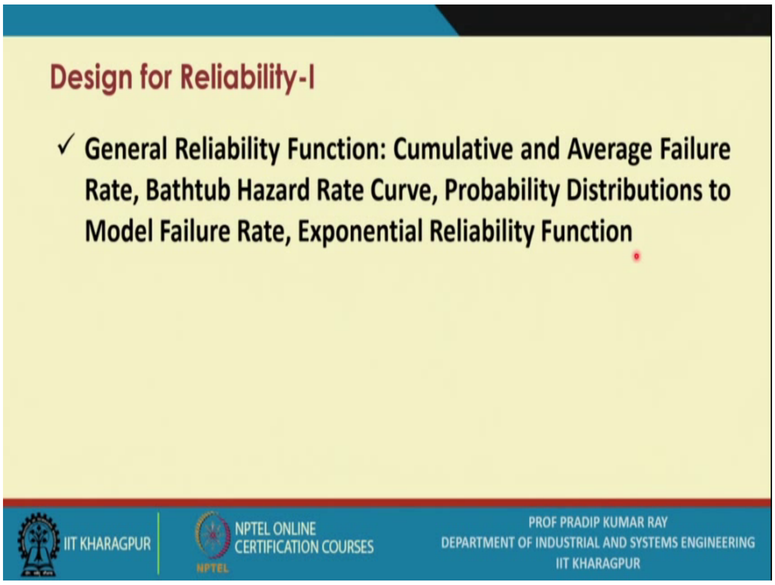
Quality Design and Control
Prof. Pradip Kumar Ray
Department of Industrial and Systems Engineering
Indian Institute of Technology, Kharagpur

Lecture - 42
Design for Reliability- I (Contd.)

During this session, on design for reliability, I will be continuing by discussions on the other important issues related to general reliability function is it ok.

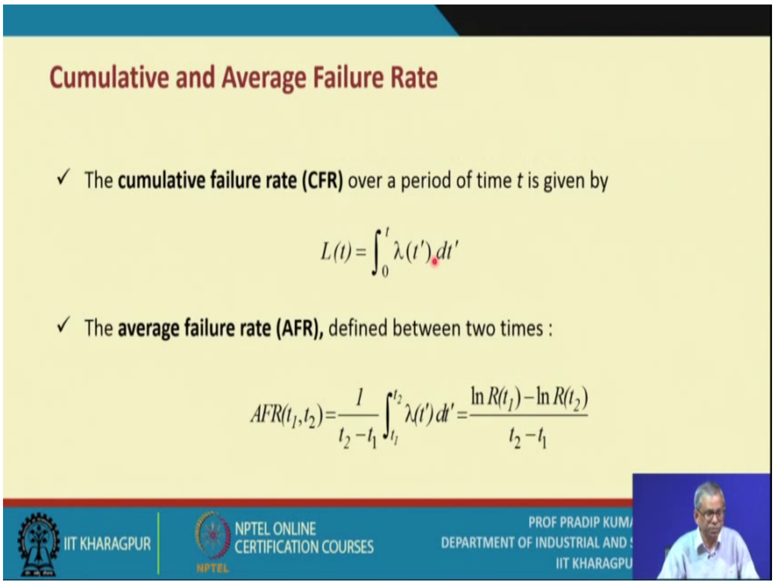
(Refer Slide Time: 00:38)



So, any exercise on reliability you know, it all depends on that how accurately, how were meticulously you know you can explained the reliability function, how you can model the reliability function. So, in this context, we need to discuss the general reliability function from several perspectives.

Now, the in this particular lecture session, during this lecture sessions, I will be discussing cumulative and average failure rate the so called bathtub hazard rate curve or the life cycle curve probability distributions to model failure rate and exponential reliability function, specifically, I will be discussing details about the exponential reliability function, there could be many other types of reliability functions, but exponential reliability function is very common and many a time, we refer to this one or modelling reliability that is why we will be discussing exponential reliability function in detail.

(Refer Slide Time: 01:48)



Now we know that for modelling reliability given a particular product or given a particular component or a systems.

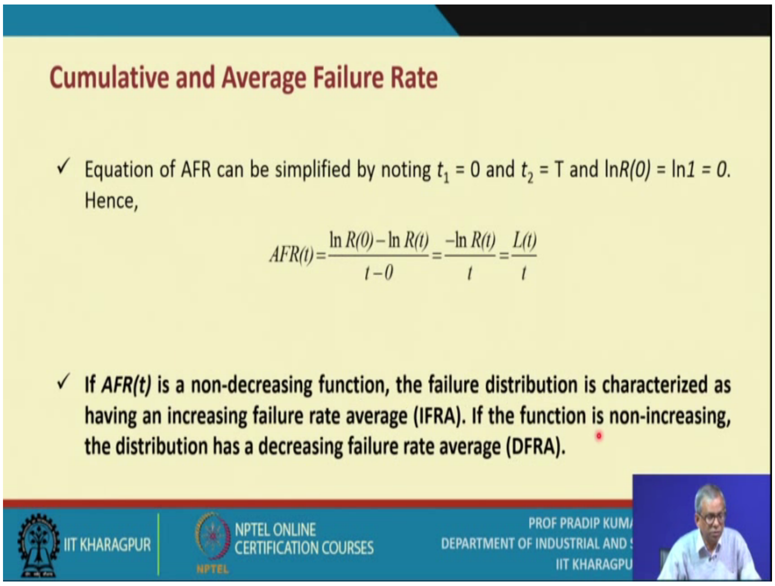
First thing you must know or you must we referring to the time to failure and you must be able to explain it properly so; obviously, the assuming that the time to failure is a random is a random variable so; obviously, I must know what could be possible distribution right now ah. So, once you have collected you collect data on the time to failure and then you refer to it is it is distribution like say probability density function, then cumulative distribution functions and then you go for reliability function and now we have already mentioned about the instantaneous failure rate and when you have the expressions for the instantaneous failure rate, in the last you know the lecture sessions, I have refer to the instantaneous failure rate; that means, the time T is equals to t.

Obviously from these instantaneous failure rate you can compute the cumulative failure rate and as well as the average failure rate is it ok. So, so how do you know get the expressions for the cumulative failure rate which is referred to as the CFR over a period of time T that is why it is known as the cumulative failure rate. So, this is given as these notations we have used that is L t is equals to 0 to t lambda t dt, is it ok. So, now, you have this expression for CFR. Now the next expression is AFR; that means, the average failure rate; that means, defined between two times is it ok.

That means that time t 1 and at time t 2, is it ok. So, failure rate I compute at time t 1 and failure rate also, I compute at time t 2 and then I compute the average is it . So, how do you compute; that means, this is the expression 1 upon t 2 minus t 1 is it ok, integration t 1 to t 2 lambda t dash into dt. So, ultimately you have these expressions so; that means, the natural logarithm rt 1 minus natural logarithm rt 2 divided by t 2 minus t 1. So, some the steps you follow and you know that I presume that there will be no problem in for you to getting these expressions is it ok. So, ah; so, alternately average failure rate between the time t 1 and t 2 is given by this expression; that means, natural logarithm of rt 1 minus natural logarithm of rt 2 divided by t 2 minus t 1 is it ok.

So, it is a rate that is why, there is a time dimension ok.

(Refer Slide Time: 05:18)



Now equation of average failure rate can be simplified by noting t 1 equals to 0 and t 2 equals to t; that means, what we are saying that at this point in time at time t is equals to 0 when the product starts functioning what could be the failure rate is it and when it with lasts say for say t time period capital T time period, it has survived evidently T time period. So, at that point in time; that means, t t equals to capital T, what is the failure rate, is it ok.

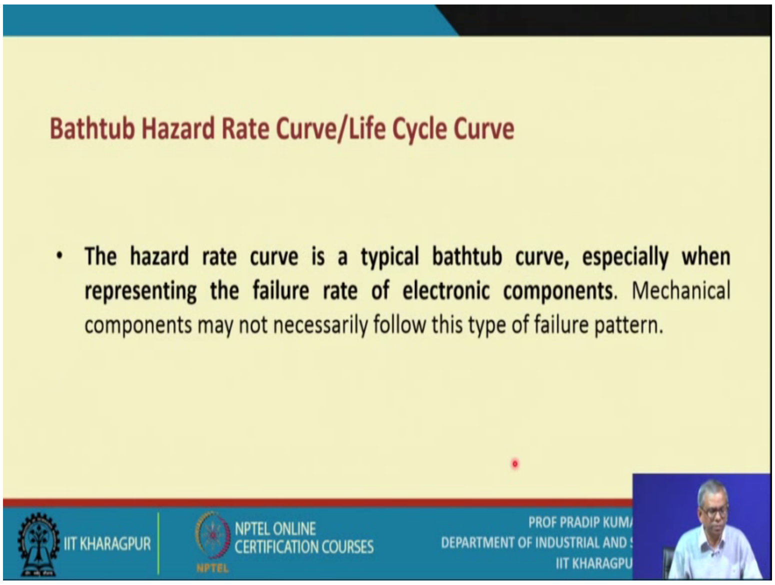
So, this is a particular case and your apply this rule. So, ultimately what you will find you know if you get these values then and this is ln R 0 minus ln R t is it ok. So, this is t minus 0. So, this could be capital t this could be capital T, is it ok. So, ln divided by L t divided by t. So, that is a particular case is it you compute when the general expression of AFR is known. So, you get the particular values of p and then you get for the corresponding in all the time period what is the average failure rate. So, if AFR t is a non decreasing function; non decreasing function; that means, a failure rate is increasing is it the failure distribution is characterized as having an increasing failure rate average.

So, we call it AFR is it a desirable situation, it means these conditions you have to check you have to control, is it ok. Now how to do that; what are the technical means with which you can control this increasing failure rate average. So, there are many ways you can arrest, you can control this, you know the increasing failure rate average is it, these are that could be engineering solutions that could be you know the technical solutions.

If the function is non increasing; that means, decreasing, what could be the possible reasons that you have to find out, is it ok, later on when you refer to the exercises, we will find that you will be asked to identify the possible causes of getting this sort of you know say the behaviour of the process or of the process or of the product; that means, increasing failure rate average, is it ok.

So, what could be the possible reasons and if you have this sort of you know you know the behaviour. So, what are the possible means with which you can control this is it and similarly, if the function is non increasing, the this is a desirable one so; that means, it maybe you can related to say the design of the product, you may related to you know the functioning of the product is a functioning of the product the distribution has a decreasing failure rate average is it ok. So, there could be many types of exercises, we can we can assigned related to this these two kinds of situations ok.

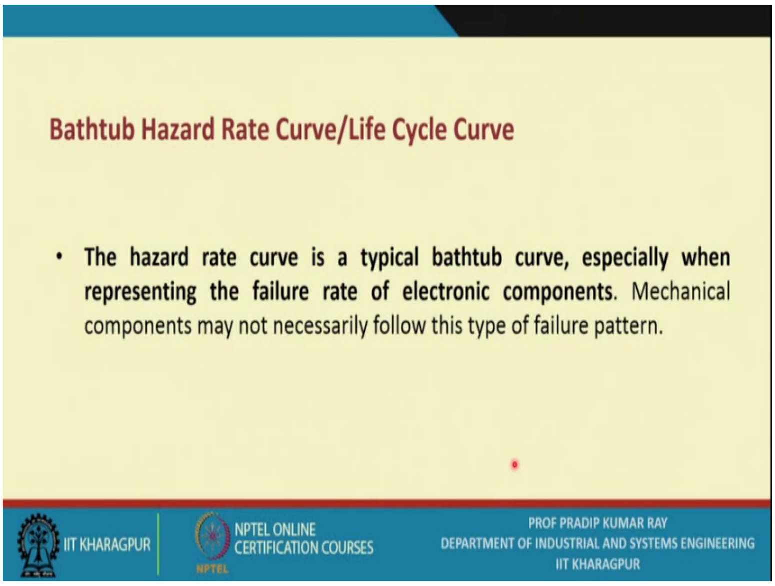
(Refer Slide Time: 08:45)



Now I will be discussing an important say curve that is always be referred to any discussions on reliability whenever we try to so, the model reliability.

We always refer to the life cycle curve and the technically, this is referred to as the hazard rate curve, but it is also referred to as the life cycle curve and as this the life cycle curve or the hazard rate curve ok.

(Refer Slide Time: 09:13)



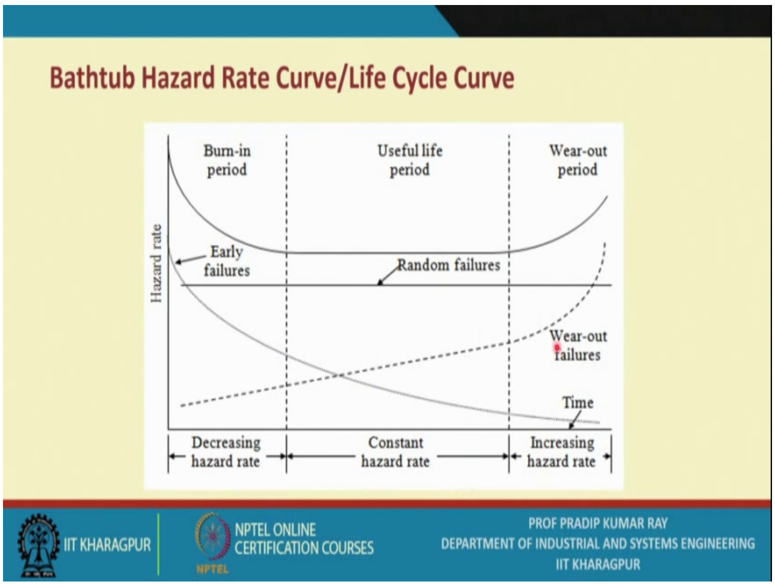
It looks like a resembles a sectional view of a bathtub. So, many a time, this is also referred to as the bathtub hazard rate curve or simply bathtub curve is it ok. So, in any exercise on reliability we check whether this the concepts of the bathtub hazard rate curve is applicable or not.

So, let us discuss this bathtub hazard rate curve in detail. Now, this hazard rate curve is a typical bathtub curve ok, I have already mentioned especially hen presenting the failure rate of electronic components, I am not saying that this other life cycle curve or the bathtub curve is you know is this concept is applicable for all types of products, but the majority of the products you may assume that this particular curve is acceptable or is valid.

So, mostly electronic components mechanical components may not necessarily follow this type of failure pattern, is it ok. So, you have to the looking to the design you have to look into the you know the characteristics of a particular product before you conclude whether the bathtub curve is applicable or not other life cycle curve, but what the researchers what the practitioners observed over the years.

For the majority of the components for the majority to the products ok, this concept of bathtub curve the life cycle curve is applicable.

(Refer Slide Time: 11:12)



Now let me explain this life cycle curve or the bathtub hazard rate curve. Now this is basically a plot of say failure rate process time in respect of a particular product. So, y axis represents hazard rate or the failure rate and x axis represents over the time. Now here when you start using the product is it ok. Now over you know during the entire life of the life of the product; that means, the product is born here you start using it and the product dies at this point in time.

So, what we say that whatever may be there is the quality of the product whatever maybe the design level of the product the product is born and product also dies. So, the entered during this entire life period is it ok. Now the failure rate actually the varies is it ok. So, at the initial stage you know you will find that the failure rate is very high and then over the time period, it starts decreasing; that means, during this phase; obviously, when you find the hazard rate or the failure rate is very high you have to take some corrective measures is it ok. So, initially you were never know you have designed the product you are started using it after it is it is manufacture and then in the in the real the situations or real conditions you start working.

So; obviously, you know when you starts working in a in a in a real environment the performance of the product may not be the you know that predictable. So, usually what happens initially as product is a new one; that means, a failure rate hazard rate is very high, but ultimately over the short period of time, it reaches a particular level that is the failure rate is it reaches over here. So, what is this phase called? So, this phase is called burn in period or the debugging phase right and then ah. So, they could be some failures over here, now what you try to do; that means, you take some you know the corrective measures over there. So, that you know the failure rate reaches over there and then the useful life period starts.

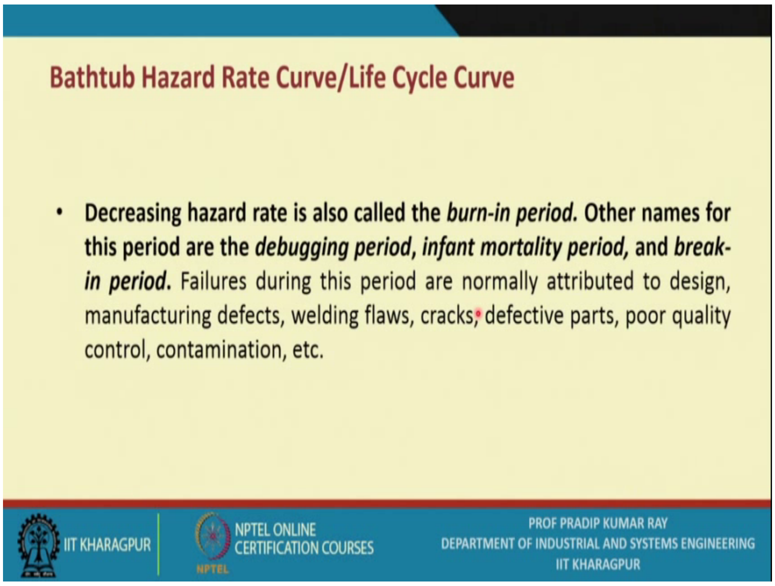
That means what you trying to do suppose a car you purchased you start you started using it initially you know you never know that whether the car will performed as per its intended function at the initial stage. So, that could be many failures. So, what you try to do you just be in touch with the you know the servicing and they will do the servicing and once the service is done then you will find that you start using the car for.

So, the next 5 years; in next 6 years without any problem; so, what we say that now once this born in period is over, now you have entered the useful life period and this will continue and during this useful life period what you have noticed that the failure rate is almost constant it should be at the minimum level and the failure rate also should be minimum, then what happens that; obviously, at a certain point in time that mean suppose between time t 1 and t 2 time periods maybe 5 years, maybe 6 years, what do will find.

Now, the aging process has started in. So, as the aging process starts what happened that; obviously, the failure rate or the hazard rate starts increasing and ultimately at some point in time what you have to do; that means, hazard rate has reached that level that you have no other alternative for to withdraw the product for that particular unit; that means, it has it has a lift with full life its full life.

So, this period is known as the wear out phase. So, whenever you refer to the lifecycle curve or the bathtub hazard rate curve; that means, this looks like a the bathtub is it that is why it is referred to as a bathtub curve. So, here three phases burn in period debugging phase is it when a child mortality phase, then you have the useful life period phase and then you have the wear out phase. So, these three phases are noticeable in any say life cycle curve or a product ok.

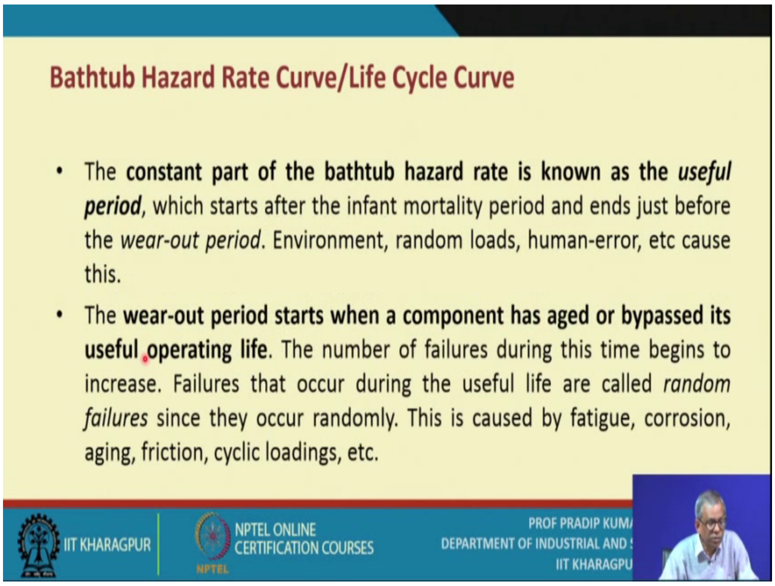
(Refer Slide Time: 16:10)



So, this for the given product, you must have the corresponding data with you, is it ok, now decreasing hazard rate is also called the burn in period that is you refer to the phase one, is it ok, burn in period other names for this period at the debugging period debugging phase, I have already mentioned infant mortality period or child mortality phase and the breaking period this is also referred to as the breaking period different names given failure during this period and normally attributed to design is it because weather your design is really you know is good or bad, you will come to know when you start using the product is it . So, the failure maybe due to the design or the manufacturing defects welding flaws cracks, say that could be several reasons.

So, I have just highlighted few reasons defective parts poor quality control and contamination, now here one particular point you should focus on that is poor quality control this also could be the main reasons of early failure ok.

(Refer Slide Time: 17:24)



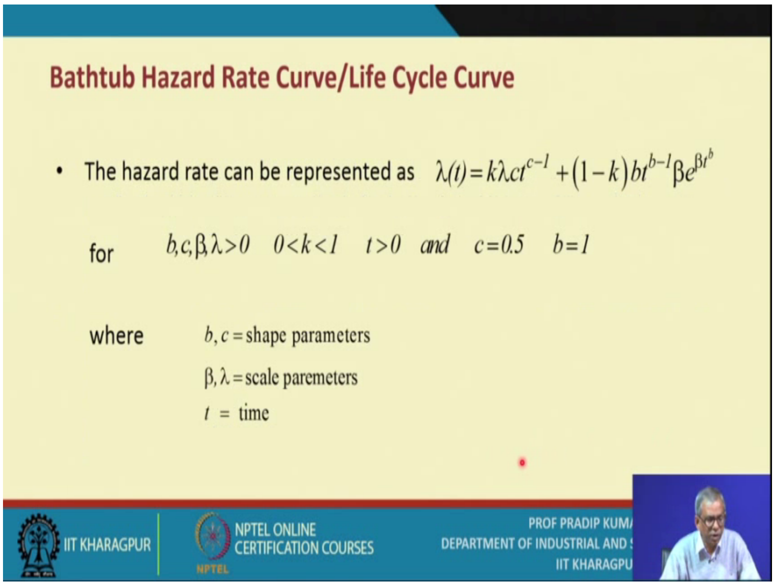
The next is when you reach the useful life phase; that means, constant part of the bathtub hazard rate is known as the useful period; that means, it is essentially the constant failure or failure rate; that means, that your almost your performance on the product is predictable which starts after the infant mortality period; that means, during infant mortality period you need to set take some corrective or the preventive measures related to design related to manufacturing.

And accordingly you know ah. So, you improve the quality of the product and hence just before the wear out period, is it ok; that means. So, when you know you declare the product to be an aged one and quiet old and the; you assure that the aging process started is it ok. So, the environment random loads human error etcetera cause this is it ok. So, you might say that why not the failure rate comes down to 0 so; obviously, there will be some uncontrollable noise factors, is it like. So, the environmental conditions wildlife shape the randomness in loading that could be human error is it ok. So, human error or unintentional human error it is absolutely you know, it says random occurrence. So, there could be such causes, ok, these the system you know in the system this causes do exist.

So, because of this there will be a a failure rate or say hazard rate, but it should come down as minimum as possible that is to be ensured now next concept wear out period. Now this period starts when a component has aged or bypassed, it is useful operating life, is it ok, the number of failures leaving this time begins to increase that is why the failure rate has increased failures that occurred during the useful life are called random failure since they are randomly, is it ok. So, you need to you know where you collect the data and particularly this exercise is very very useful in the context of quality in the context of reliability study; that is whether you can conclude that whether the failures are occurring due to some random causes ok.

So, these exercise is very very important we refer to many case studies on quality and reliability and ah. So, when you start analysing this data. So, you must be as a expert as a learner you must be able to identify the random causes also ok. So, this is caused by fatigue corrosion aging friction cyclic loading etcetera whether it is a mechanical systems or electrical systems these are the; you know the general regions ok.

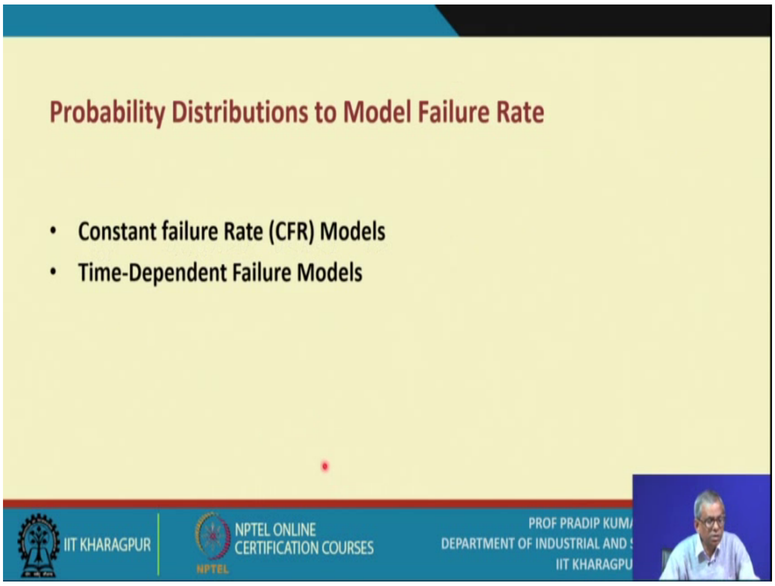
(Refer Slide Time: 20:37)



You come across now how to represent this the failure rate curves for all these three phases. So, hazard rate can be represented at this is one lambda ct c minus one plus this is this is the expressions we have is it t to the power b minus 1 beta e to the power beta t to the power b, is it ok.

So, you have three parameters alpha say the beta and gamma, is it ok.

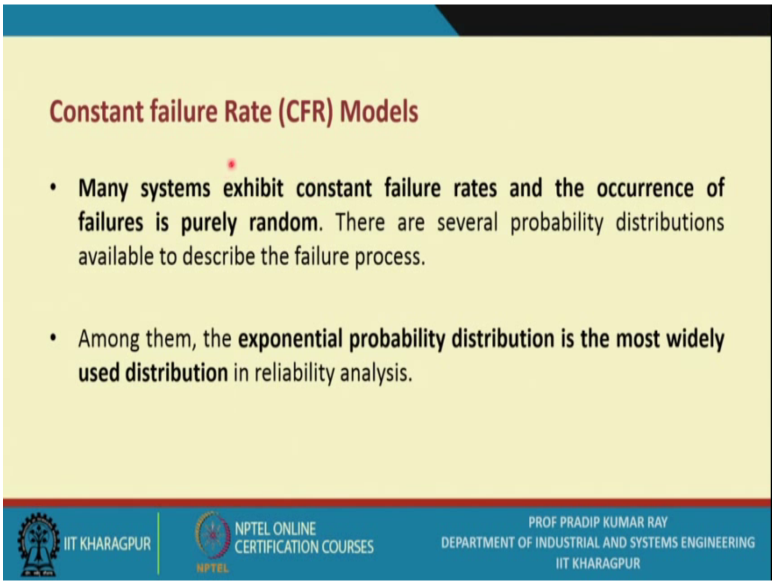
(Refer Slide Time: 21:38)



So, this is the expression lambda t and this is this is these are the bc beta and lambda these are all greater than 0 k lies between 0 and 1 and t is greater than 0 c equals to 0.5 and b equals to 1. So, where b and c are the shape parameters beta and lambda is a scale parameters k and t equals to the time is it ok. So, these are. So, this will be again referring to how to model all these three phases now let us talk about there is the modelling the failure rate and while you try to model the failure rate you refer to the corresponding probability distributions. Now there are two types of situations you come across as per as modelling reliability is concerned.

First one is the there could be cases where you assume that the failure rate is constant. So, the constant failure rate models we will be will we referring to that is the first part and another second part will be referring to the time dependent failure models is it ok. So, there are ah. So, all these models are classified under two categories right.

(Refer Slide Time: 22:32)

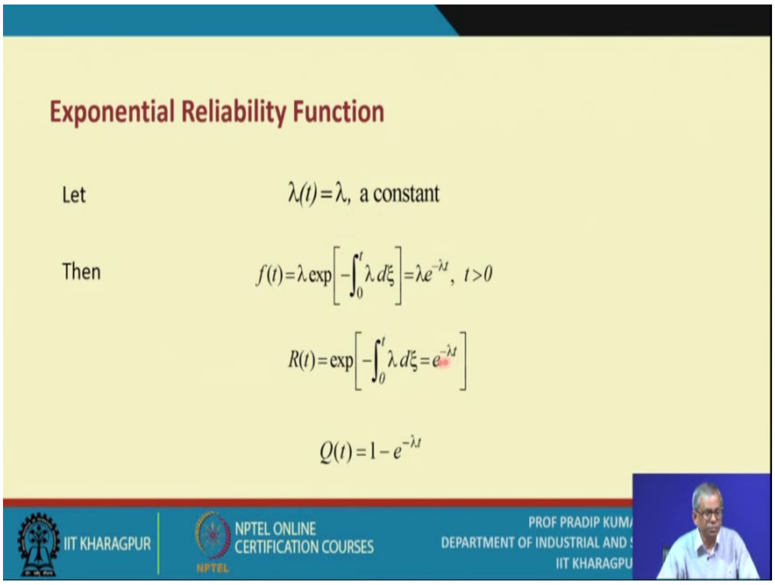


Now let us talk about I will be, just now I will be talking about the constant failure rate models; that means, what we will assume that the product or the system or the component or the process. So, there at the really useful life phase useful life phase is clear; that means, the debugging phase is over it has not yet reached the you know the burn in say it has not reached, you know or the declining phase or the wear out phase wear out phase by it is in the constant failure rate case or the phase or the useful life phase.

Many systems exhibit constant failure rates and occurrence of failure is purely random this point I have already mentioned there are several probability distributions available to describe the failure process is it ok. So, the choice is yours; that means, you have to collect data you have to analyse the data and then you check and you must know what is the what is the physical systems is it in getting the data and there must be some physical basis of assuming certain distributions or whether the distribution is an empirical form. So, that these distributions can be either in the empirical form or in the standard form is it ok. So, so that we exercise you have to do.

So, what do you saying that when you refer to different kinds of data sets you may come across or the different kinds of probability distributions to describe the failure process among them the exponential probability distribution is the most widely used distributions in reliability analysis is it ok. So, is a single parameter the distribution, you are all aware of an exponential probability distribution is a particular case of Weibull distribution, is it ok. So, I repeat that it is a special case; that means, exponential distribution is a particular it is a it is a it is basically a part of the Weibull distribution; that means, a special case of Weibull distribution is exponential probability distribution ok.

(Refer Slide Time: 24:59).

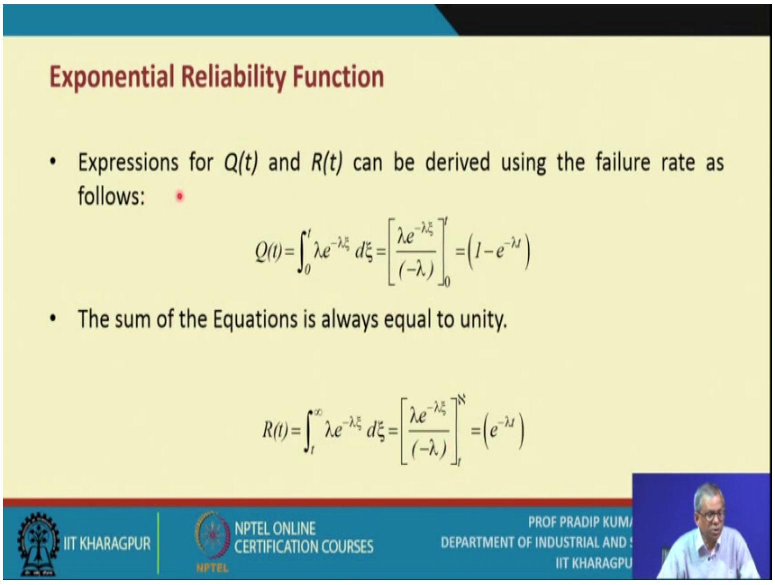


Now, let us talk about exponential reliability function; that means, whenever you try to model the useful life phase of a given product assuming that the birth of curve concept is valid now we need to model it. So, it start from this; that means, lambda t is equals to lambda is not is a function of time. So, it is a constant we are assuming.

So, what is probability density function that is ft is equal to lambda e to the power; that means, e to the power minus 0 to t lambda lambda t xi so; that means, this is lambda e to the power lambda t. So, this is dt actually lambda dt, is it ok, equals to lambda e to the power minus lambda t is it ok. So, this is a typical you know this is the density functions of probability density functions of t greater than 0 is for the exponential distribution.

So, what is Rt? Rt; obviously, you will get that is dt lambda into dt is equals e to the power minus lambda t. So, whenever you assume that that exponential say failure rate. So, you say that R t; that means, that the component or the product will last for t time period is it that is a reliability function is e to the power minus lambda t so; obviously, you know say this is Q t is distribution function there is 1 minus e to the power minus lambda t ok.

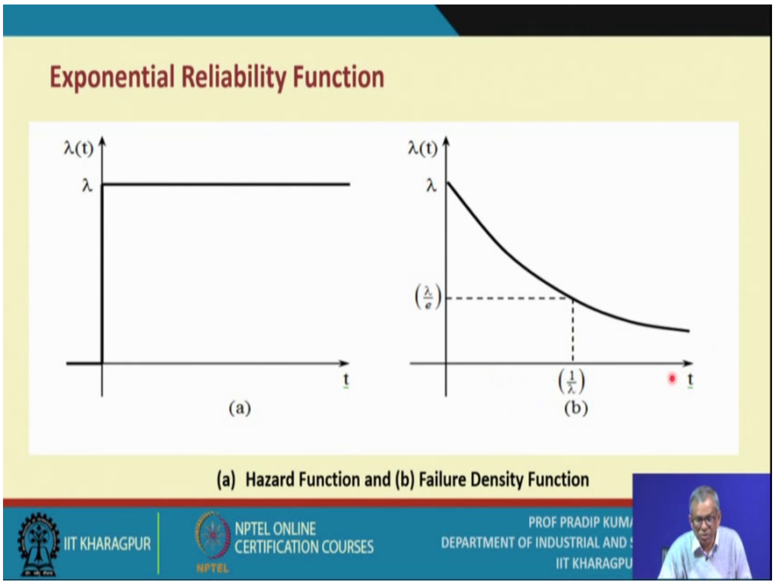
(Refer Slide Time: 26:49)



Now expressions for Q t and R t can be derived using the failure rate as follows. So, Q t 0 to t lambda e to the power minus lambda t into dt is equals to ultimately, now you go for this expressions one minus e to the power minus lambda t.

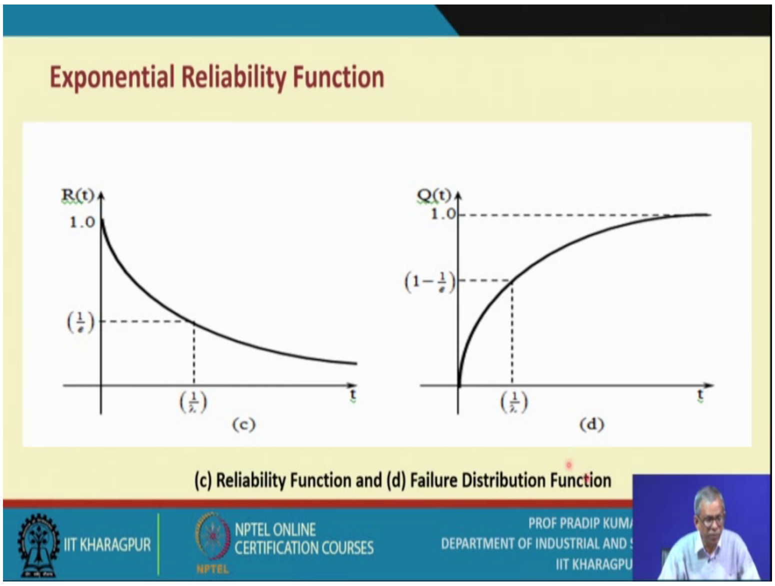
So, this part already we have derived and also the sum of the equations is always equal to unity. So, you get R t is equals to t to infinity; that means, lambda e to the power minus lambda t e to the power minus lambda t.

(Refer Slide Time: 27:24)



So, these two expressions you must remember and then this is the graphical representation; that means lambda versus t. So, you will find this is constant and then this is basically lambda t versus t ok. So, this is a function of t this is the second case is it as the time passes what do will find the lambda changes that is the second case is it ok; that means, hazard functions and failure density function ok.

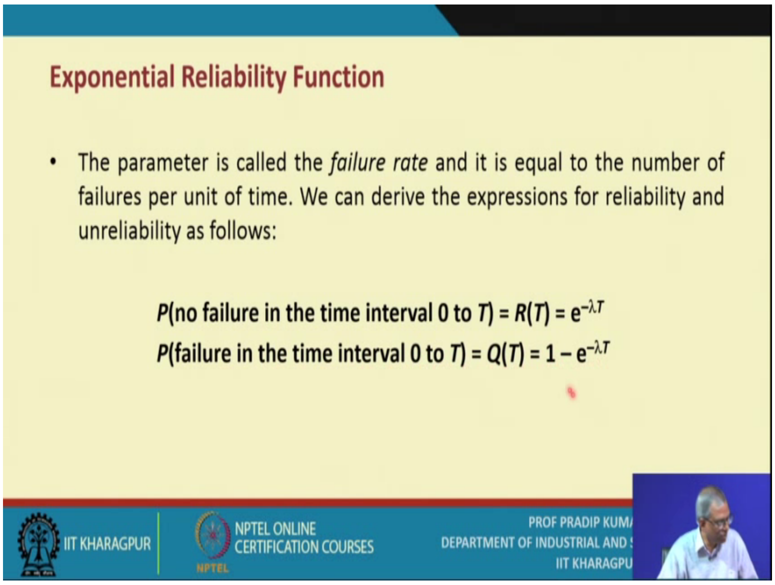
(Refer Slide Time: 27:53)



Then you have this reliability functions we have is it ok. So, as a time passes; that means, this is R t versus t.

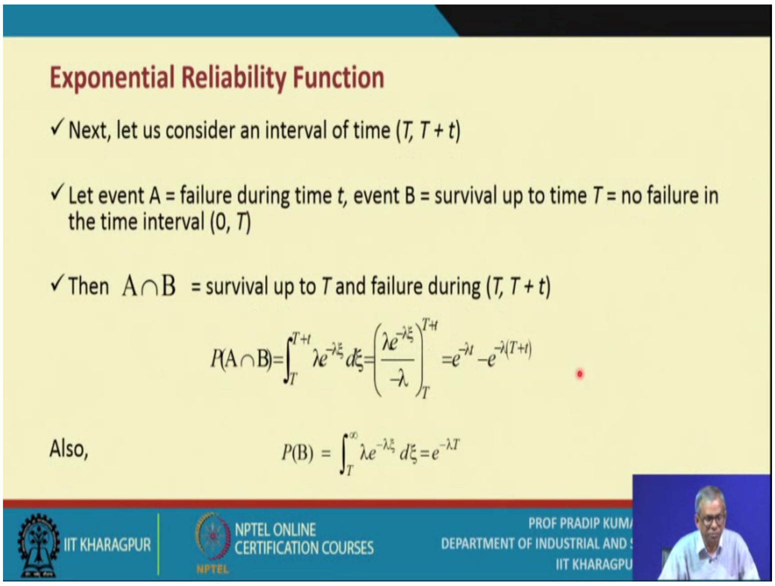
So, it is quite obvious that if a t increases the R t decreases is it and this is the cumulative distribution function specifically right. So, as the time passes the value of Q t increases and the limiting value is one ok.

(Refer Slide Time: 28:20)



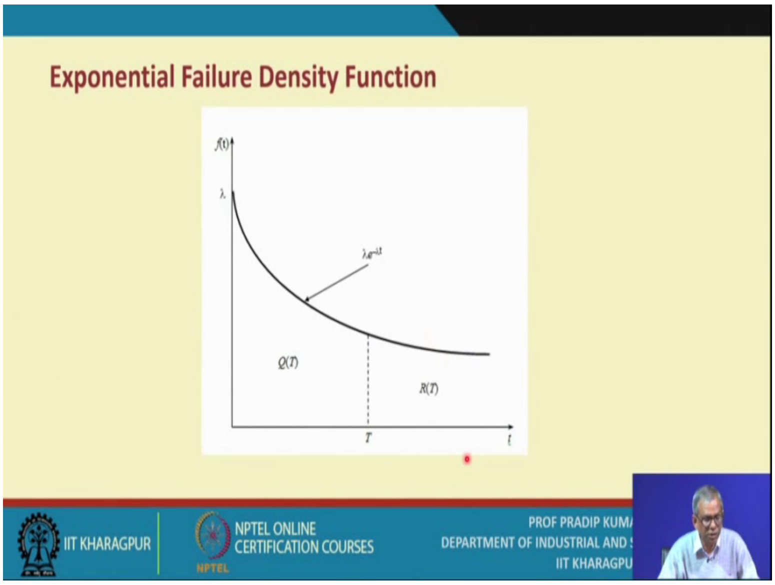
So, this is just a graphical representation. So, the parameter is called the failure rate and it is equal to the number of failures per unit of time is it that is for failure rate again we are referring to I need to define the failure rate in explicit manner. So, we can derive the expression for reliability and unreliability as follows probability that no failure in the time interval 0 to t. So, that is basically R t is equal to e to the power minus lambda t and failure in the time interval 0 to t that is Q t 1 minus e to the power minus lambda t, is it ok,

(Refer Slide Time: 29:01)



So, ; so, for the exponential reliability function, let us consider an interval of time T and T plus small t. So, let event a equals to failure during time t event b survival up to time T, no field that is no failure in time interval 0 to t; that means, between 0 to T no failure and between t and t there will be capital T and T there will be failure. So, then a you know b is equal to survival up to t and failure during T and T plus t is it ok. So, you have these expressions ultimately. So, e to the power minus lambda T minus e to the power minus lambda T plus t; is it ok. So, ultimately this is the probability that they will be b; b is the survival up to time T that is e to the power minus lambda T, is it ok. So, another way we are we are getting the expressions for us for the reliability.

(Refer Slide Time: 30:03)



So, exponential failure density function say ft versus t that is this one is it ok. So, how do you defined this density function that is lambda e to the power minus lambda T T is greater than 0 ok.

(Refer Slide Time: 30:18)

So, with this the basic say introduction to the failure rate function ah. So, ah; so, what we are trying to focus on that is how to model or the reliability. So, the reliability function must be explicitly specified for a given product and for the data to be collected and three things we have to do like. So, the first you must be able to specify the probability density function then the probability density function, you then you defined the cumulative distribution function.

And then the cumulative distribution functions now you refer to the reliability function and then you check for the different time periods is it what is other reliability value of the reliability. So, what do you observe that as the time increases the value of the reliability decreases, is it ok?

So, that is you know the general conditions in almost all cases will observe now what you need to do you need to the specified by the time period; that means, the survival time when a time when you refer to the warranty of the product; that means, you say for a given product say like an industrial products say transformer we say that you start using it you find that I am guaranty you with the ten years time period I am giving you the warranty of 10 years.

It means that already I have tested its reliability and I conclude that within a ten years time period even if it is a continuous running there is hardly any chance of his failure is it ok; that means, you have to specify certain time period where you are you are very very much sure there will be hardly any probability of failure. So, at least whenever we try to you know the study reliability of a product ultimately. So, whatever the results you get it must be reflected in assigning a particular you know the dimension of the quality that is basically the warranty is it ok. So, it is very important dimensions of quality you get an idea that how whether the product quality is excellent if you if you look at this warranty period warranty. So, it is essentially you know with respect to the reliability data we are concluding about the many dimensions of quality.