Quality Design and Control
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Lecture – 39
Acceptance Sampling-II (Contd.)

As you have already studied acceptance sampling and the different types of acceptance sampling plans we have already discussed. Now, during the next 2 sessions I will be concentrating on a number of you know the representative numerical exercises on acceptance sampling as well as some other aspects of quality design and control. Like say the reliability, like say process capability analysis as well as you know by the taguchi method.

So, during this particular session on acceptance sampling I will be discussing a few typical numerical exercises specifically on acceptance sampling and as you know that there are the 2 main types of acceptance sampling. One is the attribute sampling and the second one is the variable sampling.

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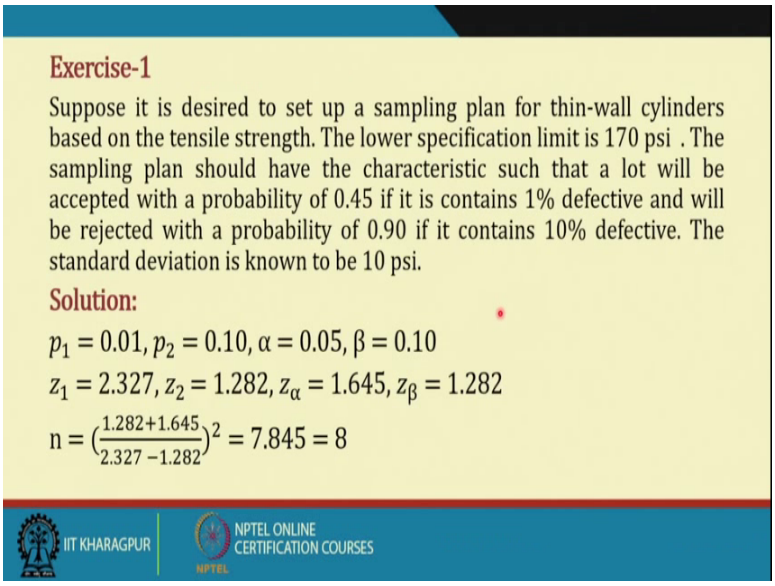

So, let us first you know the concentrate on a few problems on say the variable sampling plan. Now, the first there will be 8 exercises in this session and one by one I will be discussing all these exercises, in the first exercise let me read out the problem.

Suppose it is desired to set up a sampling plan for thin walled cylinders based on tensile strength; that means, it is a variable sampling plan. The lower specification limit is 170 psi the sampling plan should have the characteristic such that a lot will be accepted with a probability of 0.95. If it contains 1 percent defective and will be rejected with the probability of 90 it contains 10 percent defective ok.

So; that means, here the acceptance sampling plan which you are going to design, the main purpose of such a plan is to control the proportion and conforming. The standard deviation is known to be 10 psi. That means, it is a known standard deviation case, this particular case we have already discussed. Now, what is the solution, the solution is here a good quality is given by the proportion and confirming that is 1 percent; that means, p 1 is 0.01.

A bad quality is specified as 10 percent nonconfirming and so the value of p 2 is 0.10, you please refer to the derived the sampling plans; that means. So, the derived expressions for the parameter values or the parameters of the variable sampling plan, the value of alpha that is the producers risk is 0.05 and value of beta is 0.10. Now, when you refer to the standard normal deviate or the standard normal table what do you find that the capital Z 1 against p 1. That means, area under the curve beyond the beyond the Z one that you have to compute and first you compute Z one you apply the formula we will get a value of 2.327.

Similarly, against p 2 you compute the value of Z 2 and Z 2 value is 1.282 and similarly we have a formula for Z alpha and that is 1.645 and similarly the Z beta value is 1.282 that is against the tail area of 0.10. Now, in an expression for n that is the sample size and the, if you use that expression ultimately you get an x you get a value of n as 7.845 and obviously, you know the sample sizes must be an integer. So, it is rounded up to 8. So, sample size is 8 one of the parameters of such a sampling plan.

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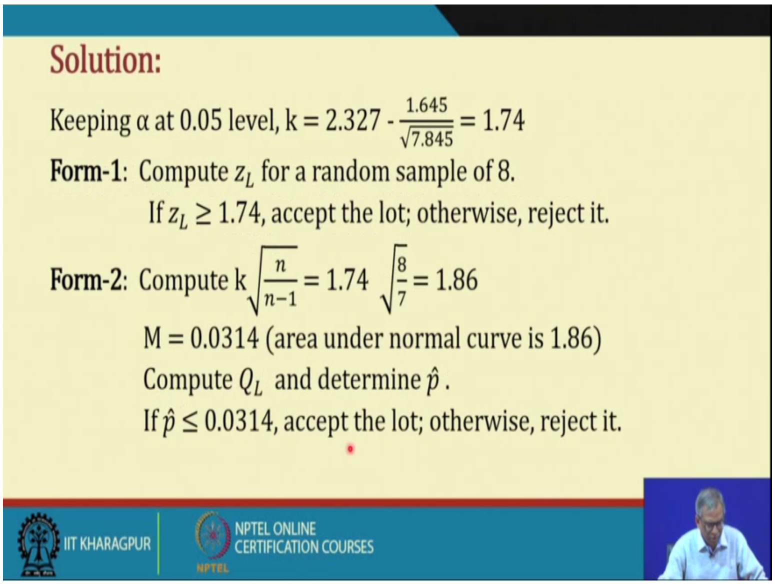

Now, as you are aware that there are 2 the methods you may apply that is the form 1 method as well as the form 2 method for such a for using such a sampling plan variable sampling plan, now here; obviously, under form 1 or the k method you must get the value of k. So, keeping alpha at 0.05 level already we have mentioned that the producers risk is at say the 5 percent. So, you know the expression for k so what will be the value of k. So, value of k is 2.327 minus 1.645 divided by root over 7.845. So, the value of small k is 1.74.

Now, if you use form 1 what do you do? First you compute Z l. So, Z L and you compute for a random sample of 8 is it. So, how do you compute Z L that is basically x bar minus capital l; that means, that L capital is the lower specification limit divided by sigma. So, value of sigma is already given, when x bar is known; obviously, you know you must get a sample of say of size 8; that means, 8 values will be getting and you compute x bar.

So, if once you compute x bar when capital L and sigma is known you can compute Z l. So, if Z L is greater than or equals to 1.74 that is the value of k we accept the lot otherwise you reject the lot. If you move to form two what do you need to do; that means, first you compute the expression of small k into root over n by n minus 1 is it ok.

So, value of k is 1.74 the value of n is 8. So, the value of these expression is 1.86 for the given problem. Now, what do you do now you refer to the area under the normal curve beyond say m equals 2.0314 is it. So, these value you will get; that means, 1.86; that means, area under the normal curve that is beyond 1.86 is it. So, that value you will get as 0.0314. So, that is the value for capital m; that means, 3.14 percent is the maximum allowable proportion nonconforming.

Now, you compute q l; that means, it is an unbiased estimate of Z l; that means, the Z L must be multiplied with root over n upon n minus 1. So, you get the value of q L and then you refer to the a d under the normal curve, say the beyond say the q L and that area you get as p hat. Now, if p hat is less than equals to capital m that is 0.0314 you accept the lot otherwise you reject the lot is it ok.

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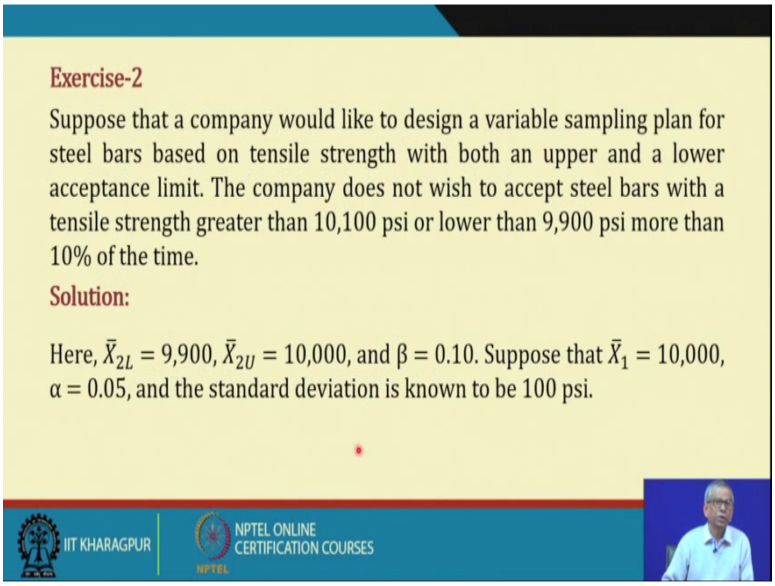

So, this is this is problem exercise number 1 and I am sure that you have understood all the steps. Now, let us move to exercise number 2, suppose that a company would like to design a variable sampling plan for steel bars based on steel tensile strength with both an upper and lower acceptance limit is it; that means, it is a 2 sided specification limit case or the double specification limits case. So, this case already we have discussed; that means, we have derived the corresponding say the formula for determining say the parameter values.

So, the company does not wish to accept steel bars with that assigned strength greater than 10100 psi or lower than 9900 psi more than 10 percent of the time; that means, from this expression from this statement you can the make out that what will be the corresponding the probability of acceptance. That means, even if the quality is extremely poor as it; that means, if it is greater than this it is considered to be extremely poor, similarly if the value is less than 9900 again it is considered to be extremely poor quality.

So, against these 2 poor quality levels what is the corresponding you know the probability of acceptance that is 10 percent; that means, it is essentially a consumer risk. So, here a x t2wo L bar is 9900 and x 2 u bar is 10100 and beta is equals to 0.10 that is the consumer risk, suppose that X 1 bar is 10000 how do you get x one bar usually unless otherwise specified you say that the midway between these 2. That means, x 2 u bar and x 2 L bar the mid way that is considered to be X 1 bar and X 1 bar is considered to be is assumed to a very good quality for which the probability of acceptance is very high is it ok.

So, here the probability of acceptance is 0.95 and that is why the producers risk alpha is 0.05 and the standard deviation is known to be 100 psi; that means, it is the standard deviation known case.

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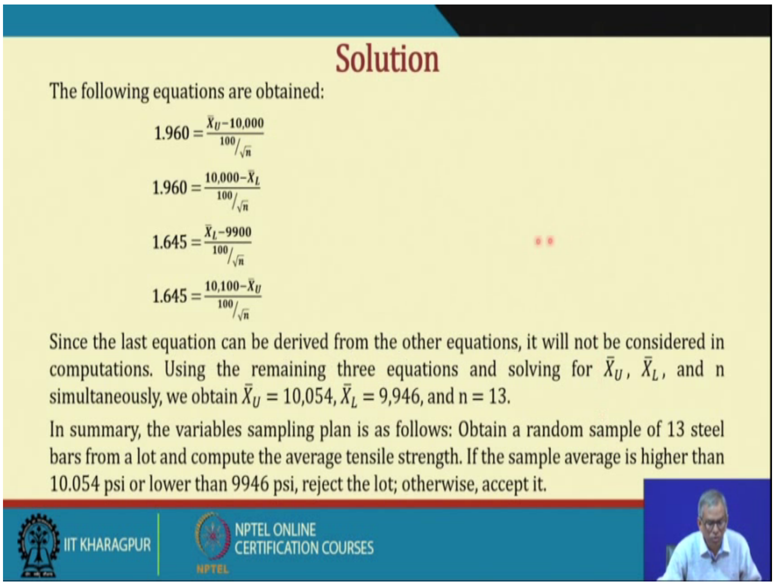

Now, what do you do, we have already derived this you know this the 4 expressions like say this is point 1.960; that means, against a value of say 0.025 that is the tail area, you will get a value of Z as 1.960 and similarly say x u bar minus 10000, this is 10000 minus x L bar this is x L bar minus 9900 and this is 10100 minus this.

So, all these 4 expressions you have. So, this is 1.960 this is minus 1.960, this is 1.645 we refer to the standard normal curve. So, against those corresponding tail areas you will get these values and again this will be minus 164 values, when 1645. So, one is positive second one is negative, third one is positive, fourth one is, fourth one is negative.

So, since the last equation can be derived from the other equations not only the last one, you choose any 3 equations from this set of 4 equations you will you will find that the fourth equation can be derived from say any 3, fourth equation any tree of from the first 3 equations. So, so what do you try to do; that means, that means one equation is a dependant one whereas, the remaining 3 are independent simultaneous equation.

So, what do you do? You just select out of this four any three equations. So, using the remaining 3 equations and then you solve for x u bar x L bar and n. So, what is this is, what is x u bar that is the upper acceptance limit what is x L bar that is the lower acceptance limit and n is the sample size.

So here, when you solve these 3 simultaneous equations. So, you get a value of x u bar as 10000 10 10054 x L bar as 9946 and n equals to 13. So, in summary the variable sampling plan is as follows, obtain a random sample of 13 steel bars from a lot that is the sample size and compute the average tensile strength. That means, the actual value of x bar; that means, 13 such values you will be getting you compute the average value and that is x bar, if the sample average is higher then this one; that means, 10054 psi all one then 9946 psi reject the lot otherwise you accept it is it ok.

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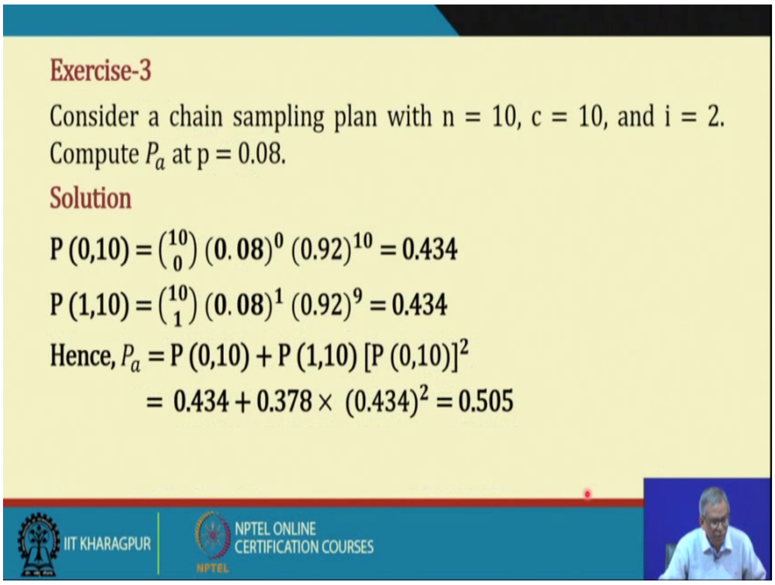

So, now let us move to exercise three and exercise 3, is related to a special kind of sampling plan you know, we have mentioned that one of the special kinds of the special purpose sampling plans is the chain sampling plan and the chain sampling plan. You use when the sample size is very very less, less than equals to 10 and if the sample size is less than equals to 10 why the sample size is less than equals to 10 because there are many cases where inspection is done through destructive testing.

So; obviously, the sample size should be as minimum as possible and if the sample size is less; obviously, you know the acceptance number is 0. So, acceptance number will be 0 and here the third important variable is I equals to 2 is it all right. So, consider this chain sampling plan with n equals to 10 c equals to 0 this is a very special case because if we use a single sampling plan with c equals to 0 as I have already mentioned that then you get an oc curve and oc curve will be a convex function. And it is very difficult to say the work with a sampling plan where the you know the oc curve is a convex function.

So, you have to change the oc curve shape from a convex function to an inverted s. So, n equals to 10 c equals to 0 and I equals to 2. So, compute probability of acceptance at an incoming lot quality of 0.08 that is p equals to 0.08. So, what do you do? That means, this is the expression; that means, what is the probability of acceptance that if you do not find out of 10 any nonconfirming unit. So, you are going to accept it you are again going to accept it, accept the lot if you get out of 10 units inspected one unit is nonconfirming provided that the last I number of the samples which you have drawn from the lot because you are dealing with the series of lots not an isolated single lot and you will find all the previous high number of the samples you do not find any nonconfirming units that is why it is 0 10 ok.

So, under this condition you accept even if in the present sample you find one nonconfirming unit. So, this is the probability of acceptance if out of 10 units you inspect 0 found as nonconfirming. So, you assume poison ah say the binomial distribution. So, it is 10 c 0, 0.08 to the power 0 1 minus 0.08 that is 0.92 to the power 10, 10 is n.

So, this is 0.434 and what is the probability that out of 10 units one unit found as nonconfirming that is 10 c 1 0.08 that is incoming lot quality to the power 1 1 minus 0.08 that is 0.92 to the power n minus 1 that is 10 minus 1 that is 9 that is 0.43 this is maybe it is different its value is 0.63 something. So, you just compute this value it just cannot be 4.434 is it ok.

So, hence probability of acceptance is actually this value is 0.378 is it. So, it is 0.378. So, ultimately where the probability of acceptance is 0.434 plus 0.378 plus 0.434 square equals 2.50 ok.

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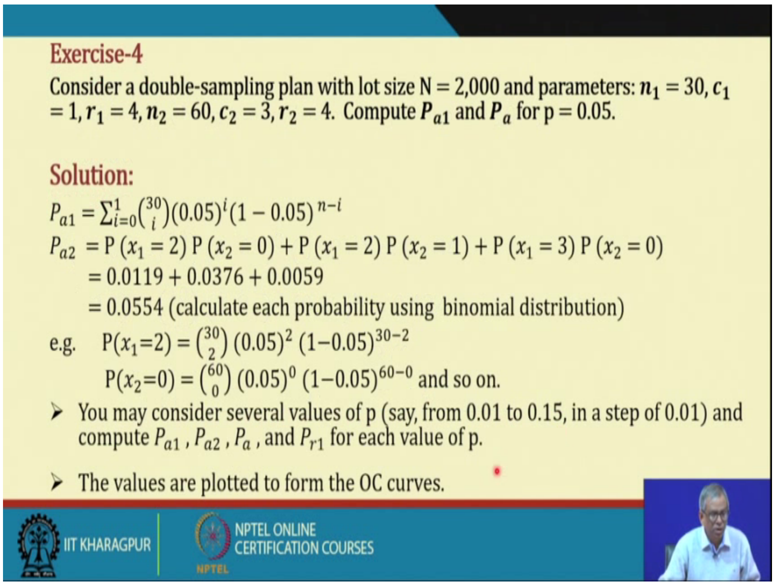

So, now you move to exercise 4 consider a double sampling plan in attributes category. Now, we refer to attribute sampling plan and one particular say the type of sampling plan that is the double for sampling plan and the double sampling plan is widely used. So, here is an example of a double sampling plan is it. So, the given lot size n equals 2000 and what are the parameters n 1 that is the first sample size is 30, first acceptance number; that means, acceptance number based on the first sample that is c 1 that is 1 accept rejection number based on the first sample that is 4 you already know what is the working of a double sampling plan we have already explained.

The second sample size is 60 that is n 2 equals to 60 then acceptance number of the combined sample that is 3 rejection number or the combined sample that is 4 compute a pa 1, what is pa 1? that means, the probability of acceptance based on the first sample and what is pa; that means, probability of acceptance are the total probability of acceptance for a given incoming lot quality p equals to 105.

So, how do you compute pa 1, pa one is say sigma say I equals to 021 is it. So, actually I equals to 0 to c. So, here as you consider the first sample size, the first sample size the acceptance number is 1; that means, either there could be 0 nonconfirming unit or one nonconfirming unit in the sample of size 30 then only you are going to accept it based on the first sample. Now 30, c I 0.05 that is p to the power I into 1 minus 0.05 to the power n minus I is it ok.

So, similarly under what condition you go for pa 2; that means, when the number of nonconfirming in the first sample is either 2 or 3 you have no other alternative, but to go for say second sample say second stage; that means, for making a decisions you have to draw the second sample of size 60. So, these conditions are mentioned; that means, in the first sample size that is random variable based on the first sample size. So, that is x 1, x one 1 to 2 you are going to draw the second sample

And in the second sample you may not get any say nonconfirming agreed that is x 2 equals to 0 and similarly when x 1 equals to 2 in the first sample and so; obviously, you know in the second sample you may get. So, you must have just one nonconfirming unit for the acceptance of the lot or in the first sample you will find there are 3 nonconfirming and if you find three nonconfirming and based on the second sample suppose you are going to accept it; obviously, in the second sample you must not have any nonconfirming unit. So, these are the conditions you must know under which you know, but the first then that the second stage you are going to accept the lot.

So, we have explored all those possibilities and then what you are trying to do; that means, we go for ah say poisson approximation to binomial and. So, you calculate all these individual probabilities and then the total probability of acceptance based on the second sample that is 1.0554. So, these values already you can compute either you can apply binomial distribution or in many cases you go for poisson approximation to binomial.

You may consider several values of p say from 0.01 to 0.15 in a step of 0.01. That means, you are considering 15 such values incoming lot qualities possible incoming lot tea values and compute p a 1, p a 2 p a, what is p a? P a is basically pa 1 plus p a 2 and p r 1; that means, probability of rejection based on the first sample for each value of p is it ok.

So, the values are plotted to form the oc curves, is it ok.

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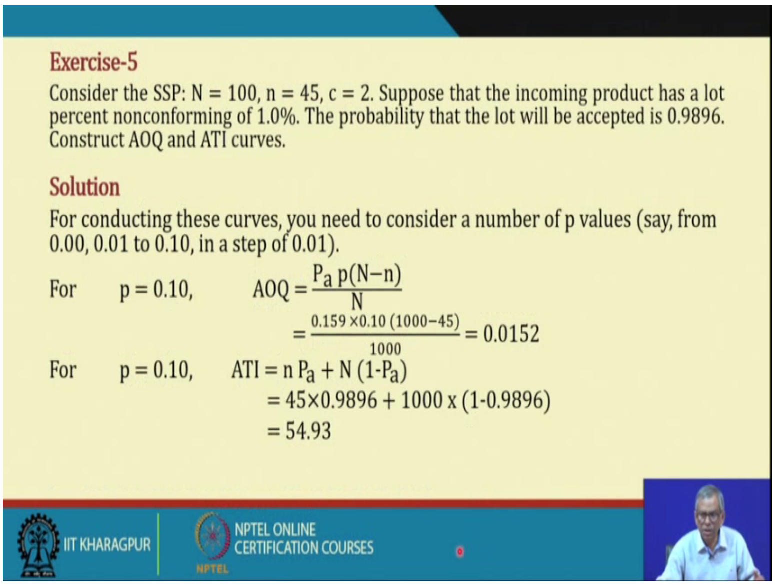

So, this is an exercise you can carry on; obviously, you know that against a particular value of p how do you how can you compute the probability of acceptance, either based on the first sample or the second sample or say the total probability of acceptance, probability of acceptance based on the probability of rejection based on the first sample and probability of reaction based on the second sample.

Now, let us move to the next exercise, exercise 5 consider the single sampling plan ssp stands or single sampling plan where n equals to say ma say 1000 small n equals to 45, c equals to 2 is it. Suppose that the incoming product has a lot person nonconfirming of one person of one percent the probability that the lot will be accepted is 0.9896 construct AOQ and ati curves. We have already explained what is AOQ is it and what is ati we have defined these 2 terms and particularly these 2 terms are applicable you have to define when you apply rectifying inspection.

So, what is the solution for conducting, for constructing these curves you need to consider a number of p values say from 0.00. So, if it is 0.00; obviously, the probability of acceptance will be 1 right. So, you can check it and then you consider the 10 values in a step of 0.01; that means, the first value will be 0.01 second value will be 0.0, true third value will be 0.03 and so on.

So, for just I am giving you one example; that means, against a particular value of p say 0.10; that means, this value what will be the AOQ for a single sampling plan p into p into capital N minus N divided by capital n we have already derived these expressions and then what is the value of pa, pa is a 0.98, 0.15 say 0.0896 and p is 0.10, 0.9599896 0.10 and this is 1000 and n is 45 and capital is 1000 is it. So, this value is 0.0151 is it ok.

So, AOQ value is this and for p equals 2.10 you calculate ati, again if we apply this formula n into pa plus capital n into 1 minus pa and. So, you apply you use these values and you get a value of say ati that is 54.93.

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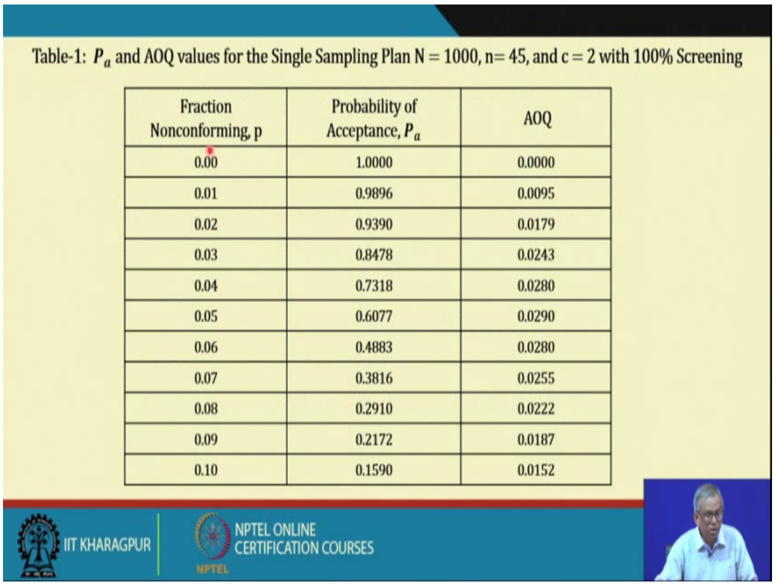

Table-1: P_a and AOQ values for the Single Sampling Plan N = 1000, n= 45, and c = 2 with 100% Screening

Fraction Nonconforming, p	Probability of Acceptance, P_a	AOQ
0.00	1.0000	0.0000
0.01	0.9896	0.0095
0.02	0.9390	0.0179
0.03	0.8478	0.0243
0.04	0.7318	0.0280
0.05	0.6077	0.0290
0.06	0.4883	0.0280
0.07	0.3816	0.0255
0.08	0.2910	0.0222
0.09	0.2172	0.0187
0.10	0.1590	0.0152

Now, what do you consider you consider several values of several values of p and you as I have already pointed out that the against a value of p equals 2.00; that means, the best possible quality; that means, no nonconfirming say nonconfirming units in the lot. So, that is the incoming lot quality; obviously, the probability of acceptance will be 1 right.

And several other values you consider for each such the values of p you compute pa and immediately you apply the formula for AOQ. So, get the values of AOQ.

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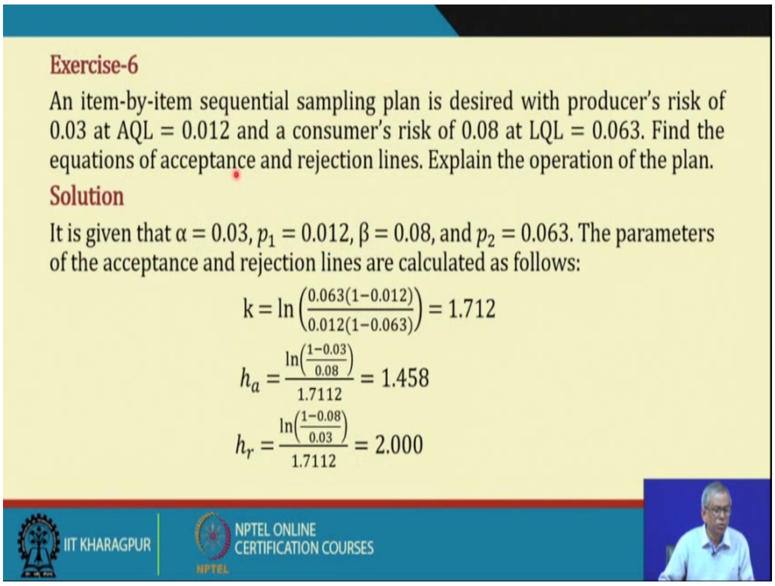

Now, you move to the exercise 6 an item by item sequential sampling plan. So, again it is a special kind of sampling plan in attributes category we use. Now, an item by item sequential sampling plan is desired with producers risk of 0.03 at aql of 0.012 and the consumer risk of 0.08 at lql are points. So, 0.063 find the equations of acceptance and rejection lines.

When we explained the sequential sampling plan it is essentially unit by unit or item by item say sampling plan not lot by lot acceptance sampling plan. So, explain the operation of the plan. So, alpha value is 0.03 p 1 is 0.012 beta is 0.08 and p 2 is 0.063. So, these values must be specified.

The parameters of the acceptance and rejection lines are calculated as follows; that means, what will be the expression for the k. So, we apply the formula, you refer to that particular formula you get a value of 1.712 and similarly this is basically for the acceptance line, acceptance line, what is the intercept that is h a and this is the expressions. So, you use and you get a value of 1.458 and this is you know the intercept for the rejection line that is h r ok.

So, this again you get the corresponding values and you get the value of h r that is 2.000 and so it is basically you know in terms of k that is. So, everywhere you will find that it is divided by small k and already you have computed the value of small k when p 1 and p 2 values are known right.

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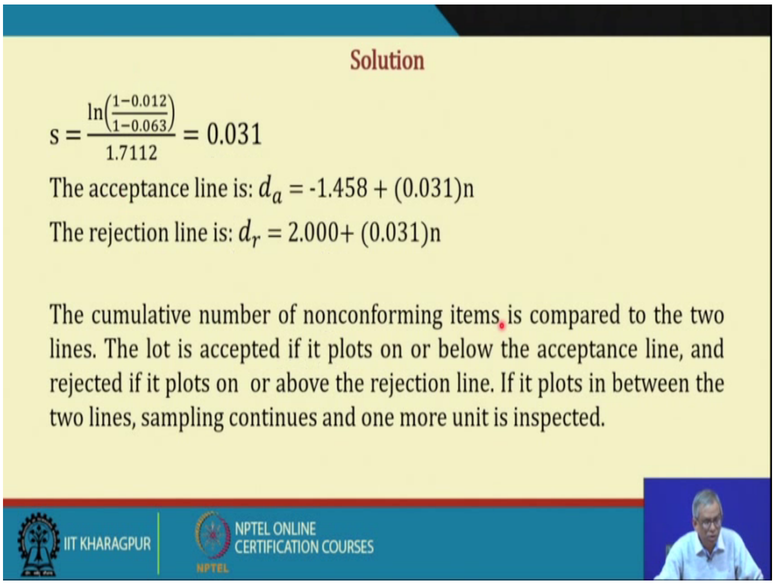

So, once you have all these values and this is the s s is the slope. So, slope component is also is a function of small k. So, you get the value of 0.01. So, now, the acceptance line is this is it minus h a plus s into n and the rejection line here is again h r plus s into n.

So, the cumulative number of nonconfirming units is compared to the 2 lines the lot is accepted if it plots on below the acceptance line and the rejected if it plots on or above the rejection line. So, that is the decision rule if it plots in between the 2 lines; that means, it is a continuing continued sampling zone sampling continues and one more unit is inspected and then you check that what is the current value of the cumulative number of a nonconfirming unit and then you decide that whether you are going to accept it or you are going to reject it or you continue sampling.

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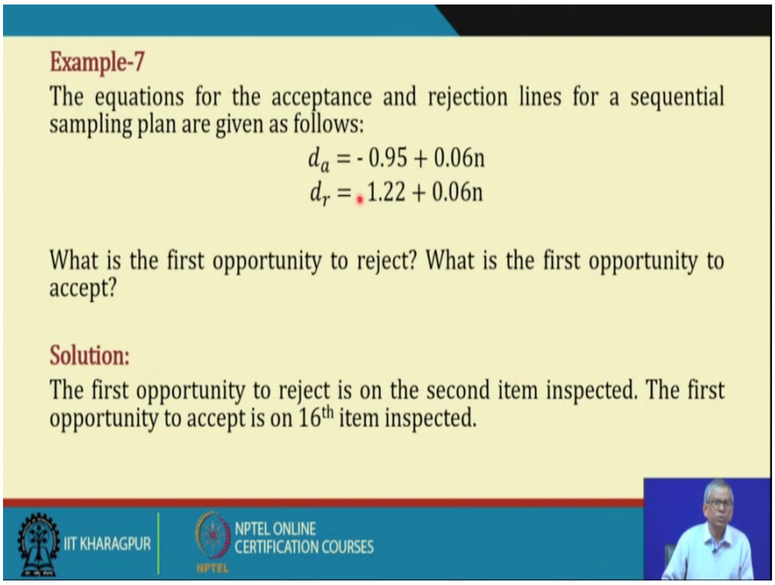

Now, 2 more examples we have example seven. So, that you work out I will just read it out it is very simple one that is the equations for the acceptance and rejection lines for a sequential sampling plan, we have already derived. So, suppose these 2 equations are given for the acceptance and rejection line for the sequential sampling plan, what is the first opportunity to reject? You please solve this problem, what is the first opportunity to accept is it ok.

So, this is for you I am giving you this exercise to solve and what is the solution; that means, you will get I will I am just giving the answer the first opportunity to reject is on the second item inspected and the first opportunity to accept is on the 16 item inspected is it. So, this is the solution.

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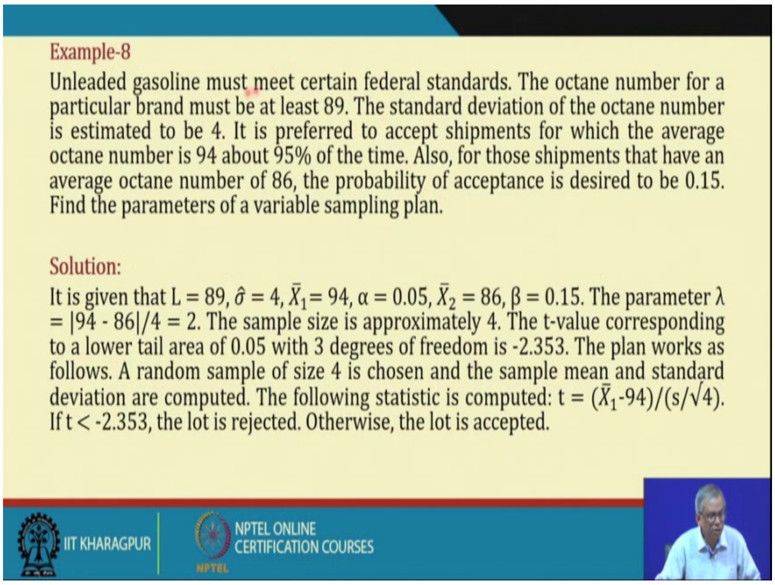

And then you move to example 8. So, this example 8 is just I will read it out and all the steps are given over here, but this is the case where you know it is a single specification a limit case. Write your purposes you will be using a variable sampling plan to control the process average and the standard deviation is unknown. So, we have already explained the procedures.

So, just I read it out unleaded gasoline must meet certain federal standards, the octane number for a particular brand must be at least 89 the standard deviation of the octane number is estimated to be 4; that means, the case of you know the lowers you know the lower specification limit case. It is preferred to accept shipments for these average octane number is 94, about 95 percent of the time; that means, your basic purpose is to control the process average also for those shipments that have an average octane number of 86 considered to be very low the probability of acceptance is desired to be 0.15 find the parameters of a variable sampling plan.

So, you know what is the value of a lower specification limit that is 89 sigma hat it is given which is 4 X 1 bar is 94 alpha is 0.05 X 2 bar is 0.06 beta is 0.95. So, what do you try to do; that means, you form the say the t distribution and you refer to that particular curve and you will find that the sample size is approximately 4 and the t value corresponding the vertical area of 0.05 with 3 degrees of freedom is minus 2.353 you refer to the t distribution table. The plan works as follows a random sample of size 4 is chosen and the sample mean and the standard deviation are computed the following statistics is computed; that means, X 1 bar minus 94 divided by s that is the sample standard deviation divided by root over n; that means, here the sample size is 4 n equals to 4 if t is less than a minus 2.353 the lot is rejected otherwise lot is accepted. So, these are the 8 problems we have discussed in this session.