Quality Design and Control
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Lecture – 37
Acceptance Sampling-II (Contd.)

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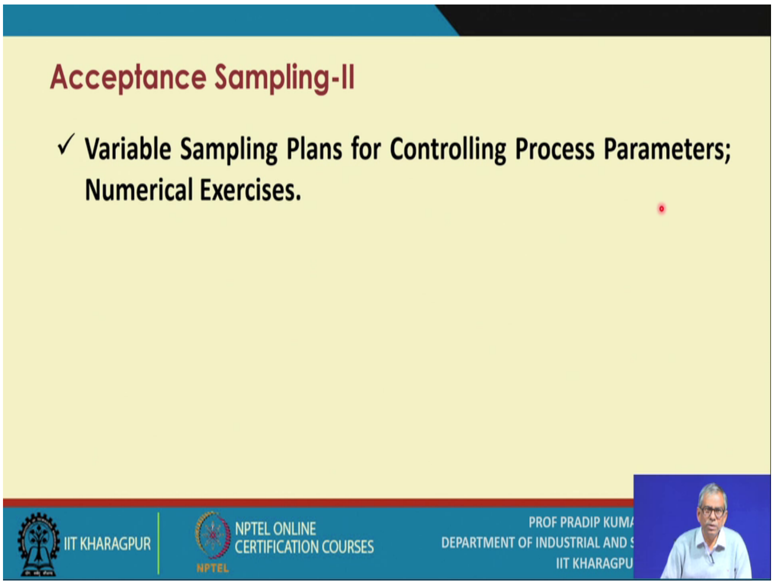

During this session on acceptance sampling, I will be discussing in detail the variable sampling plans for controlling process parameters. This point, already I have explained in the previous lecture sessions.

So, this is essentially, we are referring to type one variable sampling plans. So, we will explain the basic concepts. We will explain that why it is needed, what are the parameters of such the variable sampling plans and then how to design such a variable sampling plan.

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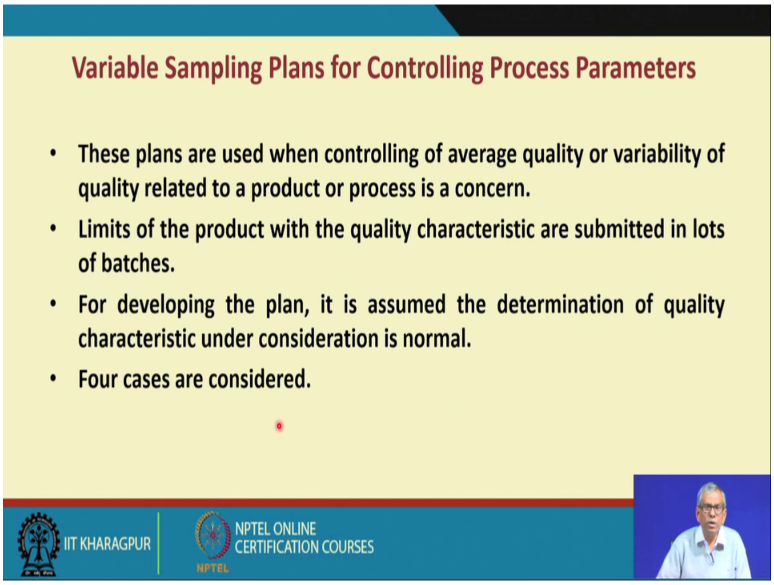

So, what are the conditions to be known and what are the criteria to be known and if you use such a variable sampling plan, what are the advantages you may have. So, you will get you know, it is a clear-cut idea that under what situation or a variable sampling plan of this category as a type one category is recommended. Now these plans are used when controlling of average quality or the variability of quality related to a product or the process is a concerned.

Average quality; that means, the controlling say by the process parameter that is the average given by the average or the process parameter may be the standard deviation and that is why you have mentioned in general a term called variability or as a dispersion now the limits of the product with the quality characteristics are submitted in lots of batches is it ok?

That means here; that means, what you have mentioned actually that is the lots of product with; that means, here the product with the quality characteristics are submitted in lots of batches; that means, these quality characteristics which we refer to; that means, the limits of these quality characteristics must be known for developing the plan. It is assumed that the determination of quality characteristics under consideration is normal.

That means it is assumed that the distribution of the quality characteristics under consideration is normal is it. So, that is we assume and here and the 4 cases are considered like I have already mentioned that what are the 4 pole cases under type one variable sampling plan.

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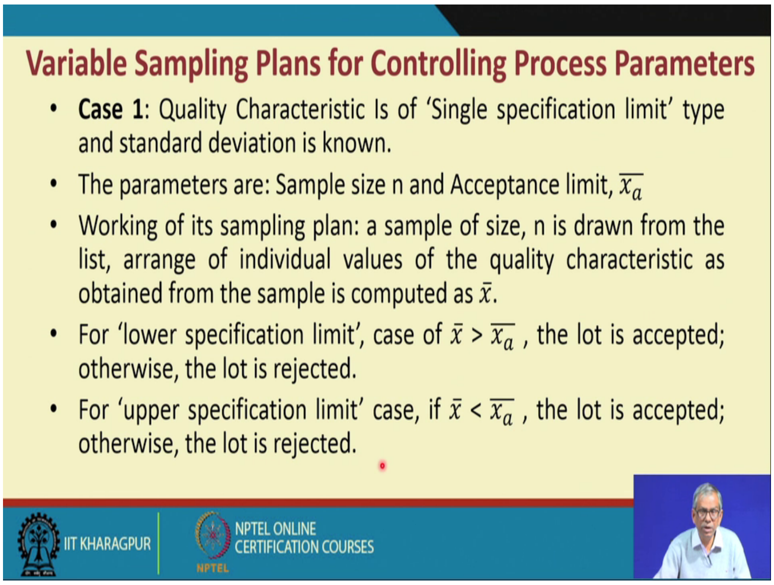

So, the case one is a variable sampling plans for controlling process parameters the case one is quality characteristic is of single specification limit type and the standard deviation is known. So, that is case one. Now what are the parameters the parameters are the sample size n and acceptance limit the notation is x a bar. So, this is capital X a bar.

So, explain the working of such a sampling plan is it. So, what do you do when the sample size is known; that means, a sample of size n is drawn from the lot arrange the individual characters as obtained from the sample arrange; that means, you measure the individual values of the quality characteristic as obtained from the sample is it; that means, you calculate the average you collect the individual values of the quality characteristics as obtained from the sample and the then the average is computed as x bar, is it clear?

For lower specification limit case, if x bar is greater than x a bar that this is basically called the acceptance limit is it. So, what will be the value of the acceptance limit you are going to determine when you design the sampling plan. So, if you come across a lower specification limit case and if you find that the sample average that is x bar capital X bar is greater than capital X a bar the lot is accepted otherwise the lot is rejected, is it ok?

So, now this is basically if x bar is greater than x a bar the lot is accepted, but the upper specification limit case if the sample average given by x bar is less than x a bar, the lot is accepted, otherwise the lot is rejected, is it ok?

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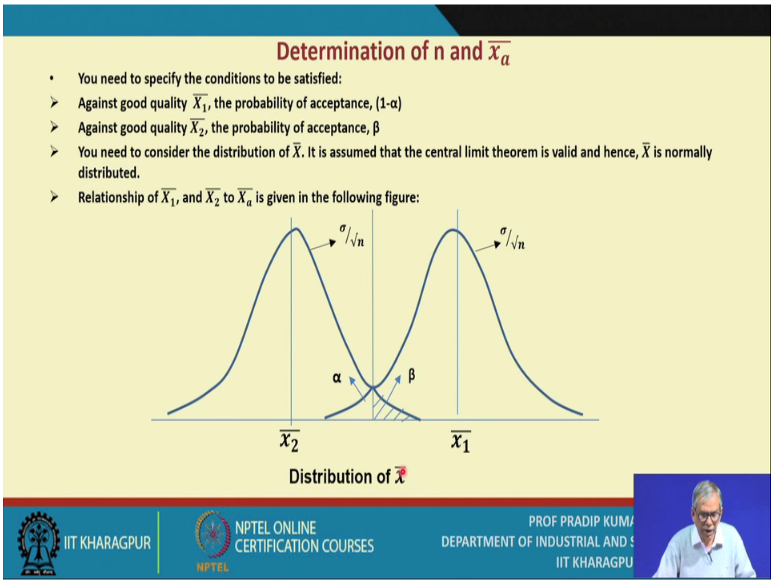

Now, the question is; that means, here it is a the single specification limit case and your objective is to control the process average as given by x bar and what you assume that the distribution of x bar is normal, is it ok?

So, here what do you find that first you have to specify the conditions and these conditions are to be satisfied and in majority of the cases you specify 4 conditions what are those 4 conditions or maybe just the 2 conditions. So, what are these conditions first you specify; that means, you design the sampling plan in such a way that these the conditions are satisfied you need to specify the conditions to be satisfied this I have elaborated against good quality x 1 bar the probability of acceptance is 1 minus alpha and similarly against bad quality x 2 bar the probability of acceptance is beta, right, you need to consider the distribution of x bar is it as already pointed out.

It is assumed that the central limit theorem is valid I presume that you know what is the central limit theorem the central limit theorem is explained a most simply as that whatever may be the distribution of x the distribution of any statistic of x say the average say x bar or the standard deviation of x is it like say s is normal or is approximately normal when n is large.

So, this is the statement of the central limit theorem and in many majority of the cases when x the value of x is dependent on several the factors it is basically is affected by it is a response variable essentially and these response value of this response variable is dependent on several other factors or these factors are occurring in the you know the upstream stage. So, if this is the condition. So, and you produce this units with large number; that means, that the physical basis of say normality assumption is there and that is why the central limit theorem is valid and hence x bar is assumed to be normally distributed, is it ok?

Relationship of x 1 bar that x 1 bar is considered to be a good quality x 2 bar is considered to be a bad quality. So, how do you represent the relationship of x 1 and x 2 x a bar? So, what is x a bar that is the acceptance limit.

Now, this relationship you can represent to with this with the help of a particular figure. So, what you have over here; that means, when you refer to the distribution of x bar you say that this distribution is a shifting is it. So, when you achieve a good quality given by x 1 bar. So, this is the distribution and this particular line is referred to as x a bar. So, this is x a bar.

So, x a bar is basically the acceptance limit right and what do you find that there is a probability of alpha like say that even if the quality is say the quality or very good quality you get, but there is a possibility of its a rejection, so that is basically the producers risk and when. So, suppose the value changes to x 2 bar and x 2 bar is considered to be a bad quality what do you find that you consider you compare x 2 bar with x a bar that is the acceptance limit you find that this value this is the area under the normal curve that is existing this area beyond x a bar. So, this line this vertical line is x a bar the acceptance limit. So, you will find that this is the probability of acceptance.

That means even if there is a there is a very bad quality you achieve on an average, you will find there is a possibility of its acceptance and that is beta is it. So, this way you explained the relationship of x 1 bar and x 2 bar with x a bar is it and here; obviously, you are dealing with the distribution of x bar. So, as for the central limit theorem; obviously, you know the variance of x bar is sigma upon root n. So, what is sigma? Sigma is basically the standard deviation of the process, is it ok?

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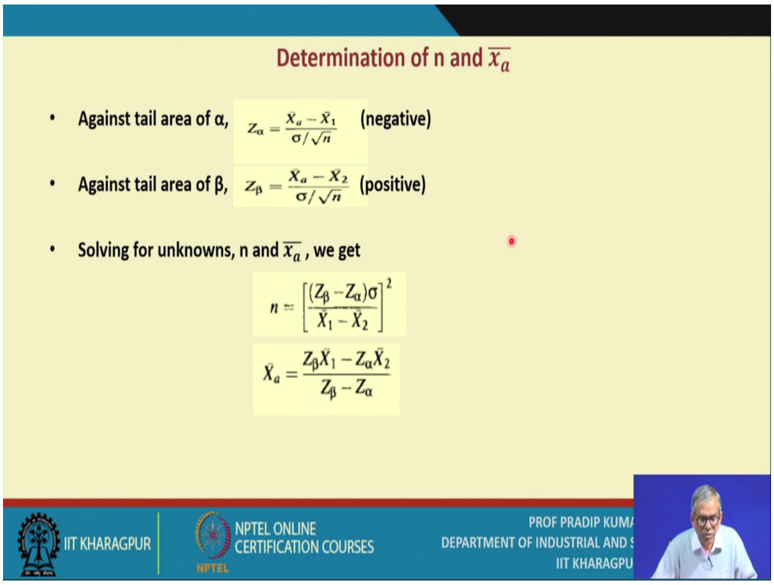

So, how do you determine n and x bar against the tail area of alpha. So, you refer to the previous figure what you find that you compute the z alpha; that means, the standard normal variate against a tail area of alpha. So, that is x a bar minus x 1 bar divided by sigma upon root n; that means, the difference from say good quality and explains these differences in the standard deviation units. So, what is the standard deviation over here that is sigma upon root n what is sigma? Sigma is the process standard deviation.

So, you will find the way it is computed z alpha z alpha will be negative is it because this is negative against tail area of beta again you refer to the previous figure you will find that the standard normal variant that is the z beta is equals to x a bar minus x 2 bar divided by sigma upon root n so; obviously, x a bar is greater than x 2 bar in the figure that is why this value is positive. So, z alpha is negative z beta is positive the way we have computed.

Solving for unknowns now there are these 2 equations you have. So, how many unknowns you have the unknown is n that is the sample size and unknown is x a bar what is x a bar that is acceptance limit for a single specification limit case and it is a single sampling plan is it we are not using double sampling plan when you when we refer to or when we use say that say the variable sampling plan, but in certain cases definitely, you can also go for a double sampling plan, is it ok?

But right now, we are now discussing double sampling plan in the variables category. So, solving for unknowns you get an expression of n, is it ok? We can solve these problems I solve for n and solve for x bar. So, what is the expression for n that is z beta minus z alpha into sigma divided by x 1 bar minus x 2 bar this is good quality this is the bad quality and this is a square to the power 2 and x a bar is equals to j beta x 1 bar minus z alpha into x 2 bar divided by z beta minus z alpha, is it ok?

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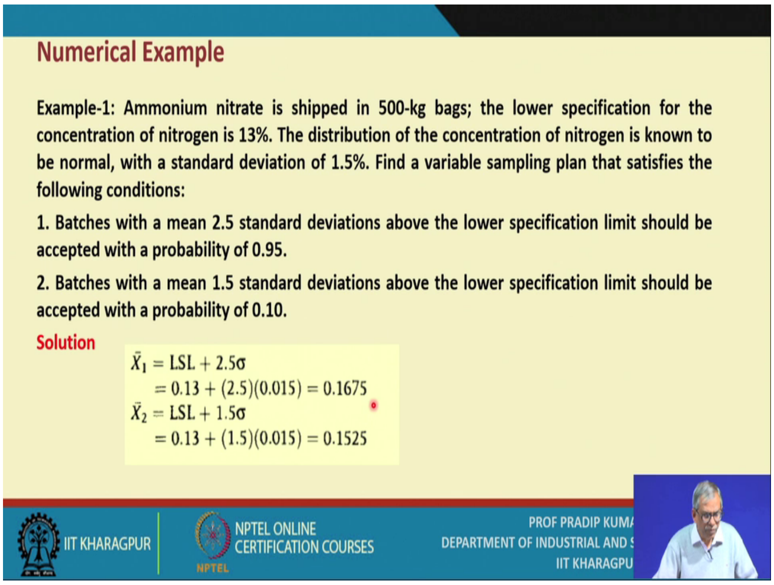

Now, we will take an example a numerical example. So, that you know the concept is made very very clear ammonium nitrate is shipped in 500 kg bags the lower specification for the concentration of nitrogen is 13 percent lower limit; that means, the value must be greater than 13 percent or more.

The distribution of the concentration of nitrogen is known to be normal that is the assumption normality assumption holds with a standard deviation of 1.5 percent, find the variable sampling plan that satisfies the following conditions, is it ok? So, what are these conditions like the batches with a mean of 2.5 standard deviations above the lower specification limit should be accepted with a probability of 0.95. So, what are the producers? Risk producer risk is 0.05 even if the quality is extremely good.

And the second condition is batches with a mean of 1.5 standard deviations above the lower specification limit should be accepted with a probability of 0.10; that means, what is this probability this probability is basically the consumer risks; that means, even if you know the quality deteriorates is it you will find that there is a possibility of acceptance. So, that is the consumer risk.

So, first what do you try to calculate you calculate the good quality you estimate that what is the value of X 1 bar. So, what is X 1 bar that is LSL plus 2.5 standard deviations above the lower specification limit? So, what is the lower specification limit lower specification limit is 0.13; that is 13 percent 0.13 plus 2.5 times into sigma. So, what is sigma that is 1.5 percent; that means, 0.015. So, 0.015; so, the value of X 1 bar is 0.1675 and similarly the value of X 2 bar is LSL plus 1.5 sigma. So, LSL is point one 3 sigma is 0.015. So, you get a value of X 2 as 0.1525, fine.

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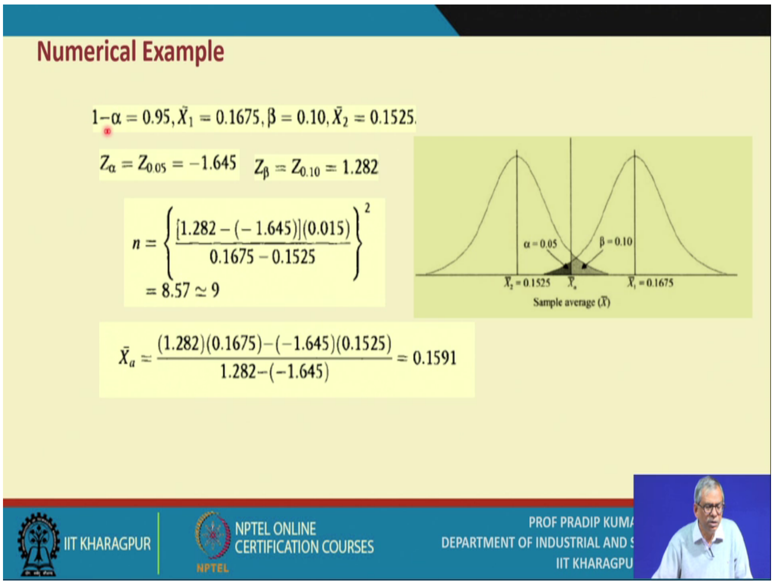

Next what do you do? What is it is given that 1 minus alpha is 0.95 X 1 bar is 0.165, beta is 0.10, I have already explained X 2 bar is equal to 0.1525. So, now, you refer to the standard normal table and in any text book on quality you will find as an appendix these the standard normal tables are provided. So, please refer to those the standard normal table and you also must learn how to read the values from the standard normal table, is it ok.

So, Z alpha is equals to Z 0.02; 0.05. So, this is producer’s risk. So, if you refer to the standard normal table you get a value of minus 1.645; is it. So, I have already mentioned that the way we have computed Z alpha. So, it is negative that is why it is minus 1.645 how do you compute Z beta again you refer to the standard normal table.

So, here the value of beta is 0.10 that is Z subscript 0.10, this is the notation you use and if you refer to the standard normal table you will find the against the tail area of beta this value is 1.282 is it. So, the value of Z.

Now, you apply that for n and you substitute these values in this particular formula. So, ultimately the value of n, you get as 8.57 so; obviously, it is a sample size. So, you go for a round it up and this is the integer value next higher integer that is 9. So, sample size is just 9 less than equals to 10 as I have already pointed out in majority of the cases the sample size for the variable sampling plan will be less than 10.

So, what is X a bar? Again you use that formula you substitute all these given values and ultimately you get a value of 0.1591. So, you just refer to this particular figure X 1 bar is known, it is these value is given X 2 bar is also you have computed that is 0.1525, X a bar that is the value is 0.1591. So, this value is 0.1591 and this is alpha that is 0.05. and this is beta 0.10.

So, we are referring to the distribution of X bar is it. So, exact values of X 1 bar X 2 bar is known.

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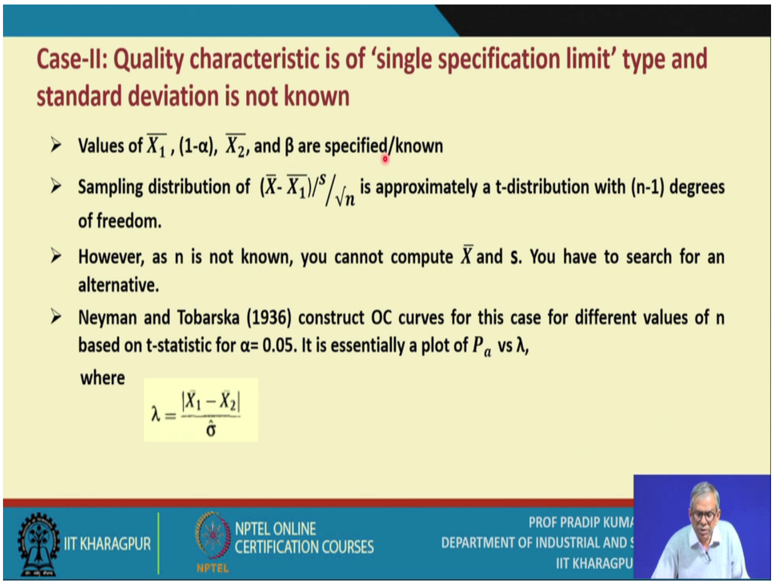

Now, we are referring to the case 2; that means the quality characteristic is of single specification limit type and the standard deviation is not known is it. So, this case also you frequently come across right particularly when you come across a situation which is a new one; obviously, you may not have a sufficient pass data. So, values of X 1 bar 1 minus alpha X 2 bar and beta are specified or known. So, this is your starting point you cannot deviate from this condition. Now what is known that the sampling distribution of this particular statistic that is X bar minus X 1 bar.

So, what is X 1 bar? X 1 bar is basically its referring to good quality. So, the averages of good quality that is X 1 bar divided by s by root n what is s? S is the sample standard deviation and the sample of specified size. Now sampling distribution of this particular statistic is approximately 80 distribution with n minus 1 degrees of freedom is it. So, this is the statement you have and this is your assumption.

And now whenever you know the standard deviation is not known; that means, the normality assumption is not valid. So, if the normality assumption is not valid. So, you have to search for you know other kind of distribution is it in non normal condition. So, that is essentially the t distributions we refer to.

But here the problem is at this point in time when you do not have you do not know what is the value of n the sample size is not known because you have not yet designed the sampling plan. So, how can you calculate the sample standard deviation? So, as s it not known you cannot compute X bar and s you have to search for an alternative Neyman and Tobarska through their empirical study through their empirical study construct o c curves for these case for different values of n based on t statistic for alpha equals to 0.05; that means, many years back in 1936, they conducted several sorts of experiments and where after the you know analyzing those this experimental the data, now they could construct the o c curves when alpha is equals to 0.05.

That means we are referring to this particular empirical study and its finding and it essentially a plot of P a versus lambda is it. So, how do you know define lambda. So, the lambda is defined as X 1 bar minus X 2 bar absolute; that means, the difference between X 1 bar and X 2 bar and this is in the standard deviation unit.

So, what is this sigma hat the sigma hat is an estimate of sigma which is going to change; obviously, there could be a starting value of sigma hat and as you collect more number of data points, you revise the value of sigma hat is it. So, this way we define lambda is it so.

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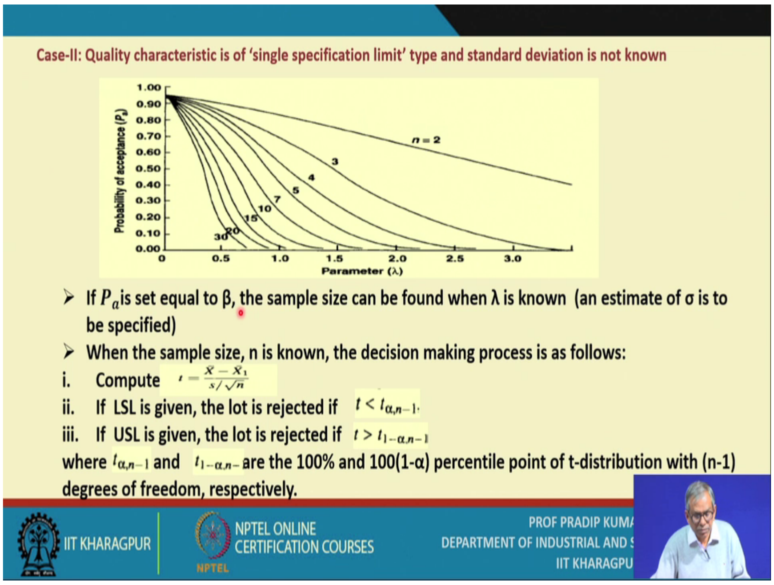

They have constructed these o c curve and what they found that the shape of the o c curve depends on the sample size like say this is the probability of acceptance. So, that is P a versus the parameter that is lambda now when n equals to 2 the curve is really flat one where as n becomes thirty is it; that means, sample size thirty they have done lot of experiments on this you will find that the curve becomes very very stiff. So, this is also an o c curve and these o c curves they have constructed several such o c curves like this one they have constructed at a value of alpha equals to 0.05.

So, if P a; that means, the probability of acceptance is said equal to beta the sample size can be found when lambda is known; that means, first what do you do when x 1 bar and x 2 bar these 2 values are known sigma hat you specified and then you get the value of lambda. So, suppose lambda is here and then you said the probability of acceptance at beta value. So, here the beta value is 0.10; that means this is one. So, then what do you do.

You just to have you know; this is the horizontal line you draw and this is the vertical line you draw from the estimated lambda is it and this is the point of intersection, is it ok? It falls on a particular curve and against that particular curve what is the value of n is it. So, this is the procedure you follow.

So, when the sample sizes is unknown; that means, with this approach you will come to know what is the sample size. So, when the sample size n is known the decision making process is as follows compute this sample statistic now there is no problem because x bar will be known because n is known s is also known because n is known because the sample standard deviation; that means, n values you have already collected. So, with these n values you can calculate the sample standard deviation x 1 bar is already specified and n is known.

So, this actual value of t you can calculate if LSL is given the lot is rejected if this condition holds and if you USL is given, the lot is rejected, if t is greater than t 1 minus alpha n minus 1. So, where t alpha n minus 1 and t 1 minus alpha n minus 1 are the 100 percent and 100 alpha percentage and 101 minus alpha percentile points of t distributions with n minus one degrees of freedom respectively, is it ok?

So, 100 into alpha and 100 into 1 minus alpha for LSL and USL case respectively.

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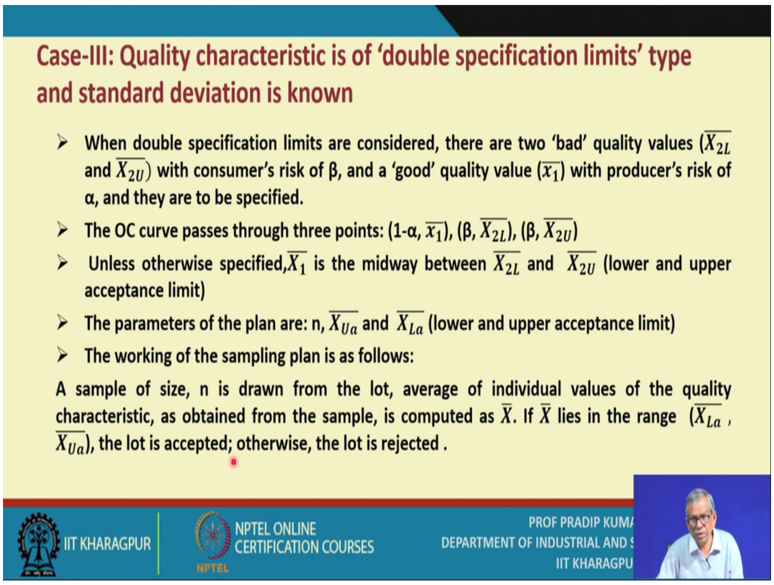

Is it ok? So, what is case 3? Case 3 is the quality characteristic is of double specification limits and the standard deviation is known. So, what you have here; that means, when it is a double specification limit case; that means, against USL against LSL there could be you know the 2 bad quality values. So, they are specified as x 2 l bar and x 2 u bar with a consumer risk of beta and a good quality x 1 bar with producer risk of alpha and they are to be specified.

So, the o c curves passes through 3 points one minus alpha x 1 bar beta x 2 l bar and beta x 2 u bar. So, this is the conditions you impose unless otherwise specified x 1 bar is the midway between x 2 l bar and x 2 u bar. So, lower and upper acceptance limit the parameters of the plan are n equals n that is the sample size upper acceptance limit and the lower acceptance limit is it because you are dealing with the 2 sided specification limits or the double specification limits.

So, what is the working a sample of size n is drawn from the lot average of individual values of the quality characteristics x bar is obtained from the sample and this is computed at x bar if x bar lies in the range x l a bar and x u a bar is it; that means, the range specified as the upper lower acceptance limit and upper acceptance limit the lot is accepted otherwise the lot is rejected.

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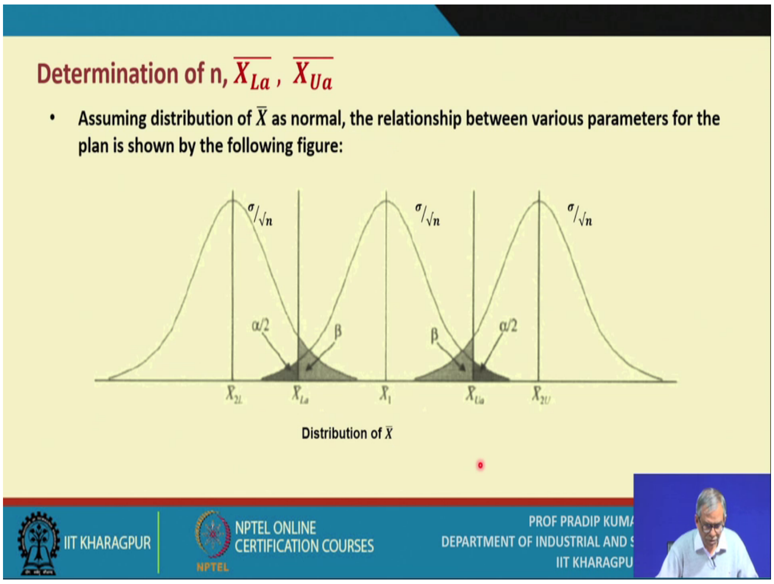

So, this is the working and this is basically a, but we are explaining the relationship between say X 2 L bar X U a bar and X 1 bar as well as X L a bar and X U y bar. So, this is just an extension of the single specification limit case double specification limit case.

So, instead of having just one bad quality you have one bad quality over here another bad quality over here is it. So, assuming distribution of X bar as normal the relationship between various parameters for the plan is shown by the following figure. So, so you are considering distribution of X bar; that means, and X 1 bar is the midway between say this one and this one.

So, when your good quality. So, you have the consumer risk as alpha; that means, one side is alpha by 2 and the other side is again alpha by 2 and when you, either you have quality here bad quality over here bad quality over there in both the cases, you will find that are the probability of acceptance is beta is it consumer risk.

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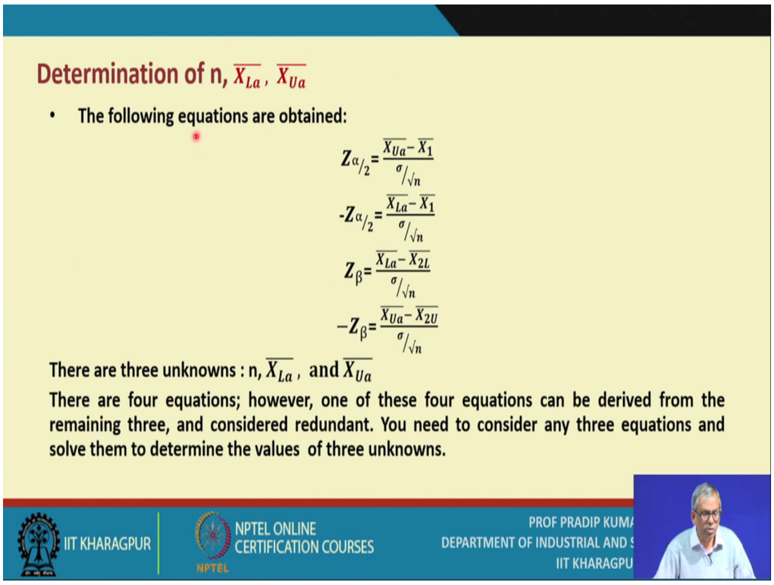

So, the following equations are obtained; that means, this is you know against the tail area of alpha by 2 on 1 side, on the other side, similarly, instead tail area of beta on one side and on other side. So, you have all these expressions. So, the standard normal deviates against the tail areas of say the alpha by 2 and beta you computed. So, you have 4 equations and 3 unknowns.

So, there are 4 equations; however, one of these 4 equations can be derived from the remaining 3 and considered redundant is it so; that means, there is the dependency; that means, if I write down the 3 first 3 equations you can derive the fourth one. So, what do you try to do; that means, all these equations are all these 4 are not independent. So, what do you do make sure that any one of this is removed and with the remaining 3 equations simultaneous equations you can definitely solve for 3 unknowns that is n X L a bar and X U bar.

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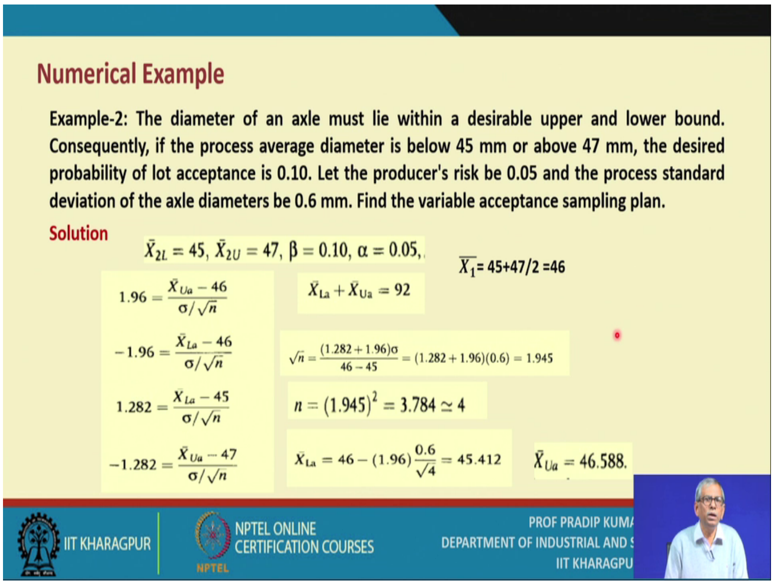

So, this is a numerical example. So, so these numerical examples you please go through and whatever the steps we have explained in deriving the expressions for the parameters for the double specification limit case when the objective is to the control the process average all these steps, we have followed with respect to these numerical examples, is it ok?

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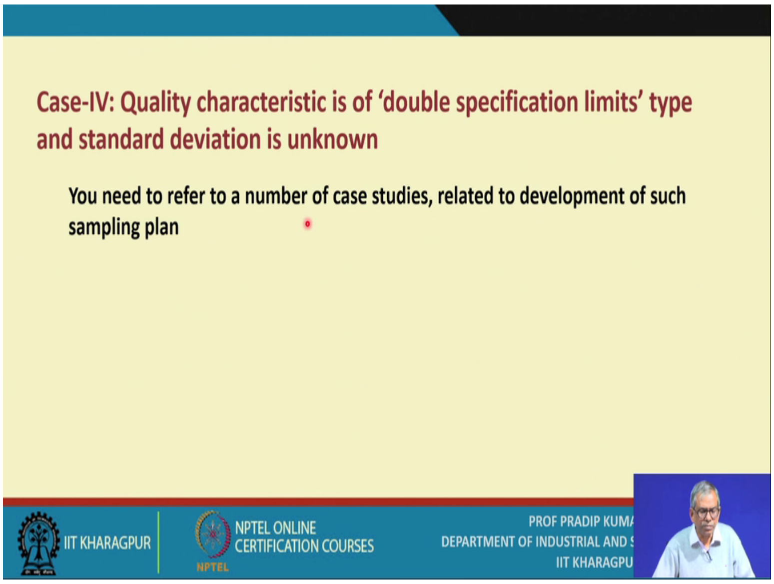

So, ultimately you have 3; say the parameters in this case and follow the step meticulously and I am sure you will be your understanding will be appropriate. So, the case 4, I just refer to the case four. So, the case 4 is quality characteristics is a double specification limits type and the standard deviation is unknown. So, you need to refer to a number of case studies related to development of such sampling plans.

So, there are many textbooks. So, that the textbooks which you are referring to. So, please go through those textbooks, you will find that the several such case studies are derived and the standard deviation is unknown; that means, you will be referring to those empirical studies and there are certain the formulations or the formulas you have that always you can use.

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Is it ok? So, these are the references we have.