Quality Design and Control
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Lecture – 40
Acceptance Sampling- II (Contd.)

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During this session am again I will be discussing a few other numerical exercises and I will not be just concentrating on acceptance sampling these exercises will be on quality control reliability as well as quality with experimental design. So, these 3 topics we you have already gone through.

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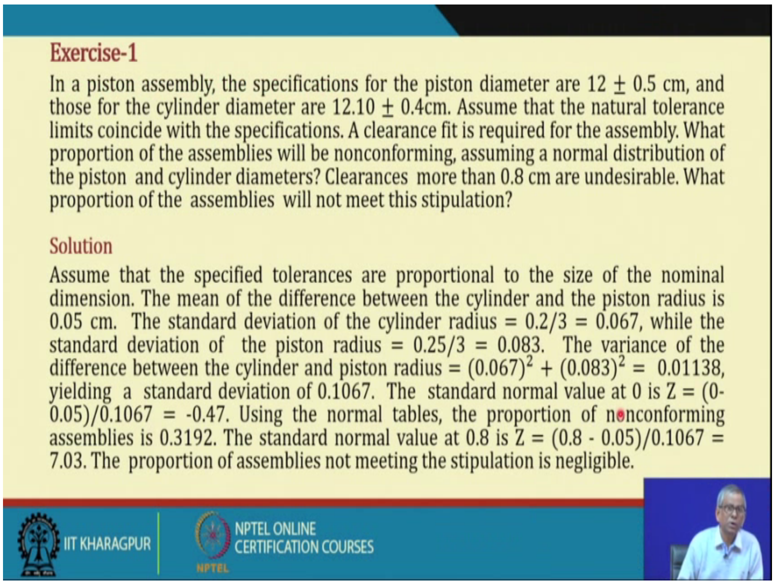

Now, again there will be 8 exercises and I will be explaining I will be discussing all these 8 problems or 8 exercises one by one.

Now, let us talk about exercise 1, this is the first one in a piston assembly the specifications for the piston diameter are 12 plus minus 0.12ive centimeter is it. So, that is the tolerance range and the those for the cylinder say the diameter a 12.10 plus minus point 4 centimeter, assume that the natural tolerance limits coincides with the specifications. So, if it happens; obviously, we are assuming that the that the process capability ratio is one; that means, the process is just capable.

A clear speed is required for the assembly you that under what conditions we say that a feet is considered to be clearance feet. What proportion of the assemblies we will be non confirming assuming a normal distribution of the piston and cylinder diameters is it so; that means, the central unit theorem holds.

Clearances more than 0.8 centimeter are undesirable is it. So, if it is more than 8 centimeter the condition called old may o may occur and that is that that is to be avoided what proportion of the assemblies will not meet this stipulation. So, this is the question. So, this is related to a particular topic we have already covered under process capability and process capability study is closely linked with the quality control. So, directly or indirectly it will have a bearing on the quality control and here the particular topic we will be referring to that is the second tolerances and the specifications of the assemblies when the individual component tolerances are specified. So, this is the domain and we will just explain that against this particular exercise what are the solution states?

Assume that the specified tolerances as proportional to the size of the nominal dimension. So, that is the first assumption we make and many time these assumption holds, the mean of the difference between the cylinder and the piston radius is it is 0.05. That means, you know the piston will have an outer dia and the cylinder we will have and say the inner dia. So, for the for the clearance condi for the clearance feet what is the condition to be satisfied; that means, before you do the assembly that this condition is that that the inner cylinder dia must be greater than the outer say the shaft dia or the piston dia. So, the standard deviation of the cylinder radius is; obviously, 0.2 by 3, spare right and so that is 0.067 while the standard deviation of the piston radius is it that is 0.25 by 3 is it all right.

So; that means, what we are assuming that the upper natural difference between upper natural tolerance limit and given natural tolerance limit divided by that is that is the 6 sigma. That means, that is means the natural tolerance limit first one is the upper tolerance limit minus lower tolerance limit or the upper specification limit minus lower specification limit divided by 6 into sigma. That means, it is essentially the actual spread; that means, the difference between upper natural tolerance limit and lower natural tolerance limit we are assuming it to be say just 1 right.

So, the process is barely capable so; obviously, this is 0.2 by 3 right and this is 0.067 that is the standard deviation of the cylinder radius, as a standard deviation of the piston radius is 0.25 by 3 that is 0.083. So, the variants of the difference between the cylinder and the piston radius that is you add the sum of the squares of the individual you know the standard deviation that is 0.067 square plus 0.83 square that 101138. So, what is the standard deviation that is the root over 0.01138 that is 0.1067 is it ok.

So, the sigma is known between the difference the standard normal value at z 0 that is z equals to 0 minus 0.05 divided by 0.1067 that is point minus 0.47. So, using the normal tables the proportion of non confirming assemblies is 0.3195 is it; that means, what you are trying to do you are getting the difference between the diameter of the shaft and diameter of the cylinder and diameter of the shaft or the piston is it and what you are assuming that this difference is also normally distributed and for a clearance condition to hold; that means, it must be greater than 0. So, that is why again 0 you compute the value of z.

Using the normal tables the proportion of non confirming assembly is 0.3192 is it; that means, the area under the curve say less than that particular value that is z equals to 0.47 minus 0.47. So, that area under the curve beyond minus 0.47 that is the value of z you refer to the standard normal table you get an idea of 0.3192.

The standard normal value at 0.8 is z 0.8 minus z it was 2.8 minus 0.05 divided by point 0.1067 we are assuming that the sigma the value of sigma does not change. So, this value is 7.03. So, if you refer to the standard normal table we will find that the proportion of assembly is not making the stimulation is negligible is it there is hardly an area beyond this value; that means, 7.03 the value of z 7.03 ok.

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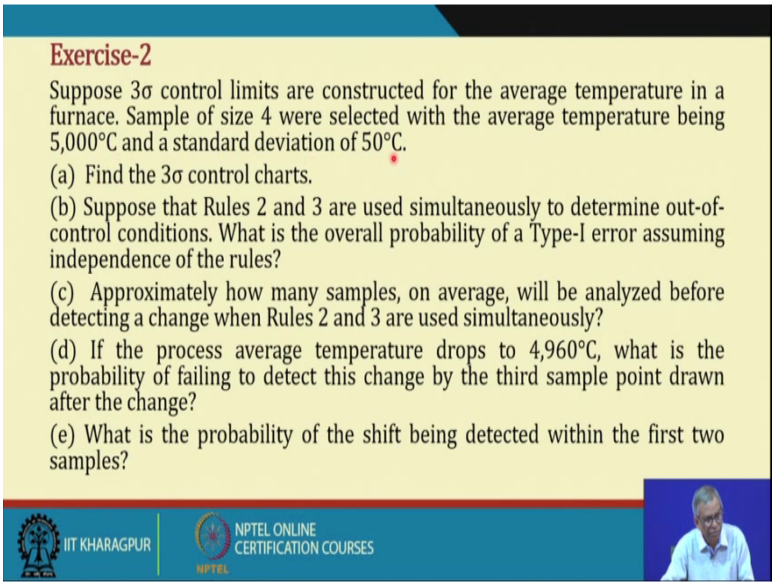

Now, let us move to exercise 2, suppose 3 sigma control units are constructed you know what is a 3 sigma control limits and this. So, the 3 sigma control limits are constructed or the control charts are constructed for the average temperature in a furnace is it. So, that is the quality characteristic of interest, sample of size 4 was selected with the average temperature being 5000 degree Celsius; that means, it is a variable control chart and the standard deviations of 50 degree Celsius find the c signal control charts suppose that rules 2 and 3 we refer to the sa the 5 specific rules for out of control conditions we have mentioned. So, the rule 2 and rule 3 you refer to, suppose that rules 2 and 3 are used simultaneously to determine out of control conditions what is the overall probability of a type of error; assuming independence of the rules?

So, that means, the second question the third question is approximately how many samples on an average, we analyzed before detecting a change. When rules 2 and 3 are used simultaneously, when we go to question number 4 if the process average temperature drops to 4960 degree Celsius, what is the probability of failure to detect this change by the third sample point drawn after the change?

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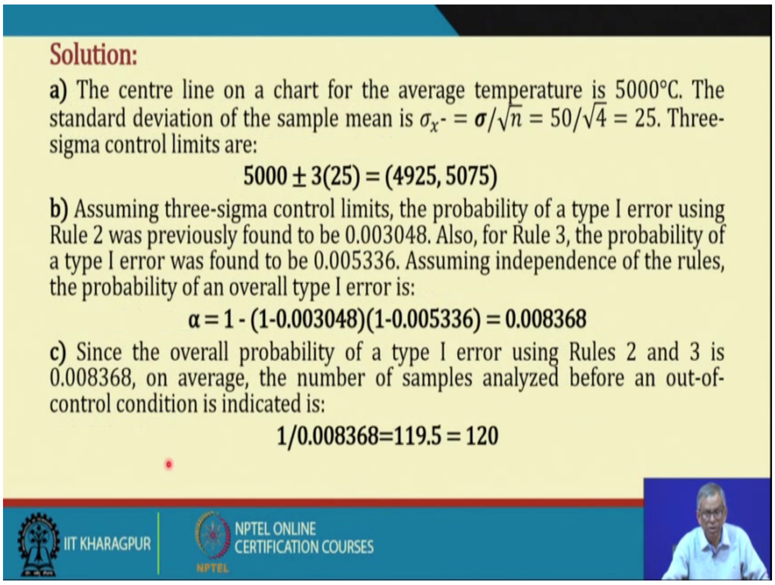

The last question is what is the probability of the shift being detected when in the first sample? So, these are the typical problems you come across related to control charting, now the central line on a chart for the average temperature is; obviously, 5000 degree Celsius right. The standard deviation of the sample mean that is sigma x bar is equals to sigma upon root n, that is as per the central limit theorem. So, the sigma is 50 that is given and n is 4. So, it comes out to be 25.

So, 3 sigma control limits are 50 plus minus 3 into 25; that means, 1925 comma 5075 fine. So, you know what are the control limits assuming the 3 sigma control limits the probability of type one error using rule 2 was previously found to be this 1.003048 also for rule 3 the probability of type one error was found to be this one is it. So, you know what is rule 1 and what is rule 2 rule 3. So, you please refer to rule 2 we have already completed the what is the as per the if you follow the if you follow the rule 2; that means, the 52 out of 3 consecutive the sample points plotted on the same sight of the the central line, but is a plotted beyond 2 sigma 1 in limit is it. So, that is basically the rule 2. So, what is this probability already you have computed.

Similarly, rule 3 is you know the 4 out of 5 consecutive sample points plotted beyond one sigma control limits on the same side of the central line. So, corresponding probability is this one we have already computed assuming independence of the rules the probability of an overall type 1 error. So, the overall type one error suppose you deal with I number of such out of control rules. So, what is the pole what is the expressions for oral type one and error that part also we have discussed. So, you refer to that particular formula and you just apply that formula with respect to just the 2 rules rule 2 and rule 3. So, ultimately you get an expression of alpha as 1 minus a this is the product come is it pi I equals to 1 to 2 and the corresponding allow is that is the type 1 error probability with respect to 2 and one minus of that ok.

1 minus alpha 1 into 1 minus alpha 2 or here 1 minus alpha 2 into 1 minus alpha 3 is it. So, alpha 2 and alpha 3 already you have computed. So, ultimately the overall type 1 error probability is 1008368.

Since the overall probability of a type 1 error using rules 2 and 3 is this one that is this value on an average the number of samples analyzed before an out of control condition is indi is indicated is 1 upon this 1; that means, 1 upon alpha so; that means, one upon overall type 1 error say the probability. So, that is around 120is it.

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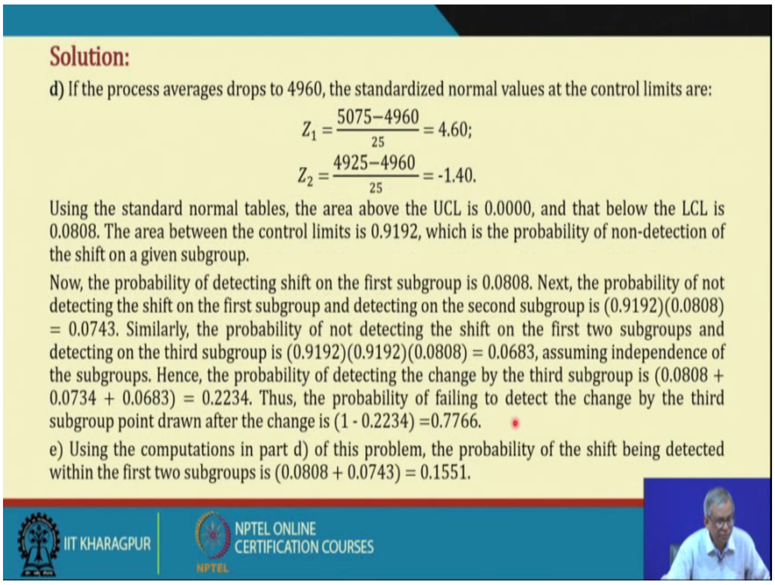

So, points then the next one, say if the process average drops to 1960. So, what do you need to do; that means, against the standardized normal values that the control limits are again you calculate actually 1960 and this is 25 that is 4.60 and z 2 is 1 minus 1.40 using the standard normal tables the area above the a c l is that is beyond the 4.60 is up to 4 decimal place it is 0. So, it is absolutely you may assume it will be 0 and that below the l c l that is minus 1.460 that is 0.0808; that means 8.08 percentages.

The area between the control limits is 0.9192 which is the probability of non detection of the ship on a given subgroup; that means, this is the probability of making type 2 errors. Now, the probability of detecting shift on the first subgroup is this makes the probability of not detecting the shift on the first group and detecting on the subgroup is 0.9192 into 0.008 0.0808 that is 0.0743. Similarly, the probability of non detecting the shift on the first 2 subgroups and detecting on the sub subgroup is this one.

So, all sorts of the possibilities we have to explore and then you get the probability of failing to detect the change by the third subgroup point drawn after the change that is 1 minus this is the overall. The till means you need to we can detect the shift by the third sample that already you have computed that is 0.2234 you follow this particular the steps I am sure that you will understand fully and ultimately the probability of non detection that is 0.7766. Using the computations all these computations which you have carried out the probability of ship being detected within the first subgroups already you have computed that is 0.1551 is it ok.

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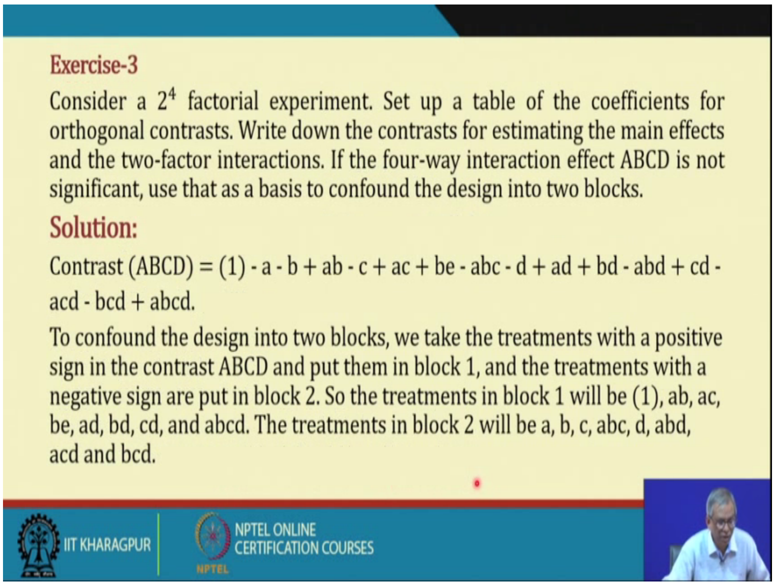

So, you have answered to all the questions set in this particular exercise, next we move to exercise 3 consider a 2 to 2 the power 4 factorial experiment, we have already discussed what is the next factorial experiment. So, 2 to the power 4 means we are considering 4 factors an each factor at 2 levels set of a table of the coefficients for orthogonal contrast, we have already referred to the orthogonal contrast. So, please refer to my the say the lecture sessions on orthogonal contrast it is a it is a part of you know ba the topic called the quality by experimental design.

Write down the contrasts for estimating the main effects and the 2 factor interactions if the 4 way interaction effect is a, b, c, d is not significant, now that is your assumption use that as a basis to confound the design into 2 blocks. So, this is the problem. So, you go through the contrast a, b, c you apply this formula is it there are 4 factors is it; that means, if you say a. That means, out of this 4 factors the factor is at the high level whereas, the other 3 factors at the low level. So, this will be follow, you follow this formula for 2 to the power 4 factorial experiment the general formulation is given to confound the design into 2 blocks we take the treatments with the positive sign in the contrast a, b, c, d and put them in block 1 and the treatments with negative sign are put in block 2.

So, this is the rule we have we follow. So, the treatments in block 1 will be 1 ab, ac, be, ad, bd, cd and ab, cd. So, this is bc, abac, bc, ad, bd, cd and a b c d at the treatments in block 2 will be abc, abc these are all negative values d a b v a c d and v c d is it ok.

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Contrast	Treatment							
	(1)	a	b	ab	c	ac	bc	abc
A	-	+	-	+	-	+	-	+
B	-	-	+	+	-	-	+	+
AB	+	-	-	+	+	-	-	+
C	-	-	-	-	+	+	+	+
AC	+	-	+	-	-	+	-	+
BC	+	+	-	-	-	-	+	+
D	-	-	-	-	-	-	-	-
AD	+	-	+	-	+	-	+	-
BD	+	+	+	+	-	-	-	-
CD	+	+	+	+	-	-	-	-

So, now you get all the values and then you create this table and this is the contrast A, B. AB, C, AC, BC, D, AD,BD, CD is it and here you have this one say the one a, b, a b, c, a c, b and c is it. So, these are the this is the solution you just go through this table right.

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Contrast	Treatment							
	d	ad	bd	abd	cd	acd	bcd	abcd
A	-	+	-	+	-	+	-	+
B	-	-	+	+	-	-	+	+
AB	+	-	-	+	+	-	-	+
C	-	-	-	-	+	+	+	+
AC	+	-	+	-	-	+	-	+
BC	+	+	-	-	-	-	+	+
D	+	+	+	+	+	+	+	+
AD	-	+	-	+	-	+	_	+
BD	-	-	+	+	-	-	+	+
CD	-	-	-	-	+	+	+	[illegible]

And, so this continuing; in fact, this table continuing. So, please refer to this table and I am sure you will understand all the steps is it.

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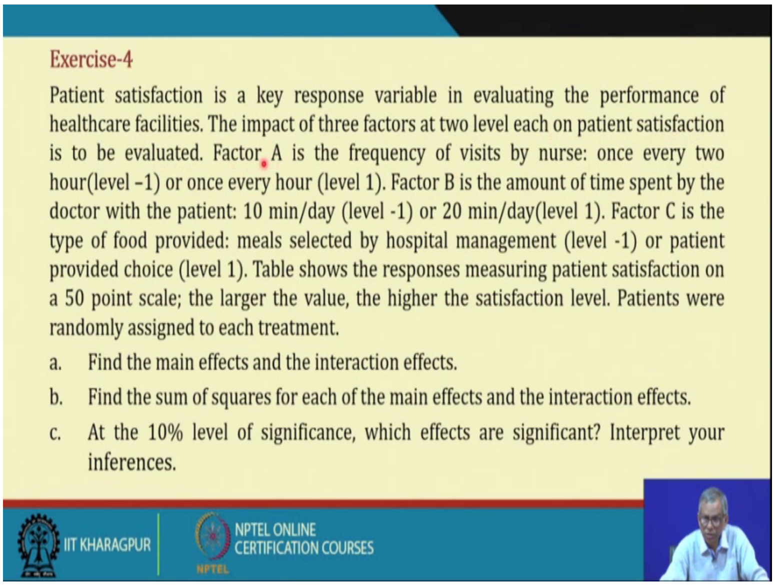

So, and you refer to my, you know the lectures and plus the portion which you are then the topic of, say the quality wise experimental design.

Now, let us move to exercise 4, the fourth one. So, here the patient satisfaction is a key response variable in evaluating the performance of the hill care facilities is it. So, that is the key response variable for such a system.

The impact of 3 factors at 2 levels each on patient satisfaction is to be evaluated. So, what are the factors? So, first factor is referred to as a factor A is the frequency of visit by the nurse, is it that is factor A, once every 2 hour that is level 1 or once every hour that is level 2. Factor B is; that means, it is the level minus 1 and this is level 1 is it; that means, it is the low level it is high level.

Factor B is the amount of time spent by the doctor with the patient. So, 10 minute per day it is level minus 1; that means, it is considered to be low level or 20 minutes per day is considered to be high level that is which is denoted as level 1.

So, factor c is the type of food provided that is the meals selected by hospital management is it that is level minus 1; that means, the low level or the patient provided choice that is level 1 is it. So, these are the assumptions and these are the values. So, we have defined, what are the factors and we have also defined say the levels in each factor.

Table shows the responses measuring patient satisfaction on a 50 point scale is it. So, essentially this is a subjective factor and you know what you try to do in many a time when you deal with the subjective factor. So, ma you need to go for making it objective. So, the scaling technique you must may use many cases we follow this. So here, if you keep on its scale, the each patient will be asked to assess his or her satisfaction level. The larger the value, the higher the satisfaction level. So, that tool also you have to specify when you go for the scaling technique.

The patients were normally assigned to each treatment, is it ok. So, random assignment parts also you know, like say we have already explained what is. So, how to apply the principle of randomizations in experimentation? Find the main effects and the interaction effects.

Find the sum of squares for each of the main effects and the interaction effects, at the 10 percent level of significance which effects are significant interpret your inferences ok.

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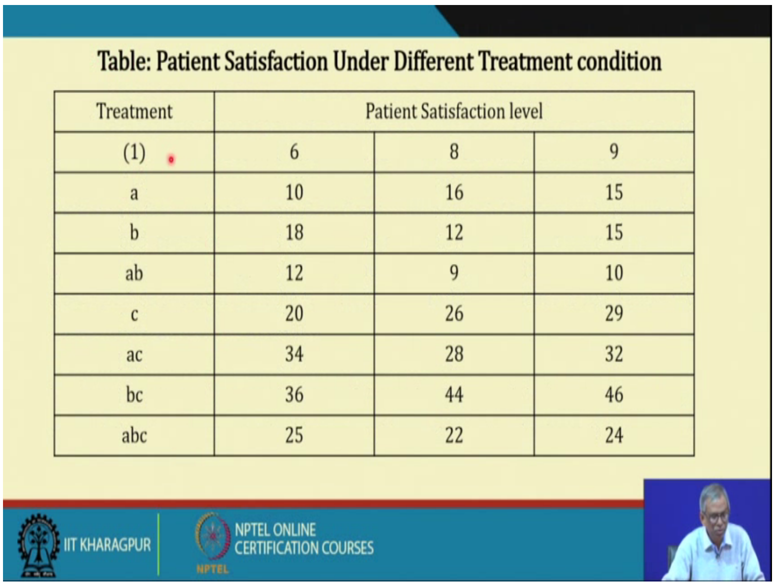

Table: Patient Satisfaction Under Different Treatment condition

Treatment	Patient Satisfaction level		
(1)	6	8	9
a	10	16	15
b	18	12	15
ab	12	9	10
c	20	26	29
ac	34	28	32
bc	36	44	46
abc	25	22	24

So, what do you try to do; that means, all the treatment combinations is it ok; that means, 3 factors we have considered and what are those treatment combinations already we have explained. So, this 8 treatment combinations you consider and the patient satisfaction level; that means, there are you know the 3 replications you have; that means, 6 8 9 similarly for these particular treatment combinations you obtain 16, 15 right. So, these are the possible say the satisfaction levels you get against each treatment combination.

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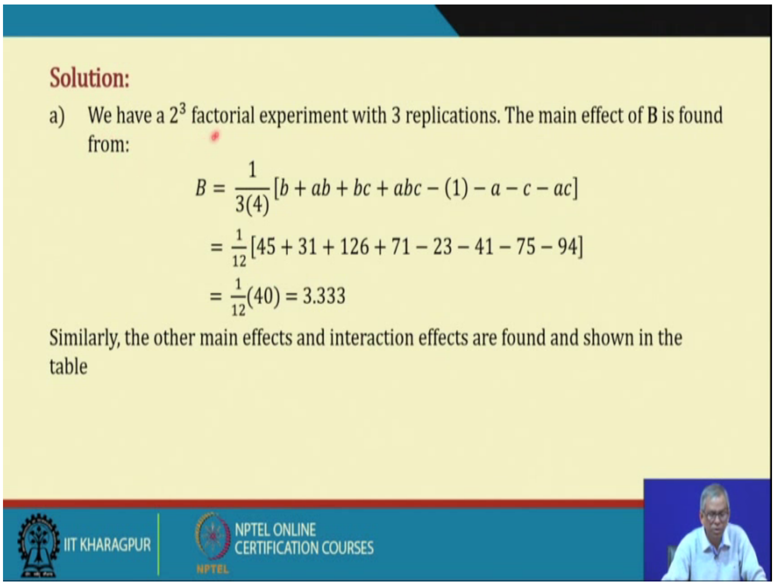

Then we have a two to the power 3 factorial experiment with 3 replications. So, you just go through I have already mentioned is a 3 replications. So, 3 values against each treatment combinations you have corrected or 3 data points. The main effect of the b is found from this one the main effect is it ok that is the formula you use right and ultimately what you find that this is basically 3 into 4. That means, this is the positive one the, those effects which where you will find this is all positives and this is all negative is it. So, you apply the formula you get a value of b 3.333.

Similarly, the other main effects are the internal effects are found and shown in the table is it ok.

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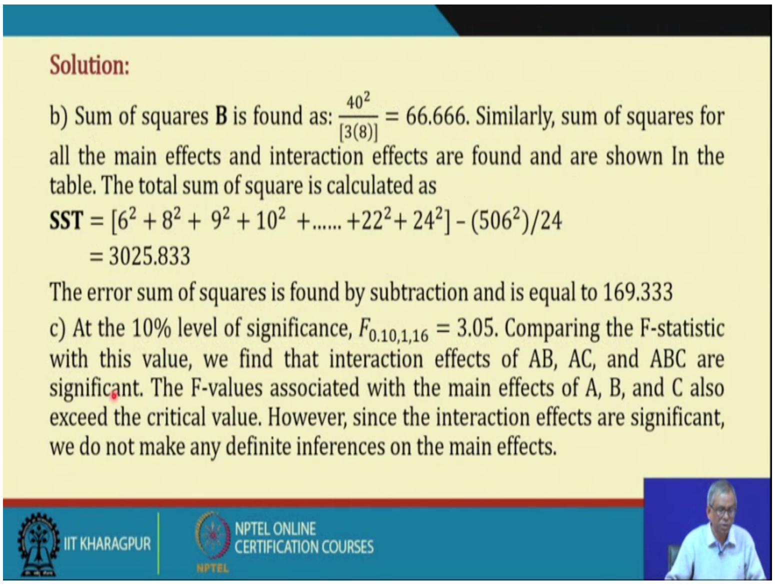

So, you just refer to the particular say the formulations and a in factorial experiments and you have the data and you compute those values.

The sum of squares is found as this, some of squares similarly the sum of squares for all the main effects and internal effects are found and as shown in the table, the total sum of squares is calculated is this. So, again we are applying the formula the either sum of squares is found by subtraction and is equal to 169.333. That means, Anova table you constructed, at the 10 percent level of significance the corresponding you know the value of the f statistics is this one that is 3.05 is it at 10 percent level of significance.

Comparing the f statistics with this value we find that the interaction effects of ab, a c, and a b c are significant, the f value is associated with the mean effects of a b and c also exceed the critical value. However, since the interaction effects are significant we do not make any definite inferences on the main effects; that means, whenever you construct the Anova table. Now, you have to interpret the values which you get as a part of analysis of variants and such explanation is to be given. So, this is the typical example please go through them.

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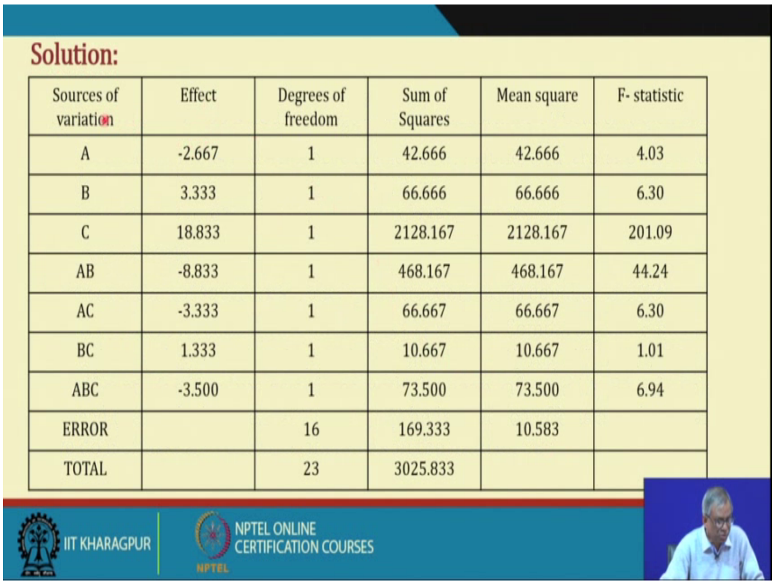

Sources of variation	Effect	Degrees of freedom	Sum of Squares	Mean square	F- statistic
A	-2.667	1	42.666	42.666	4.03
B	3.333	1	66.666	66.666	6.30
C	18.833	1	2128.167	2128.167	201.09
AB	-8.833	1	468.167	468.167	44.24
AC	-3.333	1	66.667	66.667	6.30
BC	1.333	1	10.667	10.667	1.01
ABC	-3.500	1	73.500	73.500	6.94
ERROR		16	169.333	10.583	
TOTAL		23	3025.833		

So, the sources of variations are like this the effects you calculate, degrees of freedom you all know; that means, the total degrees of freedom is 23 some of squares you compute, all the formulations are known then the mean square; that means, it is the sum of squares divided by the degrees of freedom. So, you get the mean squares and then you have the F statistics you compute is it and then you interpret them.

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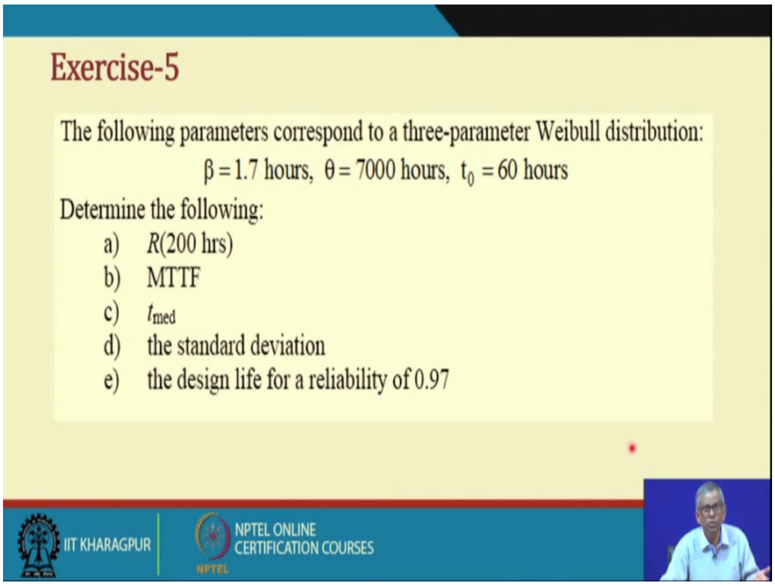

Now, next you move to exercise 5 the following parameters corresponding the 3 parameter equal distribution; that means, this particular exercise is related to the reliability. So, is it ok. So, the value of beta is 1.7 this is a is essentially it is a shape parameter and the theta is the value of theta is 7000 hours and the t is within 60 hours define the following. That means, the probability that of the of the of the system that it will run for 200 hours MTTF the medium value of T and that the standard deviation you need to compute and the design lie for a reliability of 0.97 is it ok.

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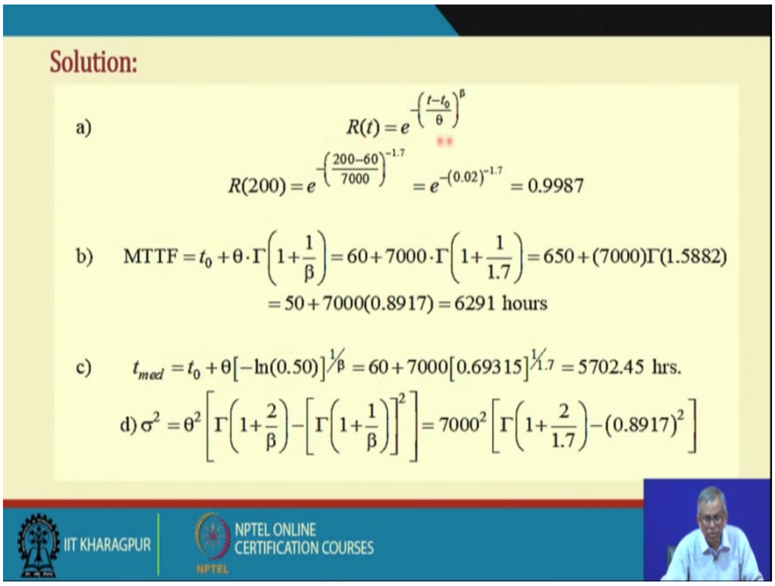

So, you apply the formula; that means, when several distribution r t is equals 2 to the power minus T minus T 0 by theta to the power beta and you know all the values over here and you get a value of 0.9987 it is very very high. Then similarly the entity of calculation the expression also you know and it is a gamma function; that means, t 0 plus theta gamma of 1 plus 1 upon beta. So, the beta value is given and then you calculate these values as a 6291 hours and the T medium again you apply this formula already which is given in the text as well as in the lecture materials as well as I have already explained it in the lecture sessions on reliability and. So, this is the value of T median and you calculate the variants applying this formula ok and then this is.

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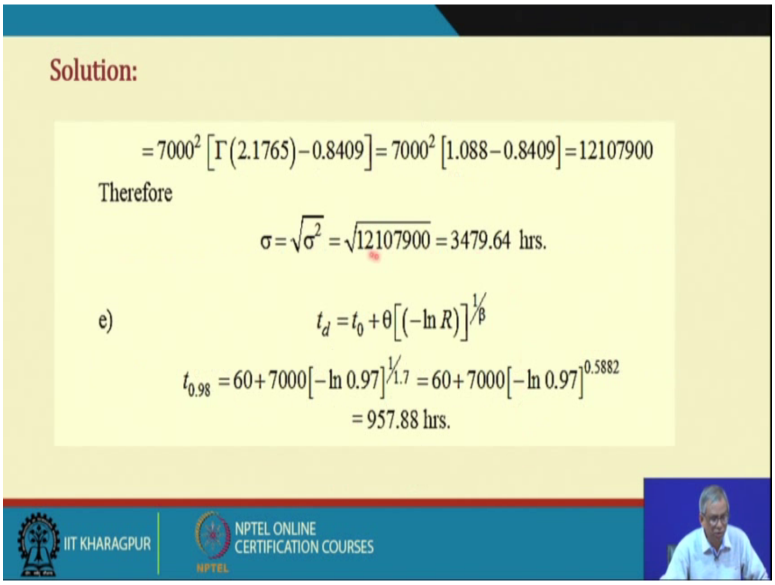

So, the expressions and you get a value of the sigma as just positive square root of the variants has 3.79, 0.64.

So, you just follow all the steps and I am sure you will be you will fully understand all the steps and this is the typical problem and this is expressions for t d; that means, when the reliability is point 9 8 what is the corresponding the life in hours of the given component. So, you just try this formula and you get a value of 957.88 hours. So, almost say 1000 hours ok.

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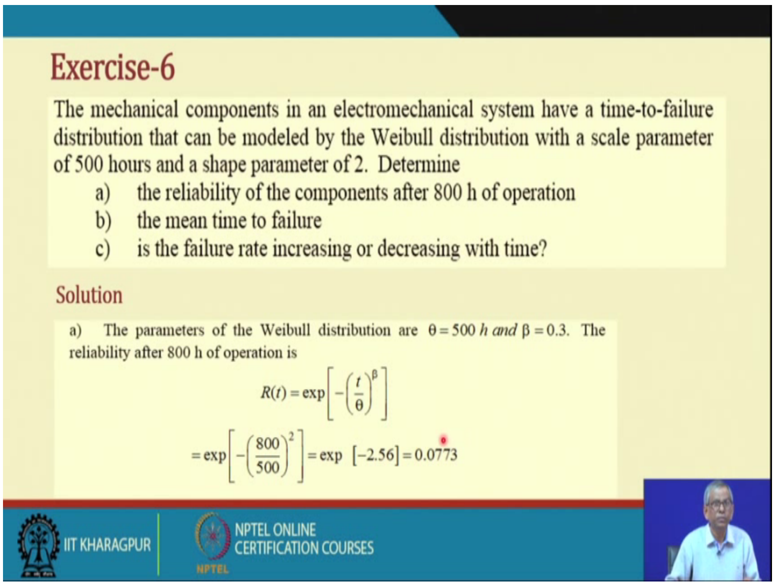

Now, one more exercise I will go through that is exercise 6.

So, here again it is related to the reliability, the mechanical components in an electromechanical system have a time to failure distribution that can be modeled by the weibull distribution with a scale parameter of 5, 500 hours; that means, the value of alpha as a shape parameter of 2 that is the value of beta.

Determine the reliability of the components after 800 hours of operation, the mean time to failure and the failure rate whether is the failure rate increasing or decreasing with the time is it ok. So that conclusions you have to make. So, what is the solution? So, a t expressions you will know is it and you know that the t is 800 theta is 500 and you get a value of 0.0773 is it. So, it is very very less.

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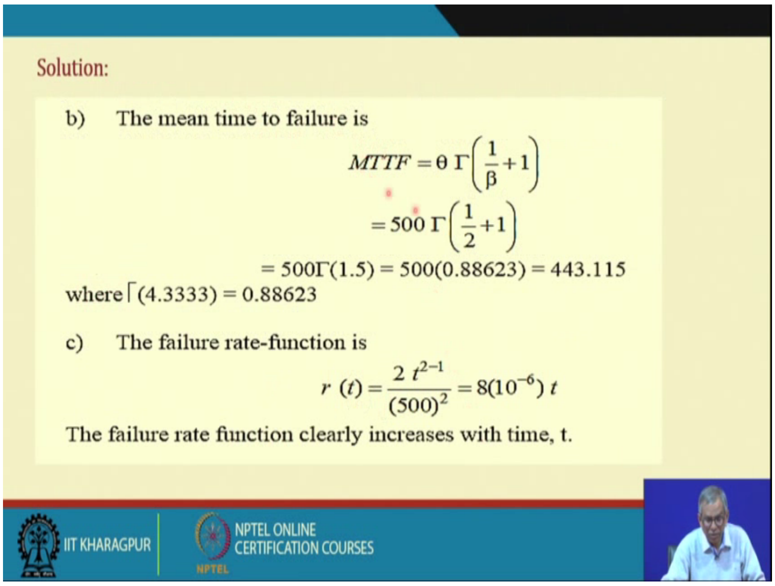

So, the mean time to failure is this theta into. So, the gamma function of one upon beta by beta plus 1. So, you get a value of 4 443.115 and the failure at function is this, is it r t is it is like say f t by r t capital r t that expressions already we have derived. So, you get a value of this one, as the failure rate function clearly increases with time is it. So, that is your conclusions.

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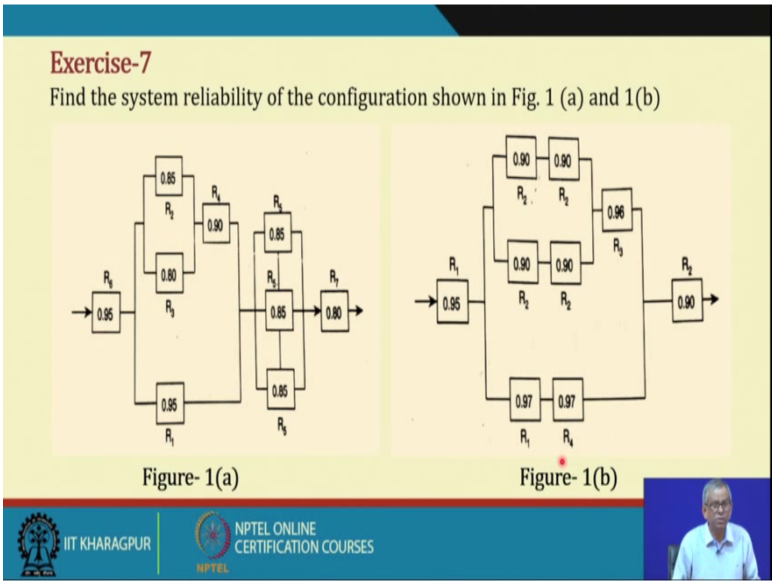

Exercise 7 is related to you know ah the two kinds of redundancy, low level redundancy as well as high level redundancy. So, when we refer to say the reliability for different types of configurations. So, one particular configuration is related to redundancy.

So, if you incorporate redundancy in the system or in the systems configuration, what do you expect that the reliability of the of the system increases. Now, we have already explained that what is the low level redundancy? What is the high level redundancy? So, here find what do you need to do, suppose these particular system structure is given find the systems reliability of the configuration shown in this one this is figure 1 and this is figure 2. So, there will be you know the series assistance, that will be a parallel systems and is essentially is a combination of series and parallel systems.

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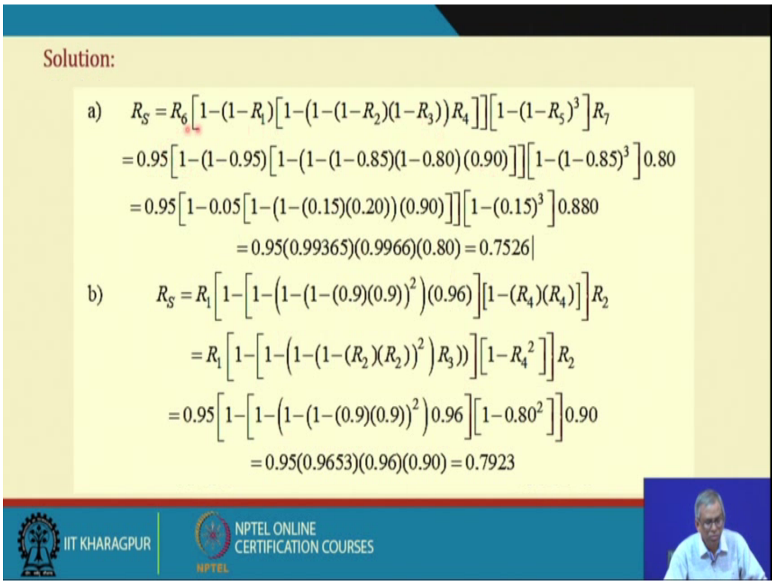

So, now we apply the formula you already know that how to write an expressions for the with respect to the first system structure, what is the systems reliability that is, it is you apply this formula the steps are involved and you get a value of 0.7526 and for the second system structure you get a value of 0.7923 is it. So, we have already explained all these details.

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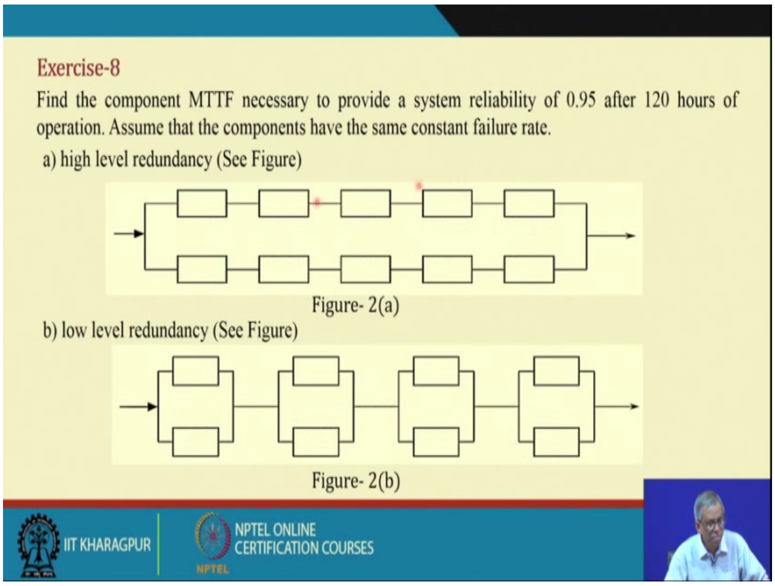

And exercise 8 is find the component MTTF, find for the component MTTF necessary to provide a systems reliability of 0.95 after 120 hours of operation assume that the components of the same constant failure rate. So, high level redundancy the figure is given and low level redundancy figure is given is it ok.

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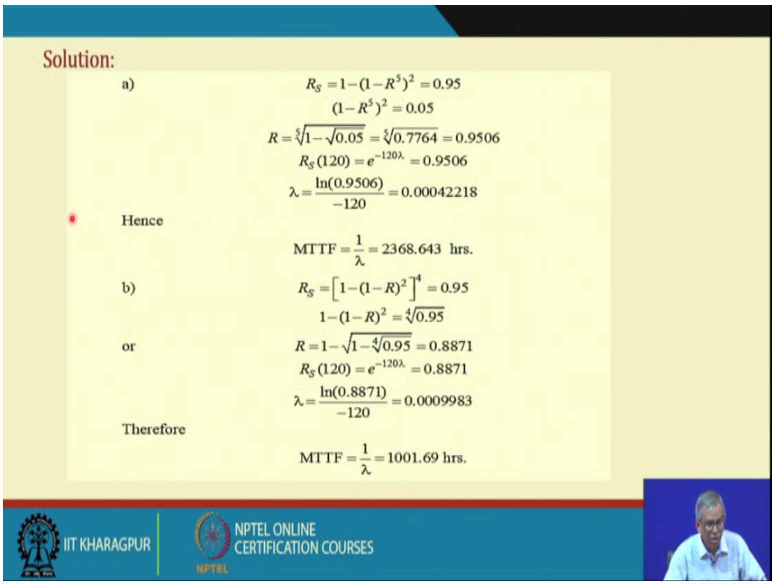

Now, you compute ah the systems reliability and for the first one and you again compute the system reliability for the second one and you calculate the MTTF for the first one and you also calculate the MTTF for the second one is it with the corresponding value.

So, these are the 8 problems we have referred to and my suggestion is that you go through all the all the exercises which we have covered against each topic and the plus definitely in course of time we have there are there will be different kinds of assignments. So, we will go through all these assignments. So, our main purpose is so, on a any topic in this particular course. So, you must have say extensive the knowledge as well as an in depth understanding of all the tools techniques and the approaches that we have discussed.