Quality Design and Control

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Lecture - 41
Design for Reliability-I

During this the ninth week, I will be discussing on a very important topic that is designed for reliability. Even you know, the next week; that means, during the tenth week, I will be continuing discussing on design for reliability ok.

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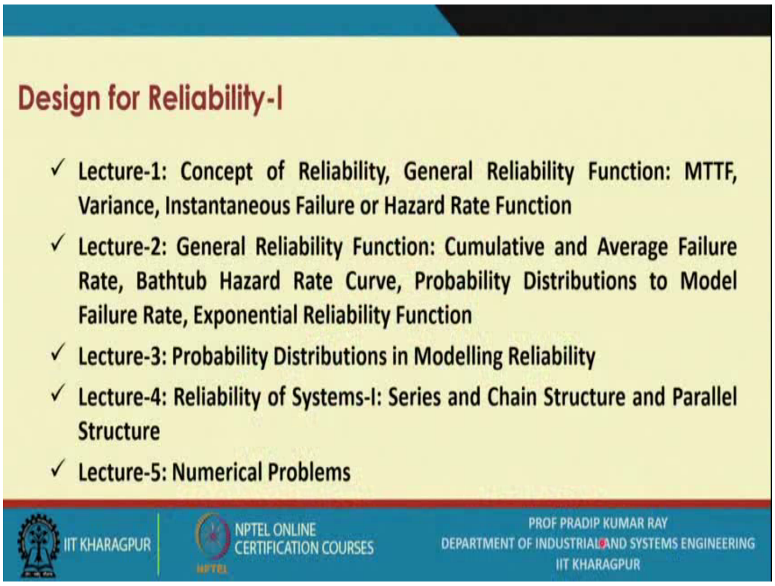

Now, during this week, there will be 5 lecture sessions and let me fist tell you, what are the topics, I have included in this lecture sessions and these topics are all related to reliability.

So, in the very first lecture, I will be discussing the basic concept of reliability, general reliability function, MTTF; meantime to failure, variance, instantaneous failure or hazard rate function. In the subsequent lecture, the lecture two, I will be discussing on the general reliability function, cumulative and average failure rate, bathtub hazard rate curve, probability distributions to model failure rate, exponential reliability function, this will be the coverage during the second lecture. In the third lecture, I will again continue discussing on probability distributions, in modelling reliability with the set of numerical problems. In lecture four, reliability of systems we will be talking about and there are different kinds of systems.

So, this will be the first set of the systems for which the reliability modelling we will be discussing. Series and chain structure and parallel structure and in lecture five we will take of several typical numerical problems related to reliability.

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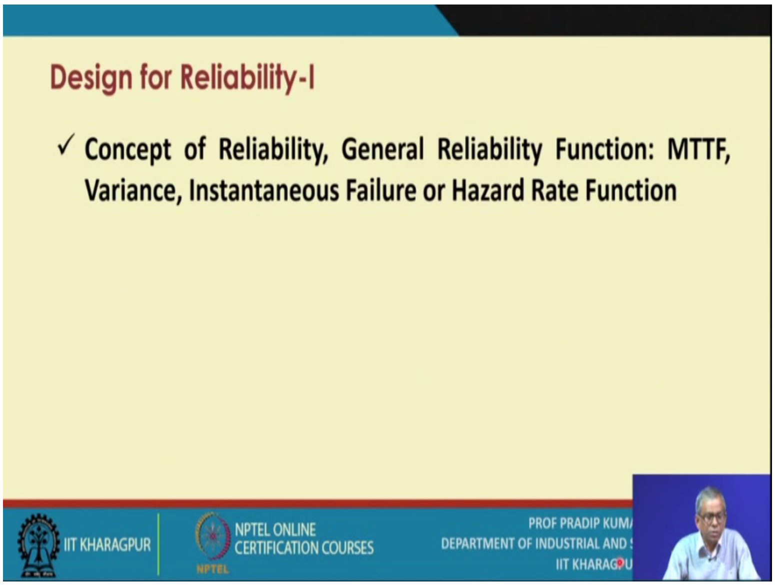

Now, let us first discuss the concept of reliability in the next half an hour or so, I will be discussing the concept of reliability, then what is this general reliability function and relay and related to general reliability function, what is MTTF, what is variance, what is instantaneous failure or hazard rate function, ok. So, this will be my coverage.

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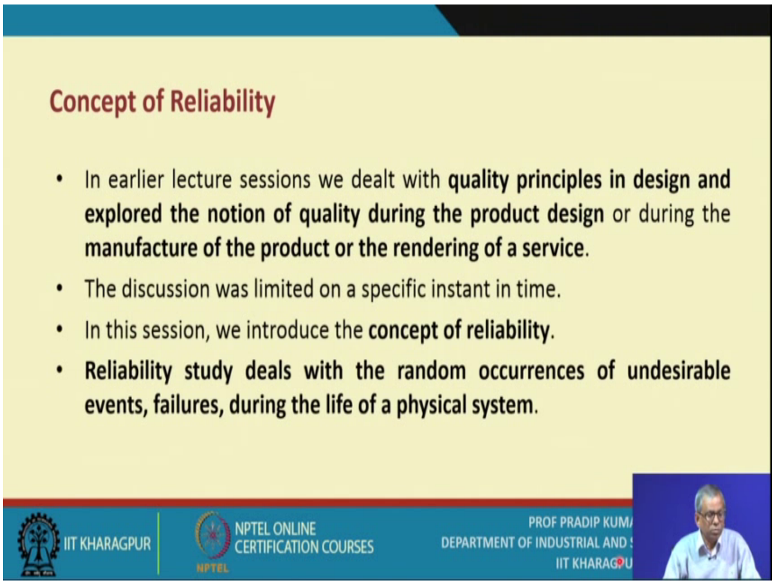

Now you know what you have done in all earlier lecture sessions.

This course is on quality essentially. So, in all earlier lecture sessions, we dealt with quality principles and design, there are many kinds of principles and explored the notion of quality during the product design or during the manufacture of the product or rendering of a service. So, this was a coverage, we have been discussing on different kinds of tools techniques for quality control for quality improvement.

Some of these tools to be to be used during the design phase of the product or many such tools and techniques, you are required to use during the manufacturing phase. Now, all this discussions were limited on a specific instant in time it means that whenever we try to define quality that we have defined quality we have measured quality, but this ah all these the measurements or valid at a particular instant of time; that means, at time t equals to t.

In the session, we introduce the concept of reliability, now what is this reliability; that means, in plain and simple terms reliability mains the quality in the long run; that means, at time t equals to t you said that a product is a quality product ok; that means, one or more dimensions of the quality are made available in inacceptable say in all the levels, but then the product is to be used for overtime period.

Now the reliability actually tells you or the reliability analysis will lead to a conclusion the; weather the product will be performing as per the as per the quality requirements ok, even after say a few time periods maybe after one month maybe after one year. So, that sort of you know the prediction you have to make; that means the quality in the long run.

So, whenever we refer to reliability study the reliability study deals with the random occurrences of undesirable events failures during the life of a physical system were concerned about the life of the product and whenever we refer to reliability study, we say the during the life of the product ok, there could be many random occurrences of undesirable events, particularly the failure, it might fail at any point in time, whatever maybe ah you know the design level of a particular product or a system ok.

So, reliability is a measure of the quality of the product or a process over the long run this point, I have already emphasized the concept of reliability is related to an extended period of time over which the expected operation of the product is considered, we have already refer to the functional value of the product. Now, in any reliability analysis, we need to conclude whether this functional value of the product or the process or the system is assured over a period of time is it ok. So, that is the basic focus on any reliability study. So, now, how do you define reliability? So, the reliability of a product or r system is the probability that when operating understated environmental conditions is it ok; that means, any product when you design you say why not you start using it so as per the design specifications of the product.

Now, this product is supposed to be used in a illustrated environmental or working conditions work environment that is to be that is to be mentioned now the product or the system will perform its intended function is it like say when you say, it is a automobile car; that means, I know that which when I start using the car which particular function, actually, you are trying to get from car. So, that is; that means, what we expect that the product will perform its intended function adequately has a specified in the design norms for a specific specified internal or period of time.

So; that means, when you try to quantify reliability there are four issues to be looked into first one it is the probability then the second one is related to a particular product is it ok. So, there must be some stated conditions in which is the product is supposed to function then the intended function of the product must be specified and the fourth one is whenever you talk about reliability; that means, you must be able to specify the time interval or the period of time. So, whether it is five hours or five hundred hours you expect the product to run ok.

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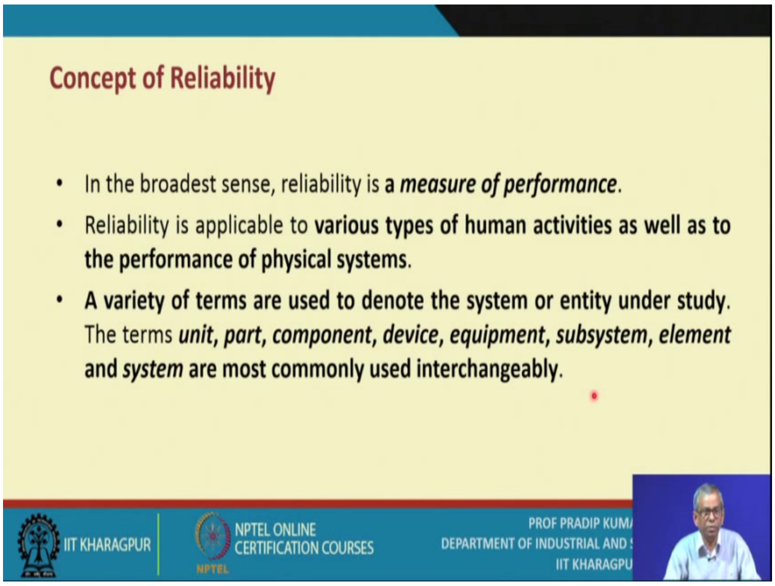

So, this is the objective definition of reliability. Now in the broadest sense reliability is a measure of performance getting my point as we have mentioned that the quality has got several dimensions. So, one such dimension; so, one of the important dimensions is the performance.

So, the performance in the long run; so, essentially the reliability is a measure of performance reliability is applicable to various types of human activities as well as to the performance of physical systems which is obvious. In fact; that means, it is a concept which is applicable not only for and say inanimate object, but also for animate object in the broadest possible you know the sense now whenever you study reliability when you whenever we try to model reliability for a system or a product or process a variety of terms you need to use.

So, a varieties of terms are used to denote the system or the entity under study the terms which we will be using that is unit part component device equipment subsystem element and system these are the terms most commonly used most commonly used interchangeably ok.

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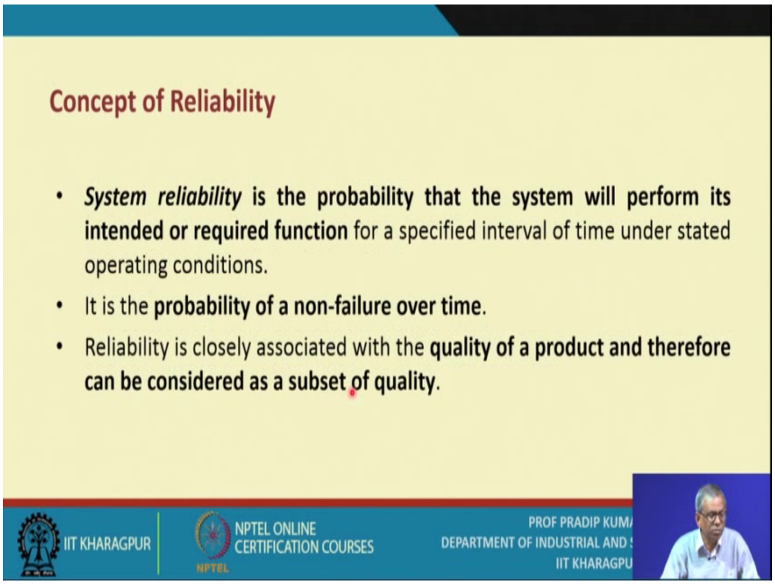

So, we will talk about will referring to solve this terms and terminologies whenever you try to you know the model the reliability; system reliability is the probability that the system will perform its intended or required function or a specified interval of time under stated operating conditions; that means, either you defined; say the reliability with respect to the part of the component or the product or in many a time, we are really concerned about the system reliability ok.

So, the same definition applies with respect to the system and you know; what is the definition of a system is it ok. So, it is the probability of a non failures overtime, this is a another way you can look at; that means, if it performs; that means, there is no failing of the component during a specified time period, you say with respect to the specific time period, the reliability is very high availability is almost 100 percent reliability is closely associated with the quality of a product and therefore, can be considered as a subset of quality.

Now the question remains like suppose the quality is excellent, we say that the product quality systems quality is excellent, can you can you assume that the reliability of other systems of the product is 100 percent; that means, that the probability is essential is a probability can you assume the value of reliability is one.

So, that is a big questions in fact, but at this point we should highlight a particular aspect like whatever maybe the quality level of a product is it ok; that means, it would be very

very excellent is the best product are created, but there is no guarantee that it will not fail, you will may not fail say within the next 5 years or within the next 10 years, but beyond 5 years, beyond the 10 years at any point in time, it might fail is it because I know at the time passes maybe the performances is significantly affected by you know the external noise factors on which you may not have any control is it ok.

So, ah we never say that the quality of the product is so excellent that the reliability is one, is it so; that means, we presume that whatever maybe the level of quality of a system of a product today or tomorrow at any point in time in future it may fail is it ok. So, that is the basic you know the assumption we make while we model reliability reliability on the other hand is concerned with how long the product continues to function once it becomes operational that is most important.

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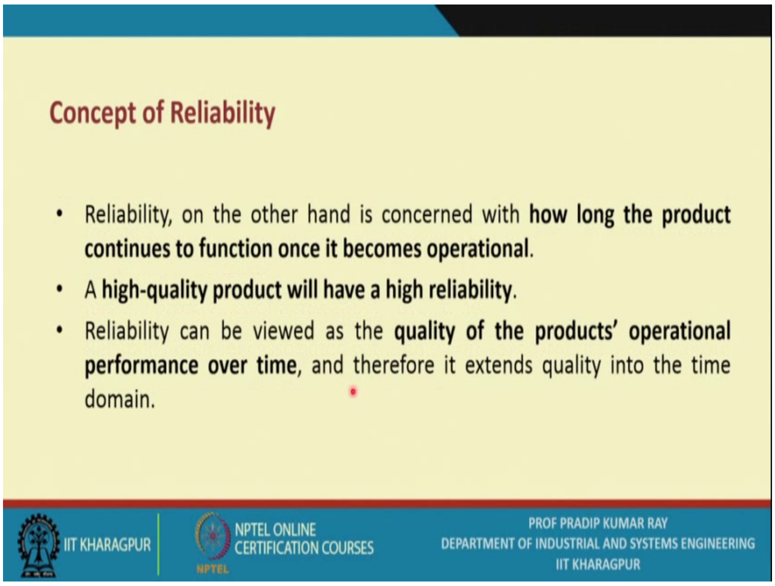

In fact; that means, suppose the machine is installed and then it becomes operational, then you have to conclude or you have to predict; how long it may perform, is it ok. So, that is also whenever you ask this question essentially you are referring to the concept of reliability.

A high quality product will have high reliability. So, this is in hypothesis, is it ok. So, that you have to prove is it and sometimes it may happen that you have defined high quality product are you have manufactured high quality product at only for the instant of time, but you are not sure whether the next instant of time, whether it will perform or as per are the requirements or not. So, there could be many such cases. So, you should we should we should know what are the; those systems existings at your workplace reliability can be viewed as the quality of the products operational performance over time and so that there is a time dimension and therefore, it extends quality into the time domain ok.

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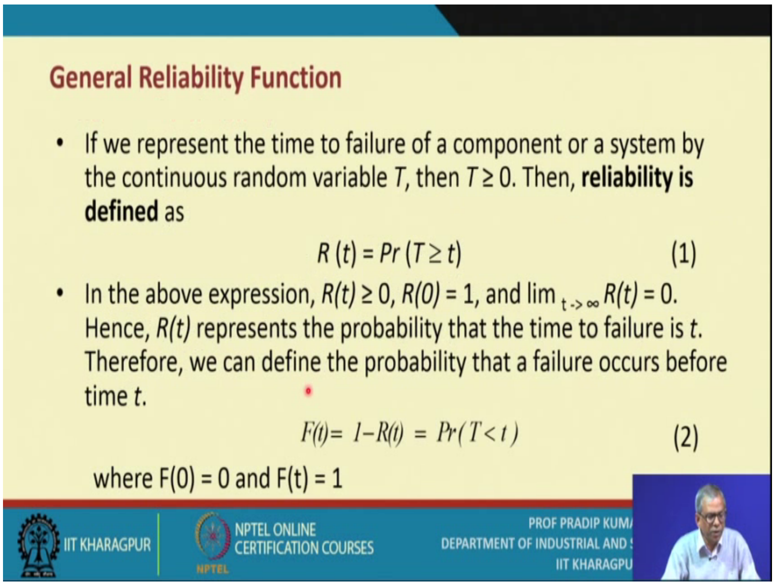

Now let us now talk about once the concept of reliability is known ok.

Then let us talk about the general reliability function. So, how do you know mathematically you represent reliability ok. So, will proceed slow, but we will try to highlight all the important issues all the important functions or the important concepts related to modelling of reliability now let us first concentrate on this; that means, how to ah represents the reliability function, if we represent the time to failure of a component or system by the continuous random variable. Now, the first thing; that means, the first you know the variable, you need to consider that is a random variable that is time to failure; that means, you start you know all the process has started working or the product has started working.

Now, it will fail either you know at a particular time period and this time to failure is; obviously, is a random is a random variable and we are assuring that this failure is occurring due to some random causes is it is not like say you know due to some assignable causes is it ok. So, that is the first assumption is it clear I think, it is clear; that means, time to failure if you assume that the time to failure is a random variable we are assuming that it is failing due to some random causes inherent in the system. So, that is the starting point suppose this random variable is T, then T is greater than equals to 0, is it ok, then the reliability is defined as; that means, reliability that that the component of the part or the systems of the product is it we will we will run for t time period and that is the probability that this ah trying to failure will be greater than and equals to t.

That means at least T time period the component of the part of the system will survive is it ok. So, sometimes, it is also referred to as a survival time now within the above expressions; that means, this is the expression; that means, the probability expressions it is showing that reliability should be greater than equals to 0 when at time T equals to 0; that means, at instant of time is it is one; that means, it starts working and; obviously, when the time in over the T tends to infinity; obviously, is reliability will be 0 is it ok; that means, a suppose it is related to the machine what we are assuming that infinite time period ok, from today from the instant of time when it is installed; obviously, reliability will be 0 is it ok.

So, that is the basic assumptions hence R t represents the probability that the time to failure is t therefore, we can we find the probability that a failure occurs before time t is it ok. So, what is F t? F t is 1 minus R t is obvious. So, probability that the failure occurs before T, is it ok, suppose between 0 and T, is it less than t. So, where F 0 is 0 is it ok; that means, it has not failed at time t equals to 0, but at time t it has definitely it has failed is it that is why it is one is it clear; that means, there is a definite conclusions, in the last equation R t is known as the reliability function is it ok.

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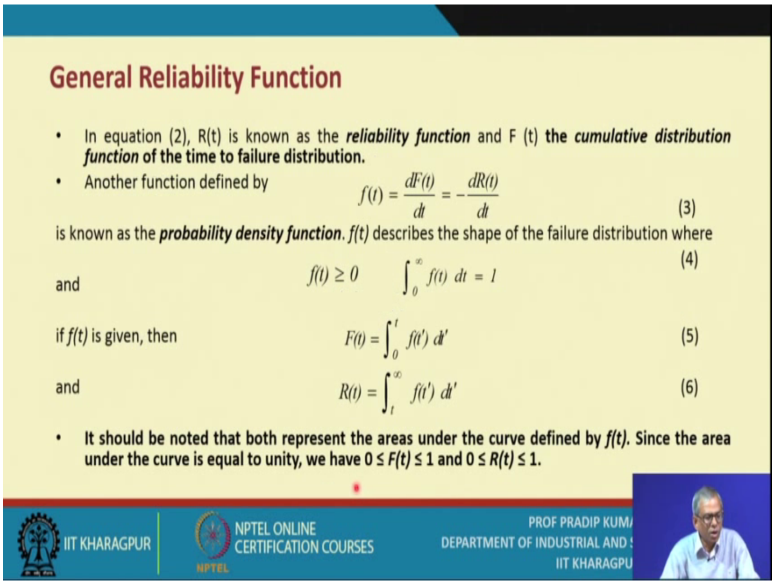

I repeat R t is known as the reliability function and F t is the cumulative distribution function of the time to failure distributions, is it ok, like related to our new started discussing quality you know we have mention if you refer to like say several kinds of the distribution we have refer to like say distribution for discrete random variable the distributions for say continuous random variable.

Now; obviously, you know for continuous random variable you have the probability density function and for discrete random variable you have the probability mass function. So, for both the cases what you need to do; that means, the cumulative distribution function of the time to failure distribution is it ok; that means, the time to failure density function you first must know. So, this is basically the density function F t is it is a continuous random variable.

So, if you differentiate ah you know, what is this is a cumulative distribution function. So, deviate of F t is nothing what minus d R t by I dt is it for this is directly you get if you look at the previous equation. So, this is the probability density function right F t describes the shape of the failure distribution is it ok; that means, distributions will have a shape where F t is greater than equals to 0 and; obviously, integration 0 to infinity ft dt will be one is it fine and if F t is given then capital F t is 0 to t is it ok; that means, of to T time periods. So, ft dt and R t is equals to t to infinity F t dash dt dash is it ok. So, ft dt, it should be noted that both represent the areas under the curve defined by F t is fine.

Since the area under the curve is equal to unity we have always F t lies between 0 and one is it 0 included one is also included and similarly rt; that means, the reliability function is also lies between 0 and one; that means, what we conclude as per is reliability is concerned say.

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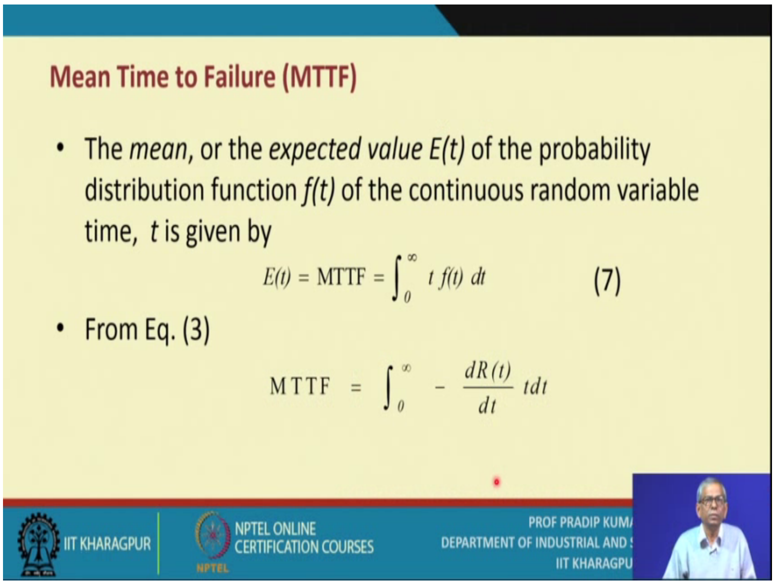

Any value between 0 and 1, it you might you may come across ok. Now one important say the important measure related to say reliability function is it that is meantime to failure. So, we say that if the meantime to failure it is one of the measures you get an idea about reliability of the component. So, if the meantime to failure is more you assume that the reliability is more, is it ok.

So, if the meantime to failure is less you say the reliability is less is it now how to compute the meantime to failure the mean of the expected value et of the of the probability distribution distribution function F t of the continuous random variable time is this MTTF; that means, t into F t that is the average value t into F t, is it ok; so, 0 to mt that is the MTTF. So, from these equations we get 0 to infinity minus dt Et about t tdt is it so; that means, is F t is replaced with this notion. So, this is just a manipulation and then if you go for integration by parts what you find that MTTF is this expressions minus trt 0 to infinity plus this expression 0 to infinity Rtdt and ultimately you get this expressions say MTTF 0 to infinity Rtdt is it ok.

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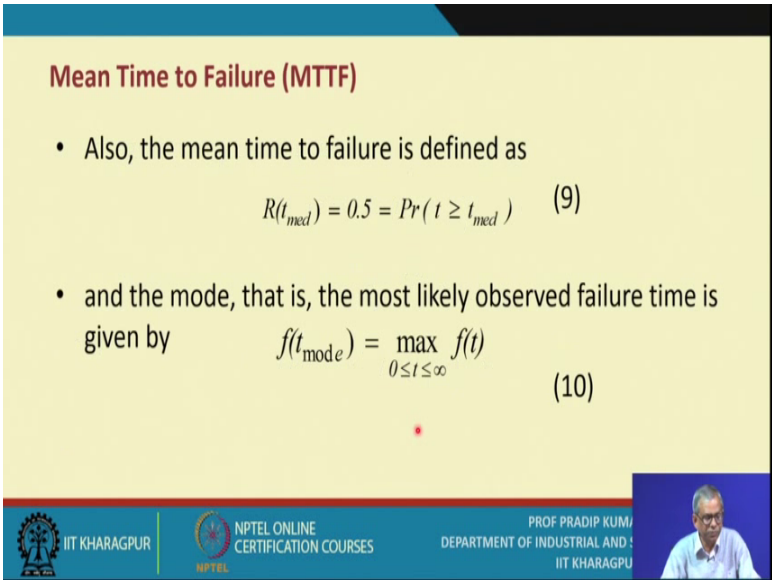

And with a note that limit t tends to infinity trdt is this expressions was to 0 is it and 0 into R t; obviously, it is 0, is it ok. So, we have the expression for MTTF also the meantime to failure is defined is this; that means, T median is it ok; that means, is the midpoint that could be the 0.5, the probability that T is greater than or equals to 2 median, so ok. So, among several values you have. So, one value at the midpoint that is called the median value with respect to the time to failure that is T and the mode that is the most likely observed failure time that is given by F t mode equals to maximum between 0 to infinity t lies anywhere between 0 to infinity F t is it ok.

So, you have expressions for you know the reliability that the component will survive is up to t median and what is the corresponding say the failure time that also you can compute is it ok.

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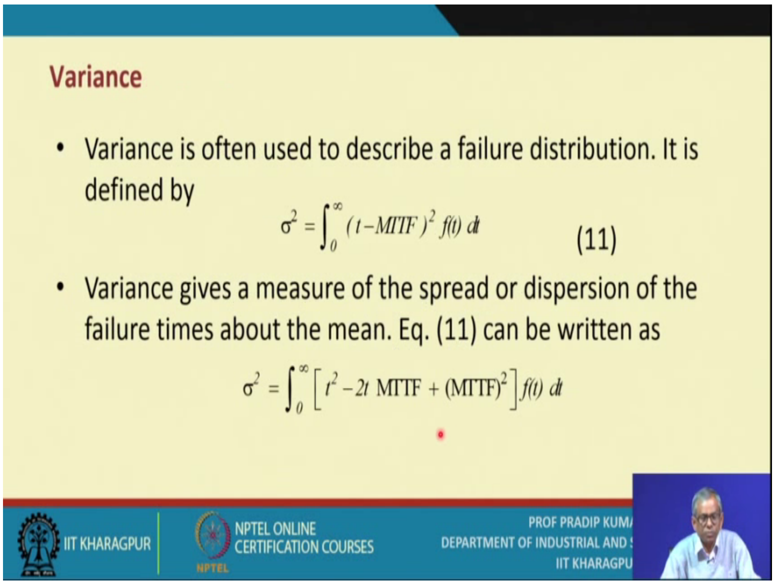

So mean median and mode then the obvious to the next one is the variance and the variance is often used to describe a failure distribution. So, it is defined as this one is it the variance; that means meantime that is the usual definition of the variance we have applied; that means, the t is basically the random variable. So, the t minus MTTF square ft dt is 0 to infinity is it and ah variance gives a measure of the spread of dispersions of the failure times about the mean is it took already, we know that the variance is one of the one of the measures of dispersion in the dispersion measures.

So, what is the expressions; for the variance is it ok, in this case so; obviously, you have these expressions ok. So, directly you can derive these expressions ok.

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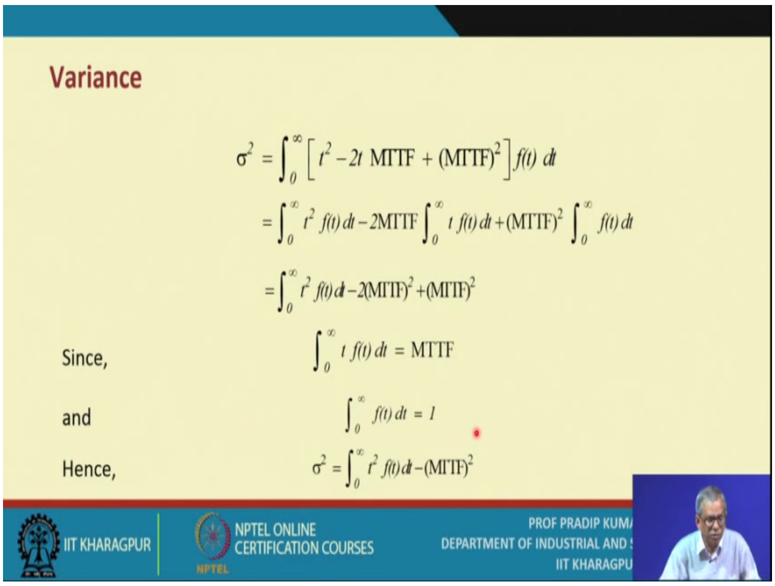

Now, so, if you simplify these expressions following these steps I have given all the steps one by one is it ok. So, it will help you understand that how do you get the final expressions and ultimately you get you can related to the MTTF and this is this is this is just one and hence the sigma square is this is it ok. So, this is all these steps are mentioned please go through them and I and if you follow the follow the steps, we will we will come to know that how a particular expression has been derived, is it ok. Now ah I will be highlighting another important you know the aspect that is the instantaneous failure or hazard rate function.

Sometimes this is referred to as the failure rate function failure rate function, but you know instantaneous failure or hazard rate functions, you need to define in any exercise on reliability.

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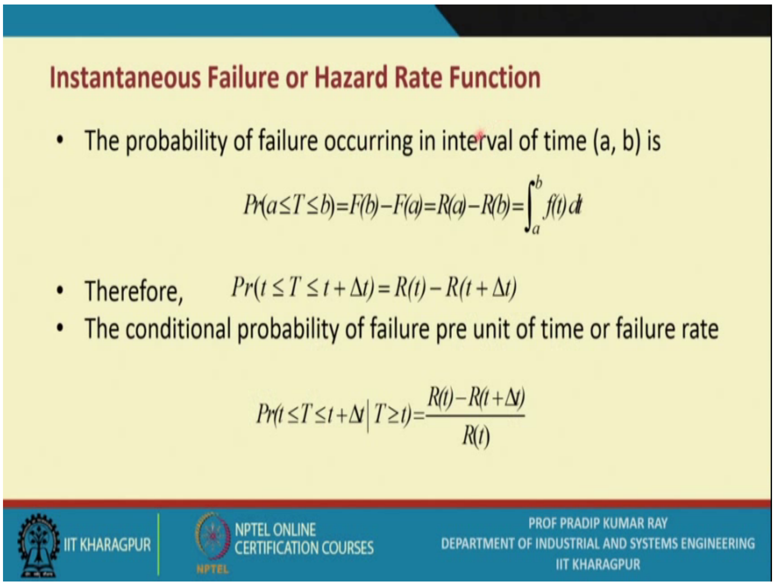

So, how do you define that ok? So, the probability of failure occurring in interval of time say between a and b is it say between say 0 to 100 hours ok. So, a is 0, b is hundred. So, how do you define it; that means, of probability that this time to failure will be between a and b. So, this way you represent. So, this is a b distribution functions cumulative distribution functions up to b minus cumulative distribution functions up to a. So, this is R a minus R b is it you just refer to the relationship between the cumulative distribution function and the reliability function.

So; obviously, it will be from a to b ftdt is it clear. So, this is very very clear. So, I am I am just following you know the fundamental principles therefore, so, what is the probability that it will fail between t and t plus delta t instantaneous value of t plus. So, R t minus R t plus t right the conditional probability of failure per unit of time or the failure rate. So, that we define like this; that means, it fails between T and t plus delta t; that means, within period and; obviously, this probability is proportional to delta t. So, R t minus R t plus delta t divided by R t is it ok.

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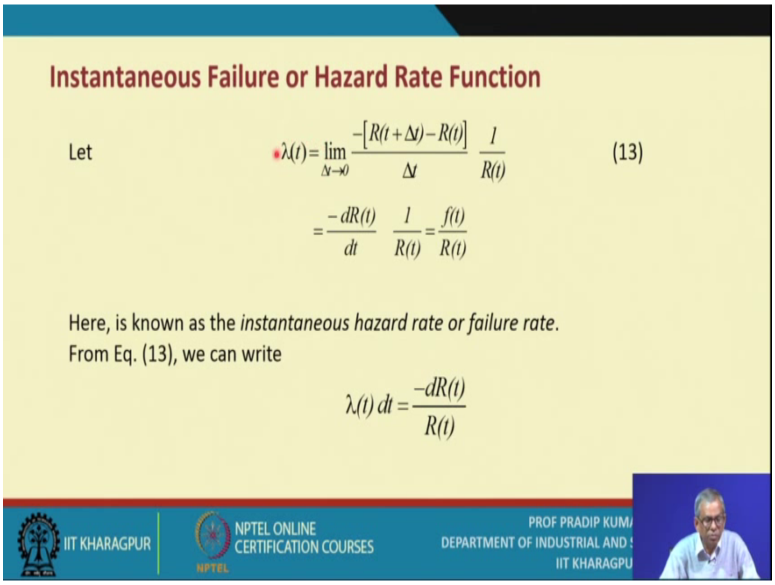

So, then we have this, the lambda t the rate functions. So, limit delta t tends to 0 and you have these functions is it ok.

So, this is minus dtt over t and the into 1 upon Rt. So, this is basically F t and F t divided by R t. So, lambda t is nothing, but F t divided by R t here it is a is known as the instantaneous hazard rate or the failure rate is it ok. So, the lambda t is known as the instantaneous hazard rate or the failure rate. So, if you were referred to this particular equation you will find the lambda t dt is equals to minus d R dt dt ok.

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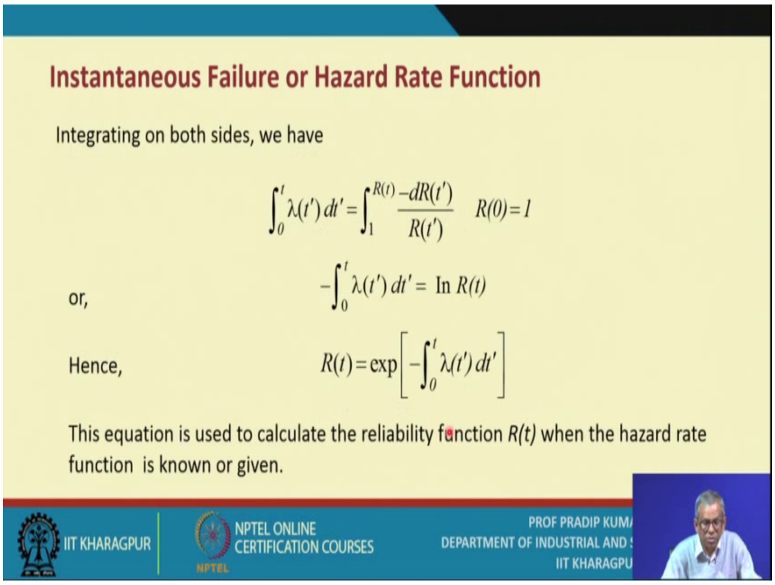

So, this is one and if you integrate both the sides you will get this expressions and ultimately you follow these steps and we will get this expressions R t is equals to exponential uk e to the power minus 0 to t lambda, t dt is it lambda t dt, this equation is used to calculate the reliability function R t when the hazard rate function is known or given, is it ok.

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So, we have defined what is the hazard rate function later on we will export this one and you know here we have defined what is MTTF meantime and subsequently we have the procedure to the variance is it now here one important say the issue I should highlight like say you know the MTTF is definitely one of the measure for assessing reliability, but another measured many time we also refer to that is MTBF. So, MTBF is essentially it is meantime between failure now weather. So, MTTF and MTBF there two different terms under certain condition is it ok. So, MTTF may be approximated as MTBF.

Now, the condition is that whenever there is a there is a failure occurring. So, what do you do you go for repairing and if it is repairable; that means, its impact is such that even if there is a failure you can definitely you can identify the fault and you go for maintaining it and you get back to it to its original state of health.

So, that is basically the repair now if it is repairable then and the repair time is almost negligible; that means, you have already streamline this one and it is done automatically suppose in extreme cases in the best possible scenario then in that case m MTBF meantime between failure may be equated with meantime to failure. So, later on when will take off examples, we will again be referring to MTTF as well as MTBF and the difference if any between these two will be will be explained further.