Quality Design and Control
Prof. Pradip Kumar Ray
Department of Industrial and Systems Engineering
Indian Institute of Technology, Kharagpur

Lecture - 43
Design for Reliability- I (Contd.)

So, continuing our discussion on Design for Reliability; let me discuss in this session the probability distributions in modelling reliability.

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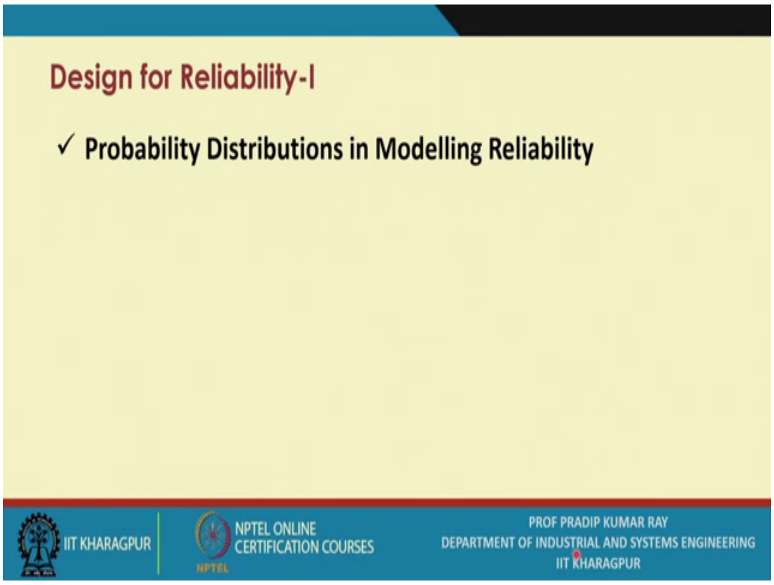

We have already referred to exponential our density function. So, that is one particular case we have we have referred to a particular situation where you know the exponential distribution is valid and that is basically during the useful life of a product if you want to model its reliability so; obviously, you know assumptions are related to exponential density functions these assumptions are valid.

Now, there could be other cases; other types of probability distributions that also you are required to assume while you model reliability.

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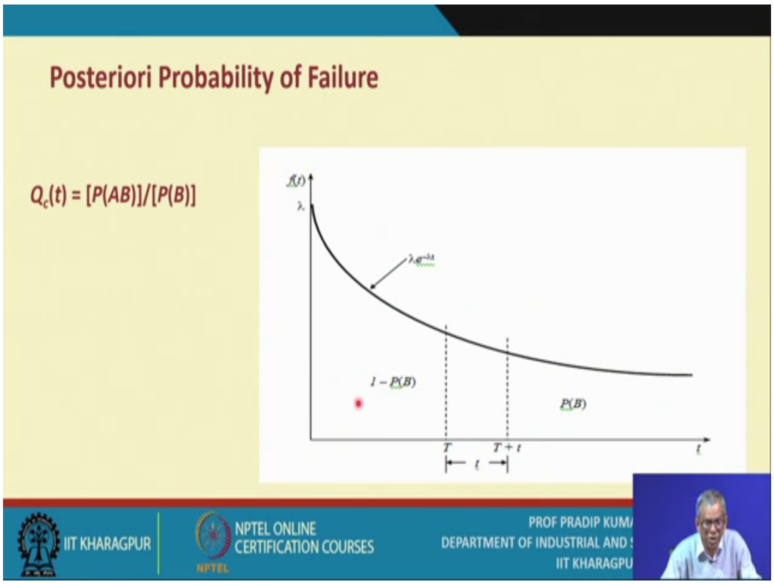

Now, you know while you try to refer to this the modelling, now let me first highlight one particular type of probability of failure that is posteriori probability of failure; that means, this is a probability of AB provided that there is a probability of B is it ok. So, this many time you need to calculate the, this posteriori probability of failure is it ok; that means, this is ft versus t, is it ok. So, this is 1 minus P B and this function is lambda e to the power minus lambda t, is it ok.

So, during this time period that is t; small t; that means, up to capital T and then up to capital T plus small t. So, this time period is the small t. So, this part is the probability for the b and this is 1 minus b, is it ok. So, this is the representation, it is very simple.

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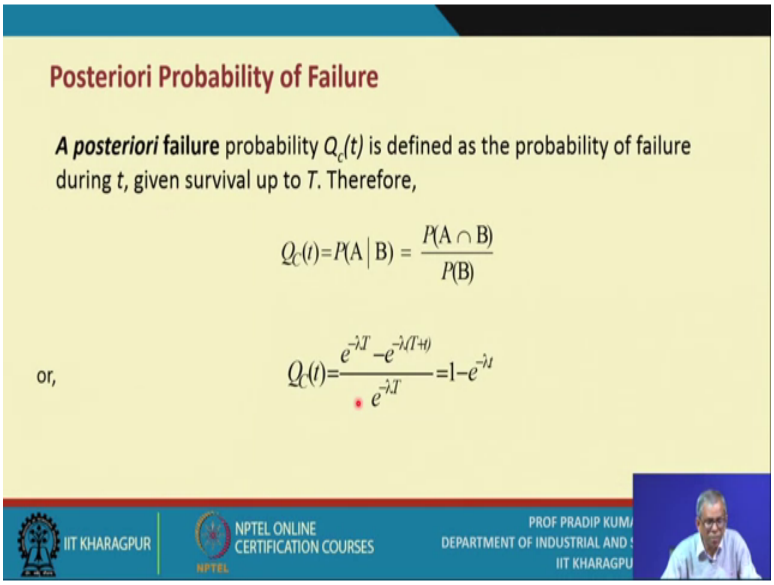

So, you can immediately you know you can calculate this one. So, a posteriori failure probability that is Q c t is defined as the probability of failure, we have used this notation ok, how it is defined it is defined as the probability of failure during T given survival up to t. So, I think it is clear, is it ok.

So, that is the probability of failure. So, this is probability a when b; that means, A union B divided by probability b probability that A union B divided by probability of B. So, now, what will be this expression that will e to the power minus lambda t minus e to the power minus capital T plus small t divided by e to the power minus lambda t, is it that; that means, 1 minus e to the power minus lambda t, is it all right.

So, you come across such terms and this way you define it, is it ok, this is the expressions for posteriori probability of failure. Now, this posteriori probability of failure is independent of the prior operating time t is it ok. So, that is assumptions we are making and depends only on the duration t of the time interval, is it ok.

That means it is almost what is happened up to capital T time period that you do not bother, is it ok, we are assuming that that this failure is dependent on dependent on you know the function of the functioning of the product during the small t time period only this time period interval, if you refer to this figure, you will always you know you say that this time period, we will be bothering about; that means, something has happened is it might happen during this time t period small t and to what extent, it might affect the performance of the product, is it ok.

So, the component does not degrade in quality with the time of operation, is it ok. So, the degrading is very very important. In fact, that maybe you know the inherent property of the material. So, the product has a material is it not only one type of material that could be different types of materials.

Now, depending on the physical property ok; so, you know there could be you may assume that the product may degrade or the product may not degrade, is it ok, but here that we are assuming that this the material property remains same and only because of the design related factors design related factors or environmental related factors there could be deterioration in the performance of the product and if there is a deterioration in the performance of the product is it ok; obviously, you know it has got a very close relationship significant relationship with the reliability of the product.

Now, we note here that is a priori failure probability of the component which is a probability of failure between 0 and t is this that already mentioned; that means, Q t is equals to 1 minus e to the power lambda t.

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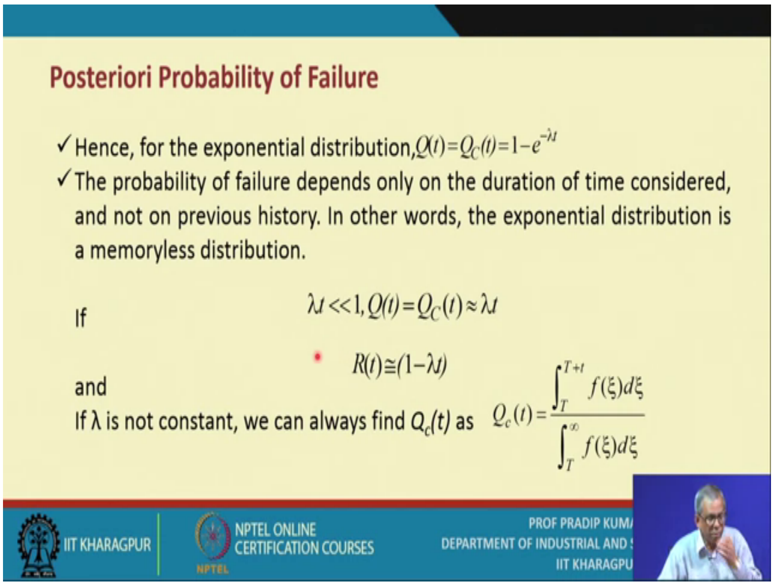

Now, hence for the exponential distribution right qt is equals to 1 minus e to the power minus lambda t the probability of failure depends only on the duration of time considered and not on previous history clear. So, this I repeat again the probability of failure depends only on the duration of time considered is it that is the basic assumptions and not on previous history in other words the exponential distribution is a memory less distribution is it memory less distribution this point is to be noted.

So, if lambda t is substantially less than 1 substantially less than 1 what do you find that the qt is nothing, but is approximately lambda into t is it is almost is lambda into t and similarly so; obviously, the reliability function will be 1 minus lambda t square.

And if lambda is not constant we can always find qc t as this one this is the general formulation; that means, on the numerator you have integration t to t plus small t f xi d xi divided by t to infinity right f xi d xi is it ok. So, this is this way, we get the expression when we say that lambda is not constant, is it clear.

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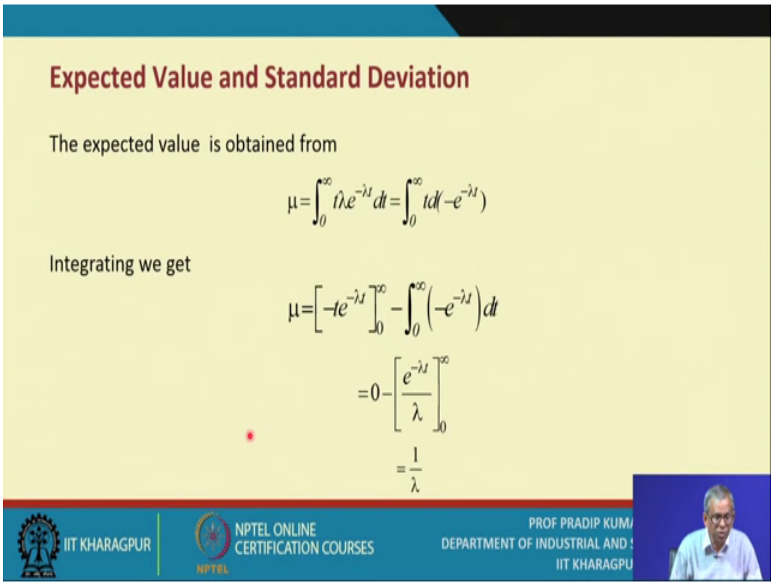

So,, so, this way you proceed and what is the expected value and the standard deviation. So, we use this is a standard procedure. So, expected value is obtained from mu equals to 0 to infinity t lambda e to the power lambda t; that means, tft dt tft dt.

So, what is ft lambda e to the power minus lambda t so; obviously, if you simplify it becomes integration 0 to infinity t there will be dt. In fact, t dt minus e to the power minus lambda t, is it and integrating we get ultimately that mu is nothing, but 1 upon lambda, is it ok. So, mu is 1 upon lambda.

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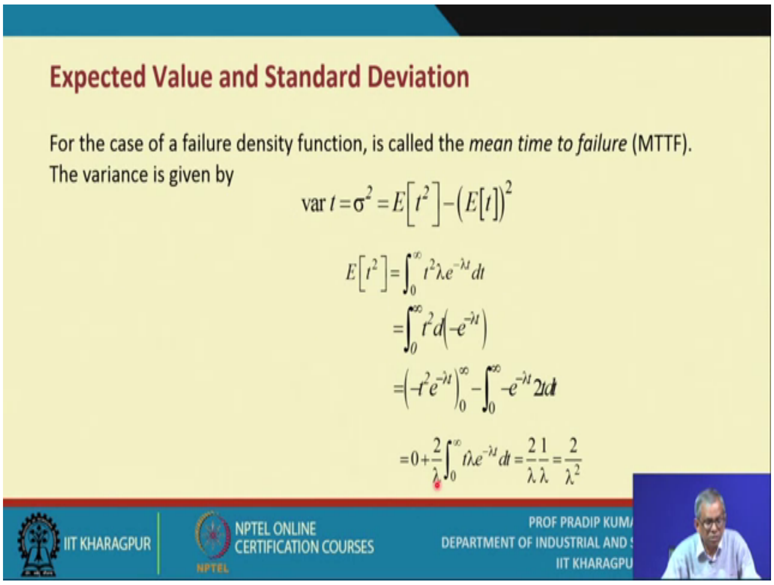

So, lambda is the parameter and so that is 1 and if you want to calculate the variance and on the positive square root of the variance is essentially the standard deviation. So, what do you do for the case of a failure density function is called mean time to failure that is mean time to failure, is it ok.

The variance is given by variance t that is sigma square then e to the t square minus et whole square is it ok, right. So, this is expected value of t square we have simplified these expressions t square lambda e to the power minus lambda t is it and then you simplify this expression integration by parts. So, you get two upon lambda square.

So, follow these steps one by one and I am sure that you will be able to follow these steps and you get these expressions of expected value of t square that is two upon lambda square.

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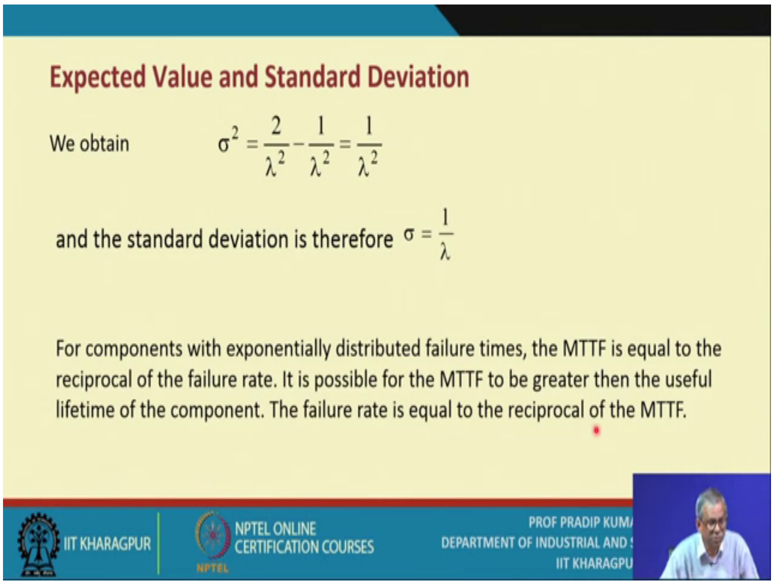

And then how do you get, we get the expression of sigma square that is 2 upon lambda square minus 1 upon lambda square that is 1 upon lambda square; that means, for the exponential distribution the standard deviation is also given by 1 upon lambda, is it ok. So, this is a single parameter distribution, right.

So, the comp components with exponentially distribution failure times the MTTF is equal to the reciprocal of the failure rate; that means, MTTF is nothing, but 1 upon lambda is it lambda is the failure rate. So, 1 upon lambda is; obviously, the mean time to failure it is possible for the MTTF to be greater than the useful life time of the component, is it that is very very important observation in many cases is it you have we specified the useful life is it ok.

So, that is the second phase in the life cycle curve or bathtub curve is it ok. So, it may so happen that MTTF is greater than the useful the life time of the component the failure rate is equal to the reciprocals of the MTTF.

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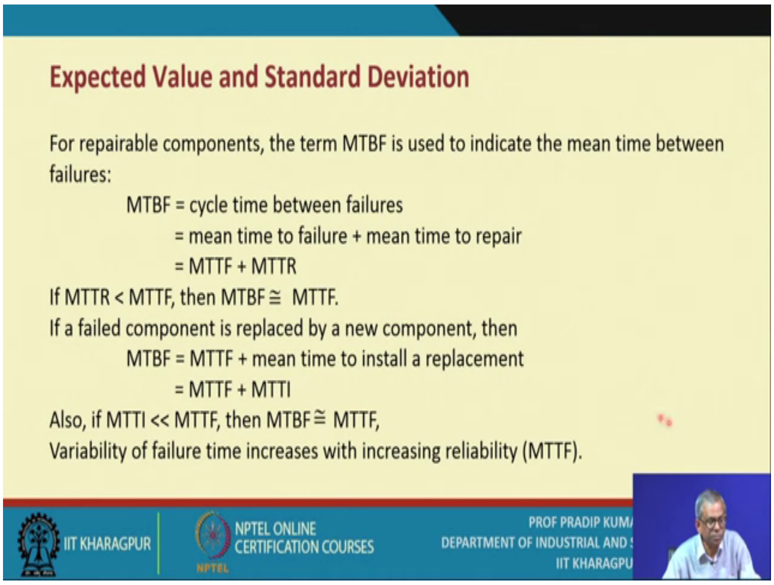

So, now I have already referred to another term called MTBF. Now let me elaborate on this for repairable components we say I have mentioned also that whenever the failure occurs what do you do you respond to the failure with the repair and under certain condition. So, that you check whether repair is possible or not, is it ok. So, if the product is a new one in majority of the cases in all likelihood that the even if there is a failure.

So, it is possible to be repaired, but as the component ages. Now, it ceases to be it may cease to be ceases to be a repairable say component or repairable product. So, what we are saying for repairable components the term MTBF is used to indicate the mean time between failures, is it ok; that means, as soon as the failure occurs then you start repairing.

So, you go for repair you take the repair time a repair time is also a random variable in majority of the cases is it by essentially you know you will be doing maintenance work and maintenance work in majority of the cases at least 50 to 60 percent of the cases or in certain situations maybe 90-95 percent of the cases or 100 percent of the cases is absolutely a human level or the manual one.

So, if it is manual one in all likelihood you know, it will be a random variable; that means, repair time is random variable. So, how do you define MTTF cycle time between failures is it and mean time to failure mean time to repair; obviously, when you consider the cycle time; that means, you start then after sometime it fails, then you repair and again you get back to your initial state of health again you start working, is it ok.

So, ah; so, there will be in the cycle time both you know the time to failure as well as time to repair. So, when you consider several cycle times ok; obviously, you can calculate the mean time to failure and you and you will have mean time to failure plus mean time to repair. So, MTTF plus MTTR mean time to repair if MTTR is less than MTTF then MTBF is almost equals to MTTF are you getting my point; that means, what we are telling you that MTBF is equals to MTTF plus MTTR now supposing this repair time is almost negligible, is it ok, this is very very less, is it ok, then what you can say; that means, approaching 0. So, you might say that MTBF is nothing, but MTTF.

But if suppose, the m this repair time is substantial; obviously, MTBF is greater than MTTF is it clear if a failed component is replaced by a new component and the replacement time is almost say is very very less and it is controllable then MTBF equals to MTTF plus mean time to install a replacement so; that means, in the replacement of a part not the entire system sorry entered the product then it is not called maintenance it is called basically the replacement.

So, at certain point in time when you know the component or the system or the product has sufficiently aged what you try to do; that means, the maintenance may not be effective and in that sense you know the maintenance may not be economic. So, you have to take a decision for replacement and related to replacement; obviously, there could be many approaches there are many models, is it ok.

So, you know. So, we are not focusing on replacement we are focusing on the maintenance part. So, the mean time to install a replacement is equals to this one. So, if MTT 1 is less than equals to MTTF then; obviously, MTBF is approximately equal to MTTF. So, you know what is MTTF you know what is MTBF how MTBF is related to MTTF and what is MTTR and then under what conditions is it whether the condition of replacement or the condition of repair MTBF may be equated with MTTF, but if it is you know under what conditions you cannot say that MTBF is equals to MTBF.

So, all these conditions you must know and accordingly you were shown that whether I will I need to say calculate MTBF and that is enough or along with the MT MTTF I also have to calculate MTBF is it ok. So, variability of failure time increases with increasing reliability failure time increases variability of failure time increases, is it with increasing reliability; that means, the variability will be more in the failure time, is it the variations.

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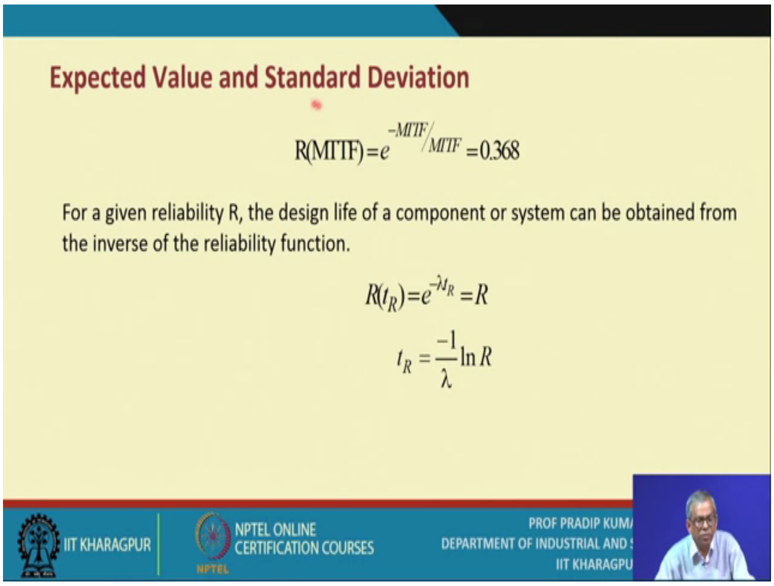

So, expected value and the continuing our discussions on expected value and the standard deviations; So, reliability that it will last for MTTF is equals to e to the power minus MTTF is it ok. So, this is say 0.368 for a given reliability are the design life of a component or the systems can be obtained from the inverse of the reliability function; that means, this is just an example 0.368 instead of data I will get.

So, the reliability that the component will last up to t R is it equals to minus lambda t into R. So, it is r. So, t R is minus ln R by lambda, is it ok?

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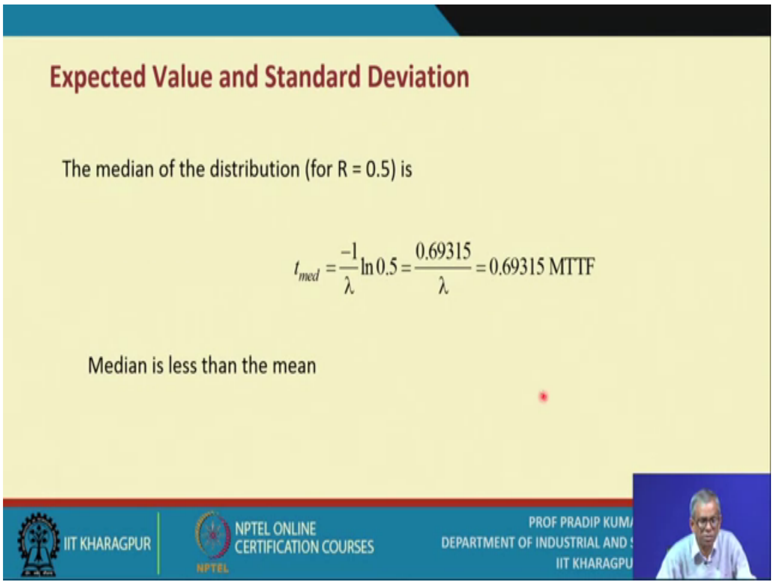

So, this is the expression. So, the median of the distributions for r equals to point five that is t medium. So, you have these expressions is it minus 1 upon lambda natural logarithm of 0.5? So, that is this value; that means, 0.69315 into m MTTF.

So, these sort of expressions you can derive and. So, what we conclude the median is less than the mean, is it clear.

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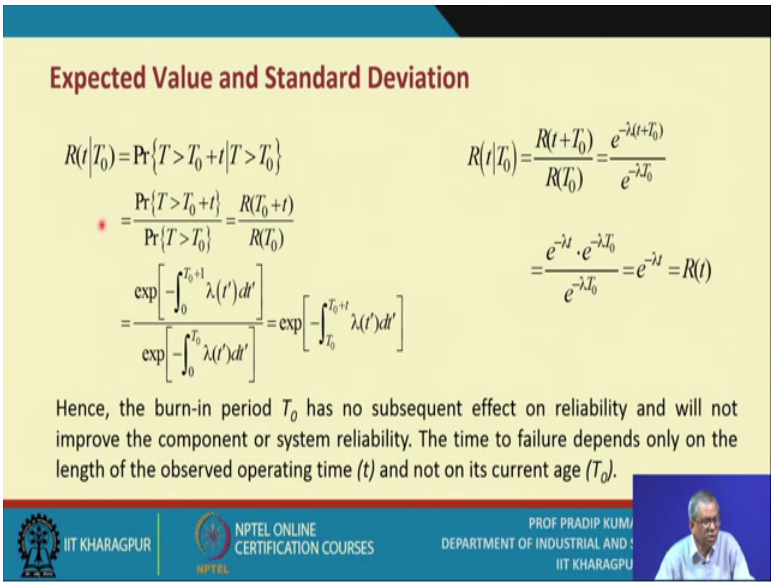

So, this is just an example we are try to do, then the reliability the t of up to T 0 time period.

So, this is the expressions we have made; that means, probability that capital T is greater than T 0 plus t provided t is greater than T 0, is it ok. So, these sort of conditions you come across and ultimately assuming you know the exponential density functions. So, you have these expressions and similarly you have computed you know this one R t T equals to 0 like this reliability that t plus T 0 after your divided by R T 0.

So, this is the expression e to the power minus t plus T 0 divided by e to the power minus lambda T 0, is it ok; that means, the component will last up to T 0 time period and this is the component will last up to t plus T 0 time period, is it ok. So, this is the conditional one already, it has lasted what is the probability that it will further you know the last or it will further survive for small t time period.

So, this way we define an ultimately if will get that it is e to the power minus lambda t and that is the reliability function is it ok. So, the several ways, you can derive and in the process your you know you get or you learn or you understand say the many you know the functions or the conditions involved or say you know the conditions, you must be able to perceive you must be able to understand while you model the reliability functions is it ok; that means, the several practical perspectives it to look into and accordingly, all the you know the aspects to be to be incorporated to be incorporate incorporated in the modelling approach hence the burn in period T 0 burn in period initially has no subsequent effect on reliability.

So, this point is to be noted is it ok; that means, you have three phases now the question is whether the first phase is impacting or the second phase performance or not and whether the third whether the third phase is dependent on say the second phase or not. So, what we are assuming that the burn in period T 0, is it that is the first phase in the life cycle carve has no subsequent effect on reliability and will not improve the component or the systems reliability, is it ok; that means, it is an almost an independent as a phase or independent event the time to failure depends only on the length of the observed operating time t and not on its current age T 0, is it ok. So, this point is to be noted and you have to prove like say whether these assumption holds or not.

If suppose these assumption does not hold; obviously, you your modelling approach will be different ok.

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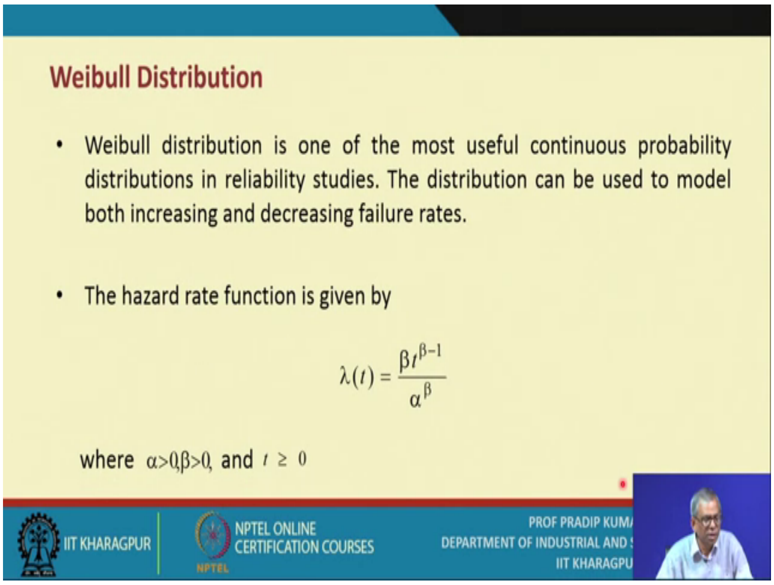

So, now as I have already mentioned; that exponential the distribution is a special case of Weibull distributions, but if you want to the. So, the model all the three phases of the lifecycle curve is it then what you need to do; that means, one single distributions you can assume that is basically the Weibull distributions is it ok. So, let me just highlight will some important features of Weibull distribution. So, Weibull distribution is one of the most useful continuous probability distributions in the reliability studies, is it ok. So, you need to study the Weibull distribution and when you assume Weibull distribution.

So, corresponding you know the reliability related parameters you must be able to say compute distribution can be used to model both increasing and decreasing failure rates; that means, it is used to model the burn in phase as well as another wear out phase is it ok. So, the first phase and the third phase of the lifecycle curve the hazard rate function already have mentioned that is you know as for the Weibull distributions is it three parameter distributions that is the gamma that is basically the location parameter then you have the beta that is in the shape parameters and the third parameter is alpha that is the scale parameters.

So, what is you; you may assume that the gamma the location parameter may be assumed to be 0 when you go for reliability modelling. So, you have alpha and beta.

So, what is lambda t lambda t is beta t to the power beta minus 1 divided by alpha to the power beta. So, this is a simplified expression where alpha is greater than 0, is it beta is also greater than 0 and t is greater than equals to 0, ok.

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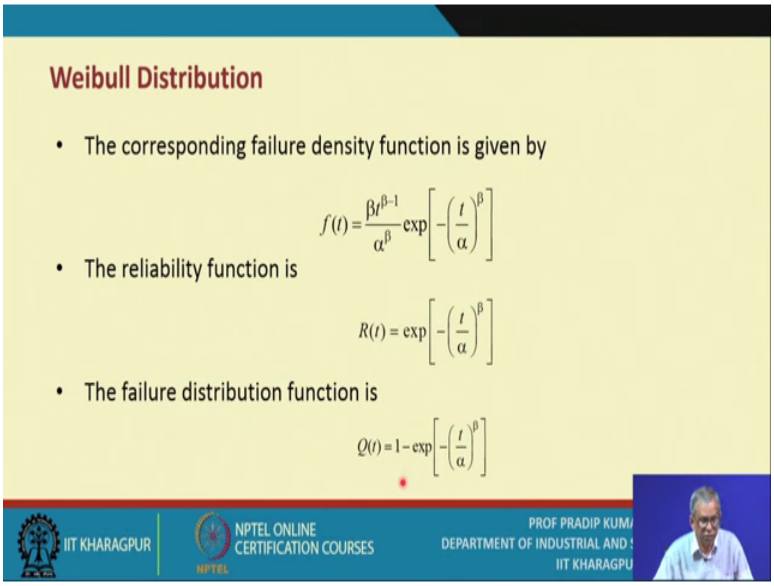

So, the corresponding failure density function is given by this; that means, ft is equals to beta to the power beta minus 1 divided by alpha to the power beta in exponential e to the power minus t upon alpha to the power beta is it ok. So, these expressions directly get from you know the Weibull density function and assuming that the gamma is equals to is 0, is it all right. So, the reliability function is Rt you follow the same approach and you get an expression that is e to the power minus t upon alpha to the power beta, is it ok.

So, the failure distribution function is 1 upon R t 1 minus these expression e to the power minus t upon alpha to the power beta ok.

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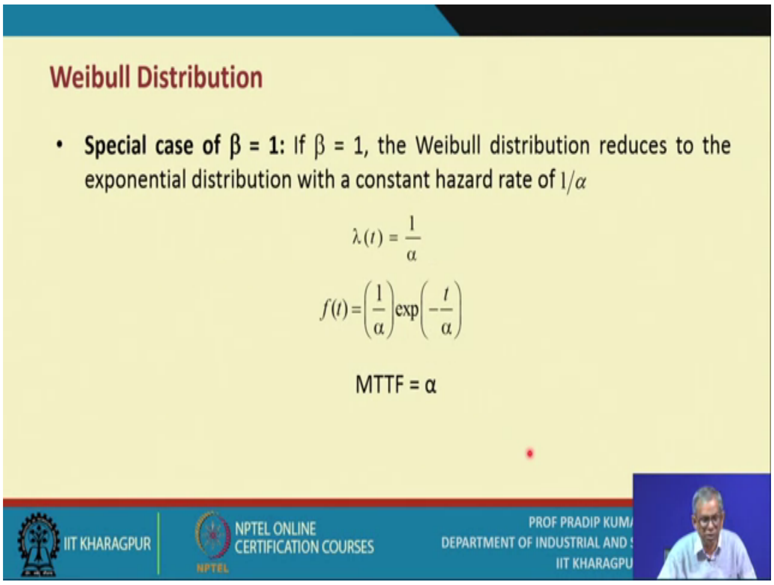

So, these are the steps you follow; now there is a special case; that means, the shape parameter that is beta equals to 1; if beta is equals to 1 the Weibull distribution reduces to the exponential distribution with a constant hazard rate of 1 upon alpha.

So, lambda t is equals to 1 upon alpha and ft becomes 1 upon alpha e to the power minus t upon alpha. So, what is MTTF obvious MTTF will be alpha, is it ok. So, and the lambda is changed to 1 upon alpha, is it ok.

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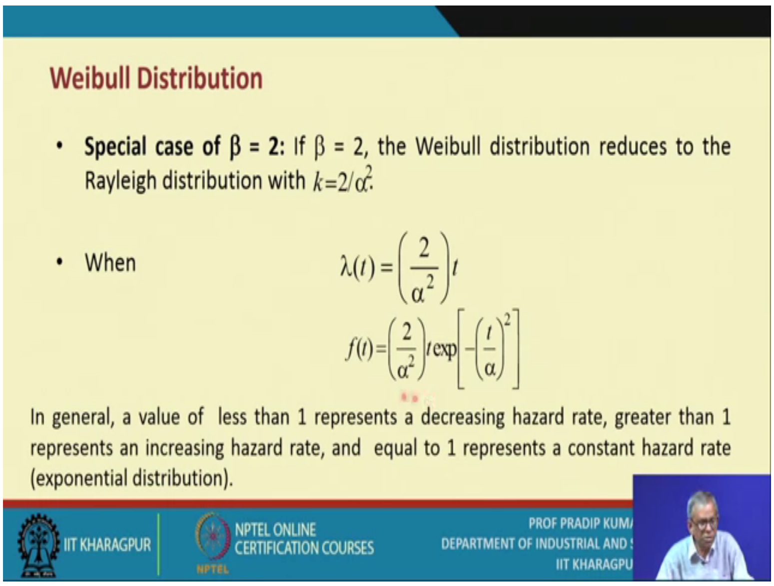

So, its constant failure special case of beta; beta equals to 2; that means, the shape will be changing if the beta is definitely greater than 0, but up to 1 it is if beta is equals to 1 you can assume it to be; that means, Weibull transforms to exponential.

If beta is equals to 2, the Weibull distribution reduces to the Rayleigh distribution, is it right. So, with k equals to 2 upon alpha square right when lambda t. So, it beta equals to 2 with beta equals to 1, it becomes exponential distribution if beta equals to 2 it becomes Rayleigh distribution when lambda t is equals to this ft becomes this 1 is it ok.

So, this is a special case in general a value of less than 1 represents a decreasing hazard rate greater than 1 represents an increasing hazard rate is it and equal to 1 represent a constant hazard rate exponential distribution; that means, the beta is very very important if beta is equals to say 0.5; that means, you can with beta equals to 0.5 you can model say you know the decreasing hazard rate; that means, burn in phase you can model

If the beta is greater than say 1; obviously, you can model say wear out phase ok.

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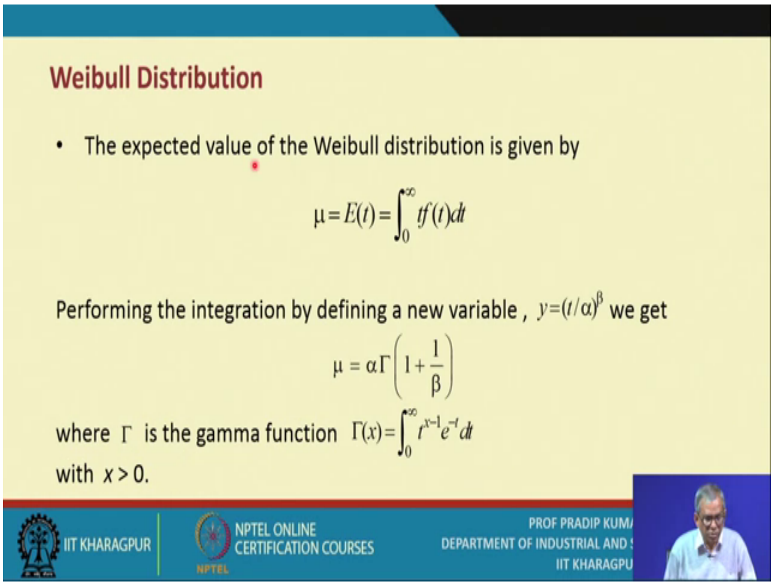

So, so expected value of the Weibull distribution this we have computed. In fact, like mu is the general notation expected value of T 0 to infinity tft dt and then ft is replaced with this one we should like say Weibull density function; that is a Weibull density function performing the integration by defining a new variable y equals t by alpha to the power beta, we get mu equals two the direct you can do this is a gamma function alpha into gamma function 1 upon, 1 upon beta. So, this is the expression; that means, mean 1 where gamma function is the is the gamma function is given by as you already know that gamma x is nothing, but it is just you recapitulate 0 to infinity, 2 to the power x minus 1 e to the power minus t dt with x greater than 0, is it ok.

So, that way we define the gamma function. So, just you remember that this mu value is alpha into a gamma function of 1 plus 1 upon beta, is it ok.

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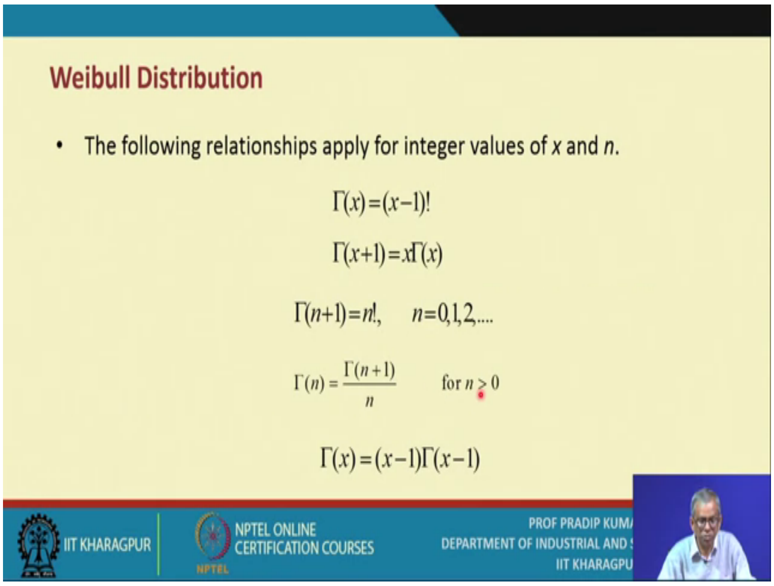

Beta; we have already defined. So, the following relationship applied for integer values of x and n like gamma x is x minus 1 where. So, these are the just the references as a references, I have given these expressions related to the gamma functions. So, just you go through them ok.

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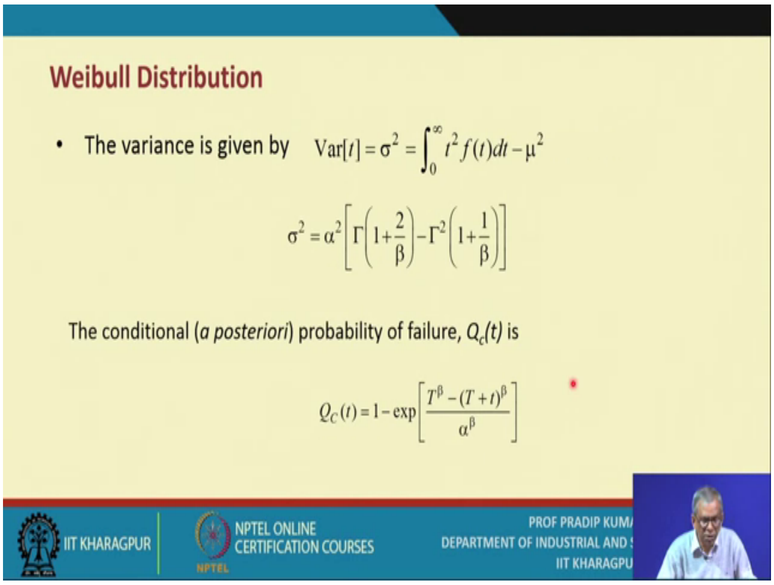

So, and then you say that the variance of the Weibull distribution is the sigma square and again you apply this you know the general expressions like 0 to infinity T square ft dt minus mu square, is it ok. So, ultimately the sigma square is alpha square gamma 1 plus 2 upon beta function minus the gamma function 1 plus 1 upon beta, is it functions.

So, the conditional or the a posteriori probability of failure Q c t again we have already explained. So, these expression you will be getting, is it ok. So, I suggest that you know you know as a part of exercise, you derive these expressions is it ok. So, I will have better understanding. So, later on we will give you several kinds of exercises on this.

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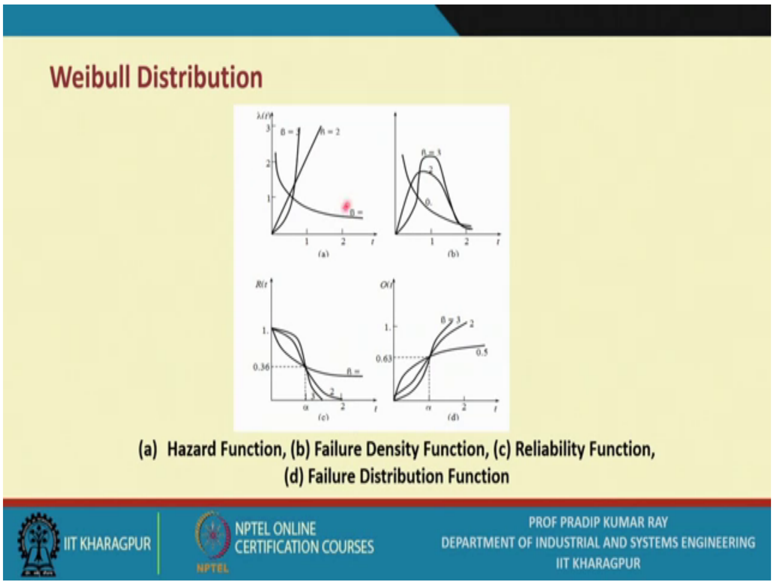

So, the under different situations like we have these four figures. So, this is with respect to say the hazard function that is figure a figure b is a failure density function this is the failure density function is it and the figure c related to the reliability function is it and the failure distribution functions that is figure d.

So, please study all these all these figures and you try to you will find that as the value of beta changes the shape of a particular curve, is it also changes. So, what do you need to do; that means, you need to know the reasons, is it like here first one is the hazard function; that means, the failure rate function is changes and beta. So, the beta equals to 2 beta equals to 3 and since a beta is equals to 1.

So, it is changing similarly this is fx versus x, is it ok. So, ft versus t; so, this is the shape, right and this is as it is changing; that means, for a certain value of beta; that means, the distribution is almost is looks like a normal distribution is it like Rayleigh distributions, we have assumed right. So, when beta is equals to two. So, that is referred to as the Rayleigh distribution.

Similarly, we are bothering about the reliability function; so, over the time period how the value of the reliability the value changes. So, that you should be aware off and alternative, but as a as alternative to reliability; that means, it is say the failure distribution functions ok. So, this is just will be the reverse, right.

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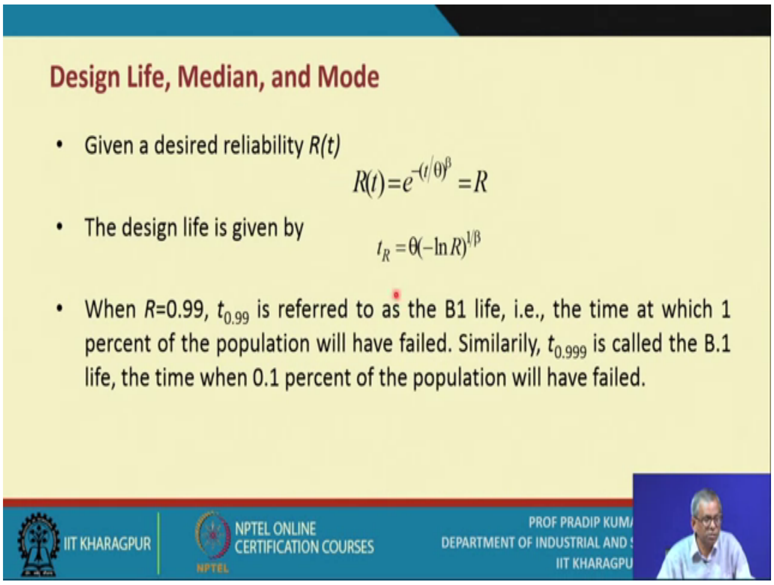

So, these are the obvious you know this sort of analysis you do. So, R t function is this one the design life is given by this and when R equals to 0.99 t at 0.99 is referred to as the B 1 life that is the time at which 1 percent of the population will have failed and similarly t 0.99 is called B 1 life the time when 0.1 percent of the population will may have failed, is it ok.

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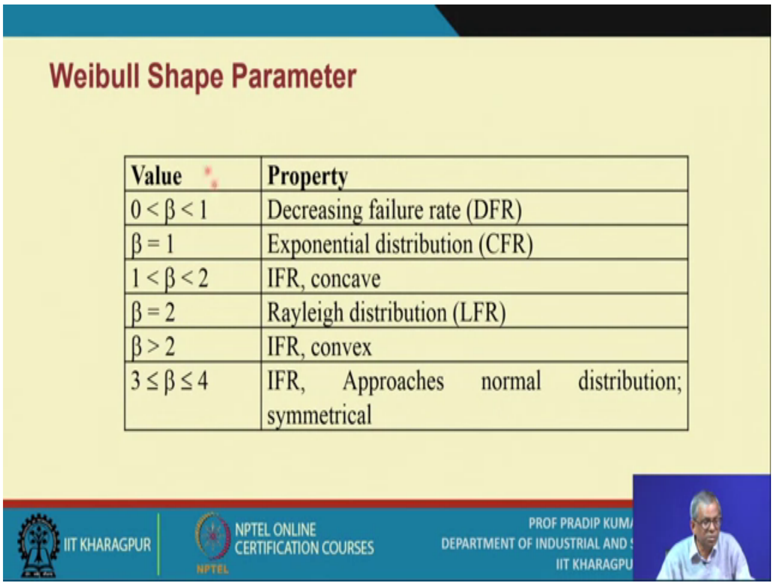

So, this is now Weibull shape parameters. So, you refer to this table I have already explained all these details. So, please refer to this table and then here is an example ok.

So, these examples we may refer to later on right. So, this is the typical examples. In fact, like say a mechanical system has demonstrated it Weibull failure pattern with a shape parameter of 1.4 and the scale parameter of 500 days determine, you know the reliability that it will last for 150 days the b 1 life MTTF the standard deviation they say the medium time and the more time is it and the design life for reliability 0.95.

So, this is a typical problem these are examples you go through and later on we will discuss several kinds of numerical problems related to a design of reliability ok. So, I conclude this session.