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# Lecture - 46 Design for Reliability-II

During this week, we will be discussing the design for reliability.

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Design for Reliability-II
✓ Lecture-1: Reliability of Systems: Systems with Standby Components, How to Improve Reliability of a System
✓ Lecture-2: Reliability and Life Testing Plans; Measurement of Reliability using Life Testing Plans assuming Exponential Distribution, Operating Characteristic Curve
<ul> <li>✓ Lecture-3: Numerical Problems on Quality and Reliability-I</li> <li>✓ Lecture-4: Numerical Problems on Quality and Reliability-II</li> </ul>
✓ Lecture-5: Numerical Problems on Quality and Reliability-III
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Already in the in the previous week, we have referred to several you know aspects of reliability modelling. Now by during this week, again there will be 5 lecture sessions on reliability.

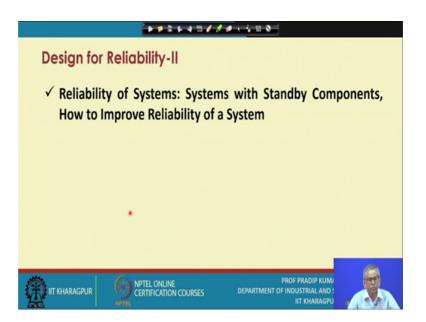
So, let me first tell you, what are these topics we are going to cover during first lecture, we will be referring to again reliability of the systems and particularly one particular say important subsystems or the systems, we will be referring to that is systems with standby components; how to improve reliability of a system. So, this you know; you must have several ideas and these ideas, first, we you know, we will be we are supposed to use those ideas to improve the reliability of a system as well as the reliability of the component. Now, what are these methods or what are the approaches which you can improve the reliability of a system.

Ah in lecture 2, I will be discussing reliability and life testing plans; that means, the main

issue is how to get an estimate of the reliability of a component or a system. So, which procedure, you should employ to estimate reliability and for which many a time, we refer to reliability and life testing plans. There are many types of life testing plans, we will be discussing all these plans one by one and essentially, we will be focusing on measurement of reliability using live life testing plans assuming exponential distribution and we will be also referring to the operating characteristics curves like we have for you know the sampling plans as well as for ah. So, the control charts ok.

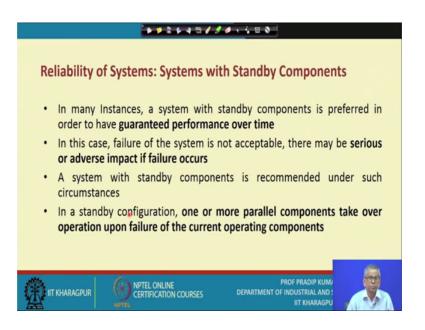
So, the OC curve is basically represents the performance measures of such a sampling plan or other, this basically a sampling plan and we in this context, we say that is a life testing plans lecture 3, lecture 4 and lecture 5, this 3 lecture sessions will be exclusively denoting solving numerical problems on quality and reliability.

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So, this is our plan. Now, let me first discuss the first topic that is the reliability of the systems with standby components and how to improve reliability of the systems, what you find that when you talk about a system.

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Now, as you are all aware of; the a systems may have different kinds of say the configuration one such configuration is a series configuration, the second one alternative to series, it could be parallel or in many a time majority of the systems, we will have both a series as well as the parallel configurations.

Now, what actually you need to do there are certain systems where you know that the failure of failure is just not acceptable and if the failure occurs, there will be serious consequences like say, suppose say the operation theatre as a system, ok. So, suppose the power line goes off the main line goes off. So, you can well imagine, what could be consequence. So, you have to think of the changing configuration of the system in such a way that even if the main power line is fails, then alternative you know the power lines you must be made available at immediately as quickly as possible.

So, when you talk about such a system, what you need to do; that means, you must have a standby component. So, in this lecture, we will be referring to how to model the reliability for a systems with standby components, ok. So, this I am going to elaborate like in many instances, a systems with standby components is preferred in order to have guaranteed performance over time; that means, there must be guarantee that come whatever may have the performance has to be assured. So, that kind of system, you want to develop.

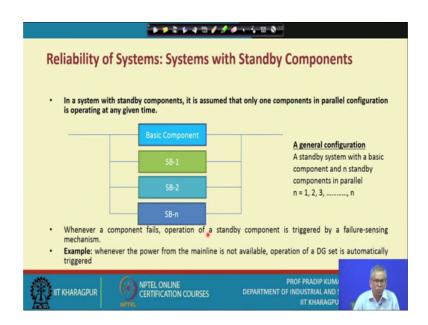
So, many a time, a system with standby components, you have to develop in this case the

failure of the system is not acceptable, this point already I have highlighted. There may be serious or adverse impact if failure course, is it ok, you can identify such systems wherein plenty ok.

So, what you as a learner, as a student, you must be able to identify those systems where the standby components are included ok, a system with standby components is recommended under such circumstances is ok. So, why you do go for the standby components so that you know when the system you allow with redundancy is it ok. So, we will be referring to the level of redundancy later on.

Ah so, in a standby configuration one or more parallel components take over operation upon failure of the current operating components, is it ok, I repeat one or more parallel components; that means, you have a basic component and basic component is linked with one or more parallel components that could be one the standby or there could be in general n number of standbys, ok, depending on you know the systems you are you are modelling. So, as soon as the basic component fails. So, one or more parallel components will take over operation and then; obviously, you know the reliability is assured is it ok.

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So, ah; that means, the services available operation is made available all the time. So, that is that is the guarantee you must provide.

Now, in a system with standby components, it is assumed that only one component in parallel configuration is operating at a given time, is it ok; that means, there could be n number of components, but what we are assuming that for the systems to run at least one component must run, is it ok.

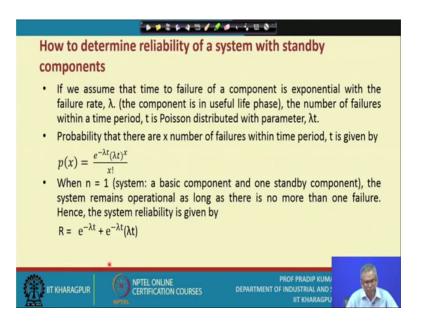
So, what is the configuration? You have the basic component and what you have added, you have standby component. So, one standby second standby and this way, you can add the nth standby components, is it ok; that means, if. So, there are at any instant of time, even if there are n number of failures suppose, n is 6 or n is 10; that means, that there is a probability ok, the probability could be very very less, but still there is a probability that suppose n equals to 10; that means, all the 10 units may fail. So, so the basic component is working or out of say you know 11, say the elements you have suppose n is equals to 10. So, out of 11 the components, you have in a system at least one systems must run, is it ok.

So, it is a general configuration a general configuration you know if you look at; you will find is a standby system with a basic component as I have already mentioned and in standby components in parallel, is it ok, the parallel systems already we have explained and n could be 1, 2, 3 up to n, is it ok.

Whenever a component fails operation of a standby component is triggered by a failure sensing mechanism; that means, the systems must be made automated, is it ok, for example, whenever the power from the main line is not available operation of a digital generator set is automatically triggered, is it ok, there are many such you know the examples you have where we say that the we have created a systems with you know standby component ok.

So, now the question is how to measure the reliability of such a system, is it ok?

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Now when we talk about say the determination of reliability of such a system with standby components, now we need to assume that the components you know existing. So, at this you are getting operation from the component and the component you know existing the which phase; that means, there are 3 phases, I have already mentioned like when you talk about the life cycle curve what you find that initially you have a burn in phase, then once this phase is over, then you have the useful life phase and then when the useful life phase is over, then you have actually the wear out phase.

Now, whenever you try to create a system make sure that the components are useful life phase, is it and if the component is in useful life phase as for as failure rate is concerned failure rate is assumed to be constant and failure rate should be as many as possible, is it ok. So, that assumption is valid in the majority of the cases; that means, constant, you know the time to failure say or say constant the failure rate that assumption is true that assumption is valid.

So, if you assume that the time to failure of a component is exponential is a particular case of binomial distribution binomial density function with the failure rate lambda, lambda is assumed to be constant. So, the component is in useful life phase as already mentioned the number of failures within a time period T is Poisson distributed with parameter lambda t, is it ok.

So, Poisson distribution is also a distribution for the discrete random variable and with

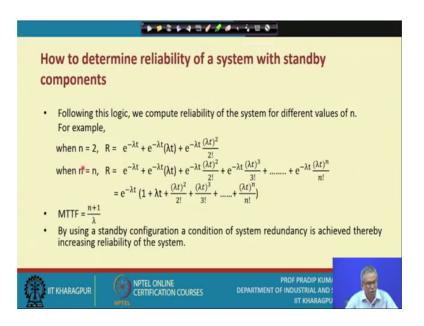
single parameter. So, that parameter is lambda t, is it ok. So, there is a proof behind this like say if the ah; that means, time to failure of a component you have to prove it, is it ok. So, this could be one of your assignments if the time to failure is exponential the rate; that means, the number of failures within a given time period say T is Poisson distributed; that means, distribution is Poisson with parameter lambda t, is it ok.

So, this is you can definitely you can assume probability that there are x number of failures within time period t is given by this is the is basically is a Poisson you know the mass function that is px that there will be x number of failures e to the power minus lambda t lambda t to the power x divided by factorial x is it when n equals to 1; that means, what; that means, you have a basic component with one standby a basic component and 1 standby component in the system, the system remains operational as long as there is no more than one failure, is it ok, you have to ensure that the operation is made available at any point in time, is it ok.

So, that is why you know you are proposed a system with standby, is it ok. So, hence system reliability is given by; that means, this is the probability that there will be 0 number of failures and this is the probability that there will be one number of failures, is it ok.

So, you have just one standby with one basic components. So, either basic component will run or say the standby component will run in parallel so; obviously, the expression for reliability is e to the power minus lambda t plus e to the power minus lambda t into lambda t, you are just applying this formula, is it ok; that means, you are assuming that Poisson probability mass function is valid, is it clear.

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Following this logic, we compute reliability of the system for different values of n as I have mentioned that the value of a value of n may vary from say 1 to n. So, for example, when n equals to 2 that are; that means, at any point in time ok, up to number of failures, is allowed. So, there are three components in parallel and to run the systems at least one component must run out of this three one is basic and 2 R 2 R you know the standby components.

So, when n equals to 2, what is the expression for R e to the power minus lambda t plus e to the power minus lambda t into lambda t plus e to the power minus lambda t into lambda t square divided by factorial 2. So, when n equals to n; that means, in general, what will be the expression, this will be your expressions; that means, you proceed up to nth term; that means, e to the power minus lambda t lambda t to the power n divided by factorial n; that means, how many components you have in the systems there n plus 1 and at least one component must run. So, that the system can run this is the condition.

So, up to n number of failures ok, you may allow, is it ok. So, what is the probability, right. So, this is the two probability so; obviously, e to the power minus lambda t is comma. So, the entire expression becomes; that means, in series e to the power minus lambda t into one plus lambda t lambda t square divided by factorial 2 plus lambda t cube divided by factorial three and then you proceed ultimately which the nth term that is lambda t to the power n divided by factorial n, is it ok.

Now, what is the mean time to failure; you know how to compute the mean time to failure and not only mean time to failure, you also know the basic expression for computing the sigma square of the variance, is it ok. So, here mean time to failure you can compute as n plus one divided by lambda is it ok.

So, what is n; that means, number of the standby components in the systems by using a standby configuration a condition of system redundancy is achieved thereby increasing reliability of the systems; that means, you know many you know many a time the question is posed that how to improve the reliability of the system, there could be many ways you can improve for the reliability of the systems will identify those ways or those methods very soon one such method with which you can improve the reliability of the system is I know by adding the system redundancy, is it ok.

So, you know; that means, you must have ah; so, the extra protection against failures is it ok. So, you need to there could be you know the different levels of redundancy in a systems. So, you refer to that.

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High-Level versus Low-Level Redundancy
<ul> <li>System redundancy may be obtained by two methods. Each component comprising the system may have one or more parallel components, or the entire system may be placed in parallel with one or more identical systems. The first method is known as low-level redundancy, and the second is called the high-level redundancy.</li> </ul>
<ul> <li>Consider a system comprising two serial components, A and B. Fig. 1 shows the system having low-level redundancy, and Figure 2 shows high-level redundancy.</li> </ul>
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Fig. 1. Two components in a Low-level redundancy Fig. 2. Two components in a High-level redundancy
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So, as for as redundancy in the system is concerned, we there could be the two level or the two types of redundancy. The first one is referred to as a high level redundancy and the second one is the low level redundancy.

Now, let me explain these the two types of redundancy for a system redundancy may be

obtained by two methods in the first method, what you do; that means, each component comprising the system may have one or more parallel components, is it ok; that means, the extra protection is to be given that is the first approach.

The second one is the entire systems may be placed in parallel with one or more identical systems; that means, here in the first case what you are trying to do; that means, you are in a system, we mention you are not adding a new system, what we are trying to do; that means, against each component in the systems you are adding one more component one or more components in parallel whether in the second systems, what you are trying to do; that means, you add that instead of having just one system with a particular component configuration same configuration, is it ok; that means, another systems with the same component configurations you are adding n series, is it ok.

So, this two types of redundancy ok, you add many a time, you do it and so the first method is known as the new level redundancy; that means, you are not adding a another; the same systems ok, you are not adding the system, what you are adding, you are adding the component in the system in the subsystems. So, that is referred to as new level redundancy, whether the second one is called the high level redundancy, is it clear, I think it is very it is well explained and it is clear, it is clearly understood.

Now, consider a system comprising two serial components a and b. So, this is component a, this is component b. So, what we are trying to do; that means, initially you have these series systems. Now in order to you know say add redundancy of the systems, what you have made; that means, against a component you have added another, you know this identical component called a and against b again, you have added you know another component called b and in the same you know you have added within the system you know the same subsystems that is a series configuration, is it ok.

So; obviously, this is you know with respect to the few components a and b we are referring to the low level redundancy; that means, within the system whereas, here what you are trying to do; that means, the two components in a high level redundancy here you have to this, this was the system you have added another systems is it ok.

So, this is basically high level redundancy; that means, the systems will run only when say one of these two will run and now when add the system what will happen; that means, one of this you know two components will run in the systems. So, we added them so; obviously, this is referred to as the high level redundancy with respect to the systems and this is referred to as the low level redundancy because it is referred to the component level, is it ok.

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High-Level versus Low-Level Redundancy
<ul> <li>If we assume that both components have the same reliability R, then the system reliability for the case of low-level redundancy is given by</li> </ul>
$R_{low} = [1 - (1 - R)^2]^2 = [1 - (1 - 2R + R^2)]^2 = (2R - R^2)^2$
For the high-level redundancy, system reliability is given by
$R_{high} = 1 - (1 - R^2)^2 = 1 - [1 - 2R^2 + R^4] = 2R^2 - R^4$
<ul> <li>Comparing the both equations it can be shown that the reliability of the low-level redundancy is greater than the reliability of the high-level redundancy.</li> </ul>
$R_{low} - R_{high} = (2R - R^2)^2 - (2R^2 - R^4)$
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Now, how to you know the compute the reliability of the systems, when there is a low level redundancy and similarly how to compute the reliability of the system, when there is high level redundancy. So, with respect to these two components, we have computed if we assume that both the components of the same reliability R, there is a particular case is in general the say component reliability may vary from one component to another, but here for you know the simplicity for illustration purpose what we assume that you know the component reliabilities are same.

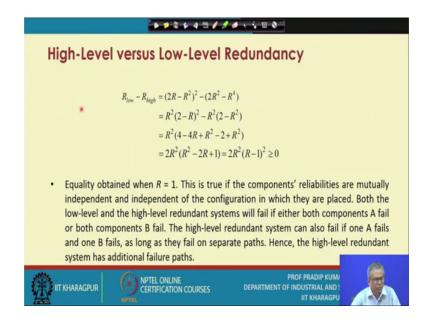
Then the system reliability for the case of low level redundancy is given by this is it is a parallel systems. So, what you have one minus one minus r square through a square so is a low level redundancy. So, ultimately you know you simplify this. So, this will be your reliability for the low level and for the high the redundancy systems reliability is given by 1 minus 1 minus R square equals to this one, is it ok. So, this is 2 to the power 2 into R square minus R to the power square ok.

Comparing the both equations, it can be shown that the reliability of the low level redundancy is greater than the reliability of the high level redundancy is clear. So, R low minus R high is equals to ultimately, you have this expressions, in this particular case, is

it ok. So, this way you compute; that means, you try with first you know high level redundancy and again, you try with low level redundancy, and then you compare so which particular system is having you know the higher reliability. So, you opt for that one is it ok.

But only thing is you make sure that if there may be you know the high level redundancy, there may be low level redundancy, but make sure that whatever the system your proposed.

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It is it is cost effective is you get what cost. So, high level versus low level redundancy when you when you refer to. So, you have this expressions the final expressions and it should be greater than equals to zero makes sure is it the differences greater than equals to zero; that means, there when you move from say high level to low level is it or you move from say low level to high level is it when the exact values of the of the component reliabilities are known so ok.

So, if you find this is this greater than or equal to 0; obviously, you know you opt for low level a low level redundancy otherwise you will opt for you know high level redundancy for the time being equality obtained when r equals to one this is true if the components reliabilities are mutually independent that that assumptions we are making is it, ok, there could be relaxation of these assumption; that means, this independence conditions may not be valid or may not be true so, but here at the initial stage as a first approximations

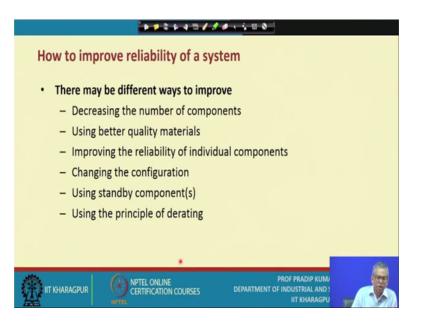
for the given systems, we are assuming that the components reliabilities are mutually independent and independent of the configuration that is very very important in which they are placed, is it ok.

This is this is the basic assumptions, we make both the low level and the high level redundant systems will fail if either both components a fail or both components b fail is it ok. So, whenever you create a configuration when you create a systems and you where look at the system from say the reliability perspectives you must know that under what conditions your systems will fail all together completely is it ok. So, those conditions, you should be aware of and normally we go to the component level ok. So, the system fails because the component a particular component or a group of the components fail at a particular point in time.

So, under what conditions they may fail. So, you must be aware of high level redundant systems can also fail if one a fails and one be fails as long as they fail on separate paths is it ok. So, this is the conditions. So, you must know. So, you must study minutely the configuration of a system is it whether it is with low level redundancy or high level redundancy or no redundancy at all and then you need to reach down the conditions in which the system may fail?

Hence the high level redundant systems has additional failure paths ok.

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So, after we considered this, now what you need to know at this level because we have covered many aspects of reliability modelling. Now at this point in time, you must know that how many different ways you can improve the reliability of a system, is it now; obviously, you know what we are assuming that the system reliability depends on the component reliability.

So, if the component reliability is poor whether you opt for a series configuration or say the series structure or parallel configuration or parallel structures or combination of both also you know you opt for by the systems with redundancy a components or this is a standby components you cannot assure high reliability for the systems; that means, the component reliability you can improve.

So, and as you might have noticed that if you deal with more number of components like say in a particular case like say is a is a missile system is having some around say 2000 components and most of this components are in series. So, what we need to do; that means, through redesign approach whether from the reliability point of view whether you can reduce the number of components in the redesigned system.

So, the first option is you have to think of ways and means you have to think of approaches with which you can decrease the number of components is it ok. So, even if you decrease, even if it is a series system, if you reduce the number of components you will find that the system reliability may improve.

So, that is the first option you have to explore there are many cases the design improvement project we opt for this one is it under one option could be the instead of having two components why do not you to have say combine component under certain conditions you can combine them until certain other conditions you cannot combine, is it ok. So, those details when we discuss the cases when you discuss numerical problems. So, we will be referring to those alternatives.

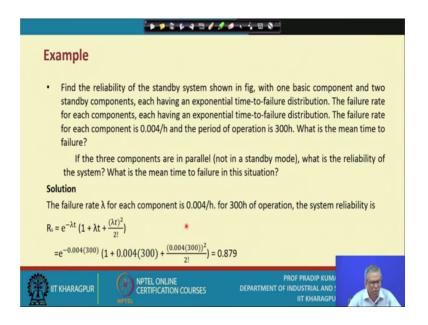
Second option is using better quality materials; that means, you just look at the physical property the chemical composition of the materials which we use in designing the component, is it ok. So, always there is an improvement possibilities or better quality material you can you can use. So, that option always you try is it ok. So, this is another option and normally you know whenever we talk about the reliability improvement we say that we must believe in the concept of design for reliability; that means, even at the

design stage you are already aware of say what are the you know the steps or what are the activities you need to carry out during the product development stage.

So, the design stage so, using better quality materials improving the reliability of the individual components is it changing the configuration; that means, from the series can you move to can you change it to parallel is it if it is parallel can you move from say parallel to parallel width say you know the standby components. So, there are many options using standby components I have already mentioned so that improve the reliability.

And the last one is using the principle of derating; that means, suppose you say that the spindle speed is say five hundred rpm is it ok. So, there is a maximum one, now if you can run it with say three hundred rpm; obviously, you know these are production that may be may be affected, but the reliability of the systems will improve.

So, there are many cases where you say that if I start working with the process with the you know the highest levels of the process parameters the display very very high; that means, the performance of the quality of the item which you are producing may be may be very very high, but there is a risk of is the failure of the systems also will be high. So, in that and those phases these principles of derating we always apply.



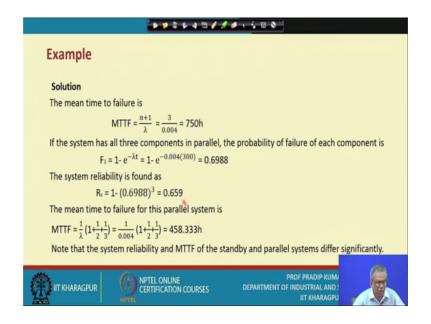
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Now, here is just one example, I will just go through like say find the reliability of a

standby system shown in figure already the figures are shown with one basic component and two standby components; that means, n equals to 2 and we are assuming that exponential time to failure distribution. So, you just referred to this particular problem just go through the failure rate for each component each having an exponential time to failure distribution the failure rate for each component is 0.004 per hour and the period of operation is 300 hours, is it always the time is to be mentioned for computing reliability, what is the mean time to failure.

Now, if alternatively if you go for three components in parallel that sort of system, what is the reliability of the systems, what is the mean time to failure; so, these two we have just applied the formula like you apply the formula with these values you get the system reliability as e to the power minus lambda t into 1 plus lambda t plus lambda t square by factorial 2. So, this is 0.879.

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Next one is the mean time to failure n plus one by lambda that is 750 hours and if the system is all three components in parallel; obviously, F 1 is 1 minus to the power minus lambda t and the systems reliability will be 1 minus 1 upon this one; that means, 0.6988 to the power 3, is it ok. So, 0.659.

So, always, we will find that equivalent parallel system will have less reliability and the mean time to failure for the parallel systems we have already you know, we know that this is the expression for MTTF, is it ok, 1 upon lambda 1 plus 1 upon 2 plus 1 upon 3, is

it ok. So, these values 458.33 hours; so, note that the system reliability and MTTF of the standby and parallel systems differ significantly.

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So, here we stop. So, we conclude this one, we you can refer to these particular reference textbooks.