**Quality Design and Control**
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**Lecture - 44**
**Design for Reliability- I (Contd.)**

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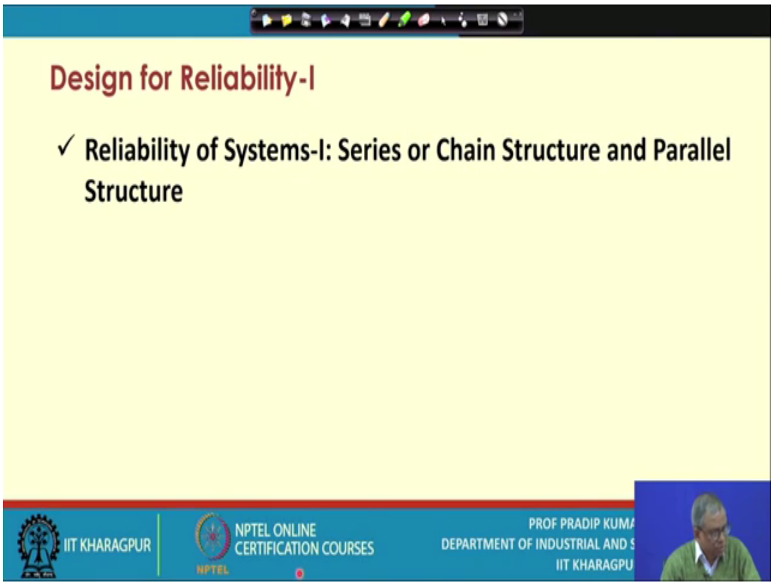


So, during this session we would like to discuss related to design for reliability or the reliability of systems. Now, though we talk about reliability of the products, but many a time no you deal with the systems. And there could be different types of systems with different configurations. And so, you need to look into reliability modelling of all these kinds of systems. Now in today's the lecture session, I will be referring to series or chain structure and the parallel structure.

So, these are the two important systems, we come across in manufacturing and other systems and we will be just referring to the reliability modelling of such systems.

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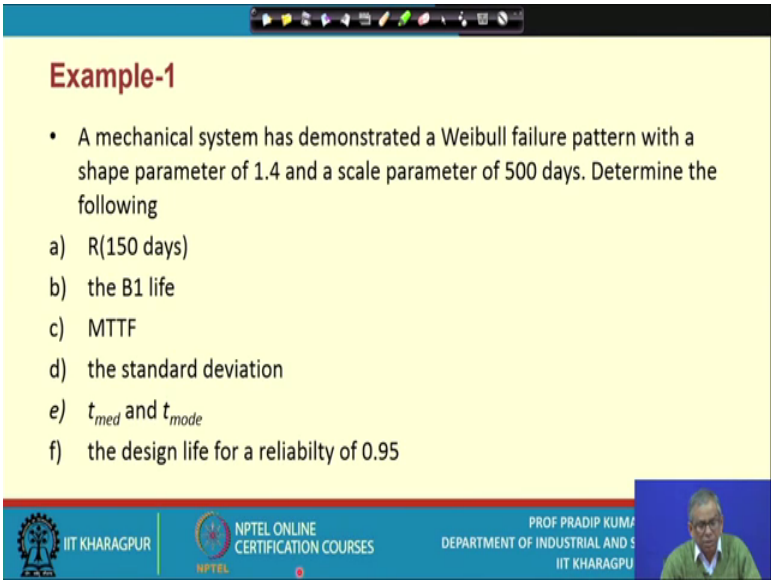


Prior to discussing these two aspects; let me refer to an example and in the previous lecture session. If you remember, we discussed at length the probability distributions for reliability modelling; there are different kinds of the probability distributions you may assume and so, we have discussed those; the typical distributions normally you come across in reliability modelling.

Now, we will just cite one example and then we will be discussing the reliability of the systems. Now, this example is a typical example. So, let me just go through this particular example a mechanical system has demonstrated a Weibull failure pattern; that means, time to failure distribution is assumed to be Weibull and already we have discussed the rural distribution with a shape parameter of 1.4; that means, the value of beta is 1.4 and the scale parameter of 500 days. So, what is the skill parameter scale parameter is essentially is referred to as alpha.

Determine the following; that means, reliability that that the system will last for 150 days, that is; the sort of notation we use the be one life MTTF mean time to failure the standard deviation of the time to failure the medium time. As well as the; more time to failure and the design life for reliability of 0.95; so, these are the you know the aspects to be looked into.

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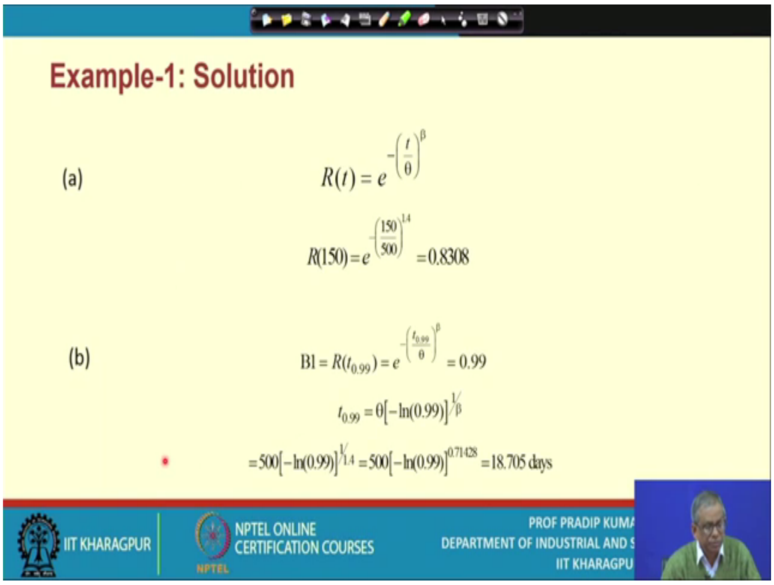


So, referring to the first, I know the question we determine the reliability with 150 days of survival of the system. So, we use this particular formula; that means, R t equals to e to the power minus t by theta to the power beta.

We have already derived this expression. So, you refer to this particular expression and you just apply this formula. So, if we apply this formula you get a value of 0.830 eight; that means, is reliability that the system will run for 150 days, that is; 0.8308. The next part is the B 1 life; we have already derived the expression for we have already explained we have defined, what is this B 1 life?

So, directly you apply this formula and you get a value of 0.99 is it ok. So, and then you say that t 0.99, that is; another expressions we have already derived please refer to all these derivations and then when you apply this formula you get this value; that means, 18.705 days ok.

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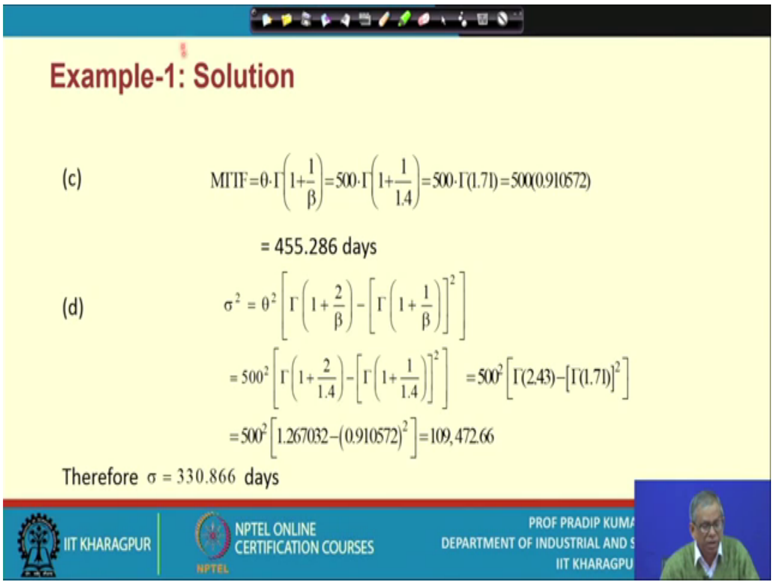


We calculate MTTF mean time to failure; this is theta gamma function 1 plus 1 upon beta. We have already derived these functions gamma functions and. So, this is alpha.

So, this is essentially 500. So, 500 gamma function 1 plus 1 upon beta; So, what is the value of beta? That is, the shape parameter 1.4, and you get a value of 455.286 days is it ok. So, that is the MTTF.

Similarly, we calculate the variance we use this formula and we get a value of 109,472.66 is it and so that is sigma square. So, the square root; that means, the positive square root of the variance, that is the standard deviation that is 330.866 days is it ok.

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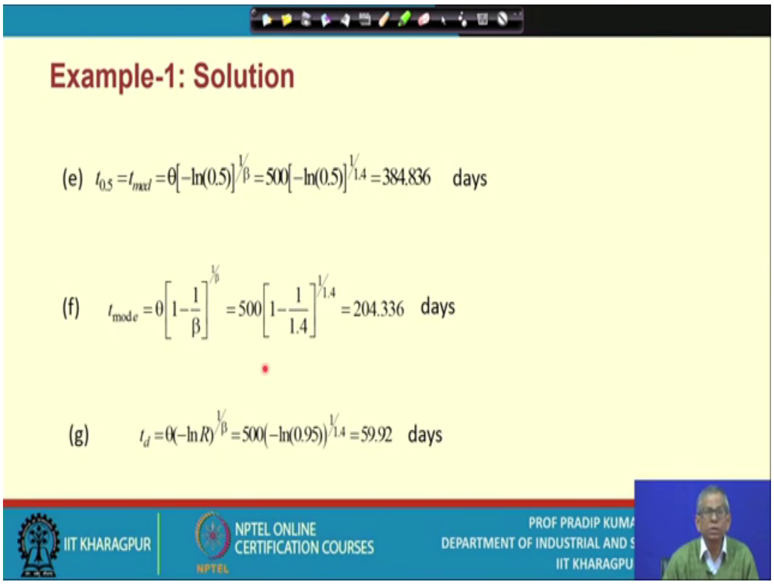


So, you know the several the formulations. So, in these examples what we are trying to do? That means, you have the corresponding data and you are trying to determine all those are the parameters ok. So, this is the t point at 0.5 that is the t median. So, again we apply this formula and the t medium is 384.836 days and similarly the t mode.

Again, we have derived this formula and when we have all these values then you get the value of t mode as 204.336 days and the td. Similarly you have these expressions for td and you get a value of 59.92.

So, this is a typical example there could be many other such examples numerical problems we will be dealing with those numerical problems in subsequent sessions ok. So, this is just one application. So, you have been using or you have been deriving many kinds of mathematical expressions.

While, you go for reliability modelling of a system. So, this is just numerical problem related to some of these the formulations. Now, let us talk about let us discuss the reliability of systems ok. So, this is the part one and again in subsequent lecture sessions, I will be referring to say the part two of reliability of the systems because you come across various kinds of systems the varieties of systems varieties of configurations and it is your attempt to go for reliability modelling for all such systems.

Now, what we have mentioned over here that the components units or subsystems within a complex system can be related to one another in series parallel series parallel.

That means, it is a combination that configuration may have or meshed structures ok. So, the you may come across several such the structures while you try to configure your system or in any combination of all these is it ok. So, this is your; you know the idea about the system the system structure or the systems configuration. Now probability concepts are applied to compute the reliability, but essentially you know whenever you try to have a numerical value of reliability of a systems or a part or a component we refer to the probability essentially it is probability.

Well, we define reliability we have mentioned that this is nothing, but the probability is it ok. So, the probability concepts are applied it is obvious to compute the reliability of the system in terms of the reliability of it is subunits; that means, a system may consist of you know several one or more of the subsystems and each system.

Each subsystem may have different sub subsystems and at a particular level maybe a particular sub subsystem or the sub systems may have several components or the parts ok; So, as per the bill of material as per the product structure code. So, you have to assemble those components you have to assemble those have assembly to get the final assembly is it ok. So, we have already referred to a particular term called product structure code and the prior to product structure code for the given system.

You must know, what is the bill of material? And the bill of material actually, this document you get from the design department is it ok.

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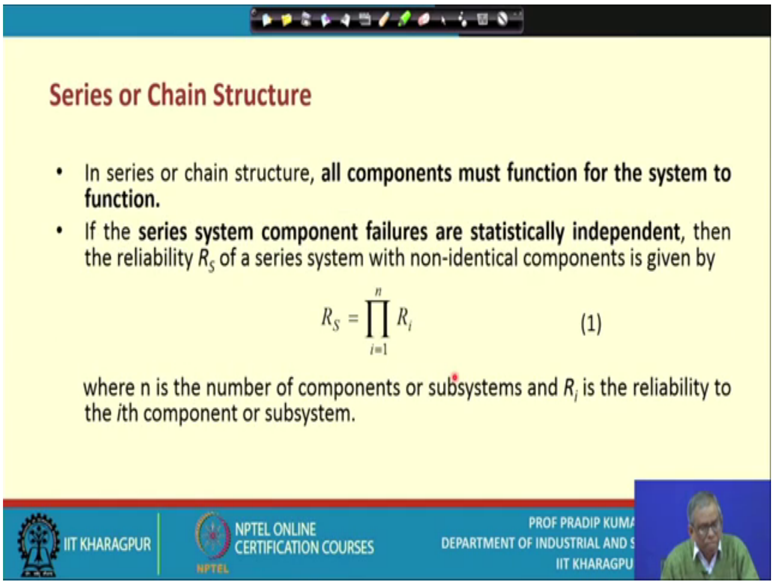


So, now the first you now with the type of systems you come across, that is referred to as the series systems or the chain structures is it ok. Now, the question is that how to measure reliability of a series systems or the so, called chain structure in series or chain structure is the type of system all components must function for the system to function ok.

That means systems must run under what condition it runs? Because, it consists of several you know the components and all the components must run simultaneously; then only the system.

Will be running like you know much missile system it may consist of some around say 4,000 components 3,000 components. So, very complex design and essentially they say that this is a series structure; that means, you can really imagine that to get the function from a missile.

What you need to do? You need to assure that all the components and these components in 1,000 numbers. So, you find that all the components must be. So, reliable that it must not fail at a particular point in time if just one component fails entire system breaks down is it ok. So, that sort of, you know the event must be avoided. So, this is the condition is it clear if the series system component failures are statistically independent is it ok.

That means what you have; that means, for each of these components it has it is own time to failure time to failure other distributions or related to time to failure it has it is own parameter values like the different of the mean value and the different standard deviation values ok. So, and we say that the function of one component no way is affected by the function of the other components ok. So, we may assume that they are all statistically independent.

Then, the reliability of the system the notation is RS of a series systems R subscript S with non identical components is given by RS product Ri is it ok; that means, it is product terms Ri i equals to 1 to n is it ok, where n is the number of components or the subsystems is it and Ri use the reliability to the ith component or the subsystems is it ok. So, what we assume that this system consists of a number of components or a number of subsystems or both is it ok.

So, you must know the individual component reliability or individual subsystems reliability. So, this is the expressions is it has to be a product term. So, this is the reliability of a system.

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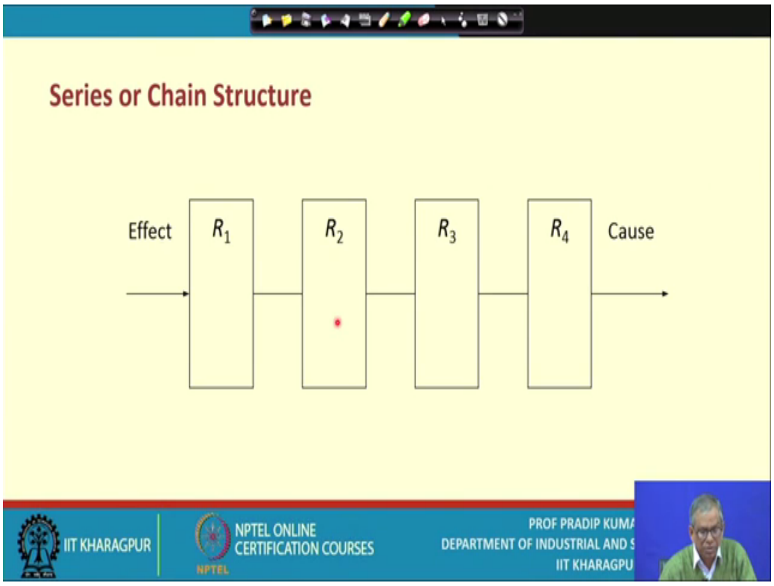


So, this is a typical you know, what you can say? The figure I related to the series of chain structures. So, you have say four components over here they are you know they are in series and this is you know we have this one like all are interrelated and you get the effect and there could be several causes there could be several types of effects so that you must be able to monitor is it ok.

So; that means, the causes of running this system, what could be the effect of running the series systems? We should be aware of is it all right there could be many problems and you need to identify that the sources of these problems. So, what you need can do; that means, for this is such a system if you face lot of problems ok, it is like say problem related to failure ok.

So, what you need to do you can you can identify the sources of the problem as well as the types of the causes. So, you may use several kinds of the tools or the techniques for identifying such causes ok.

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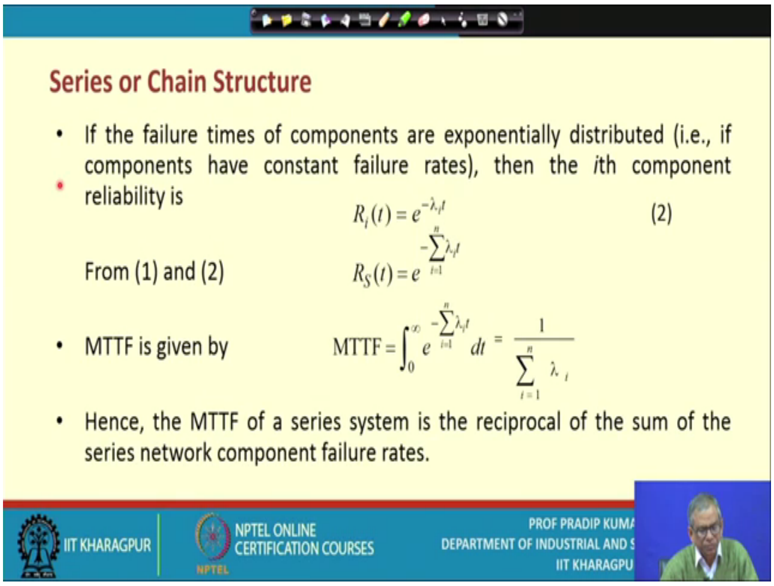


So, if the failure times of the components are exponentially distributed that what does it mean? That means, you have created the system you have you know you have included all the subsystems or components. Now, all these components you know there in basically you know useful life phase is it and if these components; that means, it has all these components they are not in the burning phase or in the wear out phase they are in the you know the useful life phase.

So, if all these components are in useful life phase; obviously, you know you may assume that the failure rate is constant and if the failure rate is assumed to be constant you may assume that the time to failure distribution is exponential.

So, this point already we have elaborated; that means, if the components have constant failure rates this point, I have already mentioned the ith component reliability is; obviously, for the ith component reliabilities e to the power minus lambda i t.

So, what is lambda i? That is basically the failure rate for the ith component and; obviously, you knew we were trying to compute the reliability for the t time period; that means, whenever you say that I compute reliability for the t time period; that means, I am assuring that the component will be surviving for the t time period. So, what do you have basically, now as the systems reliability is essentially all these for the ith items or how many items you have; obviously, there are n numbers of items.

So, the systems reliability is; obviously, to the power sigma i equals to 1 to n; n number of components in series for the systems lambda i t and e to the power minus; obviously, is it all right. So, you add all these the terms right. So, sigma lambda i into t is it all right. So, it is sigma lambda i into t MTTF is given by again 0 to infinity e to the power lambda t sigma lambda t dt. So, one upon sigma I i equals to 1 to n lambda i is it ok. So, these simple calculations you have and you can calculate the systems reliability as well as the mean time to failure.

Hence, the mean time to failure of a series system is the reciprocal this is the reciprocal of the sum of the series network component failure rate. So, this is the sum of the failure rates is it? So, how many failure rates you have in number of failure rates. So, you add them and then you take it reciprocal. So, that becomes basically the mean time to failure or a MTTF ok.

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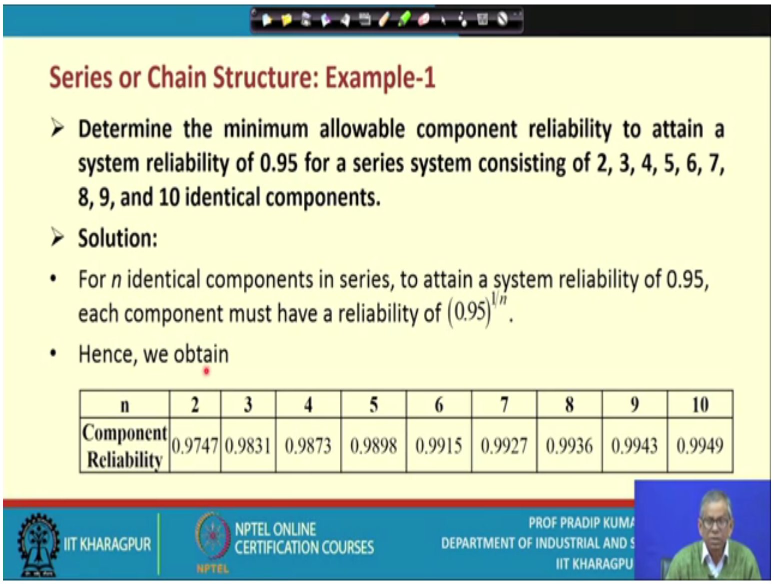


Now, here is an example. So, let me discuss this example this is a typical numerical problem determine the minimum allowable component reliability to attain a systems reliability of 0.95; that means, the systems reliability is pre specified for a series systems. So, the kind of systems that is also mentioned consisting of 2, 3, 4, 5, 6, 7, 8, 9, and 10 identical components is it clear; that means, a series systems may have 2.

So, if you have just two components two identical components, how do you define? You know identical component, because with respect to the failure rate or that first thing is identical in the sense that suppose you say that one item is identical to the next item; that means, for both the items the time to failure distribution must be same. And similarly the parameter value of the distribution that you assume must also be the same in other words in this particular case the failure rate is also same that is why they refer to us that you know the identical is it ok.

So, for in identical components in series to attain a systems reliability 0.95 ok; So, in general each component must have a reliability of this 0.95 to the power 1 upon n ok. So, that is directly you apply the formula is it the formula linking the; you know the systems reliability with individual component reliability in a series systems. So, you have these expressions, now what do you do if n is 2? What will be the value of the you know the component reliability is it if n is 3; that means, all three components are assumed to be identical ok.

In respect of their you know the failure rate. So, the value is 19831 and similarly for all other you know the numbers of components you consider; that means, 4, 5 up to 10. So, if you have 10 components in series that what you must insure; that means, for each component is it must have a reliability of 0.9949.

So, it is very very high almost tending to one is it ok. So, as you might have noticed as the numbers of you know the components in the series increases, what do you need to do; that means, individual component reliability value must also increase is it ok. So, in this case the minimum value with two components in series each component must have any reliability of 0.9747.

But when you deal with you know in the system ten identical components in series; obviously, the individual component reliability increases to 0.9949. So, the difference between 0.9747 and 0.99 for you can prove that this; the difference is significantly different ok. So, this is a typical example.

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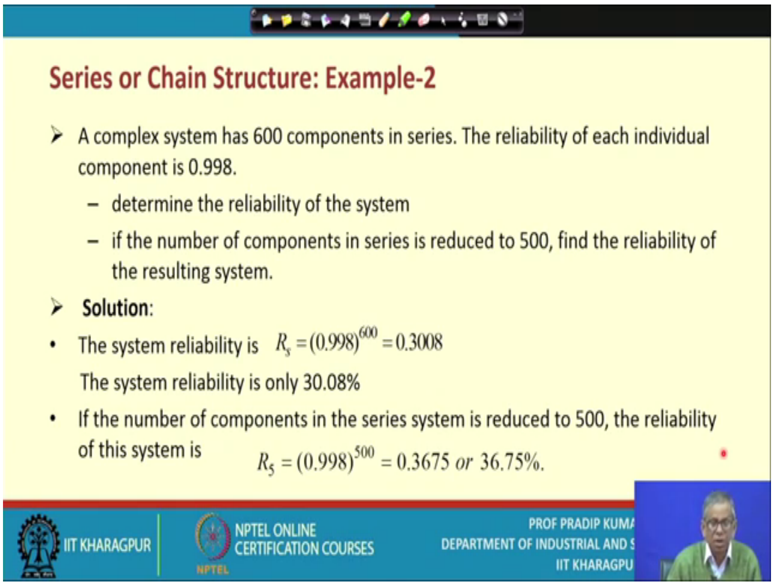


Now, the next example let me take took up the next example a complex system has 600 components in series I am telling you will come across you know the several sorts of complex systems and the complexity of your systems can be defined from several perspectives one perspective is the number of components is it ok.

So, here we are just considering the number of components indicative of the complexity of the system. So, here is a system the 600 components in series.

So, you can well imagine that how you know while you select all these components make sure that all these components must have a very high value of reliability that is to be ensured is it now; obviously, you must know that before you use a particular core component in a system ok, while you manufacture the system you also must have a prior idea that, what could be is possible reliability; that means, you must be able to estimate it.

So, what how do you estimate the reliability of a component there are procedures in subsequent lecture sessions on reliability we will be we will be discussing those aspects ok. So, for the time being we assume that the reliability estimation procedure is known and the value of the reliability of a given component is also known ok.

The reliability of each individual component is 0.998 it is very very high determining the reliability of the system is it ok; if the number of components in series is reduced to 500. Normally, you know when you try to go for say design improvement in many ways you can go for improving the design of a system the first you know the approach should be what extent you can reduce the components reduce the number of components ok.

So, as you try to the simplify the design the first thing, we try to do you try to reduce the number of components is it ok. So, here is a case you know, initially you start with a design with 600 components. Now, you go for improving the design and then with and (Refer Time: 23:55) what you can? What you can do? 600 has now become 500; that means, 100 components are reduced. So, find the reliability of the resulting system.

So, it is very easy; that means, the systems reliability is 0.998 to the power to the power n. So, what is the value of n? N is 600 in the original system. So, it has come down to just 0.3008; that means, any serious systems, if you increase the number of components in series; obviously, the systems reliability will be significantly less is it ok. The system reliability is only 30.08 percent, if the number of components in the series system is reduced to 500, you can expect that the value of reliability will increase.

And the; so what is that increment? Increment is just 6.75 percent almost is it around 6.6 percent, 7 percent; that means, it was 30.08, now it has become 36.75; that means, almost you know more than 6 percent improvement in reliability is possible by reducing the number of components from 600 to 500.

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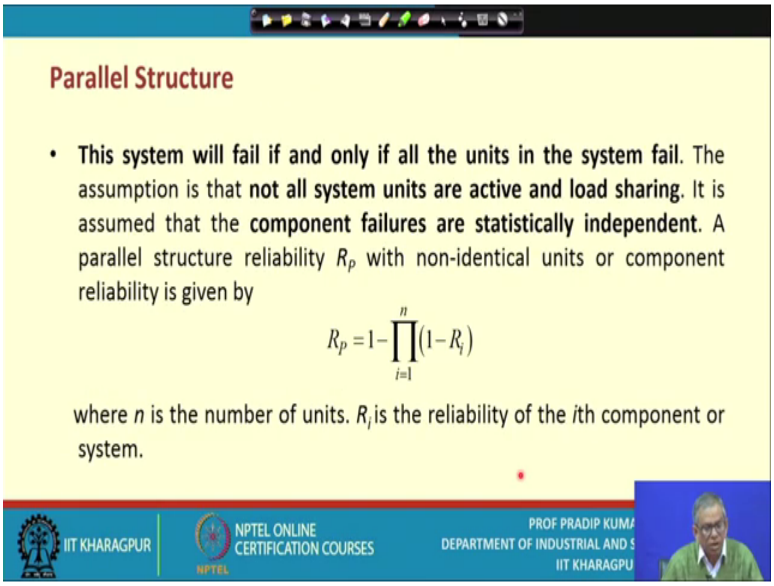


Now, what is the parallel structure? So, alternative to series structures you have again a pure structure called parallel structure. These systems will fail if and only if; all the units in the systems fail. The assumption is that not all system units are active and load sharing is it ok. So, that is the basic assumption; that means, to run the systems at least one systems on one subsystems or one component must run.

It is assumed that the component failures are statistically independent like, in the previous case a parallel structural reliability ok; that means, the notation is Rp with non identical units like in the previous cases or the component reliability is given by Rp is equals to 1 minus 1 minus Ri. So, is it ok.

That means first you check that if all the components fail. So, the system will fail is it ok? So, what is this reliability? Obviously, it will be r s the systems failure reliability ok. So, then one minus Ri is essentially that it will fail is it ok. So, all the components will fail with individual component say probability is 1 minus Ri is it ok.

So, i equals to 1 to n. So, this is the product term and when you subtract this product term from one you get the parallel systems reliability is it clear. So, here n is the number of units n is the number of units or I use the reliability of the ith component or the system. So, 1 minus R i is basically the non reliability or say it is a failure is it in a period of time ok.

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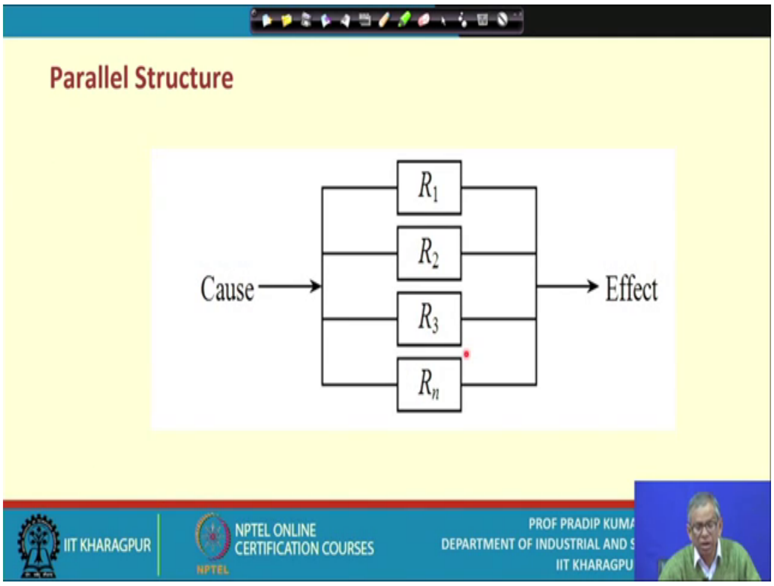


So, now here one basic assumptions we are making that is the reliability the value they remained same over time usually it is not so; that means, the reliability actually becomes a function of time reliability of a component. Here, we are assuming that the value of reliability at a particular point in time remains same for all the time period is it ok. So, I will utilize these assumptions later on.

So, this is a typical the structure, that is our cause and effect. So, this is these are the components you have, how many components you have n number of components. So, this is basically the configuration this way repairs the configuration; that means, the systems will run only when any one of these components runs is it, that is the basic condition to be ensured.

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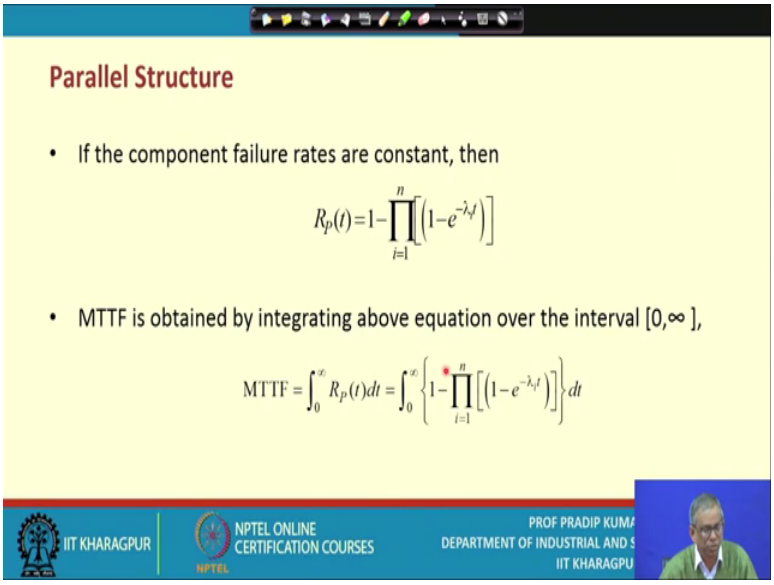


So, if the component failure rates are constant then this is your the equation this is the expressions for the reliability reliability for the t time period so; obviously, you know the failure rates are constant; that means, a lambda i; that means, for the i it component you have the failure rate as lambda I useful life phase all the components are in. So, this is the expressions ok, because for individual reliability expression is e to the power minus lambda I t is it exponential time to failure distribution and similarly we compute MTTF is it ok. So, MTTF is these expressions.

That means integration 0 to infinity Rp t dt is it ok. So, this is the expressions; that means, essentially you calculate the mean value was it starting from the fundamentals. So, these expressions you can use. So, Rp expressions is already there. So, it is replaced with this one and then you get this the you know in the value ok.

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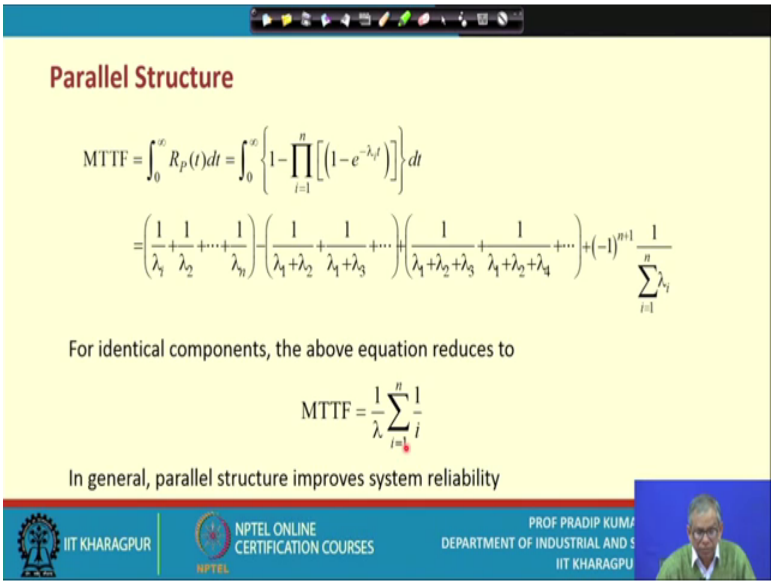


So, MTTF further you know you can you can manipulate you simplify these expressions and ultimately you get these expressions is it all right for identical components the above equation reduces to this one MTTF is equals to; that means, this is the particular case this is the generalization.

So, you just note it down this is the general expression and a particular case is this when you deal with the identical components; that means, all these components are having the same failure rate is it ok. So, MTTF expression is reduced to these expressions 1 upon lambda sigma I equals to 1 to n 1 upon I in general parallel structure improved systems reliability; that means, obviously.

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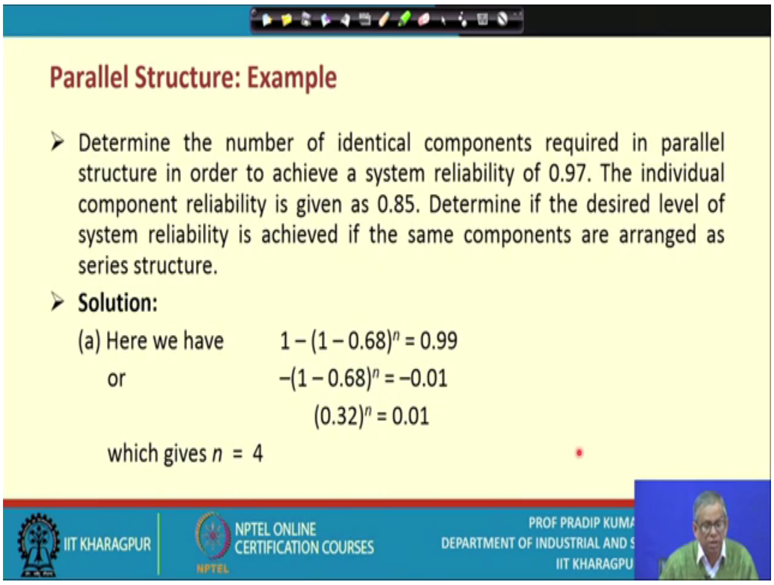


If you ask what the design improvement you go for the parallel structure from the series if it is feasible ok. So, here is an example it will mean the number of identical components required in parallel structure in order to achieve the systems reliability of 0.97, the individual component reliability is given as 0.85.

Determine if the desired level of systems reliability is achieved if the same components are arranged as series structures; that means, a comparison with the series structures we do. So, here we have this value; that means 0.99. So, ultimately what you what do you find that with 0.97. So, you get a value of say of n equals to 4 is it ok?

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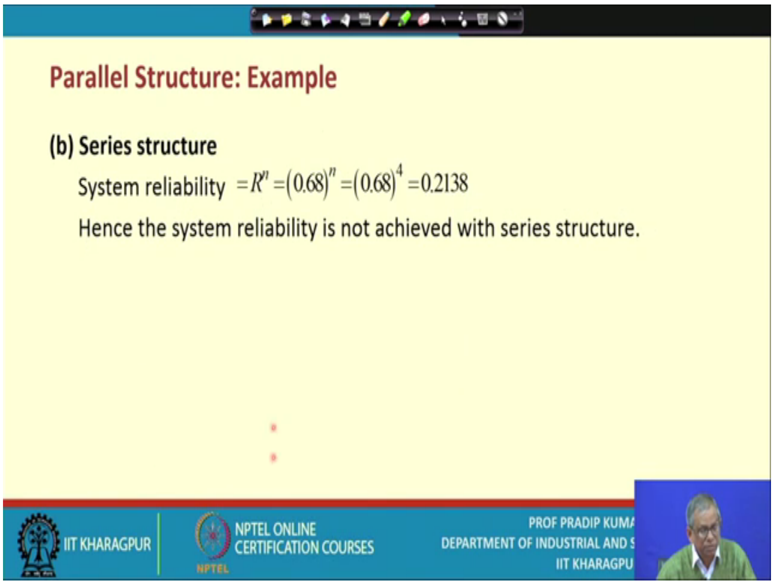


So, this is the expression and for the series structure the systems reliability that is R to the power n 0.68 to the power n, that is 0.2138. So, it is very very less.

As a systems reliability, is not achieved with the series structure. So, this is one example. So, I conclude this and just we have referred to the two kinds of systems the series systems and the parallel systems and in the subsequent lecture sessions will be taking up several examples and one particular the systems.

We always, we refer to and we come across are systems that is the systems with the standby the standby components; So, that what also we will discuss in the subsequent lecture sessions.