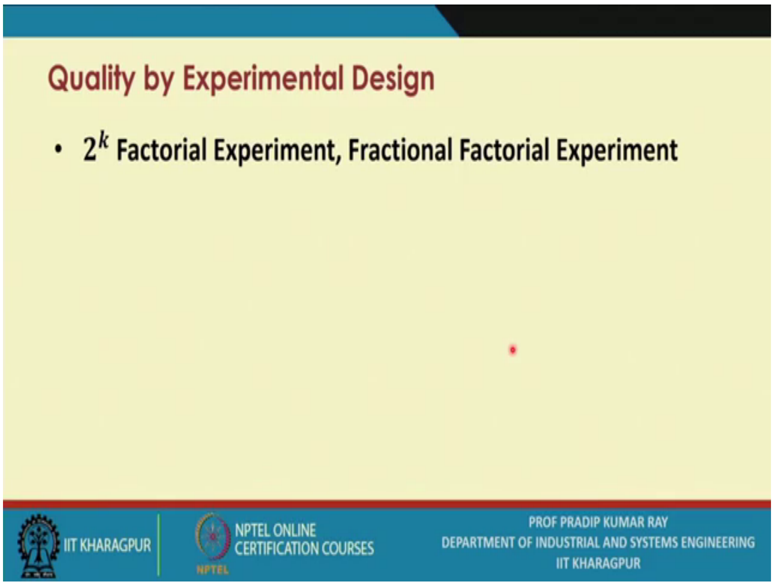
Quality Design and Control
Prof. Pradip Kumar Ray
Department of Industrial And Systems Engineering
Indian Institute of Technology, Kharagpur

Lecture – 55
Quality by Experimental Design (Contd.)

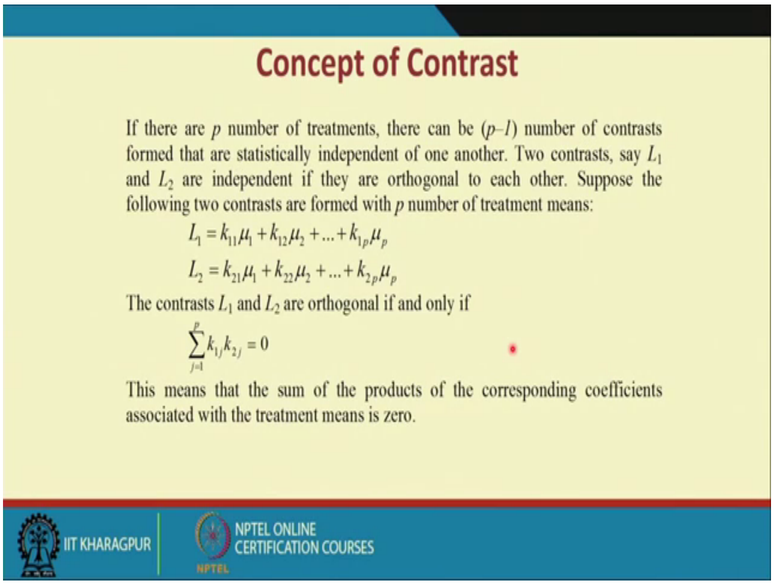
(Refer Slide Time: 00:19)



Ah during this session on their quality by experimental design, I will be discussing that 2 to the power k factorial experiment, we have already discussed the factorial experiment in the previous lecture session and 2 to the power k factorial experiment, if you know in detail you know, you will have the different kinds of insights about the design of experiment and this is widely used particularly 2 to the power k factorial experiment and along with this factorial experiment, you also must be aware of; you must also know, what is the fractional factorial experiment this experiment is also widely used.

So, these are the two topics, I am going to discuss during this session.

(Refer Slide Time: 01:10)



Now, you know before elaborate on this 2 to the power k factorial experiment as well as the fractional factorial experiments, let me highlight or let me explain one important concept that is refer to as the concept of contrast, is it like say you know, what we say that later on when we studied the Taguchi method, you will find that the Taguchi method is based on the design of experiments and a particular designed he has proposed that is orthogonal array an orthogonal array is essentially a or one type of fractional factorial experiment and we come across the term called say orthogonal say the contrast and all.

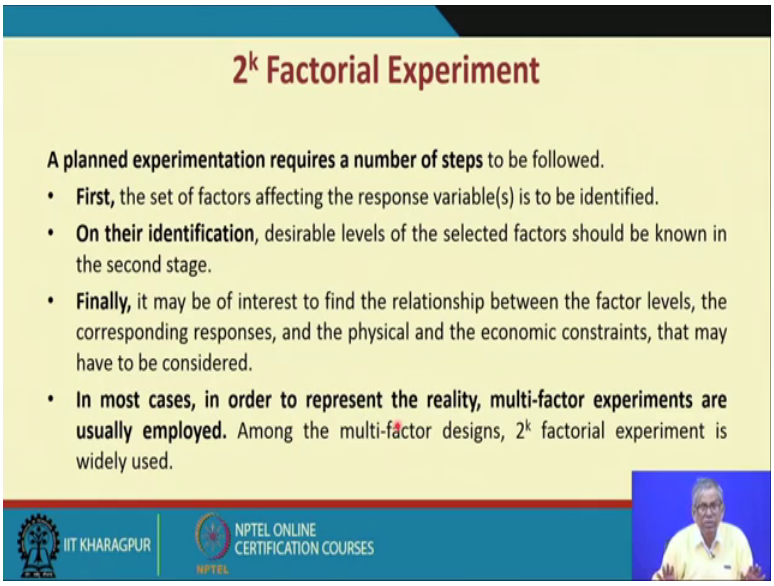
So, the basic what is a contrast you also must know. So, if; so, first let me explain what is a contrast and then I will start explaining what is 2 to the power k factorial experiment, if there are p number of treatments, you know what is a treatment there can be p minus 1 number of contrast forms that are statistically independent of one another, is it ok. So, out of p number of treatments you can form p minus 1 number of contrasts two contrast; say L 1 and L 2 are independent if they are orthogonal to each other.

Now, what is this orthogonality that we must first explain we must define? Suppose the following two contrasts are formed with p number of treatment means. So, this could be a linear combination like L 1 is one contrast with this k 1 1 mu 1, k 1 2 mu 2 plus and then you go up to the k th level or the p th level; that is k 1 p into mu p, is it and similarly the next contrast you form that is you have k 2 1 mu 1 plus k 2 2 mu 2 plus up to the p th level that is k 2 p into mu p. So, L 1 and L 2; these two contrasts are formed.

The contrast L 1 and L 2 are orthogonal if and only if that is you know you start multiplying these two coefficient values is it like k 1 one into k 2 1, k 1 2, k 2 2, like this, if you proceed and ultimately for all the p treatments you consider; that means, summation j equals to 1 to p kij into k 2 j must be 0, is it ok. So, that is the condition for orthogonality, this means that the sum of the products of the corresponding coefficients these are the coefficients you have corresponding coefficients associated with the treatment means these are the treatment means mu 1 mu 2 mu p is 0, is it ok.

So, we know what is a contrast there could be several contrast and between two contrast there could be orthogonality and we have defined what do you mean by say the orthogonal contrasts ok.

(Refer Slide Time: 05:01)



So, this is the basic this knowledge the basic understanding of contrast. Now let me explain; what is this 2 to the power k factorial experiment. Now here 2 to the power k means essentially, at these first instance, you must know that the k means the k number of factors that you are considering in experimentations, then what is 2? 2 means for each of all these k number of factors you just consider two levels, is it ok.

So, it is essentially is a k number of factors and each factor at two levels supposing each factor you consider k number of factors and each factor is at a 3 levels then what could be how do you ah. So, specify the factorial experiment, it should be 3 to the power k factorial experiment is clear. Now a planned experimentation requires a number of steps to be followed is it we have a some of the important points we have already explained the like when we started our discussions on design of experiments.

So, how to carry out the experiment what is the purpose of experimentation all these points you have elaborated. So, while you conduct an experiment certain steps you have to follow. So, what are those steps first the set of factors affecting the response variable ah. So, all the response variables; that means, either you consider one important the response variable or there could be cases where you need to consider you know a number of response variables simultaneously.

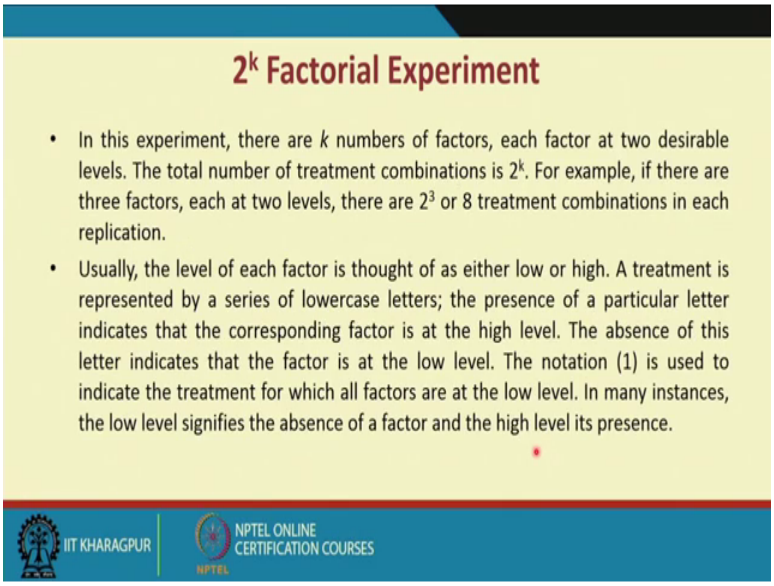
So, first I must know that what are the factors that are affecting the response variables and these factors you must first identify that is the first step the next step is on their identification desirable levels of the selected factors should be known in the second stage; that means, now you consider each factor identified and you specify their possible levels. Now, these levels can be defined subjectively or it can be defined objectively, is it and it is better that in order to simplify the experimentation as the initial stage what do you try to do you start with less number of levels ok.

So, the two is the minimum number of levels then in most of the experiments. So, you start with two number of levels for each factor finally, it may be of interest to find the relationship between the factor levels the corresponding responses and the physical and the economic constants that may have to be considered. So, many a time, what do you need to do; that means, even if you know that not only the two number of factors or 3 number of factors. So, affecting why there could be many other factors might be affecting you know say the response variable.

But the initial stage you know you go for a simplification; that means, you go for experimentations and while you carry out the experiment you should consider those factors which are which are considered to be significant initially. So, you need to the stealing them; that means, even if there are say n number of factors ah. So, you consider just a 2 or 3 factors out of n and there is a basis for selecting a few number of factors out of many ok. So, in most cases in order to represents the reality the multi factor experiments are usually employed among the multi factor designs 2 to the power k factorial experiment is widely used; that means, here as such we are not restricting the number of say the factors.

But here in the 2 to the power k factorial experiment we are restricting the number of levels, is it ok.

(Refer Slide Time: 09:30)



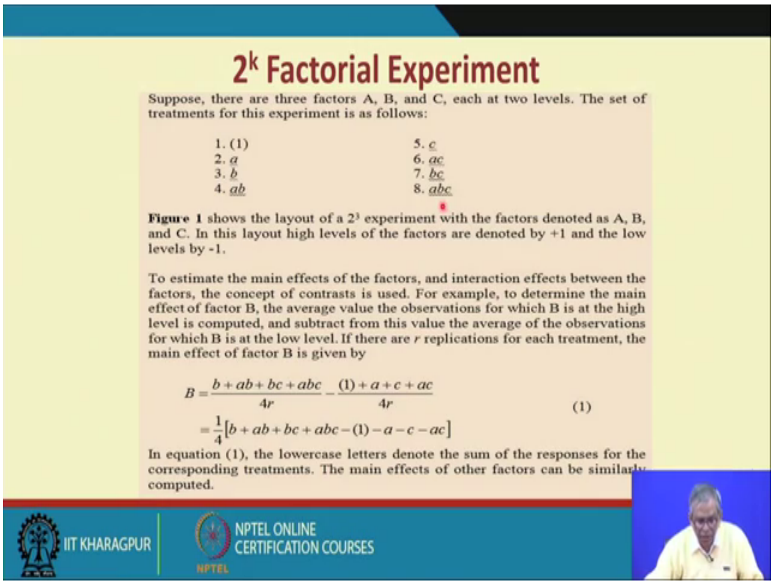
So, so, in this 2 to the power k factorial experiment there are k number of factors each factor at two desirable levels the total number of treatment combinations is 2 to the power k, for example, if there are 3 factors each at two levels, there are 2 to the power 3 or 8 treatment combinations in each replication, I think it is clear, usually the level of each factor is thought of as either low or high like in the previous examples in the earlier the discussion earlier lecture sessions, we have we have assumed that the factor a is having low level or high level.

So, these the levels could be either low or high a treatment is represented by a series of lowercase letters like say if the factor is a, we will be using the small a or corresponding to capital B factor or capital C factors, we will be using the small b or the small c. So, the lowercase letters will be using the presence of a particular letter indicates that the corresponding factor is at the high level; that means, suppose you consider 3 factors a b and c, if you use say one combination a small a, it means that the factor a is at the high level whereas, the factor b as well as the factor c there at the low level ok.

So, the absence of this letter indicates that the factor is at the low level. So, this point it just between mined while, you write the notations for a particular the combinations notation one is used to indicate the treatment for which all factors are at the low level suppose you are considering the 3 factors a, b and c and all these factors they are having two levels low level and high level. So, the low level is you know low level, suppose all these 3 factors there at low levels.

So, where do you specify you specify at one; that means, ah; that means, one in parentheses it means that I am considering one treatment combinations where all the 3 factors are at low level in many instances the low level signifies the absence of a factor and the high level its presence, is it ok.

(Refer Slide Time: 12:04)



Now, here is an example suppose there are 3 factors a b and c, capital A, capital B and capital C each at two levels the set of treatments for these experiment is as follows; that means, this is one means within parentheses it means that all the 3 factors are at low levels a, a means factor a is at the high level whereas, factor b and c at low level.

If it is b; that means, factor b is at a high level whereas, the factor a and factor c these are at a low levels ab means basically high level b at a high level and the ah. So, the this is one say the interaction, then you consider that the c and ac means another combination bc is another interaction and abc is another combination are you getting my points; that means, ab; that means, the c is at c is at the low level, is it ok.

If you say c, c means a and b all at the low level. So, at the c is the at high levels. So, these are the 8 combinations you have in each the replication. So, I am I am sure that you have understood the meaning of a these notations ok. Now if you refer to this figure one we will I will show you figure one figure one shows the layout of a 2 to the power 3 experiments; that means, there are 3 factors each factor at two levels with the factors denoted as a, b and c in this layout high levels of the factors are denoted by plus one and the low levels by minus 1, is it to estimate the main effects of the factors and the interaction effects between the factors.

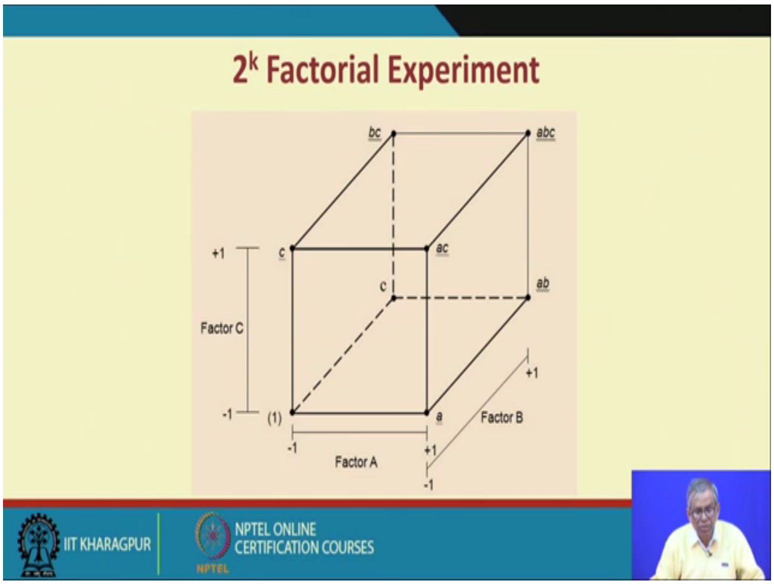
These are the two important considerations you have the concept of contrast is used is clear. So, already we have defined what is a contrast for example, to determine the main effect of factor b the average values of the observations for which b is at the high level is computed so; that means, out of all the combinations you select those combinations where the factor b is at the high level and subtract from this value the average value average of the observations for which b at the low level I think it is I made very very clear. So, if you want to estimate the main effect of factor b. So, this is the rule you follow and the same rule you have to follow when you compute the say the main effect of a and other factors of c like this.

So, if there are r replications for each treatment ok, the main effect of factor b is given by this one is it ok; that means, here what you find that this is these are the combinations where b is at the high level. So, b at the high level here also b at the high level b is at the high level here also b at the high level divided by there are these average value you consider. So, divided by 4 and how many such values you have; that means, there are r replications; that means, you have r such r everywhere you have r number of values of b r number of values of ab and so on.

So, that is why it is divided by 4 by 4 into r and similarly, here you will find that the b is at the low level b at the low level b the way we have defined the notation when you write c; that means, b at the low level when you write ac the interaction.

So, you find that b at the low level divided by 4 into r. So, this is the ultimate the simplified expressions for capital B in equation 1; that means, this equation the lower case letters denote the sum of their responses for the corresponding treatments the main effects of each factors can be similarly computed; that means, you have just one calculation for the factor b ah, but the similar sort of the computations you have for the other two factors namely factor a and factor c.

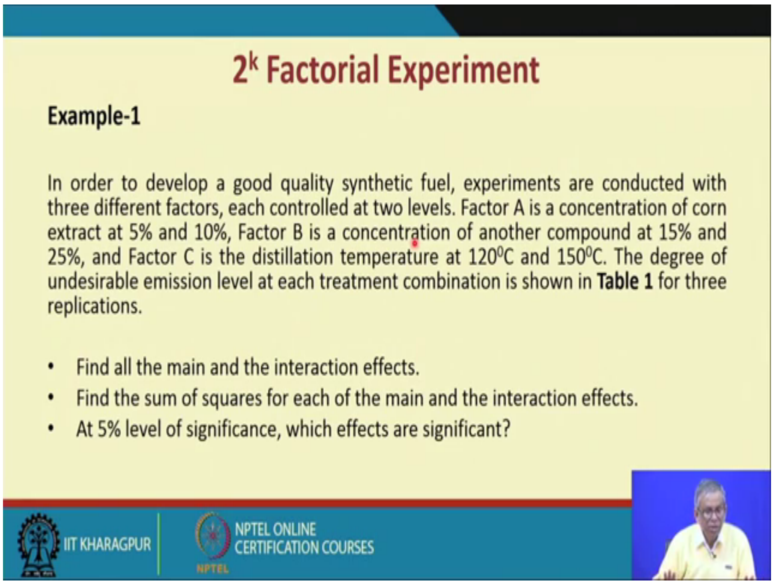
(Refer Slide Time: 16:37)



In this case; so, this is the pictorial representation of say the 2 to the power k factorial experiments.

So, if you look at this figure what you find you can identify the factor; factor a, factor b and the factor c and then what are the combinations. So, how many treatment combinations for each replications. So, this is at the low level all 3 at the low level here the c at the high level whereas, the factor a and factor b, these are at the low levels ok. So, these are the low levels you have. So, just you follow these the corners you can get an idea that how it is designed and how on what basis, what is the basis of selecting the treatment combinations and what is the basis of say they are giving notation. So, for each treatment combinations ok.

(Refer Slide Time: 17:33)



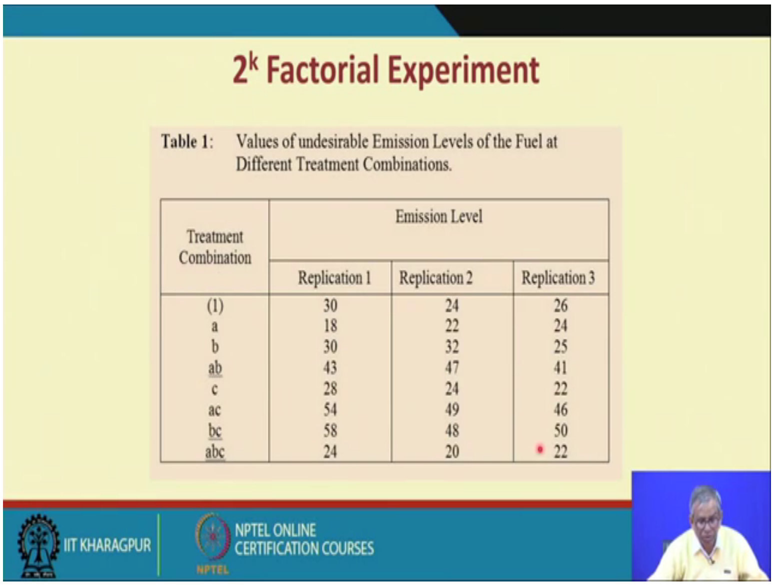
Now, here is an example. So, in order to develop a good quality synthetic fuel experiments are conducted with 3 different factors each control at two levels. So, you must have a controlling means ok, it is not that you get all these values or all these levels automatically. So, while you set the values while you select those levels you also must be aware of how to control them factor a is a concentration of corn extract at 5 percent and 10 percent, ok, factor b is a concentration of another compound at 15 percent and 25 percent and factor c is the distillation temperature at 120 degree Celsius and 150 degree Celsius, is it ok.

The degree of undesirable emission level at each treatment combination is shown in table one for 3 replication; that means, what is your main objective; that means, you try to design the fuel the synthetic fuel in such a way that that the undesirable emission level is at the lowest or the minimum level, is it ok. So, that is your objective and if you achieve this objective you say that the quality goal has been achieved. Now for such a problem, you have you have to adopt you have to use or the design of experiment methodology.

Now, here what are the questions we have there are 3 questions we have find the main and the interaction effects this sort of you know the questions are always you get in any design of experiment exercises find the sum of squares for each of the main and the interaction effect; that means, essentially, you propose a model; that means, how this the response variable is related to the individual factors and what is whether you know the best possible model, you are prescribing and whether all the important of the critical factors, you have considered while you model while or while you carry out the experiment.

So, these two values you compute and get an idea; that means, you need to create the ANOVA table at 5 percent level of significance which effects are significant.

(Refer Slide Time: 20:23)

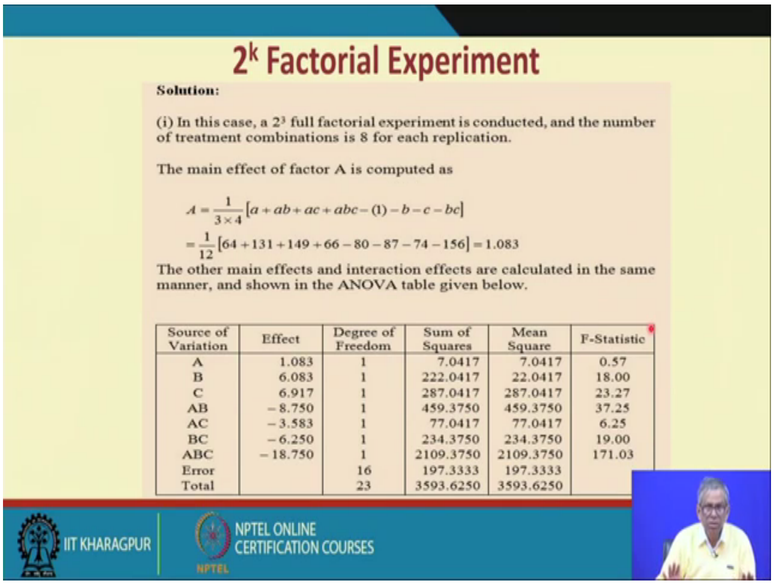


Treatment Combination	Emission Level		
	Replication 1	Replication 2	Replication 3
(1)	30	24	26
a	18	22	24
b	30	32	25
ab	43	47	41
c	28	24	22
ac	54	49	46
bc	58	48	50
abc	24	20	22

So, these are the 3 questions we have. So, what are the data you have you work; that means, you have one power an experiment and all these 8 treatment combinations with 3 factors at 2 levels, you have considered and there are 3 replications; that means, these treatment combination is the experiment is run for each treatment of combination treatment combinations 3 times that is why the replication there are 3 replications and you will find that against these treatment combinations when all these 3 factors are at low levels, you will find that the emission level the first time when you do the experiment you get a value of 30.

The next time, it is 24 and the third time, it is 26 and similarly for all other treatment combinations, you get the data the 3 times. So, the 3 times you have to carry out the tests in a particular condition or in a particular say treatment combination. So, these are the values for replication 1, replication 2 and replication 3. So, look at these values. So, this is the data you have collected next what you have to respond to the question number one.

(Refer Slide Time: 21:34)



So, how to get the solution in this case a 2 to the power 3 factorial experiment is conducted and the number of treatment combinations is eight for each replication this we have already elaborated. So, you calculate the main effect already, we have explained that which formula you need to use; that is ok. So, the main effect of a you compute as 1.083 the other main effects and the interaction effects are calculated in the same manner we have already given you a the formulations.

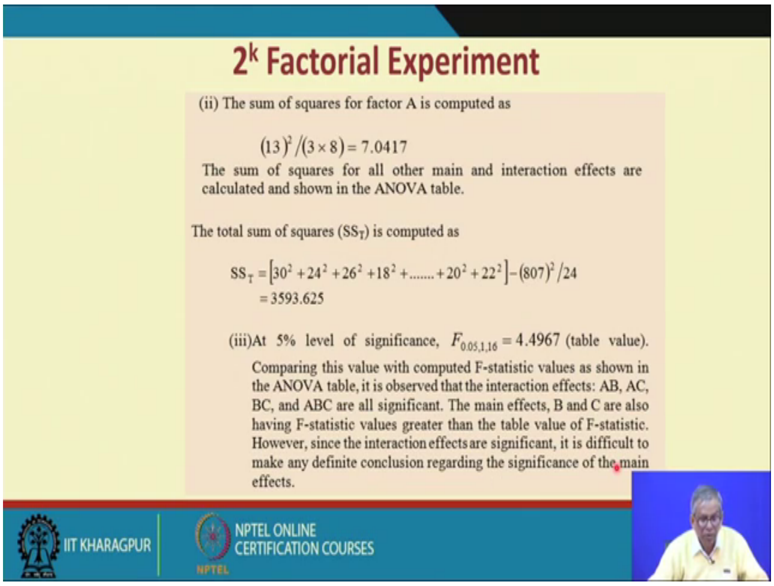
So, apply those formulations and then you create the ANOVA table like this; that means, the sources of variation in y could be a could be b could be c could be ab, ac, bc, abc or the sources of variation you are unable to explain; that means, this is a error component and this is the total say you consider; that means, all aspects you considered.

Now, you have to check if the error part is insignificant or not if it is insignificant you may conclude that your modeling is perfect if you find that the error is significant; that means, immediately you conclude that your the model is not perfect; that means, you have to make it perfect maybe you need to consider some other factors that you have a you have not considered yet.

So, this is that is why it is always any such experiment when you can when you when you conduct, this is referred to as an exploratory study you never know that whether the factors, we are considering whether those factors are all significant or not or whether you need to consider some other factors in explaining the behavior of y or say the variations in y is it ok. So, that is why it is always referred to as an exploratory study.

So, these are the effects you consider main effects the degree of freedom you consider this a total is 23 and the sum of squares, you calculate main square you consider; that means, once you consider the degrees of freedom sum of squares already you have computed. So, now, you can you can calculate the mean square right.

(Refer Slide Time: 24:13)

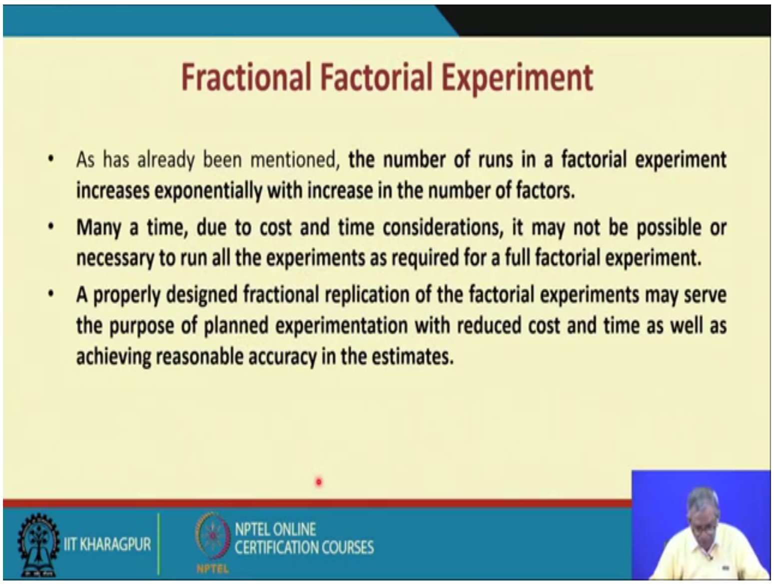


And then the F statistic you consider, is it and then what do you conclude that the sum of squares for factor a is computed like this, already I have given you the steps you have all the data the sum of squares for all other main and interaction effects are calculated and shown in the ANOVA table suggest you refer to the ANOVA table you will get all the details.

The total sum of squares is computed with this formula; that means, this is t you follow this formula these values you get from on the table and this is the factor k correction factor and at 5 percent, level of significance this is the value of F statistics 5 percent level of significance and these are the degrees of freedom it is from the table value is 4.4967, comparing this value with computed F statistic values as shown in the ANOVA table, it is observed that the interaction effects ab, ac, bc and abc are all significant.

The main effects b and c are also having F statistic values greater than the table value of F statistics; however, since the interaction effects are significance, it is difficult to make any definite conclusions regarding the significance of the main effect. So, that is the conclusions.

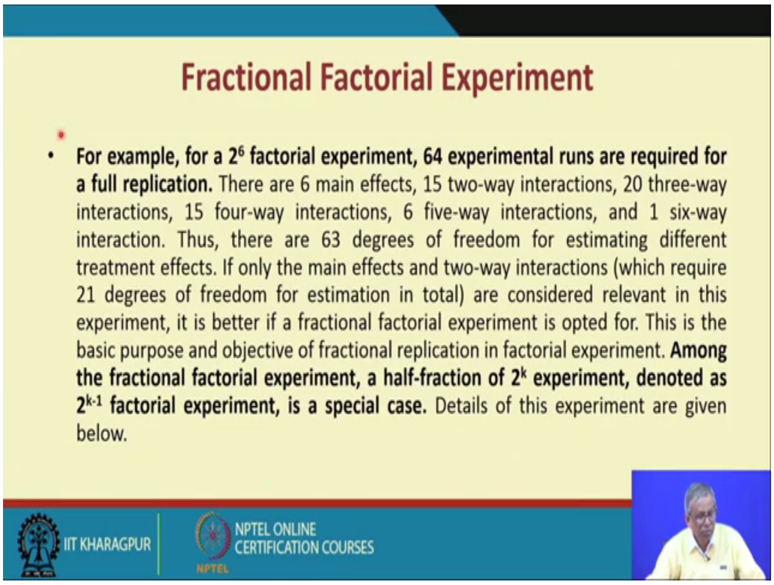
(Refer Slide Time: 25:28)



Now, what do you need to do in many a time you know the full factorial experiment 2 to the power k full factorial experiment may not be feasible; that means, if say the number of factors is reasonably very high and even if each factor is at two levels, you will find for each replication you need to conduct a several sorts of say you know conduct experiments are different a large number of a treatment combinations.

So, this may not be feasible when you have limited time and limited resource. So, you need to opt for the fraction with factorial experiment so many time due to cost on time consideration, it may not be possible or necessary to run all the experiments as required for a full factorial experiment. So, in the, that in these conditions we prefer using fractional factorial experiment.

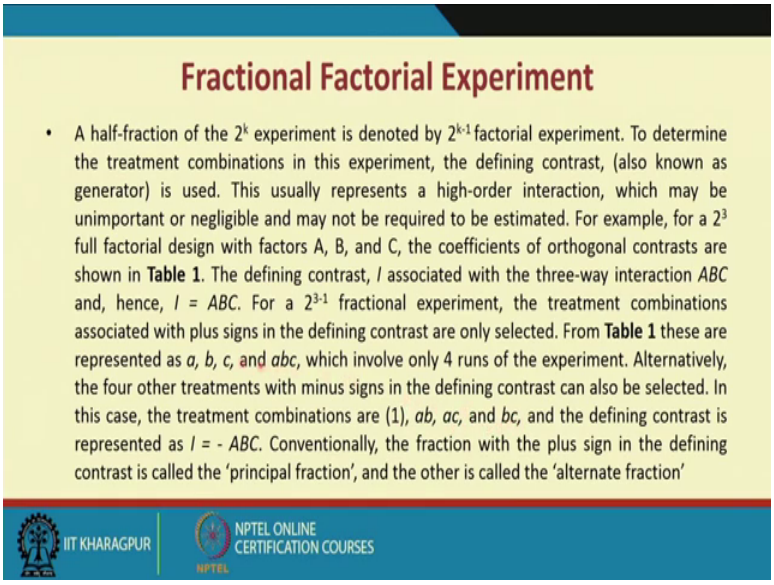
(Refer Slide Time: 26:29)



So, for example, for a 2 2 to the power 6 factorial experiment 64 experimental runs are required for a full replication just one replication. So, there are 6 main effects 15 2 way interactions, 23 way interactions 15, 4 way interactions, 6 5 way interactions and 1 6 way interactions. These are the possibilities that there are 63 degrees of freedom and if only the main effects and the two way interactions which required twenty one degrees of freedom for estimation in total are considered the element in this experiment it is greater if a fractional factorial experiment is update for, is it ok. So, this is the basis.

So, this is the basic purpose and objective of fractional replication of factorial experiment among the fractional factorial experiment a half fraction of 2 to the power k experiment denoted as 2 to the power k minus 1 factorial experiment is a special case details of the experiment are given below.

(Refer Slide Time: 27:33)

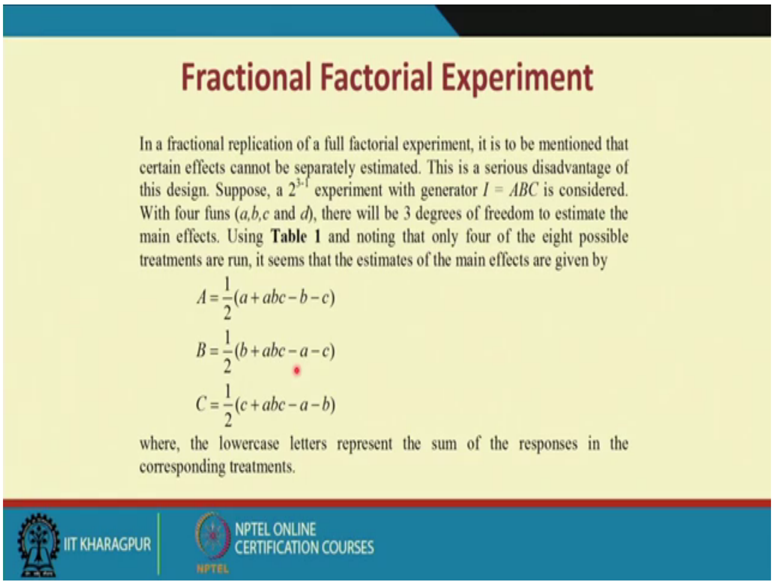


It is essentially you know the fractional factorial experiment you do and what do you try to do; that means, all the different combinations you do not consider. So, if it is a half fractional factorial; that means, you just consider half of the ah; that means, the total number of treatment combinations .

So, ah; so, here you need to define the defining contrast I associated with the 3 way interactions abc and hence I is abc for a 2 to the power 3 minus 1 fractional experiment the treatment combinations associated with plus signs in the defining contrast are only selected. So, the from table one these are represented at abc and abc which involve only 4 runs. So, in the full factor there are eight runs. So, now, you just get 4 runs; that means, only these 4 treatment combinations you consider. So, 4 other treatments with minus signs in the defining contrast can also be selected, in this case, the treatment combinations are 1 ab, ac and bc.

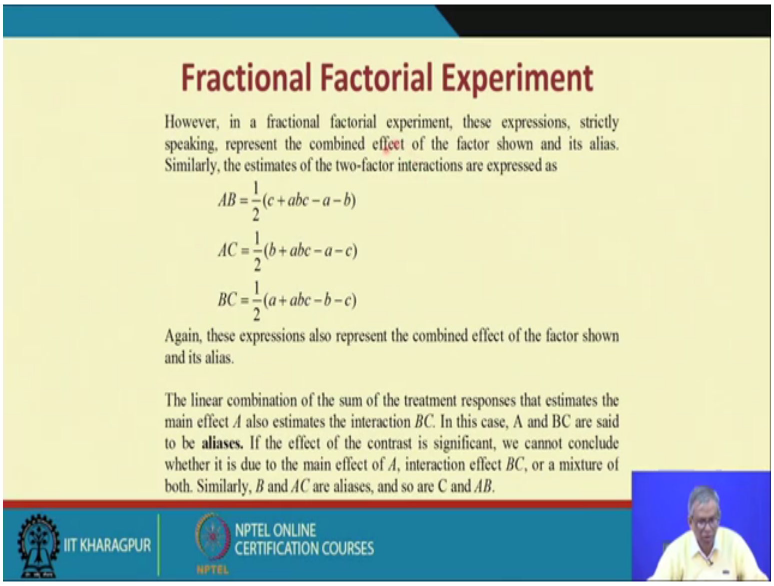
So, the choice is yours either you go you go for say the principle fraction or you may go for alternate fraction.

(Refer Slide Time: 28:58)



So, this is alternate fraction and this is the principle fraction these are the terms we use. So, when you apply this 2 to the power 3 minus 1 say the fractional factorial experiment; that means, half fractional factorial experiment; that means, the main effect is defined by this main effect of a main effect of b main effect of c. So, the lowercase letters represent the sum of responses in the corresponding treatments. So, this is the lowercase letters ok.

(Refer Slide Time: 29:27)

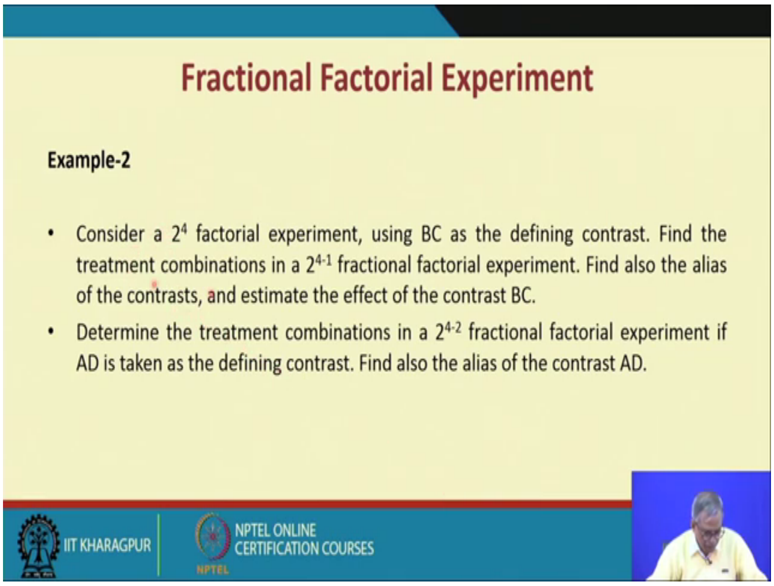


This way you conduct and; however, in a fractional factorial experiment these expressions strictly speaking represents a combined effect of the factors shown and its alias.

Similarly, the estimates were the two factor interactions that also conduct like this; that means; here say ab ac and bc against these expressions also represent the combined effect of the factor and its aliases. So, I suggest that all the steps are given and for conducting fractional factorial experiment and what is the important thing is that the linear combination of the sum of the treatment responses that estimates the main effect a also estimates the interaction bc is it ok. So, that you need to consider that a is equivalent to bc the b is equivalent to ac c is equivalent to ab.

So, so, so that is why a and bc are said to be aliases similarly b and ac are said to be the aliases and similarly the c and ab are considered to be the aliases ok. So, this particular term we use, right.

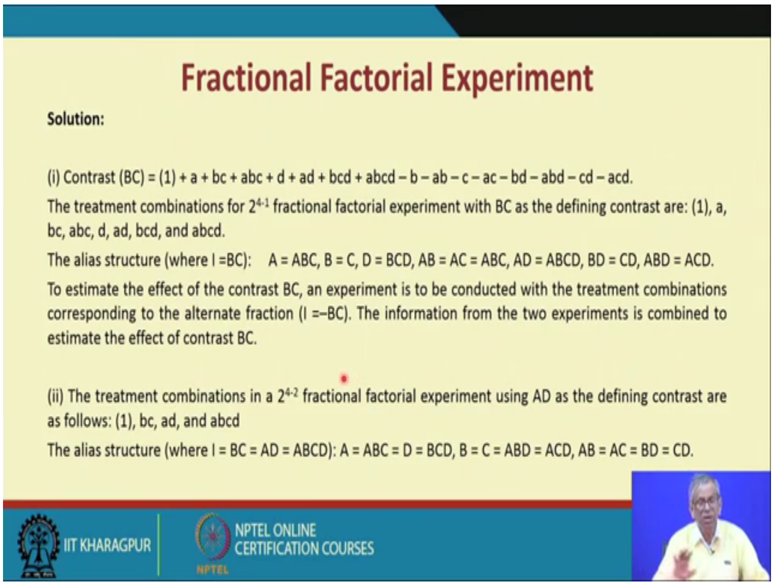
(Refer Slide Time: 30:44)



So, so, this is an example and so, please go through these examples consider 2 to the power 4 factorial experiment using bc as the defining contrast find the treatment combinations in in a 2 to the power 4 minus 1 fractional factorial experiments find also the aliases of the contrast and the estimate the effects of the contrast bc.

So, determine the treatment combinations in a 2 to the power 4 minus 2 fractional experiment, is it ok, factor fractional factorial experiment if ad is taken as the defining constant find also the aliases of the contrast.

(Refer Slide Time: 31:20)



So, these are the typical problems we said under fractional factorial experiment I just all the steps I have written please go through them because the theory part already you have explained and ah. So, you will the fine reasonings that of a computing all these all these interaction effects as well as the main effects.

(Refer Slide Time: 31:49)

And then what when you when you carry out all these exercises you will come to know that why a fractional factorial experiment is preferred in many cases and what is positive points and what are these limitations.

So, I think that we have identified all the important aspects related to the statistical design of experiments and whenever you try to improve the quality level of a product or a process a many a time, you need to look at that the design and when you try to propose a new design you need to go for the design by experiments ok. So, the design of experiments related the methodologies you have to use and in many cases for product development purposes keeping in mind the importance of quality features in the product or the quality performance always we opt for design of experiment based say the approaches ok. So, I conclude this particular session on a quality by experimental design.