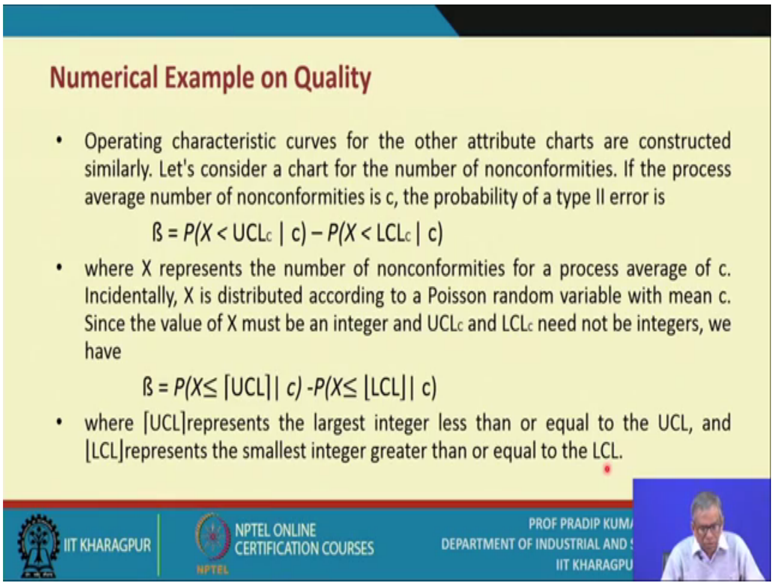
Quality Design and Control

Prof. Pradip Kumar Ray
Department of Industrial and Systems Engineering
Indian Institute of Technology, Kharagpur

Lecture - 49
Design for Reliability - II (Contd.)

So, we will be taking up several other numerical problems on quality and reliability. So, this session we will be referring to other typical numerical problems on quality and reliability.

(Refer Slide Time: 00:32)



Now in the previous lecture session we have explained with respect to one particular numerical problem, how to construct the OC curve for the P control chart ok.

Now, for constructing OC curve for the c control chart; we follow the same procedure, but need to be careful about certain notations as you are already aware that the basis of constructing a c control chart is Poisson distribution and c control chart is used to control the number of the nonconformities or the defects on a particular product.

Now, operating characteristics curves operating characteristics curves when we refer to; that means, what is whenever we refer to the operating characteristics curves for a specific control chart first what you have to consider; that means, the for this particular case; that means, for the c control chart the number of nonconformities; that means, what could be the possible the values of the number of nonconformities and the c control chart we assume that the sample size remains constant.

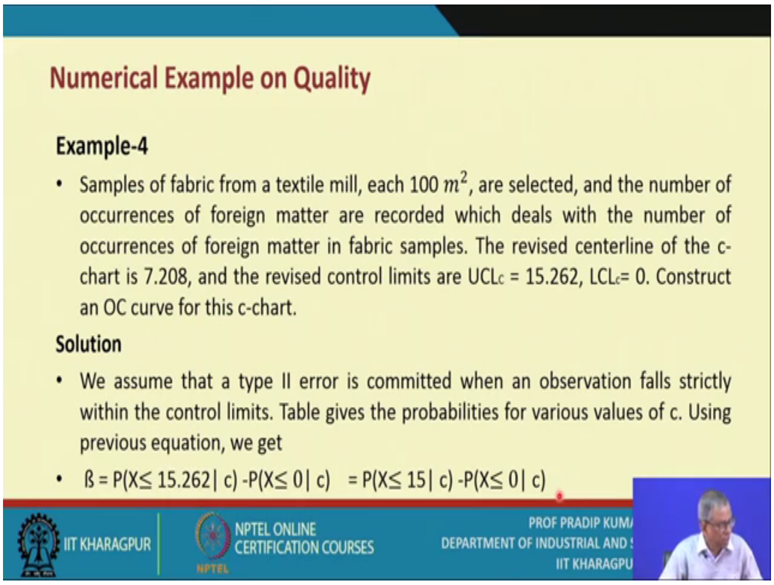
So, if the process average number of nonconformities is c is it ok. So, that is that is the parameter the probability of a type two error is given by this; that means, this is the beta function probability that X; what is X? X is basically a random variable; that means the number of nonconformities. Now, this is this value is less than upper control limit for the c control chart for the given value of c is it and this is the probability minus probability that X is less than LCL c is it ok; that means, the area between the upper control limit and the lower control limit for the c control chart given a particular value of c.

Where X represents the number of nonconformities; that is already I have I have mentioned for a process average of c. Incidentally, X is distributed according to a Poisson random variable is it ok; that is the basic assumptions we make with mean c. Since the value of X must be an integer and you will see a UCL c and LCL c need not be integers it could be 17.3 so; obviously, it is an integer and similarly LCL c a lower control limit when the c control chart could be 1.73 or 2.53 ok.

So, in all likelihood they may not be the integers. So, what to do; that means, you need to make some corrections. So, for making corrections in the formula, what you do that; we say that beta is probability that X is less than equals to UCL absolute value of UCL given a value of c minus probability that X is less than equals to LCL; that means, an absolute value of LCL given the value of c where UCL solute represents the largest integer less than or equal to the UCL. So, this rule you have to follow largest integer less than or equal to the UCL; that means, suppose it is 8.56 which has to be 8 is it ok. And similarly these LCL represents the smallest integer greater than or equal to the LCL.

Suppose the LCL is say 0.73; that means, it must be made one is it ok.

(Refer Slide Time: 05:05)



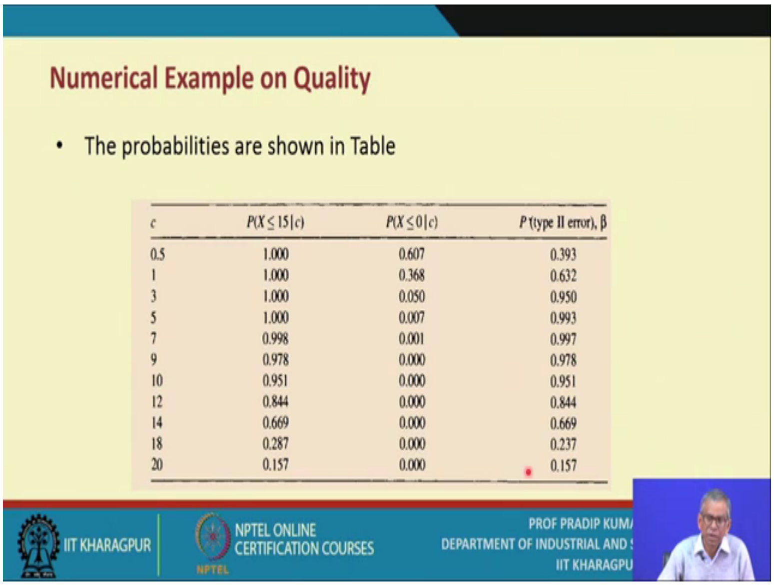
So, these corrections for integers you must you must make in the formulation and then you we take off this particular example is it samples of fabric from a textile mill, each 100 square meter, are selected, and the number; that means, the sample size is constant it is for any sample it is 100 square meter and the number of occurrences of foreign matters, that is; if you have the occurrences of foreign matters; that means, is considered to be certain defects.

So, this the occurrences are recorded which deals with the number of occurrences of foreign matter in fabric samples the revised center line of the c chart is 7.208 and the revised control limits are this 15.262 is it ok. So, and the lower control limit is 0. So, the lower control limit is 0, the central line is 7.208 and the upper control limit is 15.262 is it this is this is this is we are referring to the final control chart construct an OC curve for this chart.

So, what you try to do; that means, we assume that a type two error is committed; when an observation falls strictly within the control limits is it even if there is a shift in the process parameters value. Now, we have to construct a table and the table gives the probabilities of various values of c; that means, the probabilities of for various values of c probabilities of what; that means, the probability of making type two error given a set of upper control limit and their work and on the lower control limit for the control chart ok.

So, using you know the previous equation we get here beta is probability that X is equals to less than equals to 15.262 LCL given a value of c minus probability that X is less than or equals to 0; that means, lower control limit is 0 that is given a value of c. So, these corrections we make in for integers. So, X is less than equals to 15 is it and X is less than equals to 0 rights.

(Refer Slide Time: 07:40)



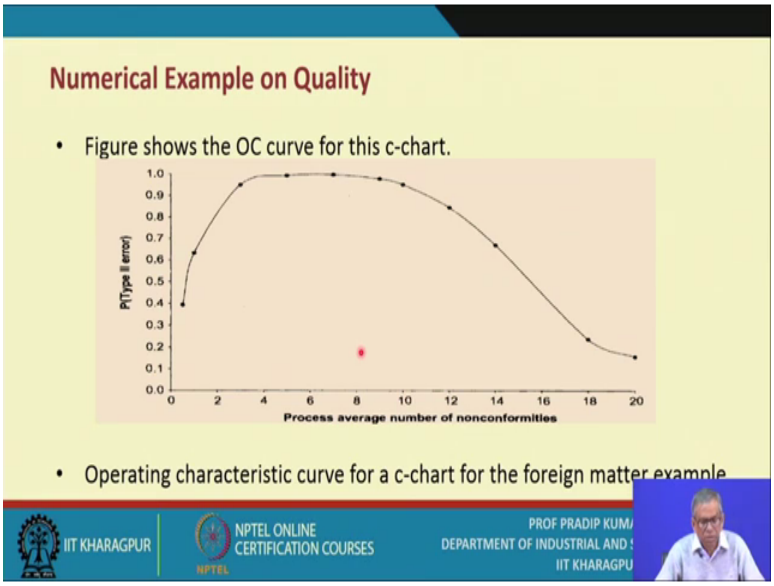
c	$P(X \le 15 \mid c)$	$P(X \le 0 \mid c)$	P (type II error), β
0.5	1.000	0.607	0.393
1	1.000	0.368	0.632
3	1.000	0.050	0.950
5	1.000	0.007	0.993
7	0.998	0.001	0.997
9	0.978	0.000	0.978
10	0.951	0.000	0.951
12	0.844	0.000	0.844
14	0.669	0.000	0.669
18	0.287	0.000	0.237
20	0.157	0.000	0.157

So, after making these corrections then for a given value of c say 0.513 and so, on. We calculate the corresponding values of this probability as well as probability of this one is it ok? So, we refer to the Poisson distribution table more specifically cumulative Poisson distribution table and this probability you compute this probability also you compute and you subtract this from this say you get 0.393. So, the same procedure you follow for all the possible values of c.

So, what is the basic assumption? The basic assumption is that this c values are the feasible; that means, there is a possibility that that the number of nonconformities may assume all these values is it ok. So, you have to be careful it just cannot be any value; that means, you must have you know some experience related to that particular the product quality and then you say that this might occur. So, suppose the c value could be as is 20.

So, what is the probability of the detection? Probability of detection is 1 minus 0.157 basically this is the beta. Beta means probability of non detection of the shift. So, as you might have noticed that as the value of c increases the probability of non detection decreases and; that means, the probability of detection increases ok.

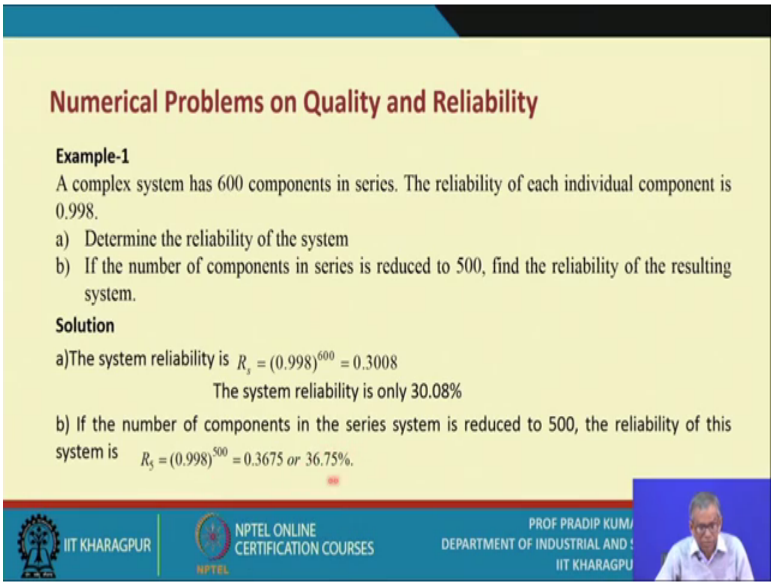
(Refer Slide Time: 09:21)



O that is the typical. So, the nature or the property of a there is a general purpose control chart.

So, when you plot all these values. So, you get a curve of beta versus c possible values of c. So, you have a typical the OC curve it looks like this for the c control chart right. So, this is this will be a good exercise for you.

(Refer Slide Time: 09:57)



And, these numerical problems; that means, whenever you use a control chart. So, you must be in a position to construct the OC curve and we look at the OC curve and looking at the OC curve you can well interpret the performance of your control chart is it ok.

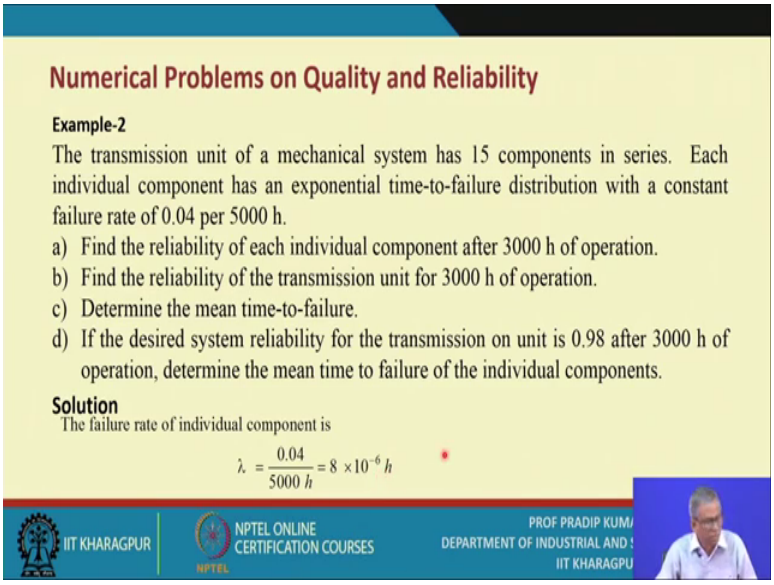
So, now the next examples we are talking about that is related to the reliability is it ok. So, whenever we discuss reliability we refer to. So, component reliability you refer to the systems reliability and whenever we refer to the system reliability. Now the system is structure needs to be explained with respect to it is configuration. So, there are the two pure configurations we have already refer to one configuration is a series configuration and the second one is as an alternative to series it could be a parallel configuration.

Now, here in this particular example what we are saying that; a complex system has 600 components in series. Now, whenever we define the complexity of a system. Now, complexity of a system can be explained from several perspectives of say the quality and reliability. Now in this case the complexity is explained or defined with respect to the number of components we use is it ok. So, there are 600 components so; obviously, in all likelihood it may be referred to as a complex system.

The reliability of each individual component is 0.998 ok. I am not assuming it to be one it just cannot be one is it ok; it could be very almost nearing one, but it just cannot be 1. So, 0.998 determine the reliability of the system and if the number of components in series is reduced to 500 find the reliability of the resulting system. So, this is here the system reliability 0.998 to the power 600 that is 0.3008. So, the system reliability is only 30.08 percent.

If the number of components in the series systems is reduced to 500 the reliability of the system is this is it 36.75 percent systems.

(Refer Slide Time: 12:29)



So, this is just a simple example right; that means, here what you will find that even if the individual component reliability is very high if you deal with a series systems in all likelihood that the overall systems will have unity will be very poor.

So, how to how to improve the systems reliability the first approach is can you reduce the number of components ok. So, this is this is one typical example I have got through next we go to the next examples on reliability the transmission unit of a mechanical system has 15 components in series this is very typical each individual component as an exponential time to failure distribution with a constant failure rate of this is it all right; that means, 0.04 per 5,000 hours,

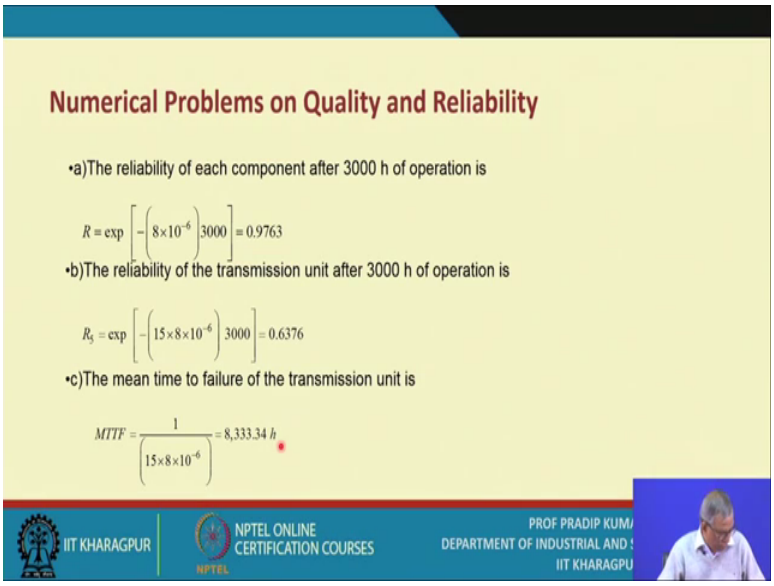
Now, the obvious question is how do you get this value? Through experimentation right; so, 0.04 that is the failure rate or if you refer to the path data right you have these estimate 0.04 per 5,000 hours is it ok. So, per 1000 hours what will be the failure rate; that means, 0.04 divided by 5 is it ok?

Find the reliability of each individual component after 300 hours of operation; that means, as you know that whenever you we define reliability is defined with respect to a specific time period the survival time.

Find the reliability of the transmission unit for 3,000 hours of operation it is for the individual component and this is for the entire the system determine the mean time to failure mttf. We have already discussed what; how do you compute or how do you get the expressions for mttf; under different distributional assumptions for the time to failure time to failure.

If the desired system reliability for the transmission on unit is 0.998 after 3,000 hours of operation determine the mean time to failure of the individual components; that means, these are the four questions we have said. Now, the failure rate of individual component, I have already mentioned that is lambda that is per hour basis if you compute you will find 0.04 divided by 5,000. That means 8 into 10 to the power minus 6 hours is it ok.

(Refer Slide Time: 15:14)



So, that is the value of lambda and we are assuming that the lambda is constant; that means, all the components as well as the systems are in useful life phase.

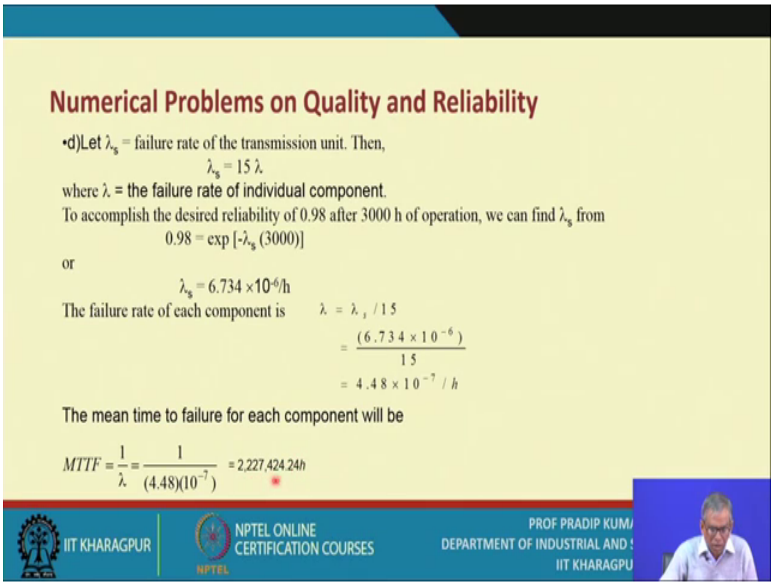
So, reliability of each component after 3,000 of operations; that means R that is a reliability e to the power minus of the lambda t. So, the lambda value already we have computed that is 8 into 10 to the power minus 6 and the value of t is 3,000 is it ok, because we are computing the value of reliability for 3,000 hours the time period so; that means, this is 0.9736.

The reliability of the transmission unit after 3,000 hours of component is basically you know R 5; that means, it is exponential 15 times there are 15 components ok; that means, it is sigma lambda i sigma lambda i into t. So, what is lambda i; that means, each one is having this lambda ok.

So, the components you have the 15. So, each one is identical to the other one that is why; it is 15 into 8 into 10 to the power minus 6; that means, the time to failure for the system is also you know the exponential is it ok. So, that is the assumption. So, 3,000 hours. So, it has come down to this value of reliability has come down to 0.6375 is it ok. So, this is R s is it ok? So, systems reliability that notation we have used 0.6375. So, it is considered to be very very poor.

The mean time to failure; obviously, MTTF is 1 upon n sigma a lambda i. So, sigma from i equals to 1 to 15. So, 15 into 8 into 10 to the power minus 6; that means, 8333.34 hours that is the mean time to repair mean time to failure.

(Refer Slide Time: 17:23)



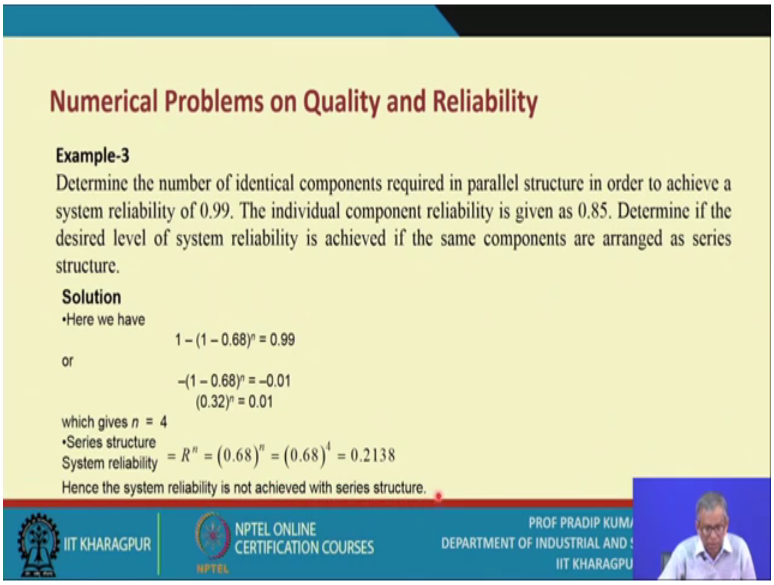
Now, let therefore, the whether the next questions what you assume that the lambda is equal to failure rate of the transmission unit; that means, transmission unit is considered to be the system; that is why you say it is lambda subscript s. So, lambda subscript is 15 into lambda where lambda is a failure rate of the individual component to accomplish the desired reliability of 0.98; that is pre specified after 3,000 of operations.

So, this is the system reliability this 0.98 and you apply this formula; that means, X e to the power minus lambda X into 3,000. So, you compute the value of lambda s that is

6.734 into 10 to the power minus 6 is it ok. So, failure rate of each component is; obviously, lambda s divided by 15; that means 4.48 into 10 to the power minus 7. So, you just follow the steps 1 by 1. So, you will get all the details you will understand you know all sorts of computations and why and how these computations are necessary.

So, similarly the mean time to failure for each component will be MTTF is one upon lambda ok. So, these expressions already you are aware of for computing mean time to failure and the value is this one ok.

(Refer Slide Time: 18:49)



Now, we will move to the next example; so example- 3.

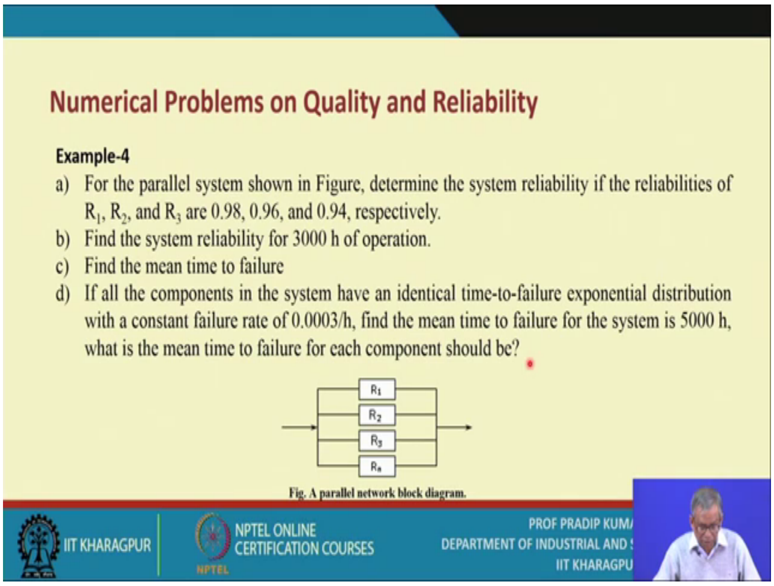
Determine the number of identical components required in parallel structure. Now we are moving from the series to parallel in order to achieve a systems reliability of 0.999. So, that is expected; that means, if the reliability is high, what you assume that; it is the survival time will be more and at any point in time getting say the function from the product is also very very high; that means, you know, but the value of the product is assured.

The individual component reliability is given as 0.85. So, it is not that high determine if the desired level of system reliability is achieved; if the same components are arranged as series structure is it ok. So, supposing you know the person concerned he recommends a series structure, but he does not know he or she does not know what is the relationship between the systems reliability and the individual component reliability is it ok.

Now, if he or she of for the series say the structure whether you can achieve a value of 0.99 of further systems reliability. So, here this is relationship; that means, the systems reliability expression for the parallel structure is 1 minus 1 minus this 1 is it ok; that means, 0.68; that means, to the power n. So, it is 0.99 is it? So, this is point minus 0, 1 right. So, 0.32 into 0.01 which gives n equals to 4.

Series structure systems reliability 0.68 to the power n ok. So, this is 0.68 to the power 4; that means 0.2138. Hence, the system reliability is not achieved with the series structure is it is very very clear.

(Refer Slide Time: 21:07)



Next problem that is example four again we refer to the parallel systems and parallel system is having this many is this typical parallel systems we have. So, for the parallel system shown in Figure, determine the systems reliability if the reliability of R 1, R 2 or R 3 this is the general you know parallel network block diagram.

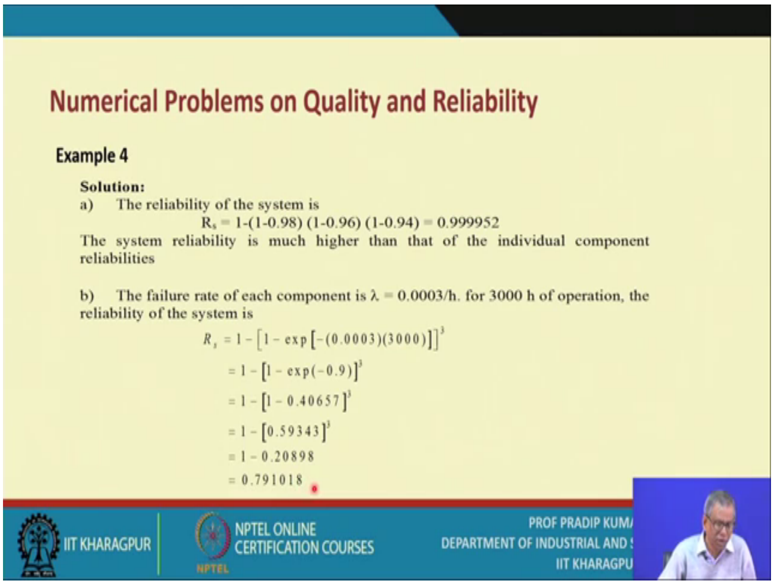
So, here what you have I have shown n number of the components in parallel, but here for the given problem you just assume that n equals to 3; that means, just 3 the components you have in parallel configuration and the corresponding the reliability component reliability; that means, for R 1 the reliability is 0.98, R 2 is 0.96 and R 3 is

0.94 is it and we are assuming that this component reliability the remains constant right that is the first assumption.

Find the system reliability for 3,000 hours of operation find the mean time to failure if all the components in the systems have an identical time to failure exponential distribution right time to failure distribution is exponential and we are mentioning identical; that means, the failure rate is same with a constant failure rate of 0.0003 per hour.

Find the mean time to failure for the system for 5,000 hours, what is the mean time to failure for each component should be is it ok. So, this is this are the typical problem. So, this is your parallel structure.

(Refer Slide Time: 23:01)



So, the reliability of the system is 1 minus 0.8 minus 0.8; 1 minus 0.961 minus 0.94. So, these 3 terms you multiply the product term and then you subtract this value from one you get the systems reliability with 3 components in parallel.

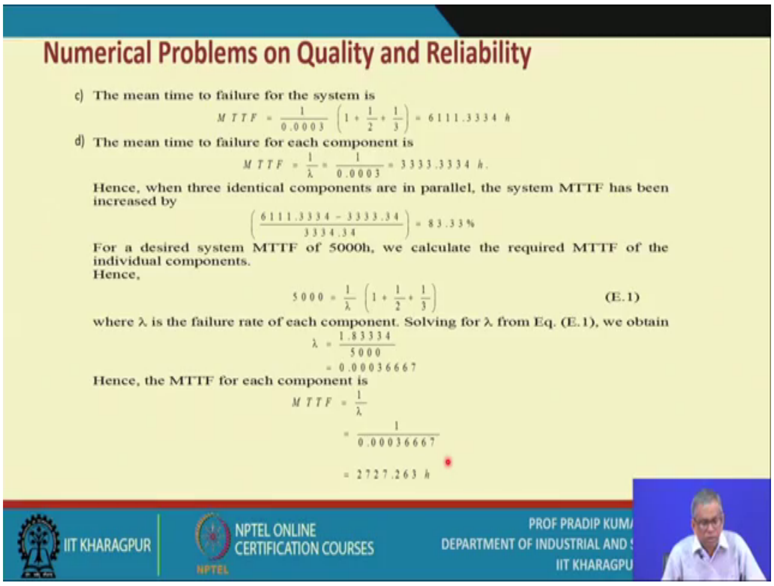
The system reliability is much higher than that of the individual component reliabilities. So, this is the advantage; that means, if suppose the component reliability you cannot improved beyond a certain value now, but you cannot compromise on the systems reliability. So, the best option is can you have a parallel you know the configuration. So, if you find that today at the present design it is a combination of series and parallel.

So, can you go for improvement in the design? So, that the series and series parallel you know the combination changes to only parallel. So, such you know the effort must be exerted and this is basically you know the redesigning effort ok. So, always whenever you opt for the better design better design we implicitly assume that it will bring in higher reliability of the system is it ok. So, this is just we exploring different options.

But the present you know the problem you must be able to quantify. So, whenever you carry out such exercises we will give an idea about how to assess the presents systems reliability or say you know the component reliability in the in the existing in the existing design.

Now, the next part is the failure rate for each component is this 0.0003 per hour; that is the value of lambda and for 3,000 hours of operation the reliability of the system is this; that means, 1 minus then 1 minus e to the power minus 0.003, that is the lambda and this is the this is t; that means, e to the power minus lambda t and there are 3 identical components in parallel. So, that is why e to the power 3 and ultimately you get the value of 0.791018 is it ok. So, this is the systems reliability.

(Refer Slide Time: 25:42)

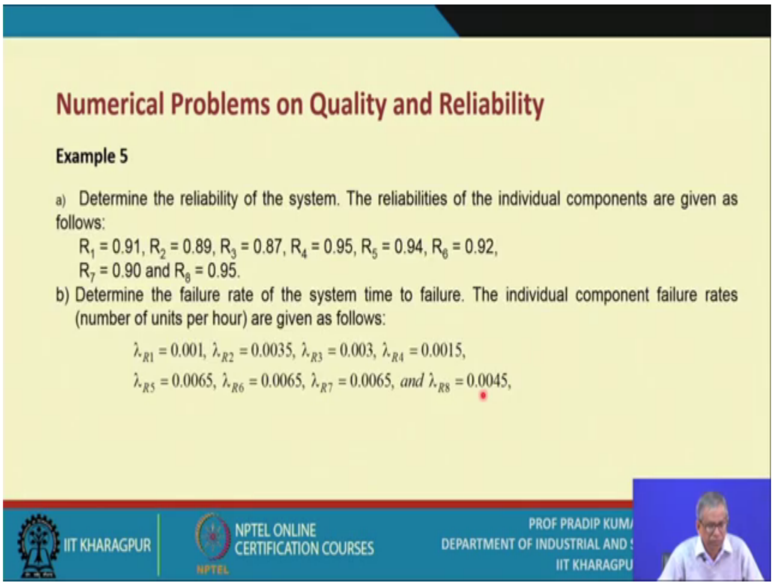


The mean time to failure that we all know that for the parallel system structure MTTF is 1 upon lambda into 1 plus half plus one-third plus up to 1 upon n; that is the general formula. So, here n equals to 3. So, you stop it here. So, this is 6111.3334 hours.

Mean time to failure for each component is one upon lambda that is ndf ok. So, one upon 0.0003 that is this value; that means, 3333.3334 hours hence when the 3 identical components are in parallel the systems mean time to failure has been increased by this one; that is 83.33 percent. So, it is a substantial increase and for a desired system MTTF of 5,000 hours; we calculate the required MTTF of the individual components and 5,000 is 1 upon lambda like this; you apply this formula and then when lambda is the failure rate of the each component. So, you get the value of lambda from here. So, lambda value is this 1.00036667.

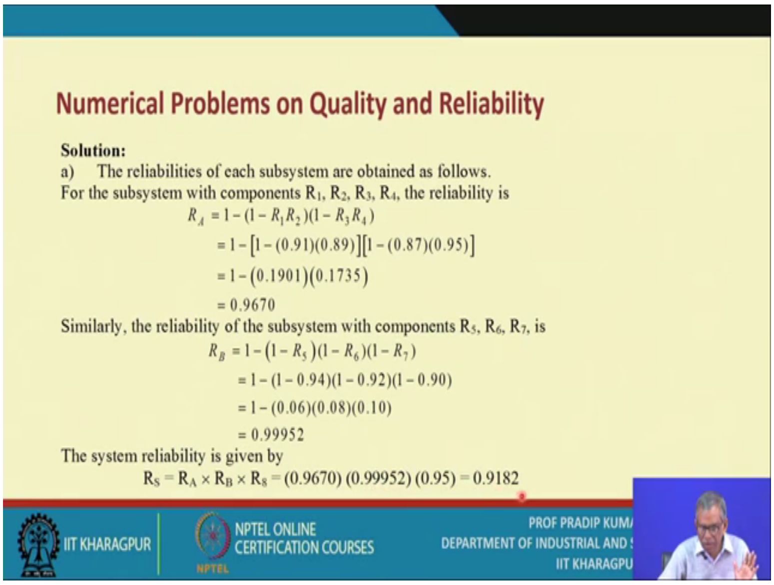
Hence, the MTTF for each component is 1 upon lambda. So, I have you know written down all the steps. So, you need to go through all the steps and you will get an idea that how many with different ways the problems can be can be explained and you have to answer to the several types of questions raised.

(Refer Slide Time: 27:28)



Now, another examples I am going to discuss, that is; example five determine the reliability of the system the reliabilities of the individual components have given as this R 1, R 2, R 3, R 4, R 5, R 6, R 7 and R 8; that means, there are 8 components determine the failure rate of the systems time to failure. The individual components failure rates number of units per hour are given as follows; that means, this is the individual for all 8 components the individual values are given.

(Refer Slide Time: 28:04)

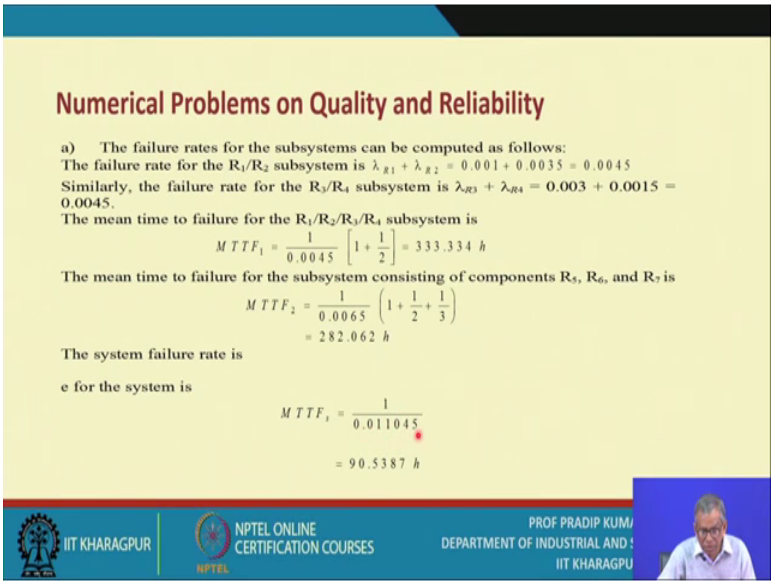


Now, how do you get a solution; that means, the reliabilities for each subsystems are obtained as follows right for the subsystems with components R 1, R 2, R 3, R 4 is it the reliability is this one that is you know you compute R 1, R 2; that means, these are in series and then they are in parallel. So, 1 minus R 3, R 4 by looking at this expression you will come to know what is the system structure; so, that is one assignment I am giving you; that means, suppose this is one configuration it represents one configuration by looking at these particular expressions for the reliability can you draw the system structure ok.

So, when individual values are given; obviously, you can calculate the systems reliability with this structure. Similarly, when you opt for this sort of formulation what is the corresponding system structure can you draw it is it ok. So, that is a part of your problem. So, RB; that means, the next systems refer to as systems B and corresponding reliability is RB. So, you have R 5, R 6, R 7 components. So, they are all in parallel is it ok? So, there could be different expressions like suppose for series you will have one kind of expressions for reliability systems reliability if it is parallel another if it is series and parallel. So, you will have an expressions by looking at the expressions can you draw the system structure is it ok.

So, corresponding values are all given for all the 8 components. So, the system reliability is given by this one; that means, the individual one; that means, RA, RB and R 8; that means, the 8th one is it 8th one is parallel it is in the series with RB and series with RA and within this RA you have another structure within this b systems P you have another structure is it and R eight; that means, this particular component exists separately is it ok. So, you have this expression systems reliability.

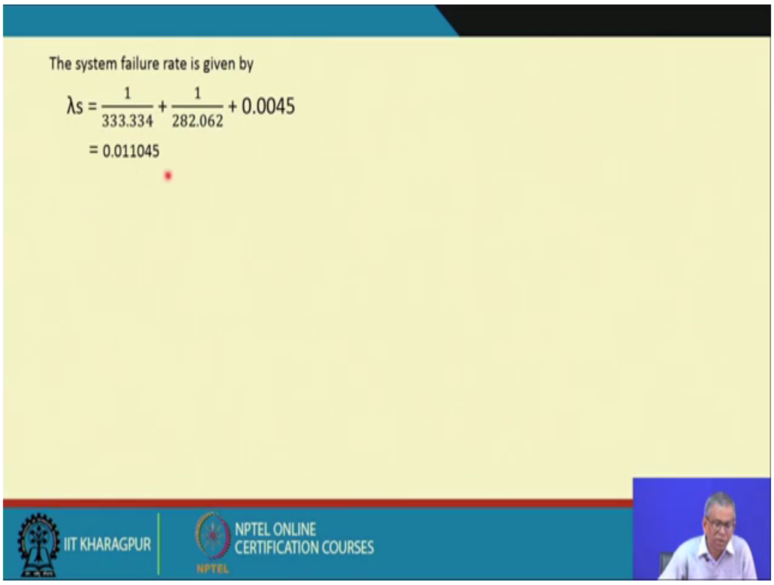
(Refer Slide Time: 30:22)



So, the failure rates for the subsystems can be computed as follows; that means, the failure rate for R one by R two subsystem is this one R 1, R 2 that is you add them. So, you get this 1.0045 you look at when you draw the structure you look at the structure you will get the meaning and why; do you go for you know adding these two values of lambdas.

Similarly, the failure rate for R 3, R 4 subsystem is lambda R 3 plus lambda 4. So, you add them the mean time to failure for the R 1, R 2, R 3, R 4 subsystems is this one is it ok. So, that is this value the mean time to failure for the subsystems consisting of components R 5, R 6, and R 7 is this one is it you just referred to a structure already you have drawn that structure and the system failure rate I will compute, but prior to that you know the systems mean time to failure that is one upon this is it ok, that is; 90.5387 hours.

(Refer Slide Time: 31:31)



And the systems failure rate that is lambda S is 1 upon 333.334 plus 1 upon 282.062 plus 0.0045 is it ok; that means, this is basically one systems you have the second systems and the third one is the component is it ok. So, when you add them then you get this value the systems failure rate ok.

(Refer Slide Time: 32:01)

So, you can refer to this particular textbook for the details.