Quality Design and Control
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Lecture – 38
Acceptance Sampling-II (Contd.)

I will be discussing the type two variable sampling plans as I have already stated there are 2 types of variable sampling plans.

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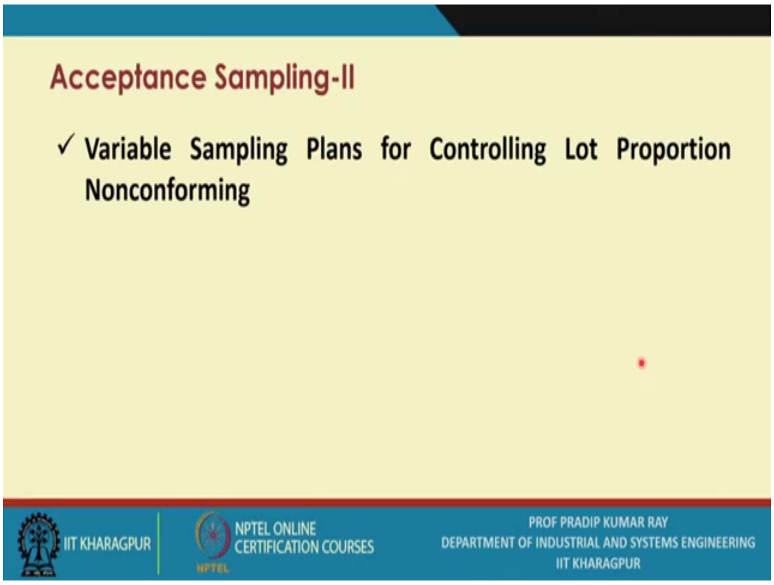

And the first type already we have discussed, now we are going to discuss the variable sampling plans used for controlling lot proportion and confirming and; obviously, you know I will be dealing with several a number of numerical exercise is it ok.

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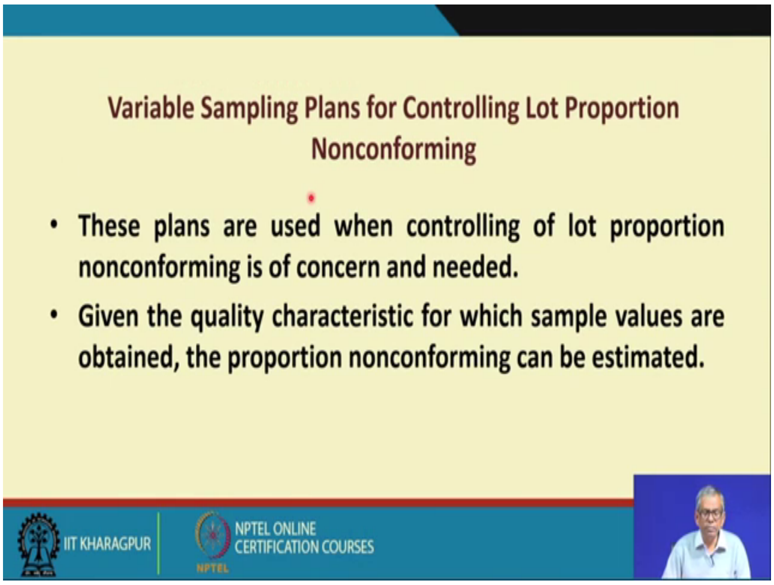

So, now these plans are used when controlling of lot proportion and confirming is of concern and needed. So, this point already we have elaborated ok.

Not only the process average, you need to control you also need to control the proportion nonconfirming and these are the proportion nonconfirming you are it is not that it is related to the attributes data many a time know whenever we say the proportion nonconfirming we say we might may assume that it is related to the attributes data. So, you may also in the defined proportion nonconfirming with respect to a quality characteristics or use the variables data are collected; that means, the exact the deviation from the standard from the nominal value, if any you can you can determine and accordingly you can determine this proportion non say proportion nonconfirming related to a particular lot.

So, given the quality characteristic for which the sample values are obtained the proportion nonconfirming can be estimated is it. So, you must not have any confusion on this.

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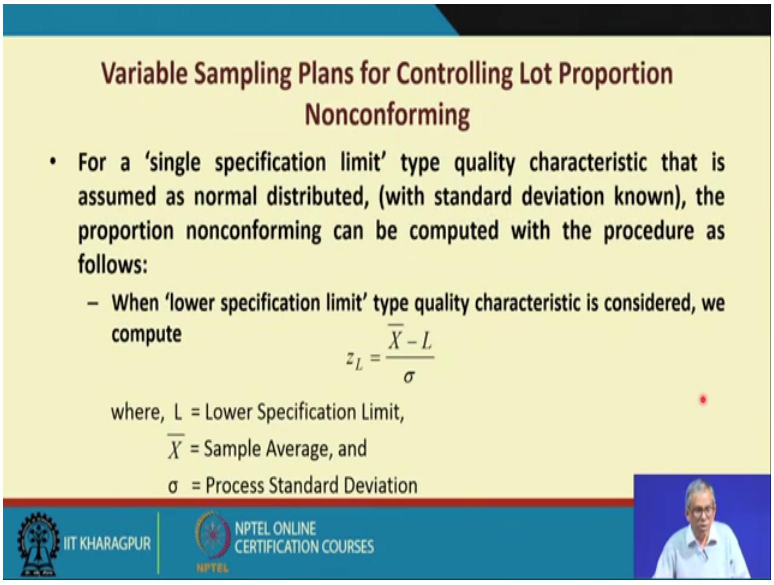

Now, so, let us talk about you know the variable sampling plan or controlling lot proportion nonconfirming when the quality characteristic is of single specification limit type. So, for a single specification limit type quality characteristics there are many such quality characteristics you come across. Now it is assumed that that this quality characteristic is normally distributed with standard deviation known is it right like in previous cases we have assumed.

The proportion nonconfirming can be computed with a procedure as follows is it. So, when the normality assumption holds. So, proportion nonconfirming you can calculate. So, how do you calculate I will just explain one procedure suppose you say word specification limit type quality characteristics? So, what do you do and this; the lower specification limit is specified as capital L. So, you compute Z L is it. So, what is Z L actually the standard normal deviate is it. So, Z L is equals to X bar minus L by sigma. Now what is this X bar? X bar is a sample average and capital L is the lower specification limit and what is sigma? Sigma is a process standard deviation which is known now.

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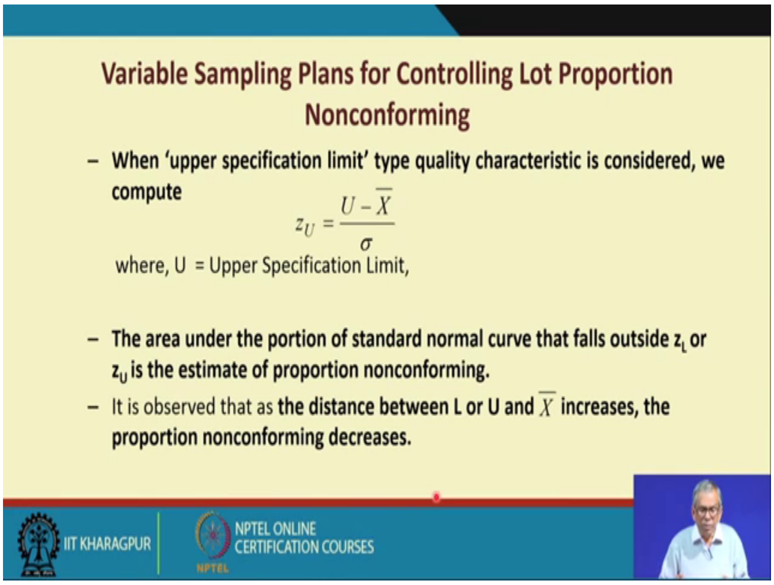

When upper specification limit type quality characteristic is considered is it. So, how do you compute Z U again, the standard normal deviate that is U minus X bar divided by sigma. So, X bar is a sample average what is U u is the upper specification limit and what is sigma? Sigma is the standard deviation as a standard deviation is known the area under the portion of the standard normal curve that falls Z L or Z U is the estimate of proportion nonconforming; I presume that you know the procedures when you start the reading the values from the standard normal to. So, we find you know the procedures said this is a way to get these values very easily.

So, here I have explained the area under the portion of the standard normal curve that falls outside Z L or Z U is the estimate of proportion nonconforming because here you know you are indexing your plan with respect to the proportion nonconforming. So, how do you define the good quality when the proportion nonconforming is very very less how do you define a bad quality when the proportion nonconforming is very high now it is observed that as the distance between L or U an X bar increases; that means, U minus X bar suppose this value increases what do you find that the value of Z U will be more and if the value of Z U or Z L is more the corresponding area under the portion of the standard normal curve that falls outside Z L or Z U also will be will be less. So, that is why you know the proportion nonconforming decreases, I think that the logic is made very very clear.

That if the distance between L and X bar or between U and X bar increases the proportion nonconfirming decreases ok.

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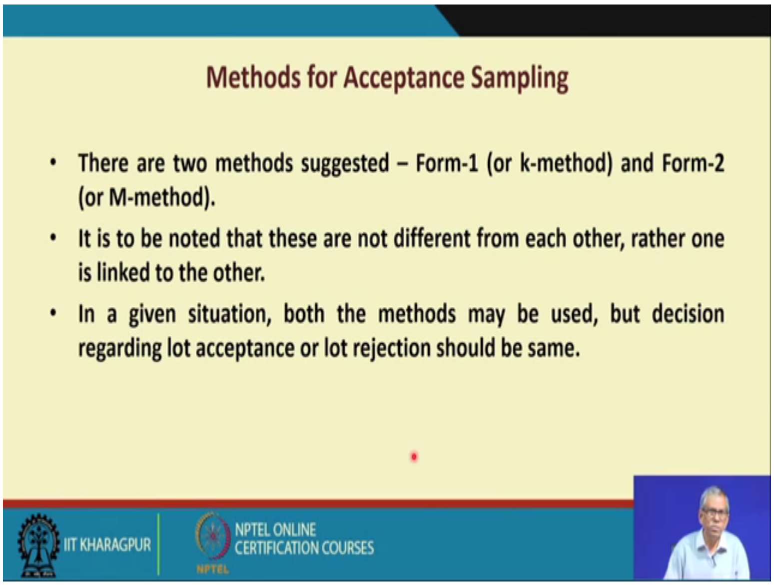

Now, with this basic understanding, then what do you do; that means, we refer to the methods for acceptance sampling when you come across or when you deal or you are dealing with the variables data and your objective is to the control proportion nonconfirming is it and; obviously, we will be referring to a particular quality characteristics just make a note that all these variable sampling plans you design you use with respect to a particular quality characteristics if the quality characteristic changes; obviously, you have to design a different you know the sampling plan.

Now, in this context when your objective is to control the proportion nonconfirming there may be 2 methods as suggested by the researches and the practitioners in this field and these 2 methods are known as form 1 method or it is also referred to as a small k method and the other method is a form 2 method or it is also referred to as the capital M method.

So, in a you know ba, we will be explaining what is this k or what is this m is it. So, these are the very standard notations we will find in any text books on quality. Now it is to be noted that these are not different from each other; that means, you know it is not with the form one, you get one sorts of result and the form 2, you get another sorts of results and then you have to make a choice is it is not like this what do you try to do; that means, first you apply the form one method and you get a decision now at the second stage you can modify the form one method with the form 2 method and you apply the form 2 method and again you take a decision.

And as you are referring to the same case is it so; obviously, the decision must be same. So, if you follow the follow the rules or follow the methods properly we will find that whatever decisions you have taken under form one same decision you will be taking under form 2 also getting my point. So, many time what you can do; that means, the form 2 method you can use for verification purpose is it. So, this is the point to be noted. So, it is to be noted that these are not different from each other rather one is linked to the other is it. So, do not start with the form 2 you start with form one and then move to form 2 it is ok.

So, while you explain the methods in details, we will understand this case. So, in a given situation both the methods may be used, but decision regarding lot acceptance or lot rejection should be the same.

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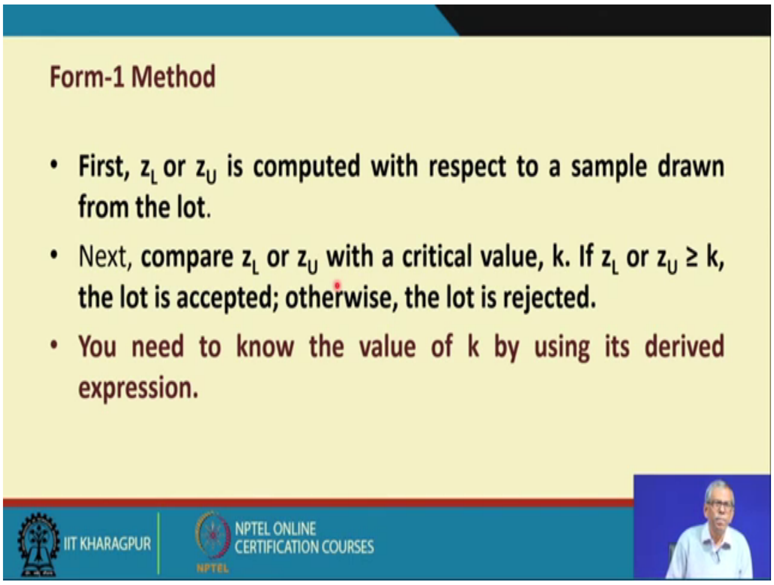

So, how do you use a form one method first Z L or Z U is computed with respect to a sample drawn from the lot how do you do it; that means, as soon as you compute Z L; that means, it is a single specification unit case and more specifically, it is a lower a say the lower specification limit case whereas, if you compute Z U; that means, it is an upper specification limit case. So, now, you compute Z L and Z U the formulas are known to you.

Next compare Z L or Z U with a critical value k is it now ba; what will be the mathematical expressions for k or how do you derive the expression for k, we are going to explain, we are going to discuss it very soon the critical value is known; that means, when you; while you have designed a sampling plan ultimately you need to know the what is the sample size and what is the value of k. So, these are the 2 parameters values in form one method. So, what do you do first is with the X bar with the L with the sigma you calculate Z L or Z U is it. So, while you compute z u; that means, you must know what is the value of U. So, either Z L or Z U compute, and you check whether Z L or Z U is greater than or equals to small k.

So, if this condition holds, then the lot is accepted otherwise the lot is rejected. So, this is a very simple procedure you need to know the value of k by using its derived expressions this point I have already mentioned.

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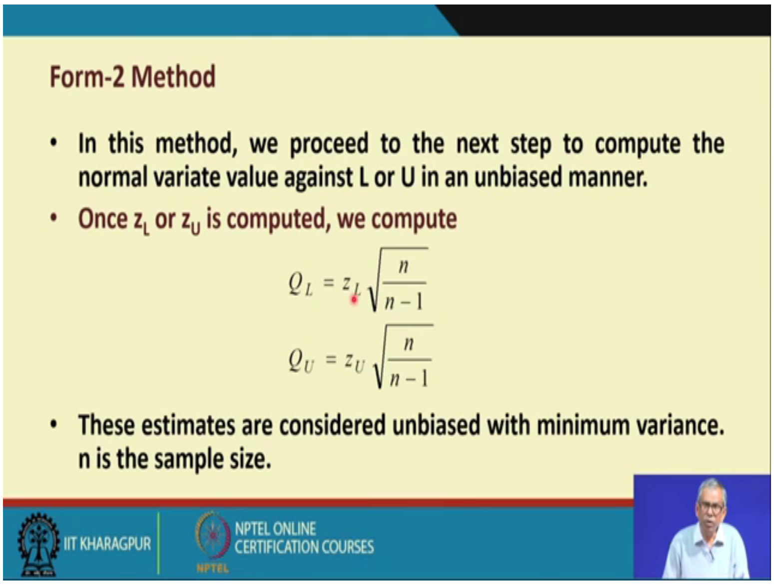

Under form 2 method what do you do in this method we proceed to the next step to compute the normal variant value against L or U in an unbiased manner now what you will you might have noticed that why do you; what when we compute Z is a Z U it may be a biased estimate is it this concept of bias can be explained from different prospective or just make a note that when a you use Z L or Z U it may be biased estimate.

So, if you want to avoid this bias and you have to when recommend an unbiased estimate for Z L or Z U what you against L or U what do you need to do you need to compute Q L or Q U as the case may be and this Q [laughter], you compute by you know the modifying the value of z l; that means, you use this multiplying factor that is root over n upon n minus one or both the cases; that means, Z L and Z U is already computed and when you compute Q L or Q U; that means, this is as a normal variative calculate what this normal variate is considered to be unbiased and with minimum variants. So, the Q L is equal to Z L into root over n upon n minus one and similarly Q U is Z U into root over n upon n minus 1.

Now, these estimates are considered unbiased with minimum variants n is the sample size is graph.

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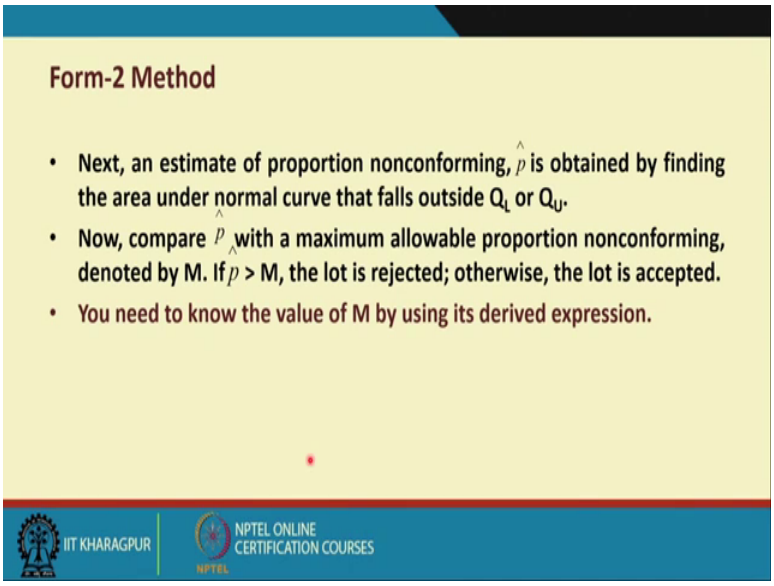

So, Q L or Q U, we compute; next what do you do? An estimate of proportion nonconfirming P hat is obtained by finding the area on the normal curve that falls outside Q L or Q U is it. So, again you refer to the standard normal table it is a normality assumption holds. So, refer to the standard normal table and you just get the area under the normal curve outside that is falling outside Q L or Q U and these proportion this area is referred to as the proportion nonconfirming and the notation is P hat is it.

So, the P hat you get these values from the table now compare P p hat with a maximum allowable proportion nonconfirming denoted by M. So, now, we have defined what is capital M. So, what is capital M. capital M is maximum allowable proportion nonconfirming is to case how do you get this value; obviously, I will explain the procedure. So, when you take up an example we will we will we will apply this that procedure or so.

So, if you find that this proportion nonconfirming that is the actual proportion nonconfirming is greater than the maximum allowable proportion nonconfirming. So, this is not a desirable situation; obviously, the lot is rejected otherwise the lot is accepted. So, the decision rule is known to you need to know the value of n by using its derived expressions like you do for the small k ok.

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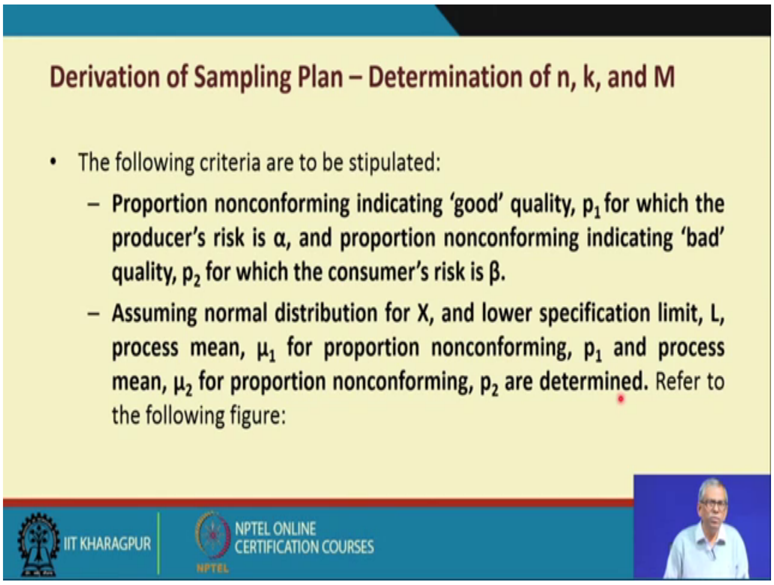

So, now, you have understood what is you know ba the form one method as well as the form 2 method. So, form one method is also referred to as the k method and the form 2 method is also known as capital M method.

Now, now what do you need to do; that means, you have to design the sampling plan; that means, you must be able to the derive the expressions for the sample size as well as the other parameter values of the given or the variable sampling plan where the objective is to control the proportion nonconfirming. So, essentially it is an exercise or determination of n k and m. So, the following criteria to be stipulated like we do for designing any other types of sampling plan.

Proportion of nonconfirming indicating good quality p 1. So, this you must be able to specify whether it is 1 percent or 0.5 percent or 0.05 percent. So, that is considered to be in a good quality in terms of proportion nonconfirming for which the producers risk is alpha. So, if the value of e one is very less what you will find that the producers risk alpha also will be very very less. So, may be 0.01 or may be 0.001.

Whether if it is 0.02; that means, this value could be just 5 percent or 0.04 or 0.05 and proportion nonconfirming indicating bad quality; that means, if the proportion nonconfirming increases; obviously, it is indicative of bad quality. So, that value you must be able to specify like say I say that 0.04 or 0.05 or 0.06; that means, 6 percent is considered to be very bad quality 6 percent proportion. So, a non co ba percent nonconfirming 6 percent nonconfirming or which the consumer risk is beta; that means, even if the quality is extremely poor very poor you might say that this is there is a high probability of acceptance. So, that is it is avoidable whether it is avoidable or not that is the different issue, but right. Now this is win the case you cannot avoid it. So, it may be 10 percent as a 10 percent.

Assuming normal distribution for x and the word specification limit L process mean mu one for proportion nonconfirming p 1 and process mean mu 2 or proportion nonconfirming p 2. Now this determine; that means, this process this process means mu one against p 1 and process means mu 2 against p 2 suppose these are determined now these are known, then what do you do you refer to the following figure.

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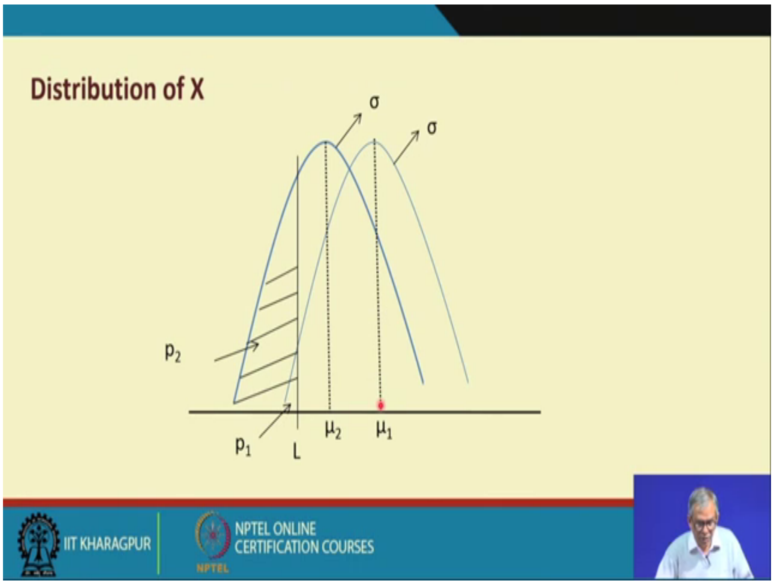

So, here what you are trying to do; that means, you are getting the distribution of x this is the distribution of X. So, what do you have over here when the distribution is here; that means, with standard deviation of sigma this is the process standard deviation that is sigma

So, this what is happening. So, this is when the proportion nonconfirming; suppose this is a lower specification limit case. So, the value; that means, this area is beyond L; that means, this is the proportion nonconfirming. So, against the proportion nonconfirming p 1 already we have specified; what is the corresponding value of mean that is mu 1. Now there is a shifting in the process. So, now, mu one has become mu 2 or in other words p 1 has become p 2. So, if p 1 becomes p 2; that means, from the good quality to bad quality. So, what is the corresponding value of in 2. So, that also you can notice; that means, the these entire area is considered to be the proportion nonconfirming; that means, pa it may be 0.01. Now it has become 0.08 or 0.10; is it clear.

So, this is. So, you must be referring to the distribution of x right when the sigma is known.

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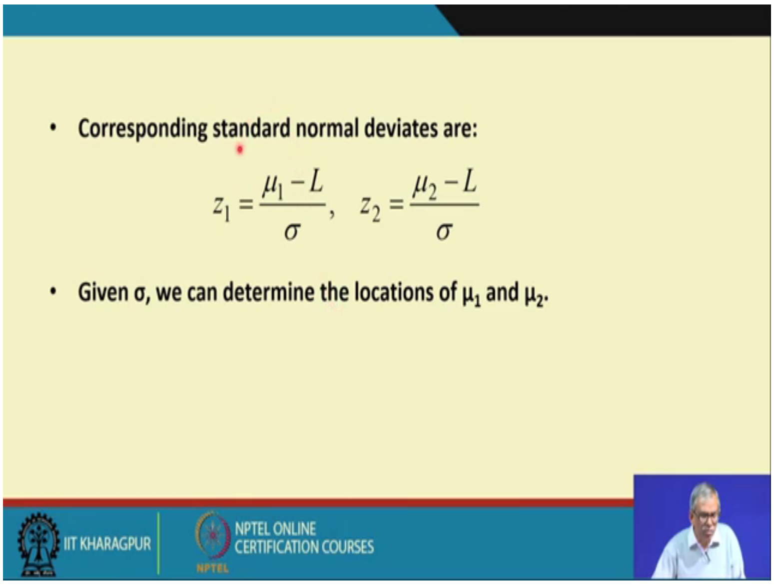

So, now corresponding to mu 1, you calculate z 1 the standard normal deviate and corresponding to mu 2 you calculate the standard normal deviate z 2. So, what is z 1? X 1 is; that means, you take the difference of new one from L this difference is a is expressed in standard deviation units. So, what is the standard deviation that is sigma and similarly you compute z 2 mean 2 minus L by sigma given sigma we can determine the locations of mu one and mu 2 is it. So, always you can do?

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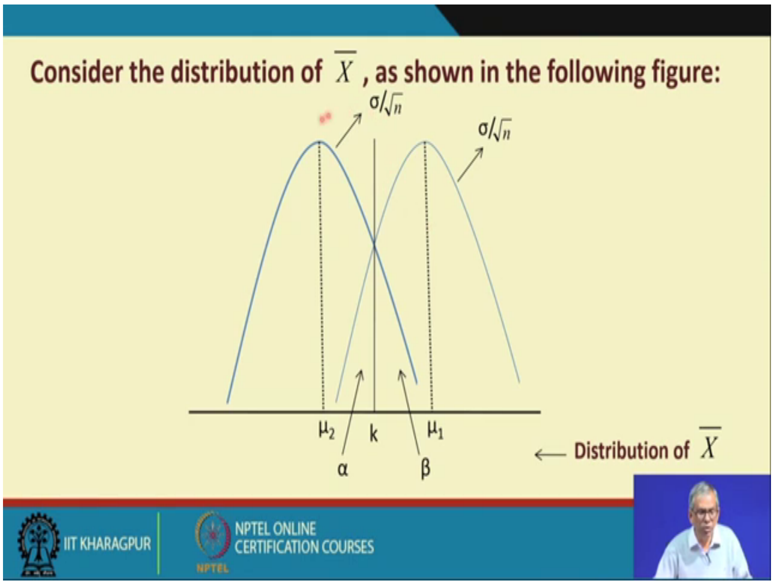

Then what do you do now you consider the distribution of X bar as shown in the following figure. So, what do you try to do now? So, you have these parameter that is small k is it now against mu 1, you have the distribution of X bar; obviously, the standard deviation is sigma upon root n as per the central limit theorem for both the cases.

So, against mu one you have again if for the quality is extremely you know the good is it what do you find that when k is the parameter; that means, this is the area that is beyond k so; obviously, this is the you know the probability of probability of known acceptance even the quality is very very high. So, this is basically called the producers risk whether when the quality deteriorates to mu 2 what do you find that you have with respect to this k. Now the area under the curve which is within k; that means, this value is greater than k.

So; obviously, you know there is a possibility of its acceptance even if the quality deteriorates is it clear. So, we are considering the distribution of X bar.

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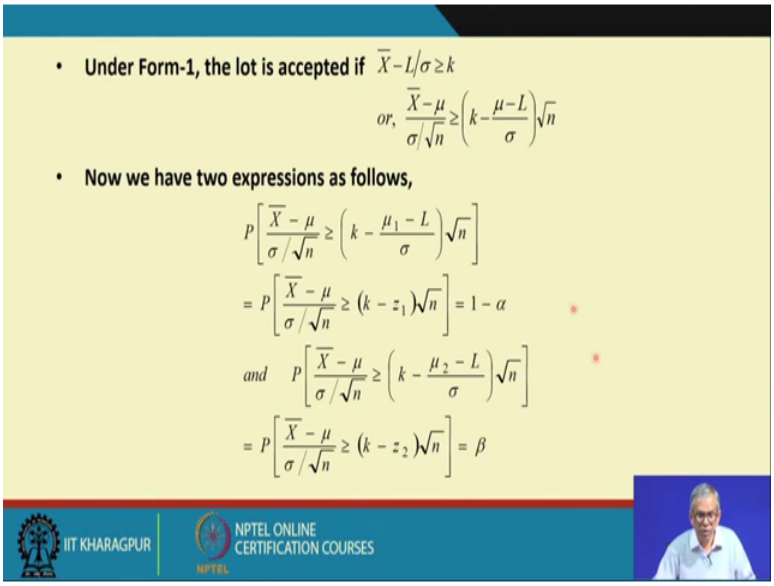

Now, what do you do you have under form one the lot is accepted if X bar minus L divided by sigma is greater than equals to k and with you know ba; this; what you can say like with these manipulations what you can do X bar minus mu by sigma upon root n is greater than equals to k minus mu minus L by sigma into root over n. So, this is the generalized expressions.

Now, we have 2 expressions like this means that the probability that X bar minus mu by sigma upon root n greater than equals to k times mu one minus l; that means, one particular value of mu is mu 1 L is fixed is it that is the lot specification unit case and sigma is the processed under deviation a root over n. So, the probability this; that means, here mu one minus L by sigma already we have computed that is z 1 is it the standard normal deviate and. So, ultimately this becomes a probability that X bar minus mu by sigma upon root n is equals to is greater than or equals to k minus z 1 into root of n this is must be one minus alpha; that means, the this is the probability of acceptance that is one minus alpha what is alpha? Alpha is the producers risk.

So, in a similar fashion when mu changes to mu 2 you get these expressions this probability expressions you get that is the probability that X bar minus mu by sigma upon root n greater than or equals to k minus z 2 into root over n that is equals to beta what is beta that is the consumer risk ok.

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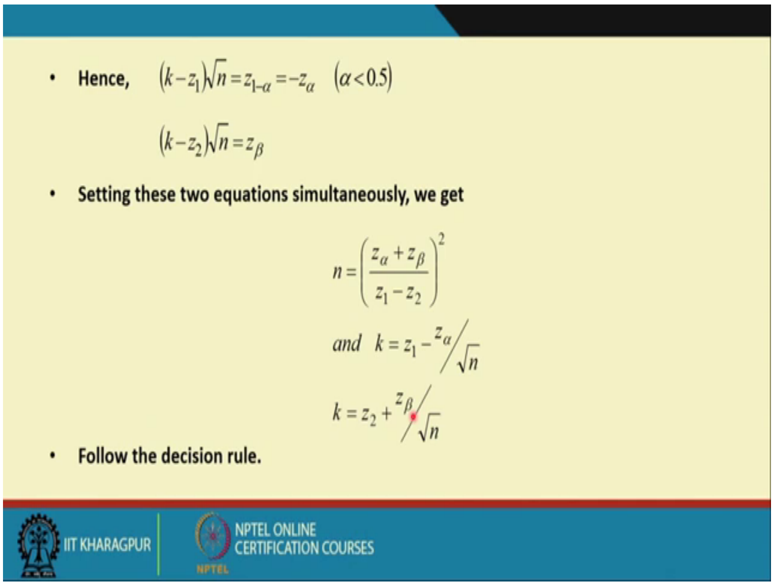

So, hence k minus z 1 root over n is equals to j 1 minus alpha equals to minus z alpha. Zlpha is less than equals to 0.5 in almost all cases and k minus z 2 into root over n is equals to z beta. So, you have the 2 equations and you have 2 unknowns and these 2 are independent equations simultaneous equations you have. Now setting these equations simultaneously solving these 2 equations simultaneously, we get n equals to z alpha plus z veta by z 1 minus z 2 square and k equals to z 1 minus z alpha divided by root over n z 1 minus z alpha by root over n.

And another expressions of k you get that is z 2 plus z veta by root over n. So, now, what do you do? You follow the decision rule when k is known; obviously, you can follow the decision root. So, what do you do; that means, z alpha z veta z 1 z 2, you compute you get the expressions for n. So, sample size is known next what do you do the either you use these expressions or these expressions now these expression when you use; that means, you try to meet the alpha conditions, but you do not bothering about a meeting the beta conditions whereas, if you use these alternative expression for computing a k for determining the value of k; obviously, you are trying to meet the beta conditions, but you are not bothering about the alpha conditions.

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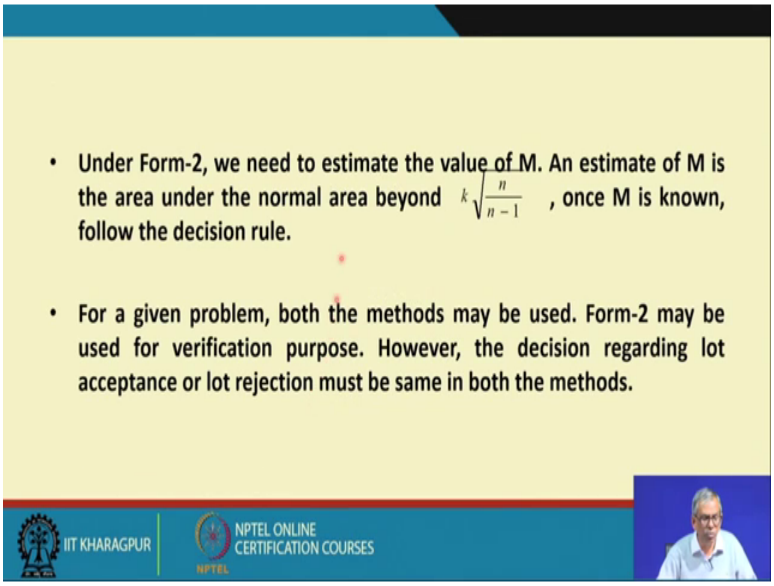

So, now you know the decision rule. So, under form 2 you need to estimate the value of m. So, we know ba the expressions for k now if you opt for form two; obviously, you must have an estimate for the value of n an estimate of n is the area under the normal a normal curve beyond k into root over n upon n minus 1 is it. So, n is known so; obviously, these expressions also will be known.

Once n is known follow the decision rule is it. So, already we have given under form 2 what is the decision rule for a given problem both the methods may be used form 2 may be used for verification purpose this point already I have explained.

However the decision regarding lot acceptance or lot rejection must be same in both the methods.

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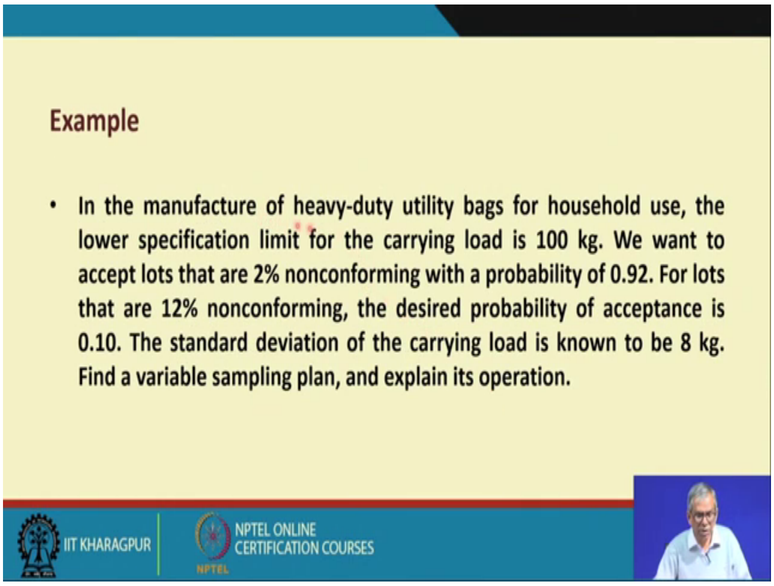

Now, here is an example, I will just go through it in the manufacture of heavy duty utility bags for household use the lower specification limit for the carrying load is 100 kg. So, it is a lower specification limit case we want to accept loss that a 2 percent nonconfirming with a probablility of 9 2 for lots that are 12 percent nonconfirming, it is a very bad quality, ba lot the desired pro probability of acceptance is still 0.10. So, this is basically the consumers risk the standard deviation of the carrying load is known to be 8 kg; that means, this is the standard deviation known case.

For a variable sampling plan find a variable sampling plan and explain its operation.

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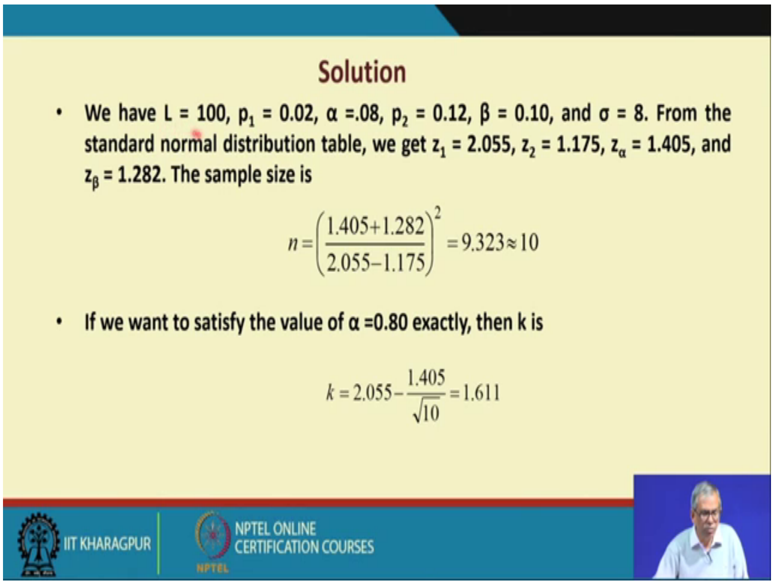

So, here we have n equals to 100 p 1 is equals to 0.02 alpha is 0.08, p 2 is 0.12 is a bad quality 12 percent nonconfirming still the value of beta is very high that is the consumer risk is 10 percent and sigma is 8.

So, from the standard normal distribution table, we get z 1 against p 1 that is 2.055 z 2 against p 2 that is 0.175 z alpha against alpha equals 0.08, you get a value of 1.405 and z beta against the value of beta the tail area that is 1.282; that means, you are always refer you are referring to the standard normal the table thus the sample size is you apply the you use the formula and you get a value of n that is 9.323 and you just make it the next high integer that is 10 and if you want to satisfy the value of alpha equal to 0.80; there are 2 the formulas for k. So, if you use the first formula; that means, if you want to meet the condition of alpha exactly then the value of k is this 1.611.

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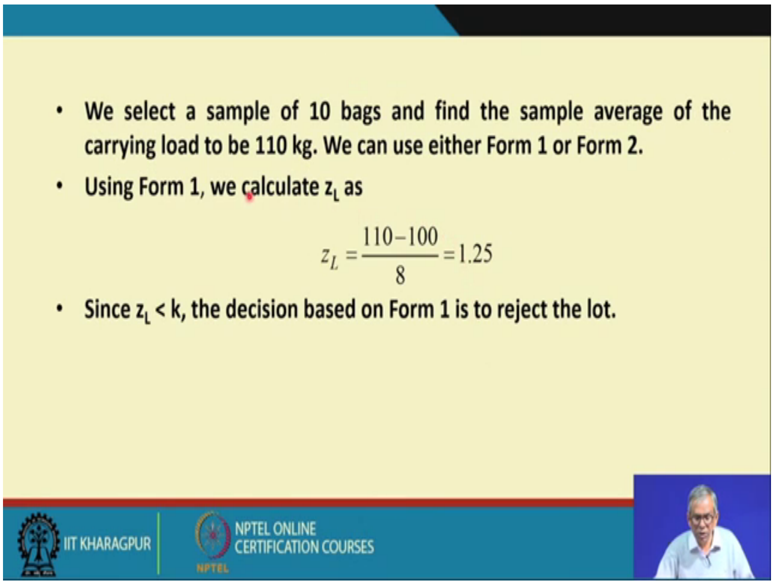

And if you want to; obviously, you can refer to the other formula for k if you want to meet the you know the consumers the risk condition exactly or what you can do? You can calculate the values of k using both the equations; that means, there will be 2 values and you may also take the average is it. So, that is the third option you can also adopt.

So, we select a sample of 10 bags and find the sample average of the carrying load 2 be 110 kg, we can use either form 1 or form 2 using form 1, you can calculate Z L as this 1.25. Since Z L is less than equals to k the decision based on form one is to reject the lot.

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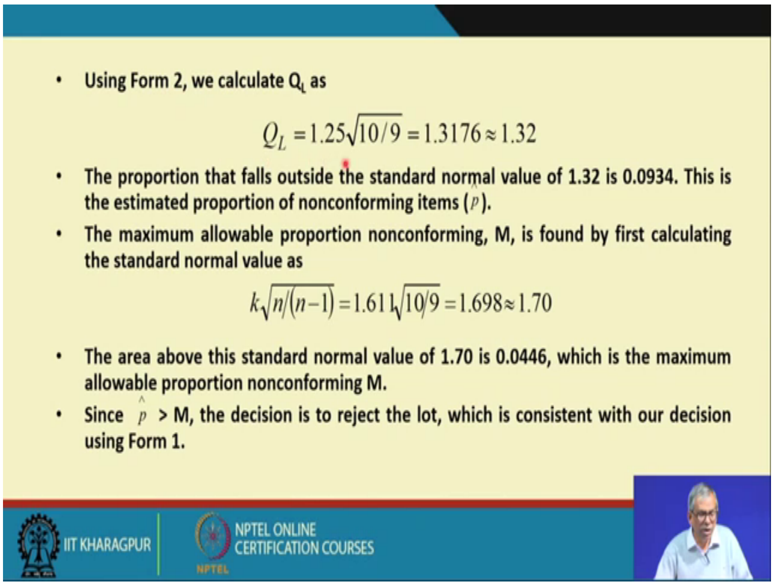

And similarly you calculate Q L is it you are moving to form 2. So, you calculate 1.32 and the corresponding the proportion that falls outside the standard normal value that is 0.0934 and then you calculate m that is the proportion nonconfirming n first you calculate the value of k into root over n into n by n minus 1 that is this value; that is 1.70 and you refer to the standard normal the table.

And against this value of 1.70 you get an area of 0.0446; that means, 4.46 is the maximum allowable proportion non confirming what you are getting that is you know ba that is; obviously, you are getting a value of say 9.34 percent.

So, since P hat is greater than m the decision is to reject the lot is it ok.

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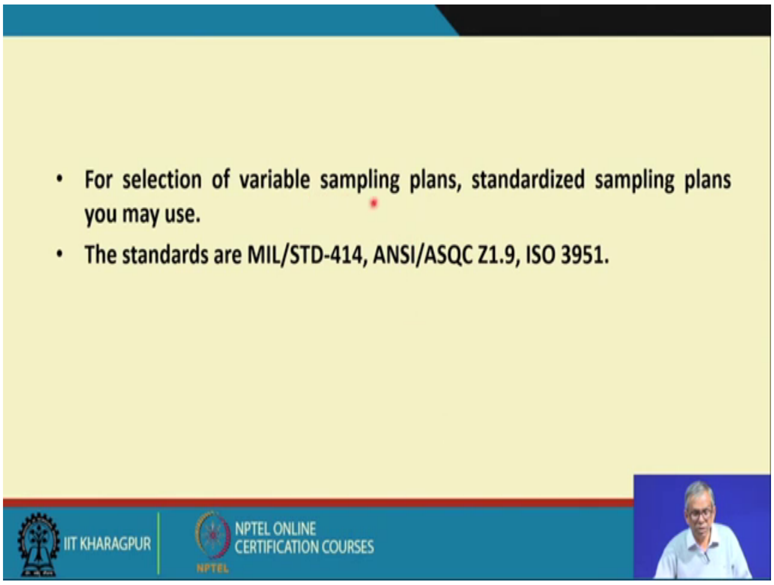

So, for selection of variable sampling plans standardize sampling plans you may use like you do for attribute sampling plans and there is a standards are as I have already pointed out mil std four one four an equivalent standard is ANSI ASQC Z 1.9 an equivalent rise. So, standard is ISO 3951.

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So, we have computed or you know the lectures on the different types of variable sampling plans in the subsequent lecture sessions we will be referring to a set of the numerical problems.