has assumed quadratic loss function, here he collected data from several say the products and ultimately we will find in majority of the cases; that means, this loss function is may be considered to be a quadratic loss function is it clear. So, this is the formulation of. So, is the loss or the societal loss or the quality loss, so quality loss function.

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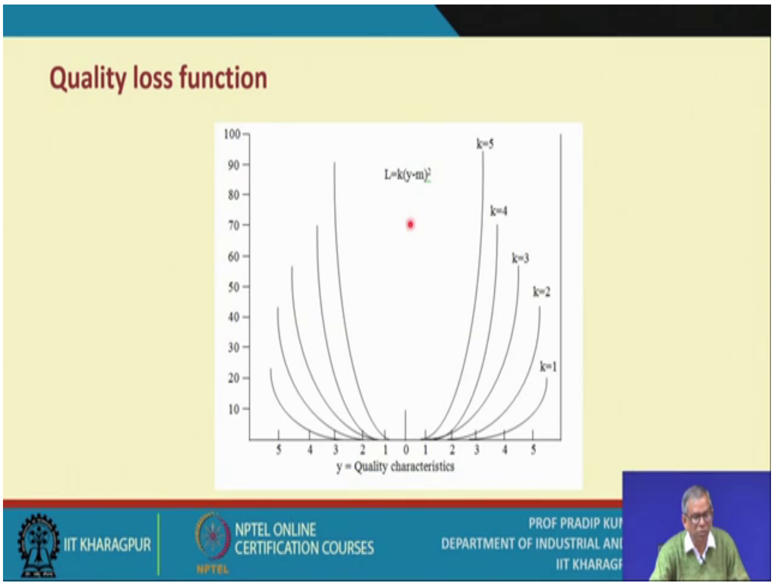

Now, how the values of say the quality loss function that is this 1 is related to the value of k is it ok. So, as we find that if the value of k increases is it ok. So, from k equals to 1 to k equals to 5 what could be the shape of the, so shape of this curve the shape of this curve is it quadratic loss function will be very very stiff; that means, if suppose this is your you know the target, so if there is a you know the deviation from the target is more; that means, the loss you incur will be very very high is it and this is a non-linear function definitely, so this is and similarly suppose on the other side, suppose you have so the deviations on the other side is it ok.

So, all these values are possible what do you find again you are incurring a loss because this is a square term. So, whether it is a positive deviation or the negative deviation the impact remains same is it ok. So, this is just a pictorial representation to know how the value of k is affecting the corresponding loss.

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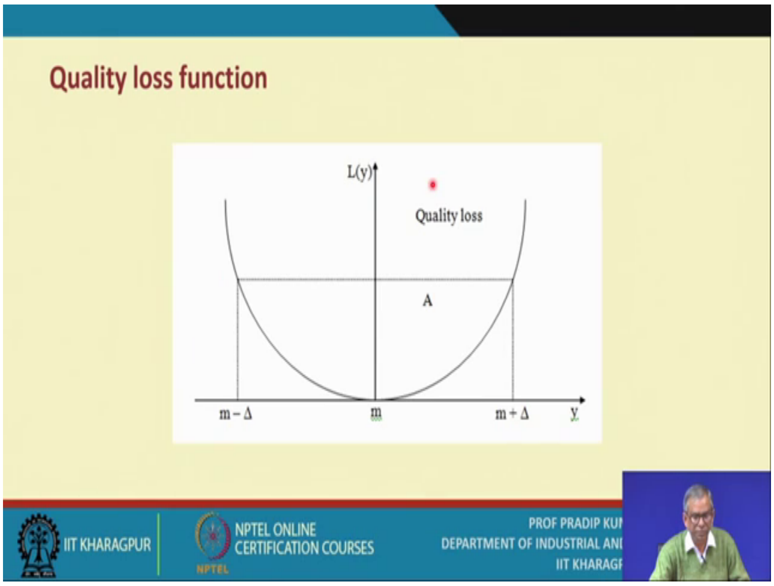

Now when whenever you say that this is the target value and the suppose the target from the target there is a deviation of plus delta on 1 side and the deviation of minus delta on the other side now this is a quadratic loss function. So, what do you expect that this is the deviation is plus delta from m then the corresponding loss is A capital A is it ok. So, capital a refers to the quality loss and the if the deviation is minus delta; that means, the value you get that is m minus delta and again you know is a quality loss function the effect remains same.

So, the corresponding loss we have incur that is a right and what the Taguchi has pointed out very clearly that whenever you try to compute it is loss, it is always better that you compute this loss in monetary terms. So, that you know everybody is interpretation oh of the loss may be almost similar is it ok. So, always there should be a money many attached to this loss function; so a convenient way to determine k.

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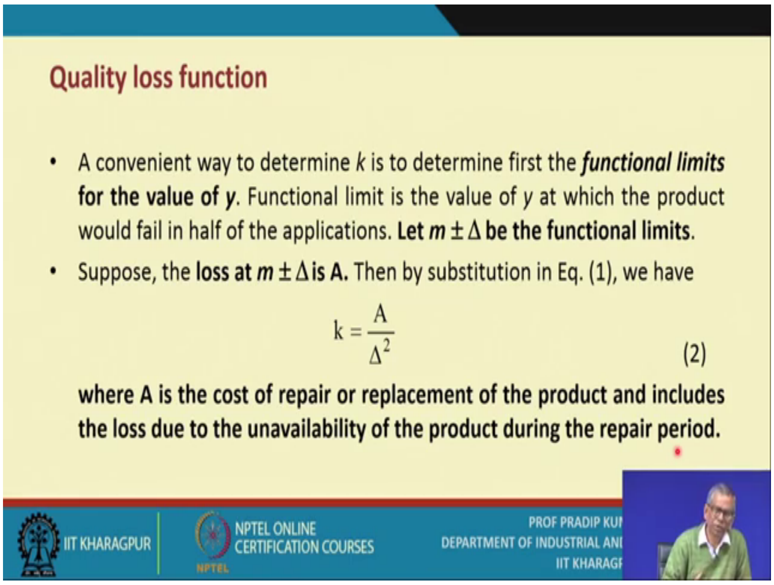

How do you determine the proportionality constant that is k. So, you determine k first to determine k what you need to do first you need to determine the functional limits for the value of y is it ok; that means, there will be range of values of y which is acceptable to you it is clear it is not that any value of y is acceptable to you; that means, you know what is it is impact you say yes with this range of value you can expect performance from the product or the performance of the component is it. So, this range is technically feasible functional limit is the value of y at which the product would fail in half of the applications, it is clear. So, let m plus plus minus delta be the functional limits as we have already pointed out.

Suppose a loss at m plus minus delta is capital A, I have already mentioned then by substitution in equation 1 we have small k equals to capital A divided by capital delta square; where A is the cost of repair or replacement of the product and includes the loss due to the unavailability of the product during the repair period is it. So, you must be able to so there could be you know the certain points to be remembered while you know with respect to a particular process, while you compute this quality loss like say you know the repair cost you must know, replacement cost you must know, similarly in the that is the cause due to unavailability.

So, these cost elements, so the corresponding data you must have is it at least there will be good estimates of all these cost elements. So, related to the kinds of loss that means, what you need to do; that means, you need to define in explicit terms in a given condition what are the loss related components, what are the loss related elements is it, so these are just an examples. So, when the capital is known and the corresponding of the deviation is also known that is the functional limit; obviously, you can have an expression for small k or the proportionality constant.

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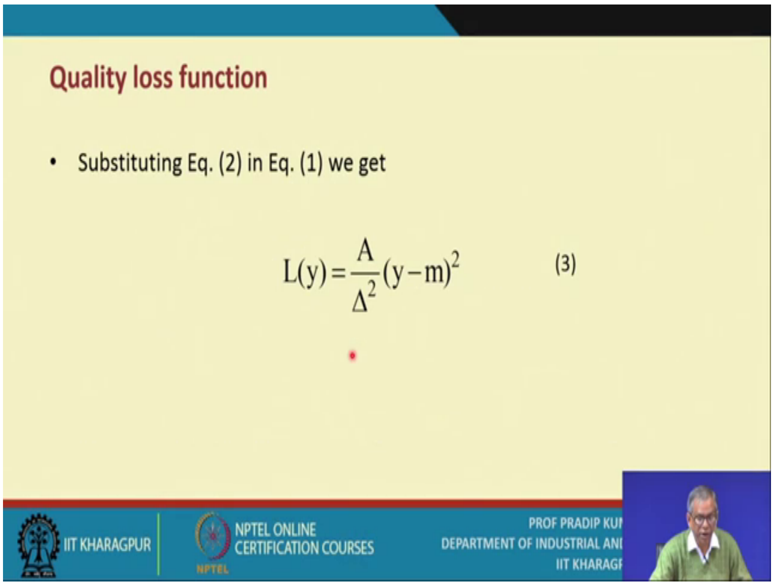

So, obviously you know, but the loss expression for loss becomes capital a divided by capital delta square into y minus m square is clear. So, this is the quality loss function.

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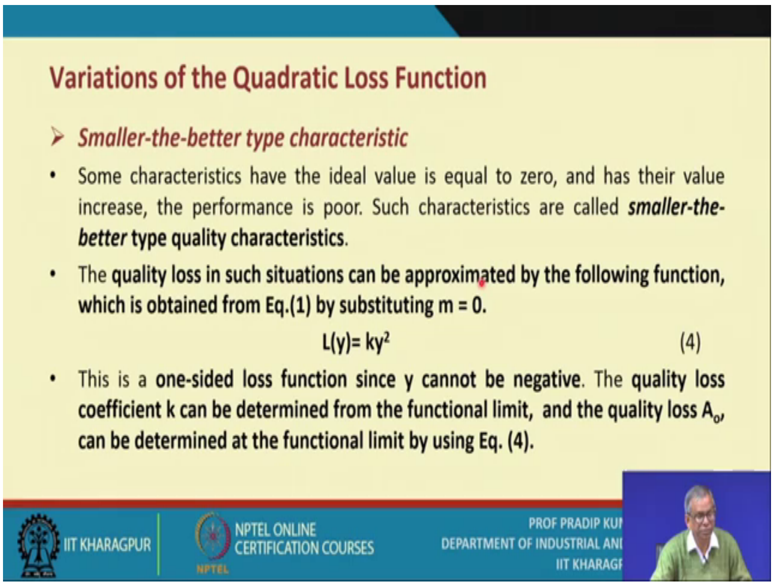

Now, I have already mentioned that as for as quality characteristic is concerned, this quality characteristic can be classified under 3 3 categories, what are they what are those first 1 is the target is best or the nominal is the best that is first type; what is the second 1 second 1 is the smaller the better type characteristics and what is the third 1? Third 1 is the larger the better type characteristics.

Now, in the previous case when we propose say that the loss function the quadratic loss functions, what we have assumed that the quality characteristic chosen is of the first type; that means, it is of say the nominal is the best type or the target is the best type is it clear. Now let us talk about the smaller the better type characteristic quality characteristics.

Some characteristics have the ideal value equal to 0 and has their value and as their value increases the performance is poor is it ok; that means, if it becomes non 0 and the positive then and as these value increases, that means the deviation from 0 what happens; that means, the loss we have incur is increases ok. So, the loss is will be very high if the deviation from 0 is large, such characteristics as called smaller the better type quality characteristic is it right; there are many examples I have already mentioned those examples. So, please refer to those examples of this type of quality characteristic, the quality loss in such situations can be approximated by the following function which is obtained from equation 1.

Again you refer to equation 1 and here your target value is 0 is it equal to 0 we have already mentioned. So, hence this is Ly is equals to k into y square this is just an application of the previous case when m becomes 0, this is a 1 sided loss function since y cannot be negative it is clear. The quality loss coefficient k or the proportionality constant can be determined from the functional limit, like we have done in the previous cases and the quality loss a 0 can be determined at the functional limit by using equation 4 is it clear; that means, the functional limit value is known and corresponding loss also you have estimated. So, these 2 values are known; obviously, you can get the value of k ok.

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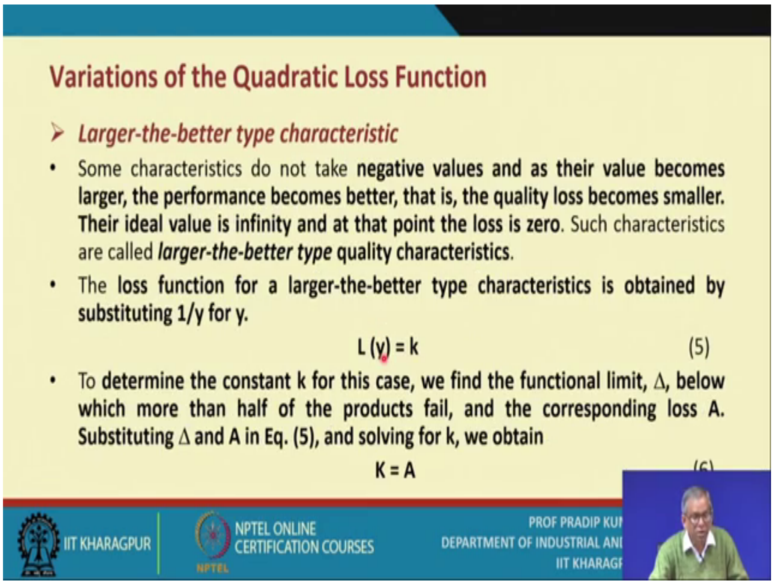

So, this is the second case, in the third case that means, when you refer to when you refer to the larger the better type characteristics some characteristics do not take negative values and as their value becomes larger the performance becomes better is it ok, like say you know the tensile strength of a component ok. So, you specify the minimum value, 3000 kg, now what do you what you may feel that any value greater than 3000kg is greater.

Whatever may be you know the square area so that is the sectional area that is the quality loss becomes smaller, their ideal value is infinity and at that point the loss is 0 is it ok; that means, it is the larger the better type right such characteristics are called larger the beta type quality characteristics it is clear; like say you know in a typical service organization their service quality should be as high as possible is it ok. So, theoretically speaking larger the beta type characteristics; that means, the value which you have in your mind of say you know the quality service could be it may go up to the infinite level theoretically speaking, so there are many such examples.

The loss function for a larger the beta type characteristic is obtained by substituting 1 upon y for y is it. So, that is why this is ly equals to k into 1 upon y square is it ok. So, k upon 1 upon y square to determine the constant k for this case, we find the functional limit delta below which more than half of the products fail and the corresponding loss is capital a same notations we have used is it; substituting delta and a in equation 5 that equation 5 is Ly equals to k into 1 upon y square and solving for k we get k equals to delta a into delta square capital A into delta square.

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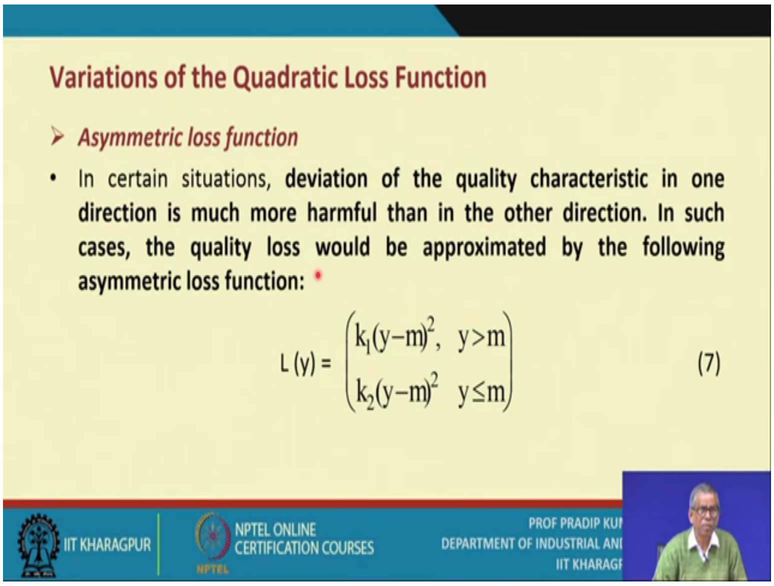

Now, there is another case like say, we are we are referring to so the quality loss function. Now there is 1 particular case that also I would like to highlight that is called asymmetric loss function, in the previous case that means, the first case when we refer to what we have assumed that even if there is a deviation on the positive side or is the deviation on the negative side like m plus minus capital delta, you will find that the impact is same and then the impact in terms of the loss that is the capital A we have assumed that means, it is a symmetric case; that means, the effect of positive deviation is same as that of the negative deviation. Now this is a symmetric case now there could be cases where you know it may not be symmetric it may be a symmetric. So, whenever you refer to that kind of quality characteristic, then we should be we will be referring to the asymmetric loss function. So, how do you define these asymmetric loss function.

So, what we have mentioned that in certain situations, deviation of the quality characteristic in 1 direction is much more harmful than in the other direction is it; much more harmful means in 1 direction with the same amount of deviation as a magnitude is same, you will find that the loss is very very high. Whereas, on the other side even if the deviation magnitude is remains same the loss may not be that high is it. So, in the other direction either in 1 direction positive or negative, so in that case the opposite will be negative or positive in such cases the quality loss, would be approximated by the following asymmetric loss function that means, instead of having 1 loss function that is k into y minus m square.

Now this 1 loss function is split into 2 is it ok? So, the first 1 is when y the actual value is greater than the target value, you have k 1 that is a proportionality constant into y minus m square. So, this formula we will be using and when y is less than equals to m is it ok, then you will you will you will be using that the second formula that is k 2 that is the proportionality constant into y minus m square is it.

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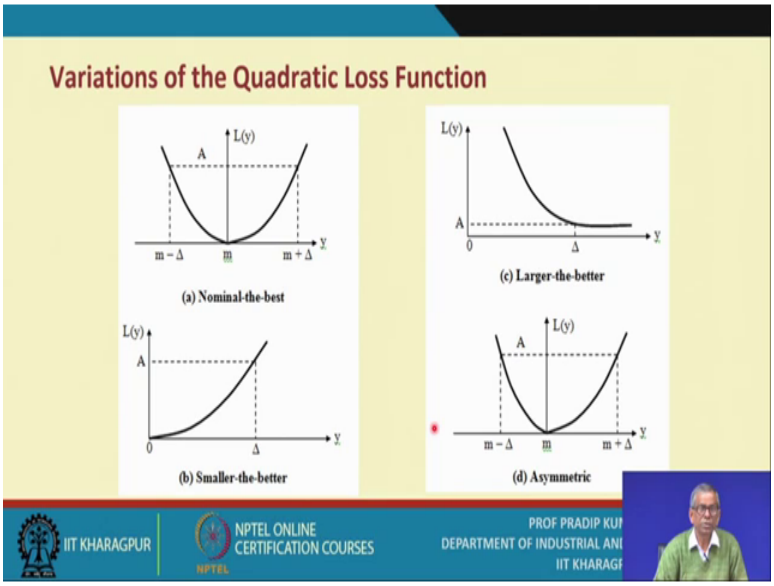

So, this the 4 cases we have explained with the help of so the figures and look at these figures we will get an idea about the variations of the quadratic loss function.

So, here the nominal is the best case, we have already explained m is the target value or the nominal value nominal value means the desired value and if the deviation m plus plus delta on this side and m minus delta on the other side, you will find it is a symmetric loss function; that means, the loss you have incur that is capital a remains same, then the smaller the better; that means, the target value will be 0; that means, this is shifted to the left is it and you reach here 0 is it.

So, if there is a deviation of delta you incur a loss of capital A, we are always sayings that better this y axis must be in I means the unit of measurement of a or the loss in general must be the monetary unit, third case is the larger is the better larger the better quality characteristics.

So, if it is very very high that is you will find that it tends to infinity, it is a it is a it is tends to the loss will be 0 loss will be 0 and if there is a deviation from the other side that is delta you will find you are incurring a loss of a is it ok. So, correspondingly you know the difference from 0, you calculate and you find that the loss is capital A. So, this is a symmetric case now you move to the asymmetric case asymmetric loss function. So, here m is the target, now if there is a negative deviation we will find that the impact is very high; that means, loss is very high whereas, if there is a positive deviation like say impact is not that high, but still there is substantial loss due to a deviation of plus delta from m is it ok.

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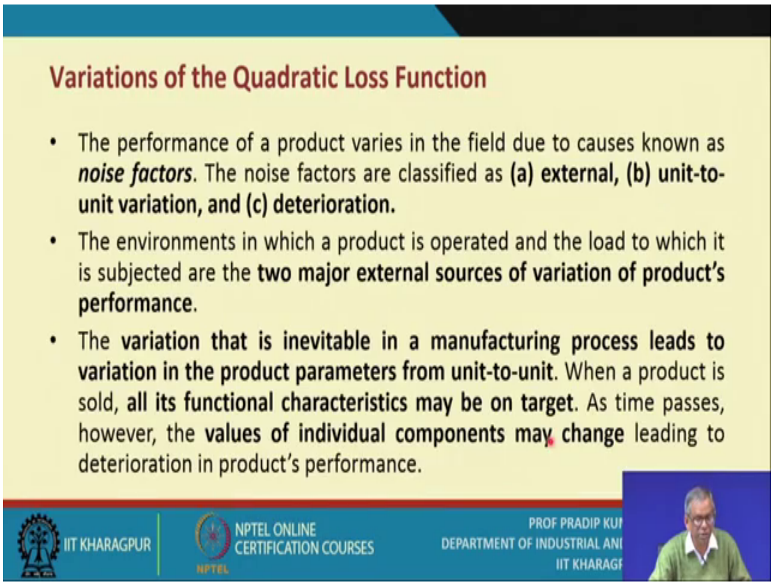

So, these are the 4 cases we come across, now the performance of a product varies in the field due to causes known as noise factors, the noise factors are classified as external unit to unit variation and deterioration is it. We have already mentioned this all these 3 factors the environments in which a product is operated and the load to which it is subjected are the 2 major external sources of variations of products performance, just we are recapitulating. In fact, and always when you know you know the Taguchi method when you work for say robust design make sure that the impact or the effect of external as well as the internal noise factors is held at minimum.

The variation that is inevitable in a manufacturing process, leads to variation in the product parameters from unit to unit these examples already we have given; when a product is sold all it is functional characteristics may be on target as time passes; however, the values of the individual components may change leading to deterioration in products performance.

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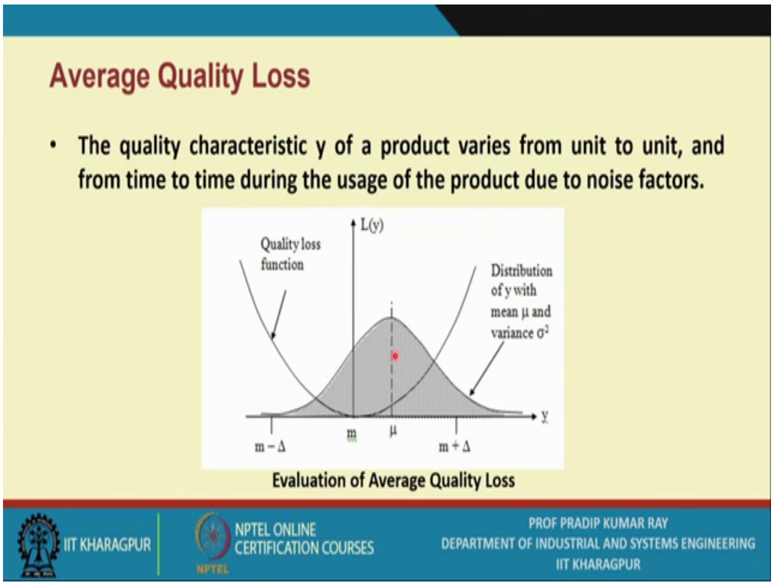

So, how do you compute the average quality loss. So, for computing average quality loss what you need to know, you need to collect data ok; that means, you need to collect data on y and when you collect data on y and you know what is m and you know the what type of quality characteristics you are dealing with. So, you will be able to calculate the average or expected quality loss.

So, the quality characteristics y of a product varies from unit to unit and from time to time during the usage of the product due to noise factors. So, this is basically at the variation, the variation this is basically a distribution of the values which you get distribution of y with mean mu and variance sigma square. So, this is mu and the several values you have collected say 1000 such data points you have collected 1000 say the values of say y you have collected and what is assume that there is a distribution; that means, there is no impact of say assignable causes. So, the system is free of assignable causes or the system is only affected by the external noise factors ok. So, this is the data you must be able to collect.

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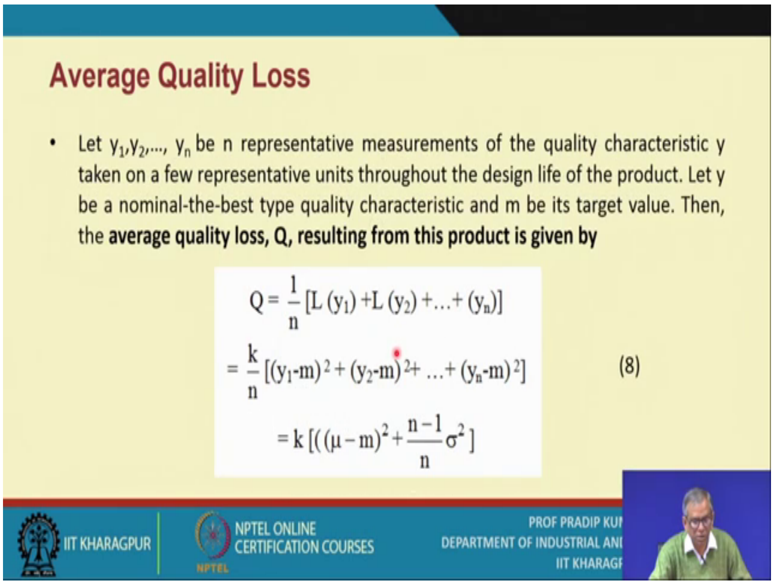

So, how do you calculate the average quality loss; that means, this is so average quality loss; that means, n number of data points you collect? So, y1 y2 yn for the quality characteristics y taken on a few representative units throughout the design life of the product, let y be a nominal the best type quality characteristics and m is the target value then the average quality loss is computed with this formula is it.

That means, 1 upon n the expected value a loss of y1 loss of y2 loss of yn and then you apply the formula that is the k is a proportionality constant and this is the square terms and when you have 2 expressions you have k into mu minus m square plus n minus 1 by n into sigma square unbiased estimate you calculated and mu and sigma square at the mean and the variance of y respectively, I have already mentioned. So, ultimately mu the expression is this and the sigma square is this 1 an unbiased estimate.

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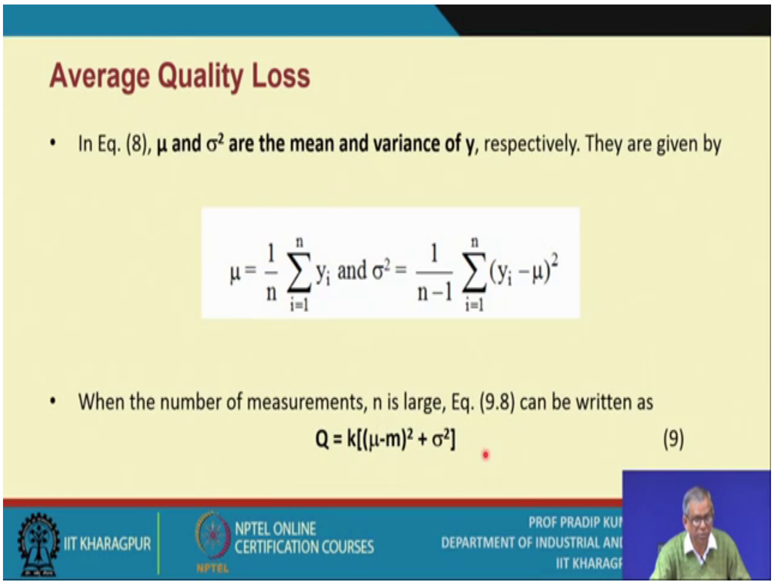

When the number of measurements n is large; that means, this is the q the quality loss is k into mu minus m square plus sigma square, so this is transformation.

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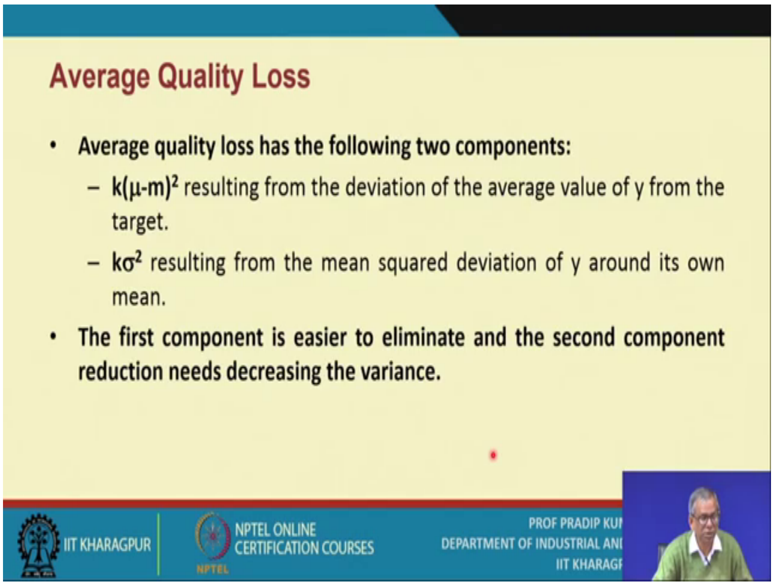

So, average quality loss has the following 2 components, the first component is this 1 k into mu minus m square resulting from the deviation of the average value of y from the target and k into sigma square, resulting from the mean squared deviation of y around it is own mean is it. So, the first component is easier to eliminate and the second component reduction needs decreasing the variance. So, that is the challenge in fact, like

I have already mentioned, that any exercise and quality is essentially an exercise on the exercise on say the variability or the variance.

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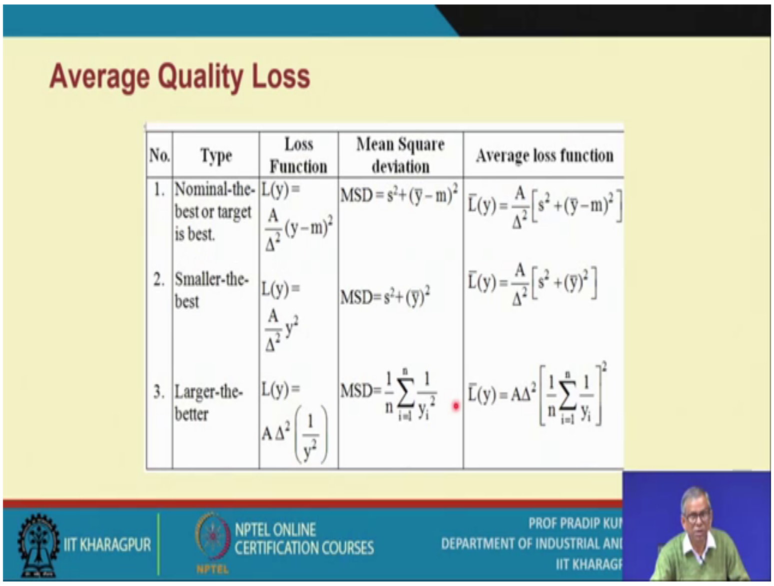

No.	Type	Loss Function	Mean Square deviation	Average loss function
1.	Nominal-the-best or target is best.	$L(y) = \frac{A}{\Delta^2}(y-m)^2$	$MSD = s^2+(\bar{y}-m)^2$	$\bar{L}(y) = \frac{A}{\Delta^2}\left[s^2+(\bar{y}-m)^2\right]$
2.	Smaller-the-best	$L(y) = \frac{A}{\Delta^2}y^2$	$MSD = s^2+(\bar{y})^2$	$\bar{L}(y) = \frac{A}{\Delta^2}\left[s^2+(\bar{y})^2\right]$
3.	Larger-the-better	$L(y) = A\Delta^2\left(\frac{1}{y^2}\right)$	$MSD = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}$	$\bar{L}(y) = A\Delta^2\left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i}\right]^2$

So, these are the again if you refer to this particular table. So, in this table what you have what you have what you have noticed that, depending on the type of quality characteristics the loss function is shown is it and the mean square deviation is also shown mean square deviation MSD. The sample variance plus y dash minus m this is the mean and this is the target value, an average loss function the expression for the average loss functions are given and similarly the smaller is the best larger the better, so all these cases we have shown.

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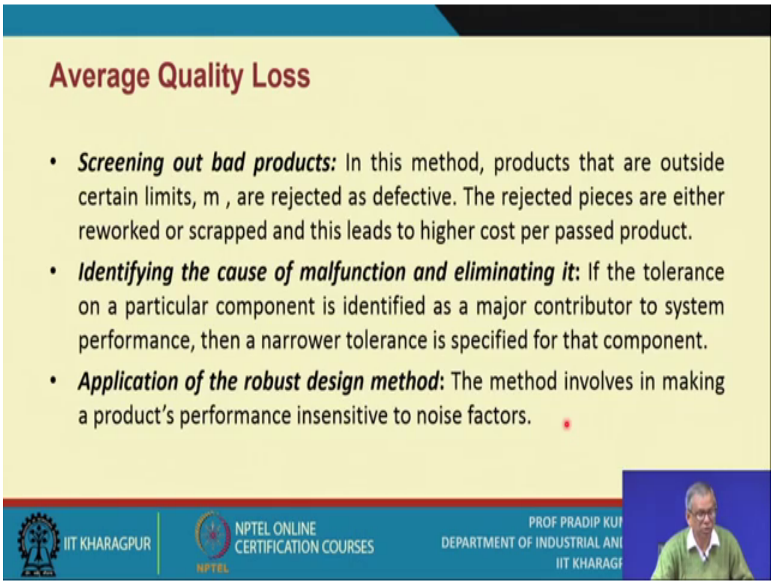

So, average quality loss. So, what you need to do you have to screen out the bad products, identifying the cause of malfunction and eliminating it, an application of the robust design method. So, the method involves in making a products performance in sensitive to noise factors is it. So, these are the actions you have to take and then what you need to do.

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That means once you are aware of these actions or the corrective measures then possibly you know you both the variability is reduced, as well as the target value you can achieve and then what you find that for the majority of the points or for majority of the values you get and the quality loss whatever the relevant formula you use with which you use this quality loss. The quality loss in monetary terms which held that the minimum level and you say that the objective of using Taguchi method is fulfilled, but this is just 1 step there are many other steps you have to follow. And those the other aspects we will be discussing in the subsequent lecture sessions.