Quality Design and Control
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Lecture - 48
Design for Reliability-II (Contd.)

In the previous sessions on the quality and reliability, we discussed a number of the problems along with the kinds of tools and techniques, we are supposed to use to address those problems so, mainly we have concentrated on the tools and techniques related to quality control related to reliability modelling, what I feel that at this stage a number of the numerical problems.

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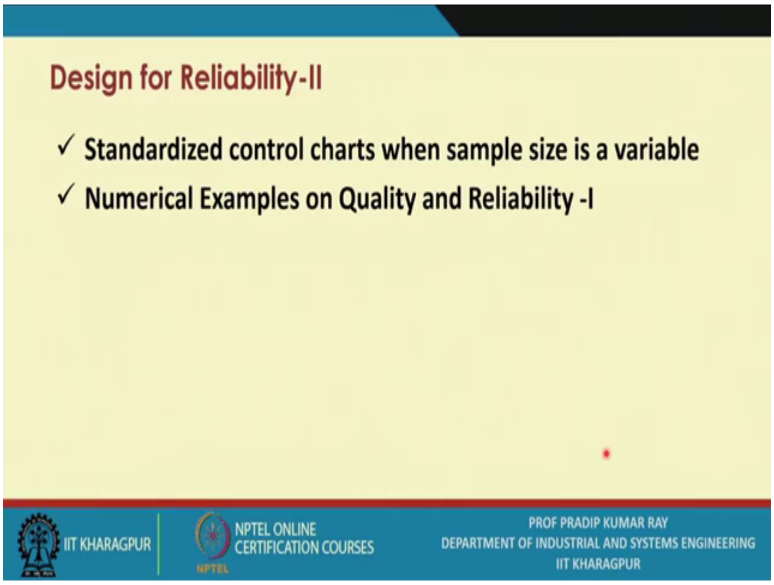

If we discuss, then your understanding will be better so, from this perspective, what I have planned that the next three sessions; we will be explicitly devoting on referring to fu typical numerical problems on both quality and reliability.

Now, in this particular session, standardised control charts which are widely used when sample size is a variable. So, this particular control chart will be referring to and a set of numerical problems typical numerical problems on quality and reliability those problems are also I am going to discuss.

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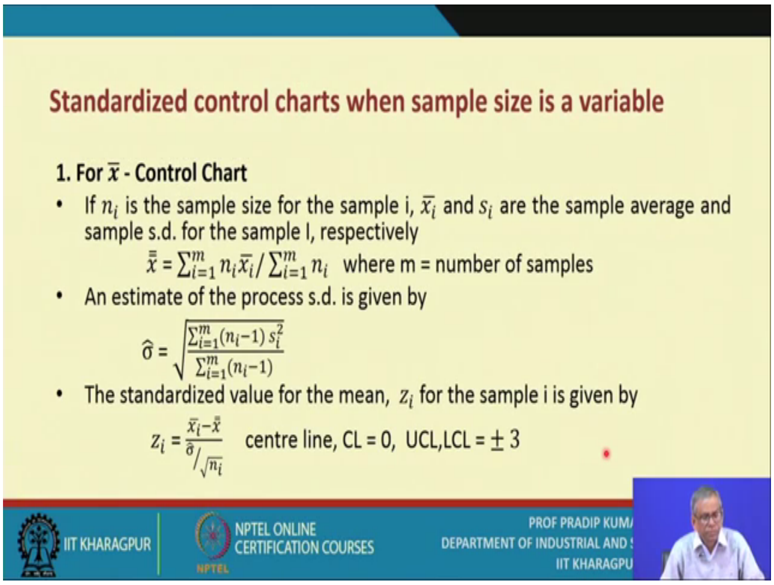

Now, let us first talk about the standardised control charts when sample size is a variable, if you recollect when we discussed the control charts for variables as well as so for attributes, we have refer to a particular case and that cases when sample size across the samples maybe a variable, there could be many reasons we have already you know highlighted those reasons.

Now, when the sample size is a variable; obviously, you know the kinds of the control charts, we use which for which are basically designed for any sample size constant that may not be applicable. So, some special procedures; you must bring in to consider the case of variables sample size.

Now, you know under variables category, we have discussed two kinds of control charts, one is x bar control charts along with r control chart. So, first you use r control chart to control the variability in the process, once you achieve this condition; that means, the variability is under control, then definitely you try to control the process mean for which x bar control chart will be used as I have already pointed out that both these charts are to be used simultaneously.

Now, let us talk about the x bar control chart when the sample size is a variable ok. So, the reasons could be that the production rate changes from a say one particular time period to another and for each time period you have to draw your sample and you say that the 10 percent of the number of say the units you produce say will form the sample so; obviously, if the number of units or the population size varies ah. So, the sample size also we will be varying. So, this is just one of the reasons.

Now, when sample size is a variable; that means, we say that if n i is a sample size for the sample i x i bar and s i; s i is basically the sample standard deviation ah. So, this are mentioned; that means, x i bar is the sample average and s i is the sample standard deviation for the sample i, ok.

Then you have considered m number of such samples. So, when you considered m number of such samples; obviously, you compute x double bar that is the mean of the means or the grand mean that is sigma i equals to one to m n i x i, is it ok, x i bar; that means, this is the sample average for that sample and the corresponding sample size is n i divided by sigma i equals to 1 to m n i; that means, you is you sum all the sample sizes. So, your m is the number of samples and estimate of the process standard deviation is given by that is an unbiased estimate that is sigma hat that is root over i equals to one to m n i minus 1 s i square this with the terms, already we have explained divided by i equals to 1 m sigma n i minus 1, is it ok. So, this is the expression unbiased estimate of sigma are given by sigma hat. So, you must have this estimate.

Then, what you try to do there are many alternatives you have for the designing the control chart. In fact, I have already mentioned that there four alternatives and one important or preferred alternative is the standardised control chart so; obviously, what you need to do. So, if you preferred this sort of control chart that is referred to as a standardised control chart; that means, the standardised value for the mean that is z i for the sample i is given by this one; that means, z i is equals to x i bar minus x double bar, divided by sigma hat by root over n i, is it ok.

So; that means, against each value of x i bar, you need to compute z i ok. So, and for the z i that is this sample the standard deviation that is sigma hat already have computed divided by root over n i; that means, we are assuming a normality and so once you have this value; obviously, the centre line will be 0 upper control limit as well as the lower control limit will be at 3; that means, plus minus 3; that means, centre line will be 0 in any sort of standardized control chart is used, it is to decided; that means, centre line will be 0 and upper control limit is at plus 3 and lower control limit will be at minus 3, is it ok.

So, this will be follow assuming that the normality condition holds ok.

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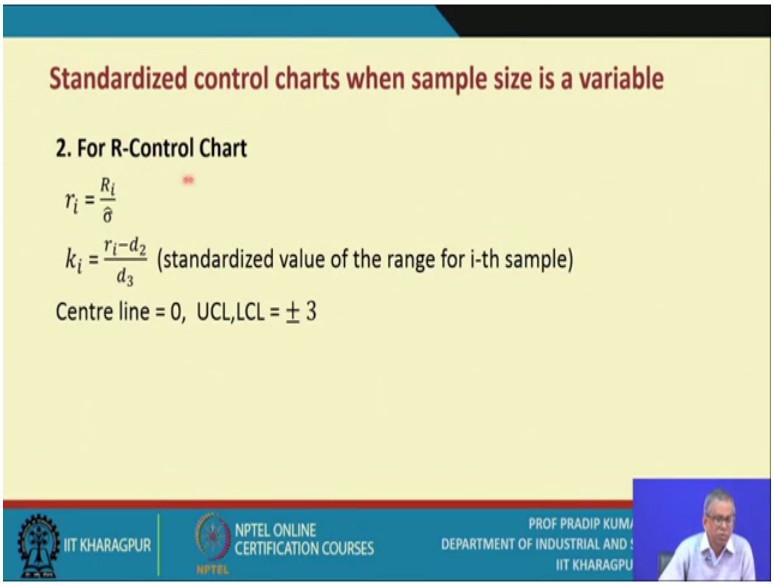

Now, then we refer to the r control chart. So, what you try to do; that means, for individual ith sample you compute R i, how do you compute R i; that means, in the sample you first identify the maximum value you also identify the minimum value and the difference between these two is basically the range that is R i for the ith sample already you have estimated the sigma that is sigma hat. So, this the range in terms of sigma that it is referred to as the relative range we have already refer to when you write the expressions for ah. So, the control limits of x bar x bar control chart we have refer to you know say relative range that is right.

Once you have these R i the for small ri for the ith sample. Now you can calculate that the standardised value that is referred to as the k i r i minus d 2 divided by d 3 so; that means, this is basically the main of the relative range and this is basically a function of the sample size and d 3 is standard deviation of the relative range, in the in the in the we sometimes we refer to another notation that is w. So, the standard deviation of the relative range that is d 3 is also proved to be a function of the sample size.

So, you convert this r i values into k i values and these values are basically the standardised values of the ranges for all the samples and then; obviously, the centre lines will be 0 and upper control limit lower control limit will be a plus minus 3, is it ok, like you have in the previous case, is it ok. So, this basically is whenever you find that the sample size is a variable this may be of preferred alternative and there could be many applications of the standardised control charts ok.

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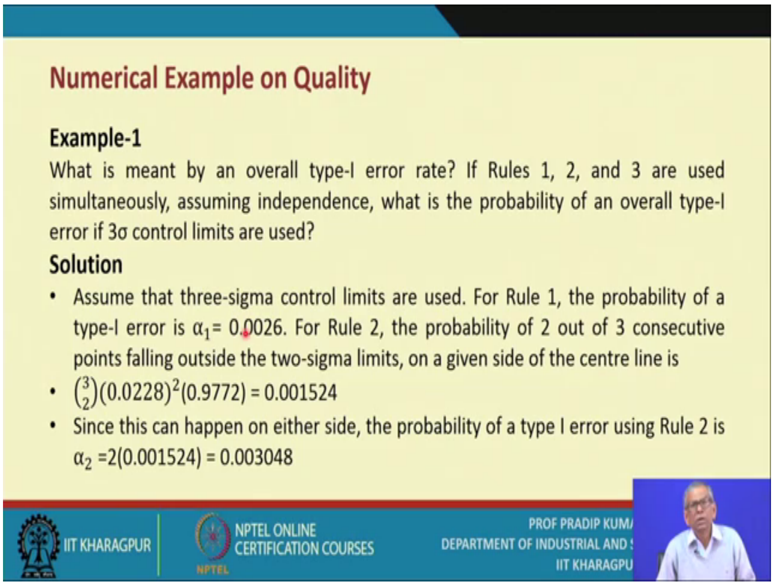

Only thing; one important point, you must keep in mind that the standardised control chart, suppose you deal with the variables data and you use the standardised control chart; obviously, you know this sort of control chart you may not be able to implement at the shaft floor; that means, you know instead of plotting the x i's values or say you know r i values or x i bar values what you need to do you need to plot z i values and then looking at the you know the patterns we have to interpret that what could be the possible reasons of say out of control points.

Now, it may be very difficult for an operator to interpret the value of z i. So, that is why what you say that this particular control chart must be handled by the process analysts is it otherwise you know, it may be very difficult for an operator to identify the possible you know the assignable causes against an out of control conditions ok. So, again we will be referring to such cases when deal with other the kinds of you know the numerical problems ok.

Now, let us concentrate on a few typical numerical examples on quality, is it ok. So, and then; obviously, in subsequent sessions lecture sessions I will also be referring to numerical problems on reliability. So, first example what is meant by an overall type one error rate already we have referred to the type one error or the probability of making type one error, is it ok, we have already explained this concept and the probability and the location is alpha any control chart is used invariably, there will be the probability of making type one error you cannot avoid it, what you can do; that means, when you design a control chart you try to minimise this value, but implicitly it is known that this type one error probability exists.

There are several rules if you recollect, we have already discussed a 5 specific rules like rule 1, rule 2 and rule 3. Now in a given case, suppose if rules 1, 2 and 3 are used simultaneously assuming independence so; that means, in rule 1 if you use it is independent of rule 2, is it ok. So, that is sort of assumptions we are making, what is the probability of an overall type one error if 3 sigma control limits are used, what we have mentioned that instead of using say one say the rule for out of control condition, is it ok.

If you use more number of such rules simultaneously it is expected that overall type one error probability overall type one error probability will increase. So, ah; so, what is that value is it ok, like say if you use rule 1 rule 2 rule 3 all these 3 rules, if you use simultaneously and if you use 3 sigma control limits right. Now, I need to compute the probability of an overall type one error. Now this I will just explain assume that the 3 sigma control limits are used for rule one the probability of a type one error is alpha one that is 0.0026, is it ok; that means, what is the rule one rule one is that that what is the probability that a particular sample point, we will plot outside of the control limits, is it ok.

So, if you have plus minus 3 sigma control limits of; obviously, you know the probabilities 0.0026 on one side, it will be 0.0013 on the other side. So, the beyond over control limit, it will be again 0.0013 assuming, you know the centre limit theorem holds and that is why the normality assumption holds. So, it is 0.026.

For rule 2, the probability of 2 out of 3 consecutive sample points falling outside the 2 sigma limits, is it ok. So, sometimes the 2 sigma limits are also referred to as the one in limits on a given side of the centre line; that means, on the same side of the centre line. So, what is the probability the probability is the 3 combination 2 3 c 2.0228, that is area on one side, is it ok, the area beyond two sigma control limit that area under the curve, the beyond 2 sigma on one side, it is 0.0028 on the other side again 0.0028.

So, the two values on one side of on of the warning limits the outside warning limits that is why it is 0.0228 square into the third point may be anywhere. So, this is 0.9772. So, this is the probability since this can happen on either side that is the rule the probability of a type one error using rule 2 is; obviously, twice of this value; that means, alpha 2 is 2 into 0.001524 that is 0.003048, is it ok.

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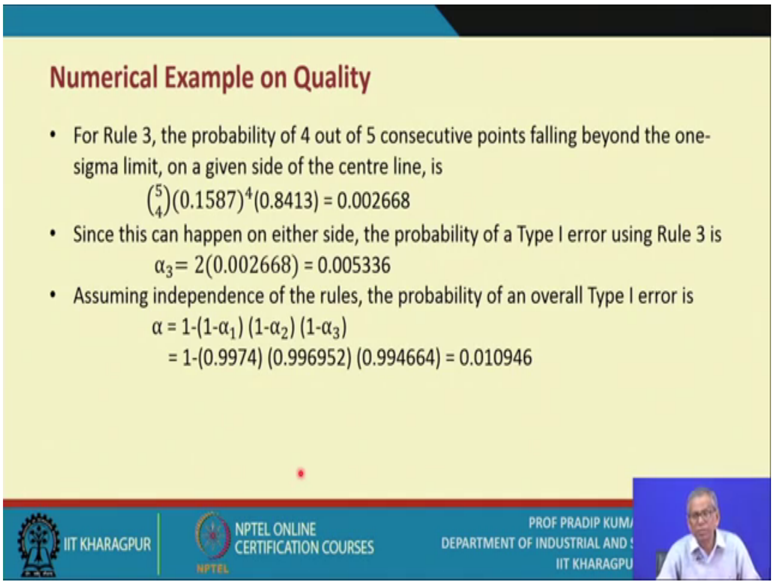

So, same approach we follow for rule 3. So, for rule 3 the probability of four out of 5 consecutive points falling beyond the one sigma limit on a given side of the centre line is again 5 combination 4 5 c 4. Now what is the probability that a sample point falls beyond one sigma limit, is it from the centre line. So, on one side, it will be 0.1587 and if you consider the other side it will be 0.3174. So, we are considering on one side of the centre line that is why it is 0.1587, is it and four points out of 5 consecutive points; that means, 0.1587 to the 4 and the fifth point may be anywhere; that means, 0.8413, is it ok.

So, this value is this one and since this can happen on either side of the probability of a type one error using rule 3; that means, twice of this value; that means, two times of 0.002668; that means, alpha 3 is 0.005336 assuming independence of the rules that; that means, as a first approximation we say that the independence condition holds the probability of an overall type one error is; obviously, this is 1 minus alpha 1 into 1 minus alpha 2 into 1 minus alpha 3; that means, this is the product terms, is it ok, 1 minus of this one; that means, you compute and ultimately what you find that the overall type one error probability is 0.010946, is it ok, what is this probability 1 minus alpha 1; that means, the point will not be falling, is it ok, what is the probability that this error will not be made.

What is the probability that the second type of error will not be made, what is the probability that the third type of error also will not be made, is it and all 3 you know there all independent. So, the; obviously, it is a product term and then what is the probability that there will be error; obviously, one minus of not making the probability of not having probability of making overall type one error, is it ok. So, that is the probability of making an overall type one error.

So, that is why this the product terms are used ok, all these individual probabilities are multiplied. So, this is overall probability 0.010946; that means, it is very very high; that means, in the 10 percent of 10 percent of the cases, is it ok, you will be making you know the type one error probability; that means, it is basically is a false warning ok, you are not going to lose, but what will happen that unnecessary will be spending time on identifying the assignable causes and in many a time find that this assignable causes are basically not there; that means, the process is not has not gone out of control, is it ok..

So, we if you have this high probability of making type one error then you become the entire process when you talk about online real time control of the process. So, entire the systems control may be may become very unproductive is it ok.

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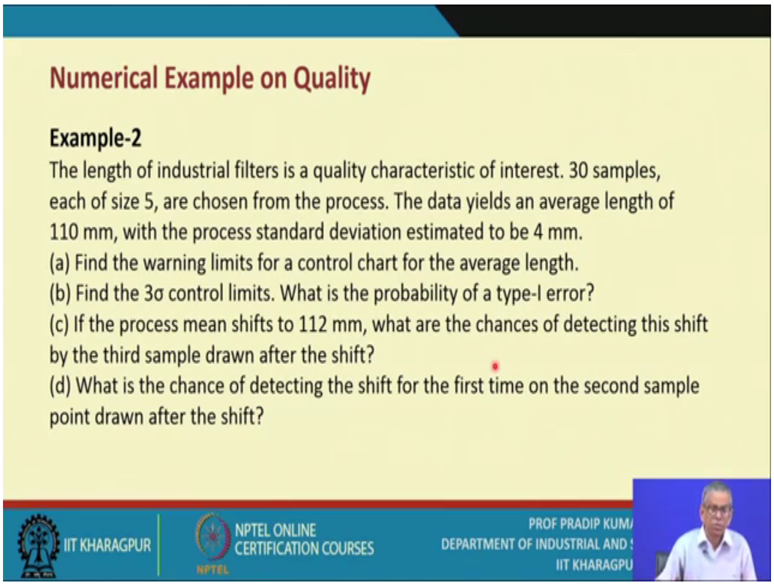

Now, let us talk about the next example, is it ok, the length of industrial filters that is say particular product is a is a quality of characteristics of interest, is it ok, 30 samples each of size 5; that means, whenever you design a control chart you have to specify the value of the sample size, ok, there are certain rules there are certain guidelines, we have already discussed and you also must be able to identify or you say specify the number of samples. So, here thirty samples we have decided each of size 5 the sample size is 5 are chosen from the process.

The data is an average length of 110 millimeter with the process standard deviation estimated to be 4 mm; that means, the process average is known as well as the process standard deviation that is also known, find the warning limits for a control chart for the average length, it is very very simple. So, usually the warning limits are set at plus minus 2 sigma limits on the centre line.

Find the 3 sigma control limits what is the probability of a type one error if the process means shifts to 112 millimeter; that means, as soon as you say that the process mean shifting to another value it is not 110; that means, you are saying that the process has gone out of control; what is what are the chances of detecting the shift by the third sample drawn after the shift, what we have mentioned there is a typical control chart the shewhart control chart if the shift magnitude; that means, is small; that means, 110 and with respect to 110 millimeter, you are constructing control chart assuming that the process exist at 110 mm around, ok.

Now, suddenly there could be shift there could be many reasons ok, the sudden shift if you refer to you know the control chart patterns, we refer to the sudden shift continuous shift continual shift and the possible reasons you need to identify, is it ok. So, suppose those reasons do exists and there is this small shift.

Now, usually what happens if you use the traditional Shewhart control chart is a this probability or the probability is very very less that this, the shift of small magnitude will be detected now. So, what we are going to calculate; that what is the probability that this shift may be detected by the third sample drawn after the shift.

Then the last question is; what is the chance of detecting the shift for the first time on the second sample point drawn after the shift. So, this is another case.

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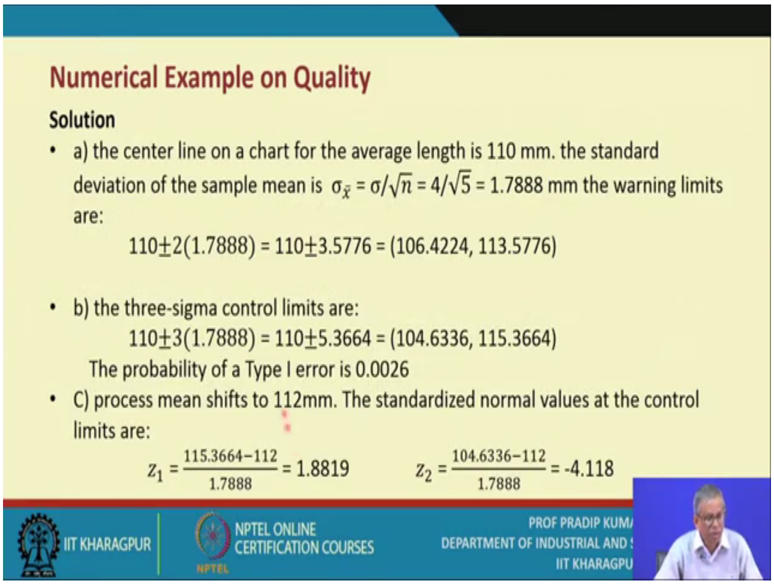

So, how do you get a solution we follow the steps the centre line on a chart for the average length is 110 millimeter already this value is given.

The standard deviation of the sample mean is this one; that means, the sample standard is population standard deviation or the process standard deviation is four that is sigma and the sample size is 5 so; obviously, sigma x bar will be four upon root over 5 that is 1.7888.

The warning limits are this is 110 that is a centre line plus minus 2 sigma. So, 2 sigma x bar that is 2 into 1.7888; that means, this you have these two values; that means, warning limit on one side and this is the warning limit on the other side 106.4224 and 113.5776, is it ok, essentially these are plus minus two sigma limits.

The 3 sigma control limits are; obviously, you know 110 that is process mean sample mean and this is basically the process the sample standard deviation; so, 3 sigma x bar. So, ultimately the upper control limit; that means, that plus 3 sigma from the centre line 115.3664 and the lower control limit; that means, on the other side at a distance of 3 sigma from the centre line that is 104.6336.

Now, what is the probability of a type one error that always you can compute assuming normality that is 0.0026 is it ok; that means, what you need to do you need to calculate z values z values, is it at the lower control limit and you just compute area under the curve beyond z values and you note down. So, it will be 0.0013 and similarly you compute the z value at UCL that is 115.3664. So, you will find the z value will be 3 plus 3 and if you refer to the standard normal distribution, you will find that the corresponding the area under the curve beyond z equals to plus 3 is 0.0013, is it ok.

So, the total area if you compute this will be 0.0026. So, the process means shifts to now the next problem is that the process mean now has shifted 212 millimeter there could be many reasons. So, the standardised normal values are the control limits are. So, z 1, it is 115.3664 minus 112 that is the change value of the mean the standard deviation remain same that is the assumption.

Similarly, at the lower control limit that is this one that is 104.6336 you calculate z 2. So, you take the take it is the difference from the change value of the mean that is 112 and standard deviation remain same so; that means, it is minus 4.118.

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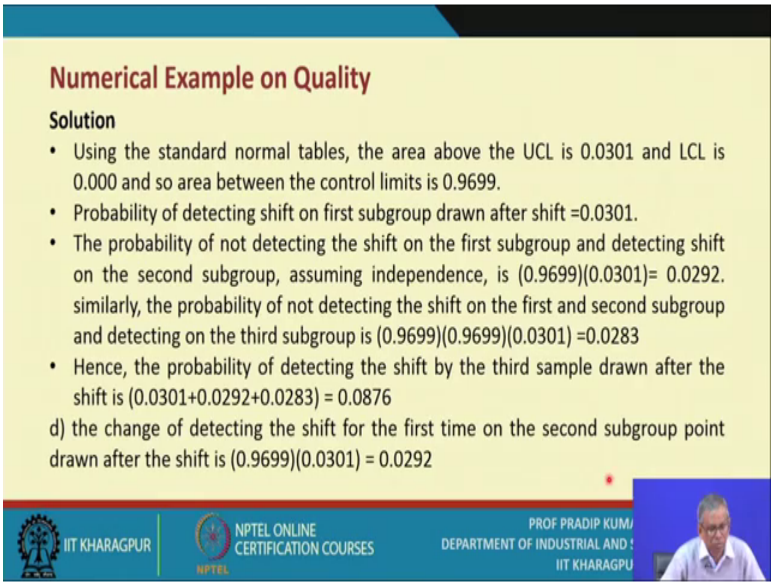

Then you refer to the standard normal table and the area above the UCL is this one that you can read this value in a standard normal table and area below LCL is almost 0 up to third decimal place, it is 0 and so, area between the control limits is this; that means, 1 minus 0.0301 plus 0.0000. So, 0.9699.

So, probability of detecting the shift on the first subgroup drawn after shift that is this one; 0.0301 the probability of non-detecting the shift on the first subgroup and detecting the shift on the second subgroup assuming independence is this one; that means, in the first sample drawn after the shift there is no detection that probabilities 0.9699, but at the second sample you could detect this the shift and that probabilities 0.0301.

So, by second sample what is the probability of detection that is 0.0292 similarly the probability of non-detecting the shift on the first and the second subgroup that is this one 0.9699, 0.9699, ok, you are unable to detect the shift, but as soon as you reach the third one ok, you are third sample you are able to detect the shift the corresponding probabilities 0.0301, what we are assuming that all the sample point. So, the samples are all independent with one another.

So, the corresponding probabilities this hence the probability of detecting the shift by the third sample drawn after the shift; that means, it is there are 3 types of events; that means, it can be detected at the first sample point that is 0.0301, it can be detected by the second sample point that corresponding probabilities this one and it can be detected by the third sample. So, all possibilities you have to explore. So, if you add these 3 probabilities then you have the total probability is 0.0876.

The change of detecting the shift for the first time on the second subgroup point drawn after the shift is this one already you have calculated; that means, by the second sample only ok, you are detecting by the second samples.

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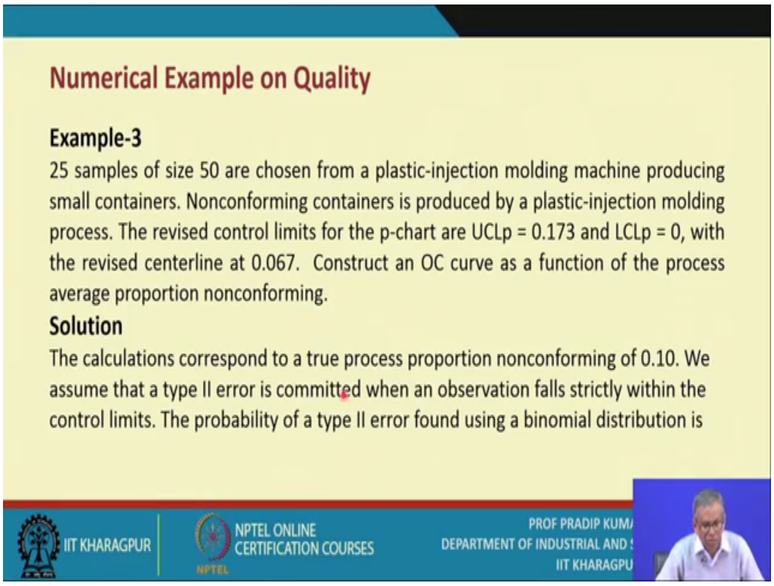

So, corresponding probabilities 0.0292 ok, now one more problem, I am going to discuss that is example 3 within the next few minutes of time 25 samples of size 50; that means, sample size is 50 are chosen from a plastic injection moulding machine producing small containers nonconforming containers is produced by a plastic injection moulding process the revised control limits for the p chart are this one, is it ok; that means, the final control limits are this upper control limit for the p control chart that is 0.173 and the lower control limit for the p control chart that is 0 with the revised centre line at 0.067.

Construct an OC curve OC curve already we have mentioned that with respect to the control chart, you know we refer to OC curve and OC curve is considered as one of the performance measures of a control charts. So, the other performance measure is here we will curve average round length.

So, now we are we need to construct the OC curve as a function. So, of the process average proportion nonconforming that is essentially is a plot of beta versus the process parameter. So, how do you calculate this.

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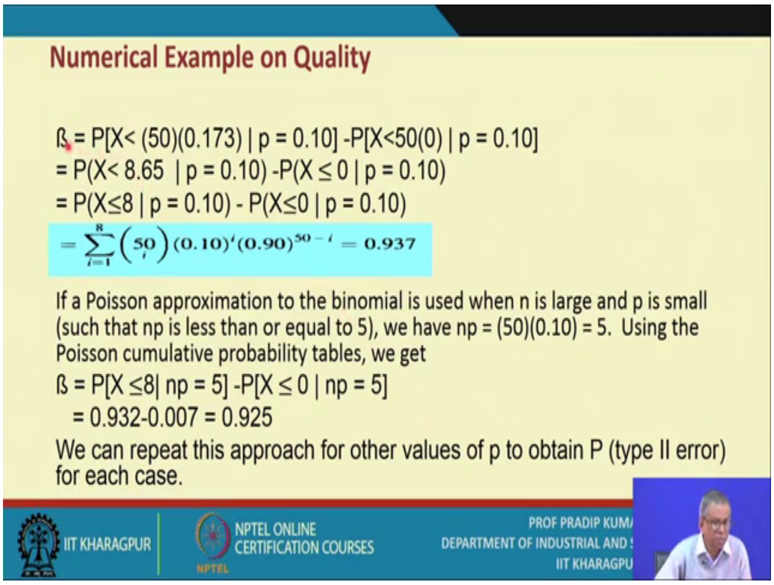

So, so, this is the expressions for the beta; that means, the value of x is this one, is it ok, it is 15 2.173 UCL for a given value of 0.10, you need to consider several values of P minus this one; that means, x is less than equals to 15 to 0; that means, what is the probability that even if there is a shift in the process parameter value; that means, suppose the current value is 0.10.

What is the probability that the sample point will be plotted within the area defined by the upper control limit and the lower control limit is it ok. So, this is the say the beta function for a specific value of p you compute this one you follow the steps and then assuming that that nonconforming with proportion nonconforming is binomial. So, this is the expression; that means, the probability you calculate as 0.937 you just follow the steps and when you try and go for Poisson approximation for the binomial.

So, what you try to do; that means, you say that I will convert this into from say you know certain rules you prescribe the later on we will refer to; that means, 8.65; that means, Poisson distribution is for the discrete random variable so, but this is not discrete this is a fraction. So, you say that less than equals to 8 whereas, this is 0. So, that is an integer. So, it is less than equals to 0. So, I refer to the cumulative Poisson distribution table and corresponding value of beta again that is 0.925, is it ok.

Many a time as you are using the cumulative Poisson distribution table is easier than using ah. So, the binomial distribution table and many a time the Poisson approximation to the binomial the condition is such that is approximation is allowed or it is it is well assumed then you refer to the Poisson cumulative Poisson distribution table to compute such probabilities.

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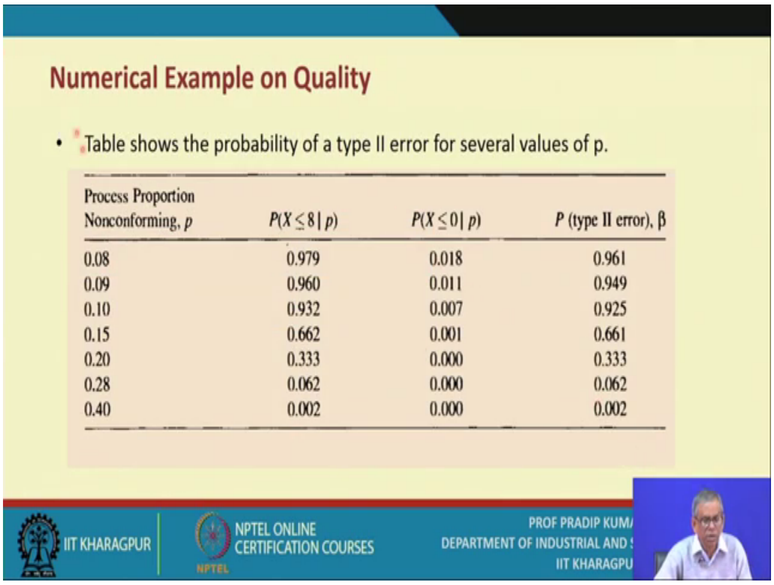

So, we can repeat this approach for other values of P to obtain P for each one. So, this way you construct for the values; that means, these are the possible values of P and then we compute the values of beta.

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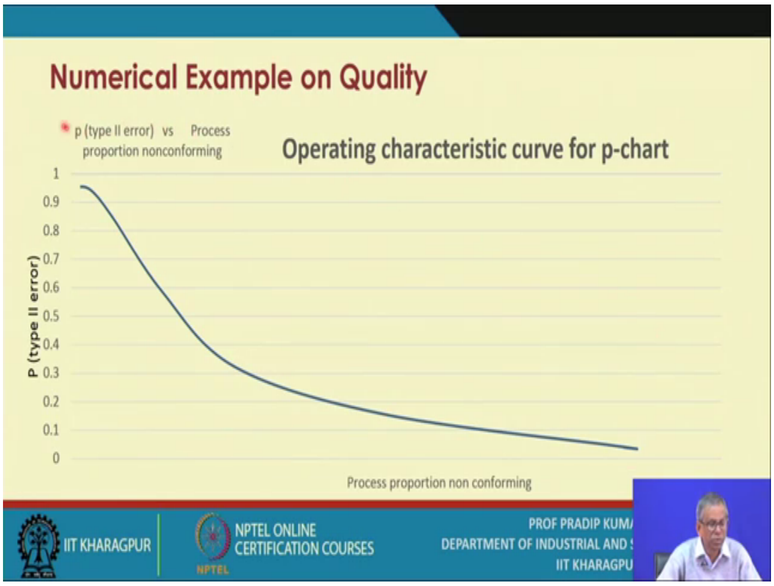

And once beta is computed then you say this is the probability of making type one type two error; that means, this is a probability of non-detection and you consider on the x axis several possible values of P. So, process proportion nonconforming which is visible.

So, suppose 10 or 15 such values you consider on one side. So, is a typical OC curve you will get operating characteristics curves for the P chart ok.

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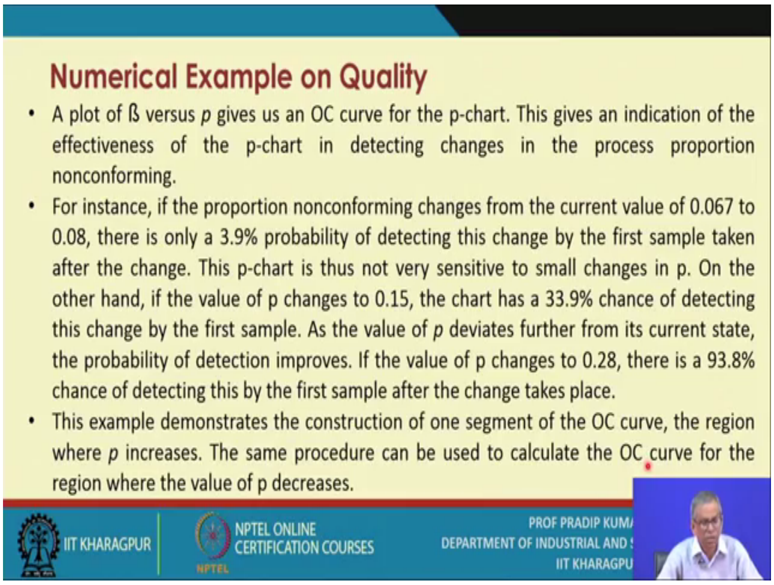

So, the other examples we will we will refer to in the subsequent classes. So, a plot of beta versus P gives us OC curve for the p chart ok. So, now, you need to interpret this control chart please go through all these details, is it ok.

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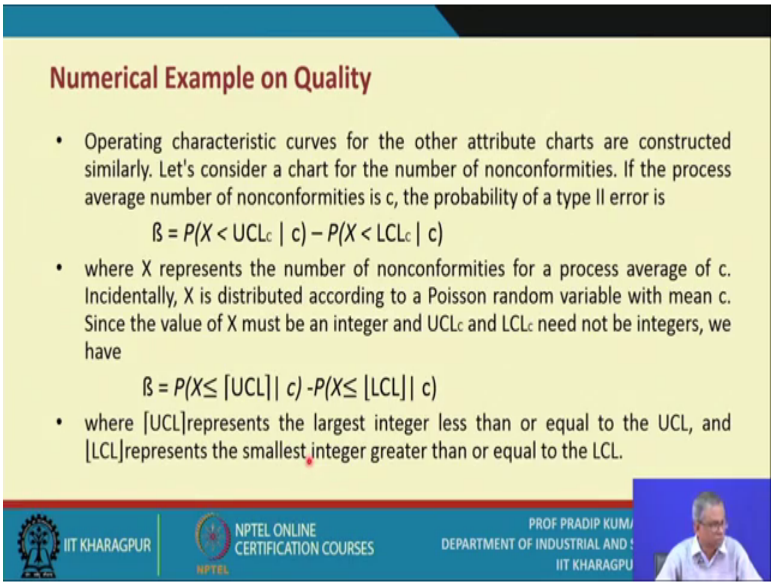

So, this way you interpret the OC curve for the p control chart is right. So, with the other you know the examples, we will be referring to in the subsequent the lecture sessions.