Quality Design and Control
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Lecture - 45
Design for Reliability-I (Contd.)

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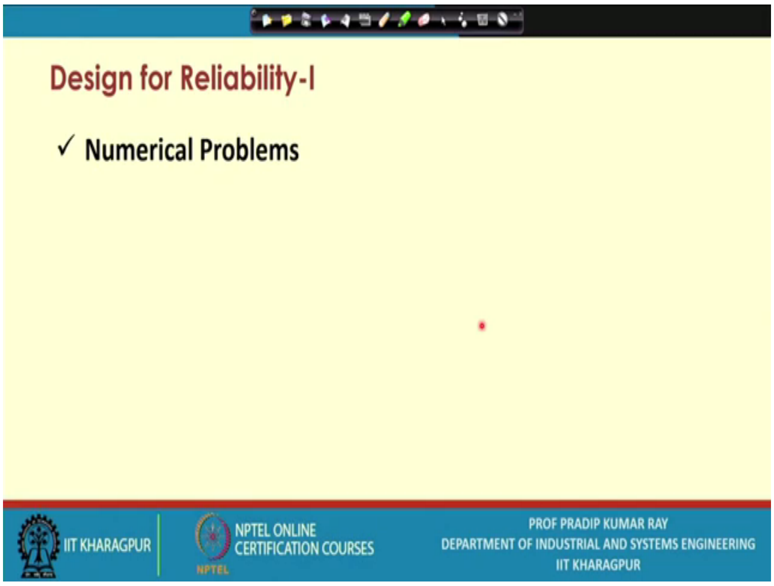

So, in this session, we will be referring to several kinds of numerical problems and we will try to solve them, we have already covered the many aspects of reliability modelling and we have considered a five typical problems related to all these aspects of reliability modelling and we will be discussing this numerical problems one by one ok.

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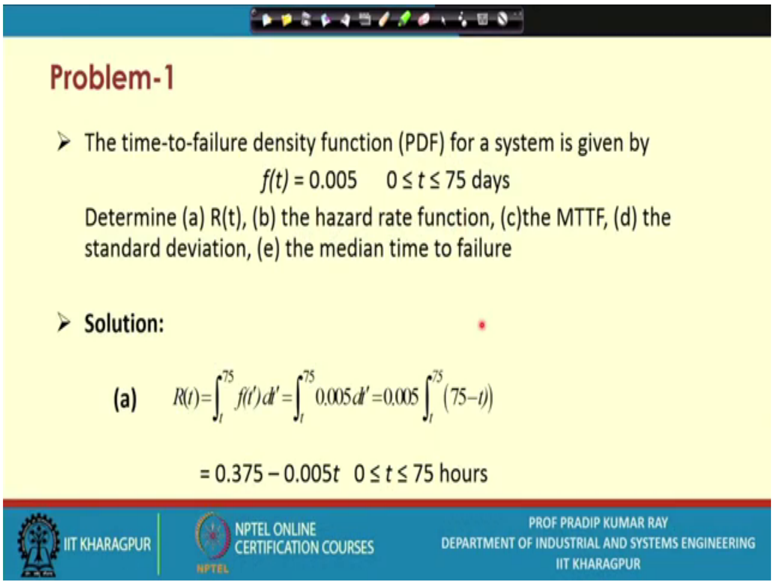

So, similar such problems, you can expect for your, you know the home assignments later on, we will give you the home assignments and so, first you must go through this typical numerical problem. So, I am going to discuss a 5 typical problems related to reliability modelling.

Now, let us talk about the first problem the time to failure density function ok, we have already refer to what is the time to failure density function in respect of reliability modelling, is it ok; that means, this density function must be known first and for knowing this density function you need to collect data, is it ok.

So, later on, we will we will referring to the kind of data you are supposed to collect for for concluding on say time to failure density function. So, time to failure density function for a system, we have already defined the system is given by ft equals to 0.005 0 and t varies between 0 and 75 days, it is fine.

Determine R t the hazard rate function, the MTTF mean time to failure the standard deviation the median time to failure. So, what how do you solve it; that means, that R t is equals to t two up to 75; that means, 75 days it must last it must survive, is it ok. So, that is ft dt t to 75 0.005 dt. So, this is a constant. So, 75 minus t t 75 minus t. So, that is 375 minus 0.005 t. So, this is the expressions for R t; clear.

So, when the t varies between 0 and 75 hours, is it ok.

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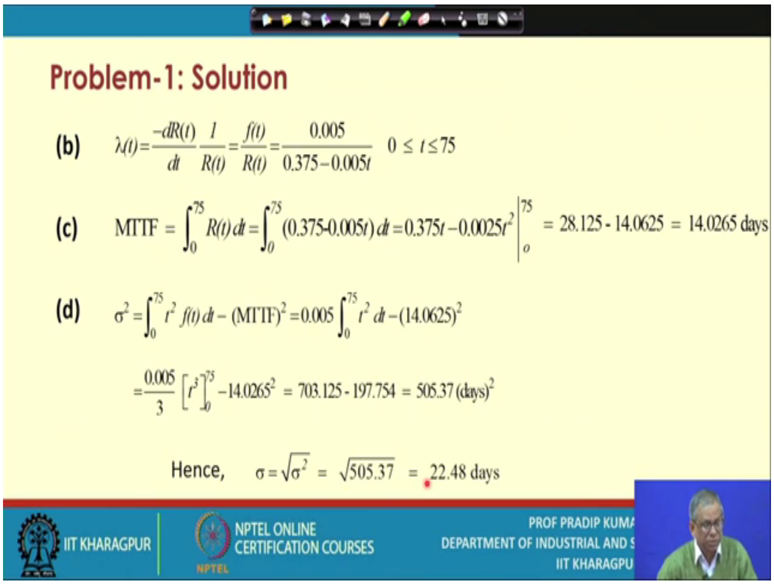

So, this is the first one, then what is the lambda t? Lambda t, if you collect the lambda t that is a failure rate function is ft that is you know the probability density function divided by R t that is reliability function. So, 0.005 divided by now, R t already derive this expression 0.375 minus 0.005 t when t lies between 0 and 75, I think it is very very clear.

Now, how do you compute mean time to failure again 0 2 75 that is must survive up to 75 hours R t dt, is it ok; that means, this is basically the mean value you calculate. So, the that that particular formula you use and R t expressions already we have derived that is 0.375 minus 0.005 t in to dt, is it ok.

So, now you go for integration and you get a value of 14.0265, it is clear. So, this is ah; that means, the mean time to failure is just greater than 14 days is it and what is the variance again you apply this formula t square FTDT minus MTTF square, is it ok, like say mean square mean use. In fact, in general formula; so, ultimately you follow this steps and you say that the sigma square is that is the variance 505.37 are; obviously, the standard deviation is 22.48 days, it is clear. So, that is standard deviation as well as the variance.

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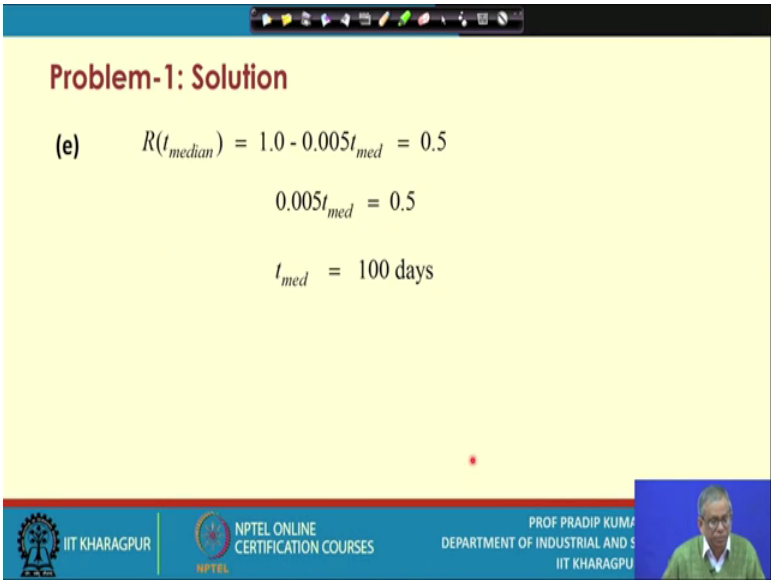

Now, what is the medium time to failure again we apply this formula; that means, one minus 1005 t median. So, t median is already given that is you know the t median is to be calculated and this is 0.5 is right and so, what is the t medium; that means, median time to failure that is hundred days, it is clear. So, this is typical say an example of the numerical problem.

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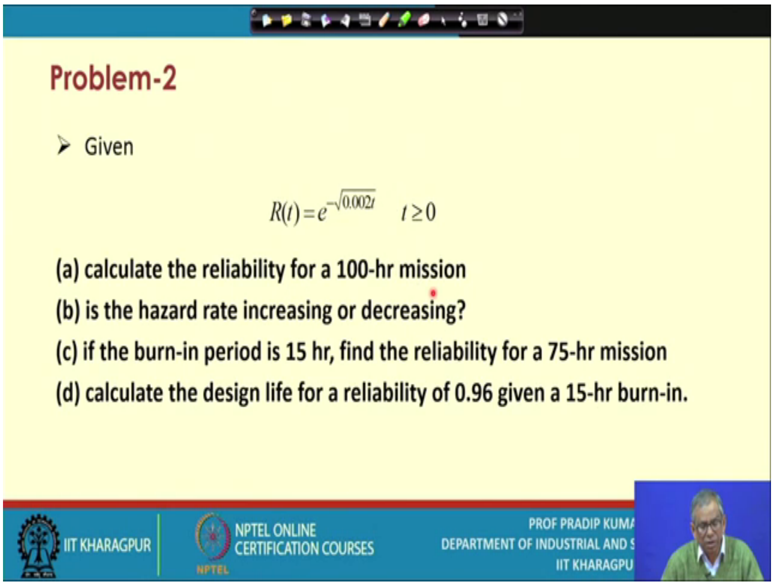

And we have derived the corresponding you know the expressions already, is it ok.

Now, second problem is very is also a typical one like given R t this expression is given e to the power minus root over 0.002 t ok, you know, what t is greater than 0, calculate the reliability for a 100 hour mission, is it ok; that means, what is the probability that it will run for 100 hours, is it or it will survive for 100 hours.

What is the hazard rate is the hazard rate increasing or decreasing, is it ok, we have already explained that what is this implication if the hazard rate increases what does it mean and if the hazard rate decreases, what does it mean; whether under what conditions or what are the possible reasons of hazard rate hazard rate increasing or what are the possible reasons the hazard rate decreasing.

If the burn in period we have already explained what is this burn in period; that means, this is also referred to as oil failure phase or you can say that the child mortality phase. So, if the burn in period is 15 hours find the reliability for a 75 hour mission, is it.

Calculate the design life for a reliability of 0.96 given a 15 hour burn in is it ok. So, ah; so, these are the four the questions we have said.

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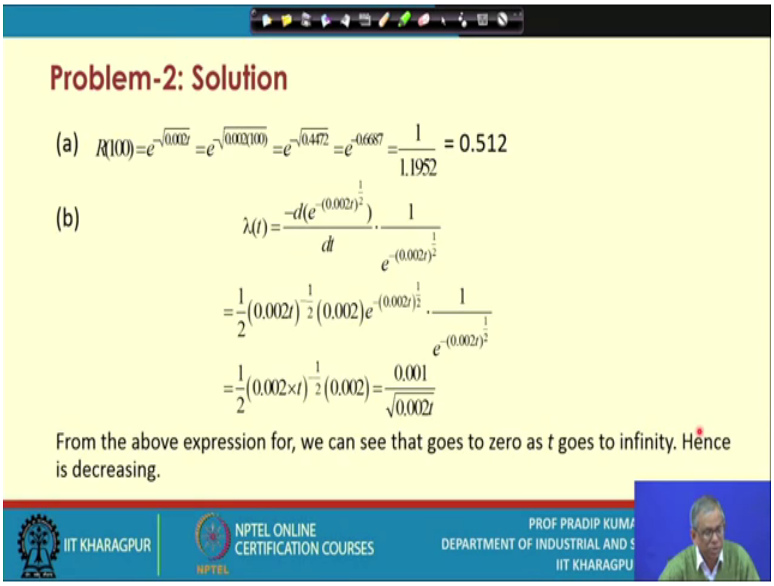

Now, we need the solution. So, against question number one what is the answer is that the reliability that it will run for hundred hours e is to the power minus root over 0.002 into t. So, the t is 100. So, you get a value of e to the power minus 0.6687; that means, it is 0.1512; that means, so, probability is just greater than fifty percent, it is considered to be very very less; that means, you may conclude that this reliability is poor it is may not be acceptable. So, that interpretation you also must have is it you must be whenever you get some result, you try to you know the interpret the value of the result from the reliability perspectives.

What is lambda t first you calculate lambda t ddt of e, is it so; that means, it is root over root over 0.002 t that is why it is written as minus 0.002 t to the power half and lambda t is essentially ft by ft by rt. So, R t is already given and ft is essentially ddt of capital F t, is it ok.

So, ah; so, this is half the ultimately you apply this formula, is it and you get the expressions like this; that means, lambda t is given by 0.001 root over divided by root over 0.002 t, is it ok. So, you follow just you follow this steps and definitely you know I am sure that you will able you know the calculate this value, is it ok. So, you just follow the steps or you just try to you know, get this expressions independently without looking at these steps.

From the above expressions for say lambda t, we can see that that it goes to 0 as t goes to infinity, hence, it is decreasing hence, it is decreasing, is it ok. So, so, the t is in the denominator. So, if the t increases; obviously, you know the lambda t decreases, is it ok. So, failure rate failure rate is decreasing over the time period ok.

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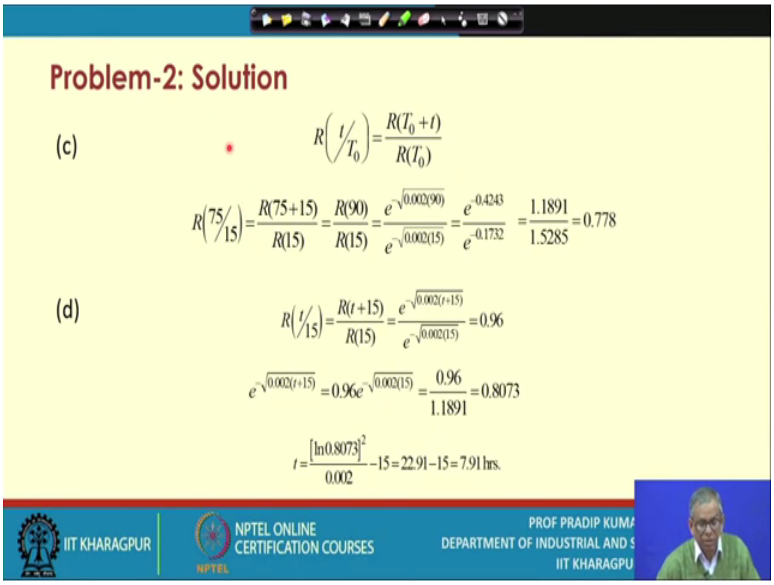

So, that is your observation. So, that is your interpretation right.

Now, this is the expression that is the conditional on the reliability. So, R. So, reliability t plus T o plus t provided that it has survived for T 0 time period is it ok. So, what is T 0; that means, it is burning inter phase, is it ok. So, what is a burn in burn in time that is the 15 hours. So, it has already survived; that means. so, 15 hours, it has survived it has not failed and then only, it reaches in the useful life period and what you are trying to conclude; that means, 75 hours in the useful life phase..

So, the total timing is 75 plus 15, is it so; that means, it is 90 hours or the 90 days whatever may be the unit of measurement and then you apply the formula assuming the constant failure rate. So, this is the timing is 15 and this timing is 90. So, just you have this one; these value one point one eight nine one in the numerator and 1.5285 in the denominator. So, you get a value of 0.778. So, it is clear. So, your reliability is just you know its approximately 7.7 or 78 percent, is it ok.

Coming down coming to the next question; that means, a 15, the t time period t plus 15. So, this 15 hours actually spend in burn in phase, is it ok; that means, R is failure phase 15 hours. So, again you apply this formula. So, this becomes 0.96. So, ultimately you know the value of t, you try to determine; that means, you want to increase the value of the reliability to 0.96.

So; obviously, you know how long for which time period this is applicable. So, what you find that this is just with 7.91 hours ok, 7.91 hours; that means, you have just you know you have come out from the burn in phase now you in the useful life phase; that means, the component is in useful life phase and the failure rate is increasing at a very fast rate and that is why when you just you know that working or say you are running the systems running the component for just 7.9 hours up to that the time period the reliability is 0.96 and as you start using this component for continuously for the time period during other useful life phase we will find that there will be as a sharp decrease in the reliability value. So, that is your observation that is your interpretation ok.

So, what my you might conclude that this is not acceptable as for as this component is not acceptable as far as this reliability is concerned. So, sort of conclusions your required to make when you carry out such you the analysis ok.

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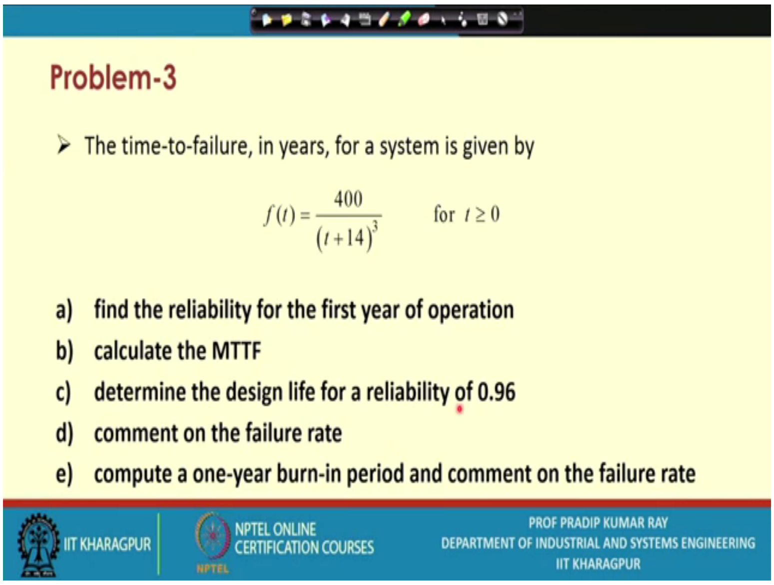

Now let us you know discuss the next problem; problem 3 the time to failure in years for the system again we are referring to a system is given by ft is this, is it ok, 400 divided by t plus 14 to the power 3 for t equals to greater than 0, is it ok, find the reliability for the first year of operation, calculate the MTTF determine the design life for a reliability of 0.96 comment on the failure rate, is it ok, when you are trying you have to do that comment means you have to interpret the value of the failure rate.

Compute a one year burn in period and comment on the failure rate, is it ok. So, these are the 5 the questions you have.

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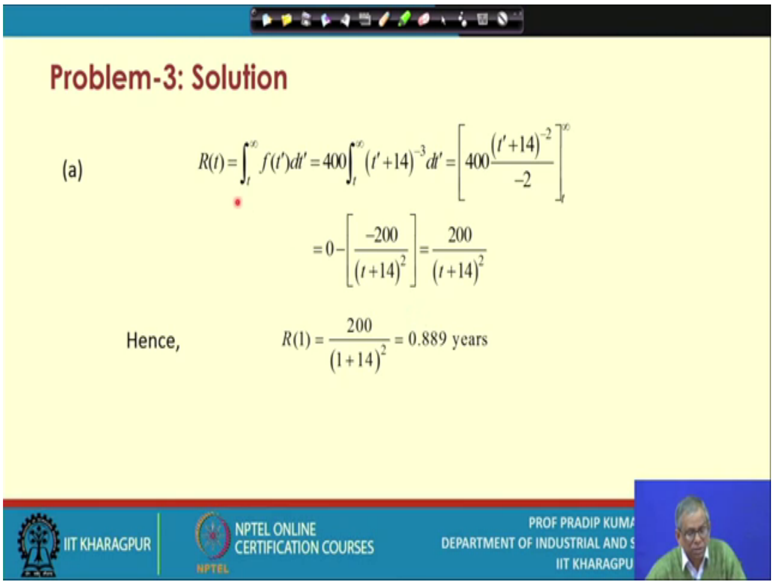

And then what you have actually the R t is given by t 2 infinity ft dt that is 400 ok, t 2 infinity right t dash plus 14 to the power minus 3 dt, ok, this formulations you have and ultimately when you compute these values, ok, you follow these steps you get an expressions like this 200 R t divided by t plus 14 to the power 2, is it ok, 2 square terms evidence is the function of t and the t is in the denominator, is it ok; that means, if t increases; obviously, the reliability decreases, is it ok. So, that is the relationship.

Now, if the t is just one one here one here what happens that you get the value of reliability for one year and the this is 0.889, is it ok. So, 0.889 is a reliability value is it now.

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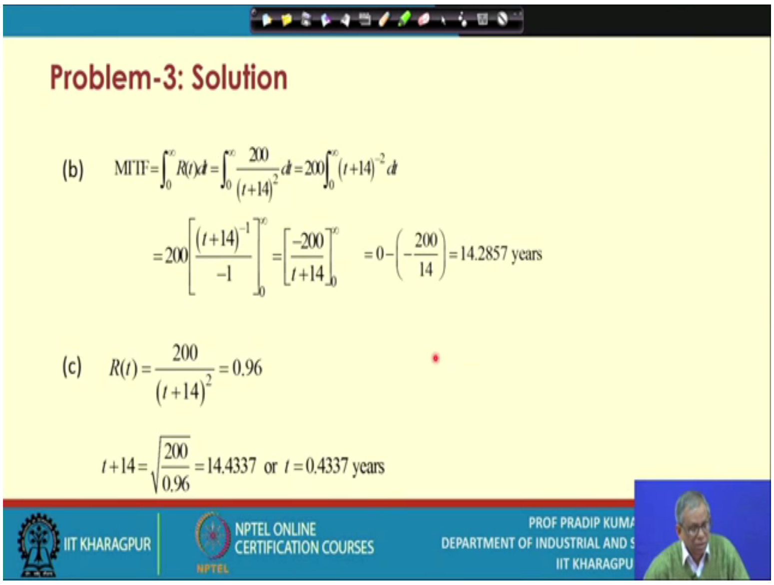

So, the next part is the your computing MTTF is it again we apply this formula is it you have these expressions 200 divided by t plus 14 square into dt you just carry out this you know this computations and ultimately you get the value of 14.2857 years. So, that is on an average is it the mean value of the time to failure is it ok.

And then we go to the next part that is the reliability this is the expression that is 0.96. So, what will be the corresponding the time period? So, t is 0.4337 years, is it ok. So, you just apply this formula and R t value is specified that is 0.96, is it ok. So, this way you compute the value of t ok, you just follow the steps.

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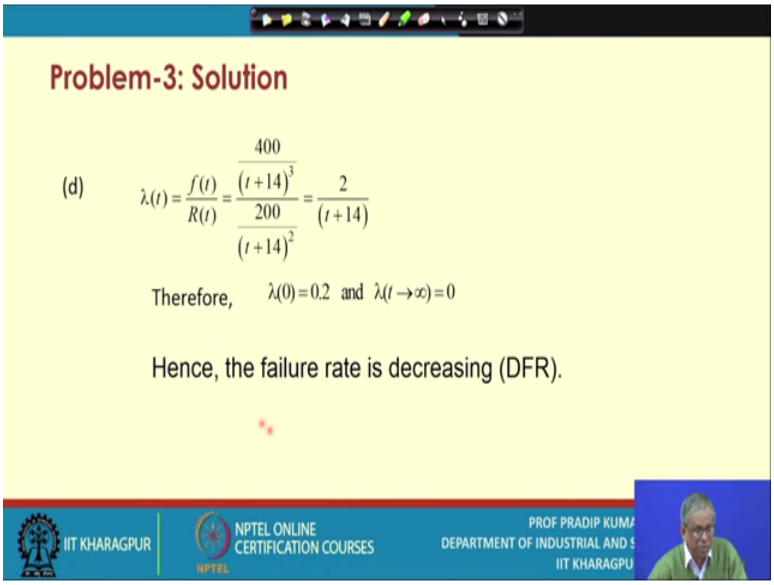

And the last one, the next one is that is the failure rate function that is ft divided by R t; that is the basic equations you have the basic expressions we have already derived in the previous lecture sessions, we have derived this expression. So, you refer to this particular expression and then you know what is the value of what is the expression for ft given by this expression and similarly R t, already you have computed. So, ultimately you get a value of 2 divided by t plus 14. So, therefore, when lambda 0 is equals to 0.2 and lambda t to the power t tends to infinity it becomes 0, is it hence the failure it is decreasing; that means, decreasing failure rate case, is it ok.

Now, this is the. So, with this is a failure rate. So, it may not be acceptable to you now you have to think of what is the means to to improve the reliability, is it ok; that means, what are the ways in which at least you can make the failure rate is it constant, is it ok. So, if you can do that; that means, when you are trying you opt for the technical solutions unit when you are trying you need to change the technology when you are trying you need to change the materials, is it or in certain cases you need to change the configuration of the system if you if you are dealing with the systems reliability, ok. So, these are the options.

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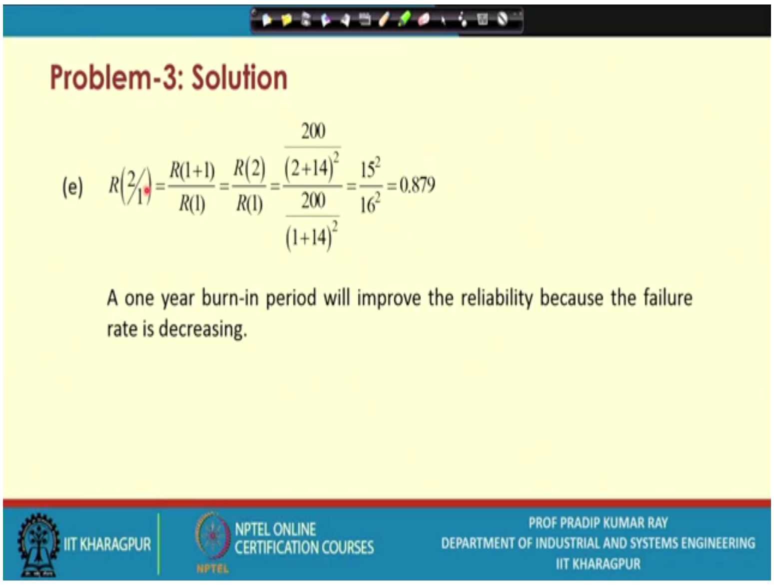

Now, the last related to the fifth questions we have this one, is it ok; that means, it is already survived one, here what is the probability that it will survive for 2 more years, is it ok. So, this is the expressions and you apply the formula and what is its reliability; that reliability is 0.879 ok.

A one year burn in period that is this one year burn in period you have successfully you know overcome all the problems that you might face during the burn in phase and then you now you were the component starts working in the useful life phase, is it ok. So, one year burn in period is over and; obviously, this will improve the reliability because of failure rate is decreasing, is it failure rate is decreasing, right. So, 0.879. So, if the if you have ah, if you really you know, during the burn in phase, you might face several kinds of problems, is it and with when you face.

This kinds of problems you have an in depth understanding of the design features and; obviously, you try to improve the design, is it ok. So, such 6 exercises you do routinely and it is expected that when the burn in period is over; that means, the quality of the product has improved the design of the product of the component has improved and that is why you know you may assume that that failure rate has come down to its minimum level and then the this failure rate with its reduced level will continue during the entire you know the life period or say you know the useful life period of the product ok. So, this is your interpretation.

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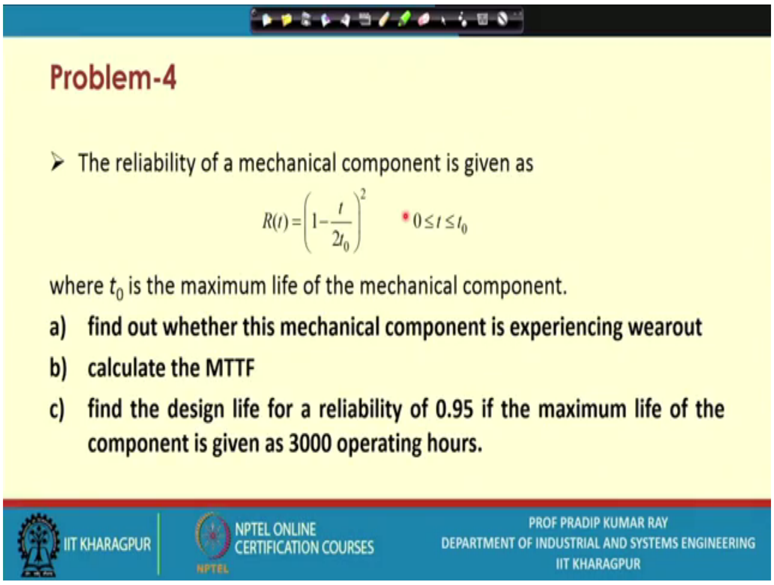

Then you move to the next problem the reliability of a mechanical component is a component wise, it is not a system. So, the reliability of a mechanical component is given as this another expressions you may raise questions this another how do you get this expression ok, later on we will we will discuss this part then how do you get this these expressions, right. So, there is a method, you apply to get these expressions is an empirical form, is it ok.

So, here what is the expressions for reliability that is R t is equal to 1 minus t upon 2 T 0 square, you will expression square it and the value of t the maximum value of t is T 0 is it ok. So, this is the condition where T 0 is the maximum life of the mechanical component, it is clear find out whether this mechanical component is experiencing wear out that is very important; that means, now we are referring to the third phase. So, where is a third phase third phase is wear out phase, is it ok.

Now, what you need to conclude that whether you know the component is in the wear out phase or not is it during wear out phase. So, what is how why do you say you are in the wear out phase because in during the wear out phase the failure rate increases at a very high rate? So, from this analysis when you carry out analysis; when you try to solve this problem, whether you will get a an expressions for the failure rate and you conclude that the failure rate is increasing at a very high rate and then only you say that possibly in all likelihood the component is experiencing wear out, is it ok.

Calculate the MTTF mean time to failure, is it ok; obviously, if it is in wear out phase what you expect that the mean time to failure will be very very less, is it ok, if suppose during the useful life period suppose the mean life mean time to failure is around say twenty hours in all likelihood if the component using wear out phase the mean time to failure could be significant less than 20 hours; that means, it could be as low as 3 hours, 4 hours, more hours; that means, constantly you are facing a failure event and then you are seriously thinking whether you know, I will continue with this product or not or I will just withdraw the component I will ask for a replacement of the component at least, is it .

Find the design life for a reliability of 0.95, it is pre specified if the maximum life of the component is given as 3000 operating hours, is it ok.

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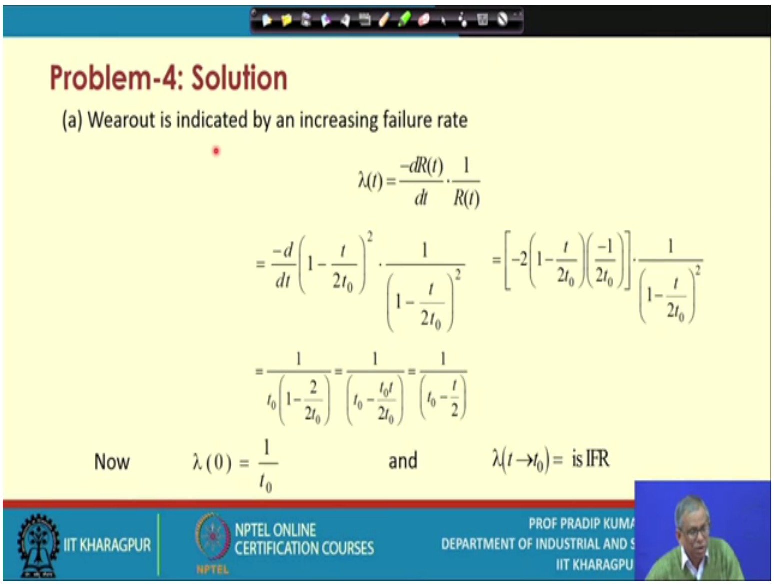

So, this is another dimension of the problem. So, wear out is indicated by increasing failure rate, I have already pointed out this one. So, lambda t, if this is the basic expressions you have, is it ok; that means, this is ft by R t, is it ok, ft by R t; now you have this expression already it is given R t expression is given. So, here you have this R t expressions here you have minus ddt of R t this is ddt of R t. So, ultimately you follow this steps you simplify and you get this expression that is one upon T 0 minus t by 2, is it

Now, lambda 0 is 1 upon T 0; that means, when lambda 0 you have one upon T 0 and lambda t tends to T 0 is it that is increasing is increasing failure rate. So, it is increasing failure rate right.

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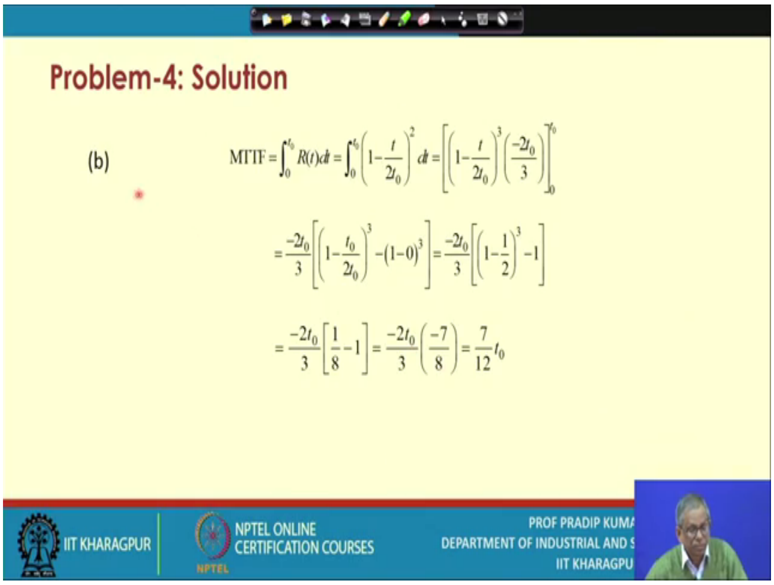

So, MTTF; that means, you have proved that you are in the your failure rate is increasing and that is why in all likelihood the component using wear out phase is it ok. So, that is your conclusions.

Now, the next part is that is how to compute MTTF with given problem again you use the same formula. So, that is R t dt say a integration 0 to T 0 and then again you compute this one; this R t is known you follow all these steps, is it and you get this expressions in terms of T 0, is it ok; that means, 7 upon 12 T 0, ok. So, this is the expressions for MTTF so ok.

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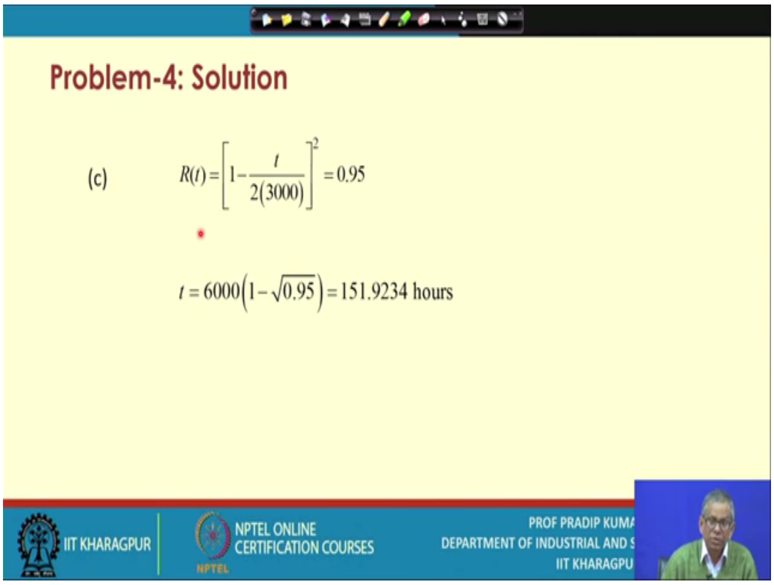

So, if the T 0; that means, that is the maximum value, is it ok, last up to T 0. So, T 0 what is your MTTF that you can always calculate, is it all right.

Now, what is the expression for reliability that is these expressions of the 3000 hours working hours, the reliability is 0.95. So, now, you calculate that what is the time period. So, you just from this expressions you get the time period the time period is 151.9234 hours, is it ok. So, this is a just a straight you apply this formula together and such.

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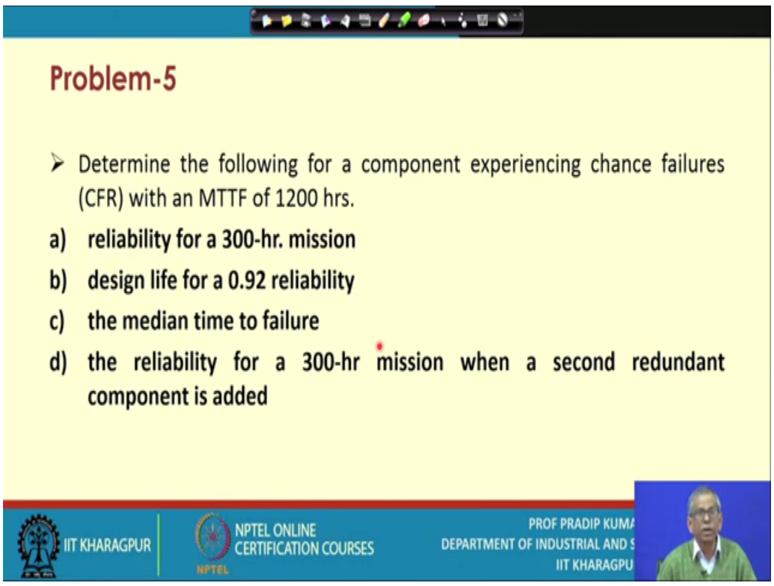

Now, we will be referring to the next problem that is problem 5 .

Determine the following for a component experiencing chance failures what is this chance failure; that means, it is where when; that means, the chance failure creates a useful life phase, is it ok; that means, when the you know the failure rate is assumed to be constant. So, the component experiencing chance failures; that means, there is a numbness you, it might fail, but there are there is no as such there is no assignable causes this is there is some because of some natural causes it is failing, is it and these natural causes do exist in the system .

So, chance failure fails with an which MTTF of 1200 hours; that means, the MTTF value mean time to failure is specified and that is around 1200 hours is a point estimate reliability for a you need to determine reliability for a 300 hour mission design life for 0.92 reliability, the median time to failure the reliability for a 300 hour mission when the second.

Redundant component is added. So, that is you know the problem number d.

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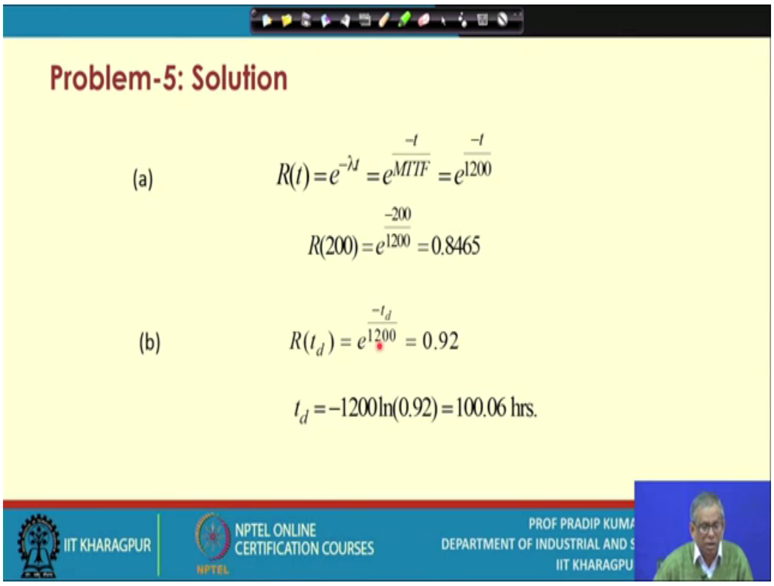

So, will be answering the first the first question that is reliability R t to the power minus lambda t is a chance failure phase; that means, useful life phase. So, we assume that exponential time to failure; that means, constant failure rate. So, e to the power; that means, t upon one upon MTTF why your used to one upon MTTF because lambda is one upon t or MTTF is it lambda. So, this is the expressions for R t.

So, when t is 200 hours you get a value of reliability as 0.8465 or 84.68 percentage now this is reliability for td. So, again you apply this formula that is e to the power minus 2 d divided by 12; 1200 that is 0.92 and this is already specified you refer to the question. So, what is the value of td, td is 100.06 hours, is it ok.

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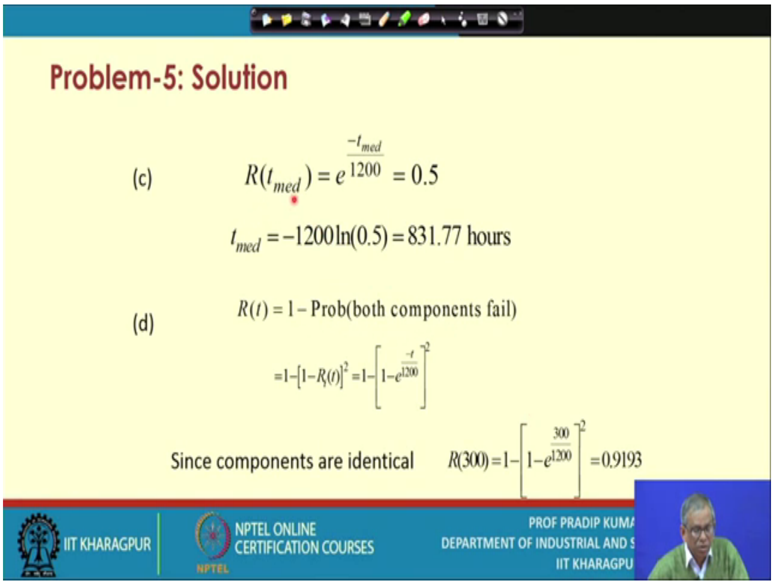

Next we use this formula for determining the reliability with median time; that means, it will last for the median time what is the reliability; that means, e to the power minus t median divided by 1200 that is 0.5, already it is known 0.5. So, t median the way, we define the median time to failure, ok. So, the reliability is 0.5. So, corresponding hours is 831.77 hours and this one is R t is 1 minus probability the both components fail, is it ok, is referred to the part d of the problem.

So, how do you know express the reliability that is one minus probability that both components fails simultaneously. So, for each one is one minus R i t, is it ok. So, this is the failure probability; obviously. So, it is one minus the reliability, then both the components we are using; that means, this into this; that means, 1 minus R i t to this square and so, for R i t expressions, you have e to the power minus t upon 1200. So, these expressions already you have derived and since the components are identical. So, what you say that if the value of t is 300. So, corresponding reliability is it is R 300 is 0.9193; that means, here you use a value of 300 ok.

So, this is this problem is also typical problem. So, these five problems we have discussed today and in this session and similar such problems will be discussing in the subsequent lecture sessions and we will be referring to not only reliability related problems as reliability is closely linked with the quality. So, there will be few sessions as lecture session is exclusively on the numerical problem. So, where we will be we will be solving problems one both reliability as well as on quality because the till date we have covered a large number of issues related to the quality control as well as the reliability modelling. So, there must be certain the, you know sessions where; I will be discussing you know a number of a set of numerical problems ok. So, all right.