Quality Design and Control
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Lecture - 33
Acceptance Sampling-I (Contd.)

So during this lecture session, we will continue discussing Acceptance Sampling.

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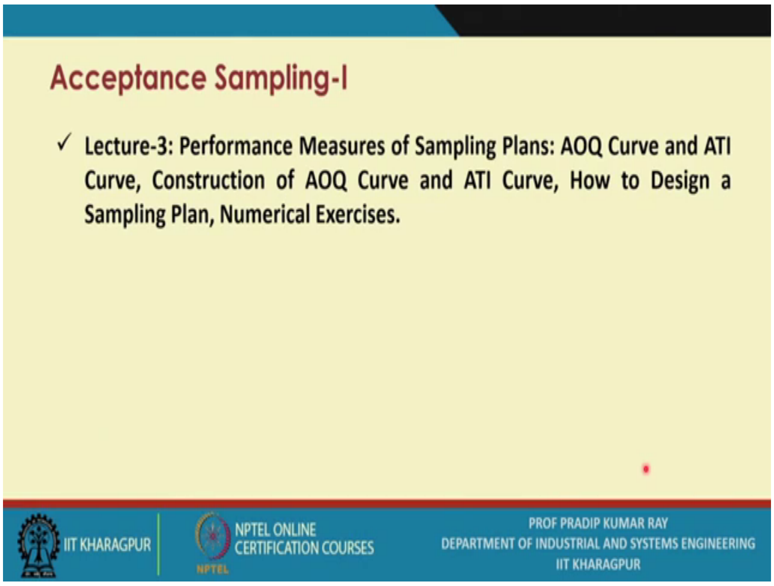

Specifically during this lecture session, I will discuss performance measures of the sampling plans and whenever, we talk about performance measure of sampling plans; there are different types of curves, we need to draw and two important curves in this particular, you know the case that is AOQ curve and ATI curve. We will be discussing in detail and how to construct AOQ curve as well as the ATI curve, we will discuss. And how to design a sampling plan with numerical exercises; we will be discussing during this lecture session.

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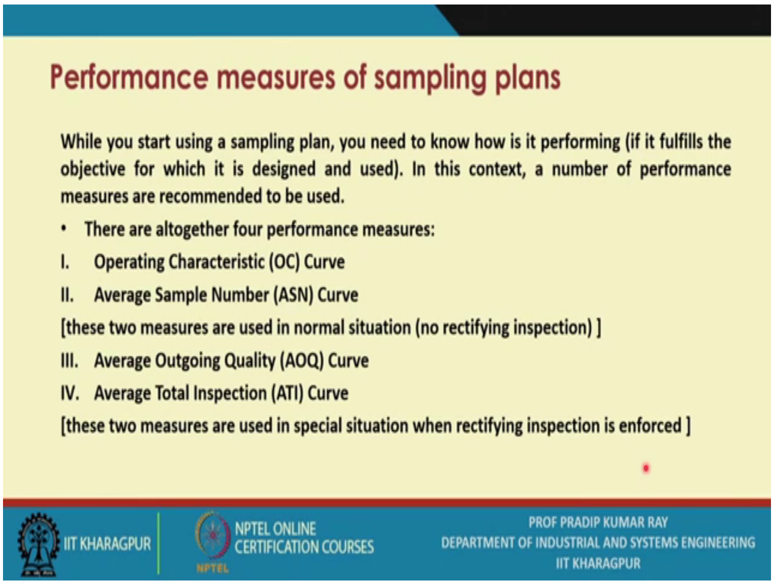

While you start using a sampling plan, you need to know how it is performing; that means, whether it fulfills the objective for which it is designed and used. We have already discussed the objectives of using a sampling plan and while, you start using a sampling plan, you must check; you must verify whether these objectives are being fulfilled or not and how is it performing? Whether it is serving the purpose or not?

Now in this context, the numbers of performance measures are recommended to be used. There are altogether four performance measures. For the first one is the Operating Characteristics Curves; that is the first performance measures we always referred to. The second one is Average Sample Number Curve. Now these two curves or these two measures are used in normal situation.

What is a normal situation? The normal situation means there is no rectifying inspection. We have already explained that what is this rectifying inspection and why rectifying inspection, you may use. And two more performance measures also we referred to. The third one is the Average Outgoing Quality Curve and the fourth one is the Average Total Inspection Curve.

Now these, the two measures are used in special situation when rectifying inspection is enforced. So, you say that or you conclude that whenever or the rectifying inspection is in vogue or is enforced, the situation is a special kind.

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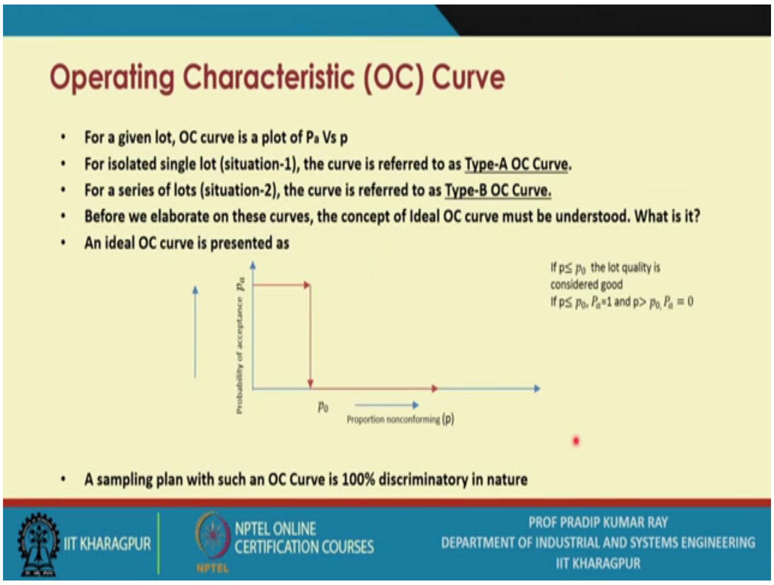

So, there are four measures; now, what is an Operating Characteristic Curve, OC Curve? For a given lot, OC curve is a plot of probability of acceptance versus you know the p. What is P? P is basically the incoming lot quality and when you deal with attributes data these incoming lot quality is measured with proportion and conforming; that is why the notation is p. So, essentially is a plot of Pa versus p.

For isolated single lot that we are saying this is situation 1, the curve is referred to as Type-A OC Curve. Is it ok? In all texts on a quality, you will find that this is referred to as the Type-A OC Curve and when you deal with a series of lots or the stream of lots, this is situation 2; the curve is referred to as Type-B OC Curve. Is it ok?

So, in a, for constructing Type-A OC Curve, your the distributional assumption is a Hyper Geometric. This point already we have discussed for isolated the single lot case. And for a series of lots or the stream of lots case, you know while you construct the OC Curve, we assume that the distributions of the random variable is assumed to be Binomial.

Before we elaborate on these curves, the concept of Ideal OC Curve must be understood. What is it? An Ideal OC Curve is presented as this one; that means, it is a plot of probability of acceptance versus proportion and conforming; that means, proportion nonconforming is essentially is the incoming a lot quality.

Now here, what you say that we specify the value of proportion nonconforming that is p 0 and any value of p less than or equals to p 0, the lot is, lot quality is considered very good. So, that is your decision. If p is less than equals to p 0; that means, the value of p proportion nonconforming is less than or equals to p 0; that means, these are the values possible values. What we assume that the probability of acceptance is 1. Is it ok? So, this value is 1, probability of acceptance is 1.

But, if so suppose, the proportion nonconforming is greater than p 0; then, the probability of acceptance is 0. Is it ok? So, this is the Ideal OC Curve; that means this is the curve the horizontal line and there is a break and then again, you move around this the line. Is it ok; when the probability of acceptance becomes 0.

So, a sampling plan with such an OC Curve is 100 percent discriminatory in nature. So, you also must know that what is the Ideal OC Curve.

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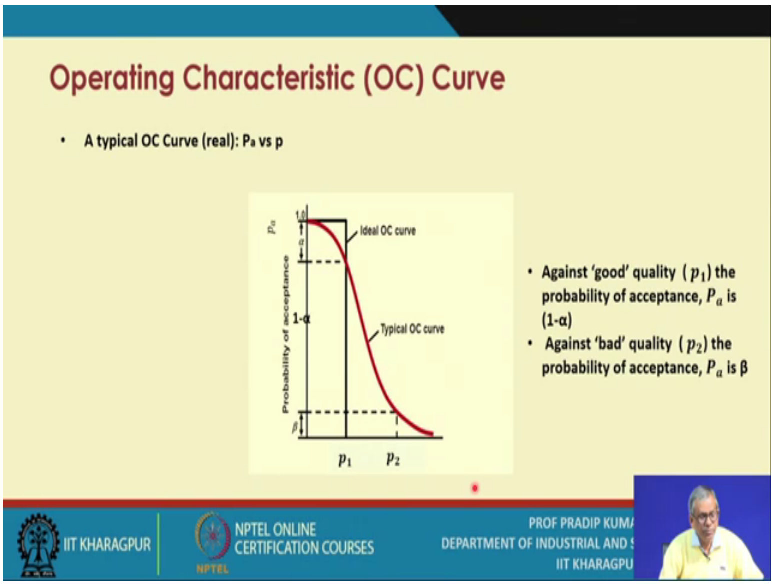

But when you draw, a typical OC Curve; obviously, you cannot get on you cannot get the Ideal OC curve. So, in the Ideal OC curve, what do you; what we assume that, it is 100 percent discriminatory. Whereas, when you draw the actual OC Curve or when you get the real OC Curve, what do you find? That the curve nature is like this; that means, against good quality that is p 1, against good quality. The probability of acceptance Pa is very high and that is 1 minus alpha. So, what is Alpha? Alpha is the producer's risk.

Against bad quality that is p 2; any value of p greater than or equals to p 2 is considered to be very very bad. The probability of acceptance Pa is just Beta; that means, this is the consumer's risk; that means, we have already mentioned.

Now, when you draw the OC Curve, this is it looks like an inverted s shape and; obviously, on these OC Curves, these are the 2 points; that means, this is 1 point that is related to producers risk and this is another point on the OC Curve, that is related to the consumer risk. So, that you need to identify; these 2 values on the OC Curve.

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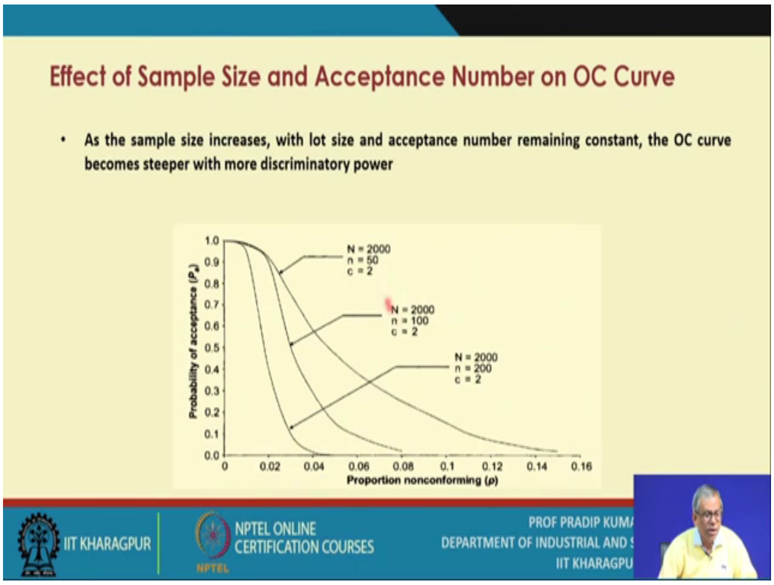

So now, what is the Effect of Sample Size and Acceptance Number on OC Curve? Like for a single sampling plan or a single sampling plan, there are 2 parameters given capital N; that is the lot size. There will be you know the sample size and there will be the acceptance number.

That is N, small n and small c; as the sample size increases with lot size and acceptance number remaining constants like say 3 cases, we have considered over here, like in this case what happens? So, you know for all these 3 cases, the lot size as well as the acceptance number remains same that is 2000 and 2; 2000 and 2 and even for the 3rd case, it is 2000 and 2.

Now, what is changing? That means, you are changing the sample size; that means, as the sample size increases, initially it was 50; the curve nature is like this. When it becomes 100; that is becoming steeper. And when it becomes 200; that means it becomes further steeper. Is it ok? That means, if the curve becomes steeper, it becomes more discriminatory. So, so this is the effect of say the Acceptance sample size on the OC Curve. Is it ok?

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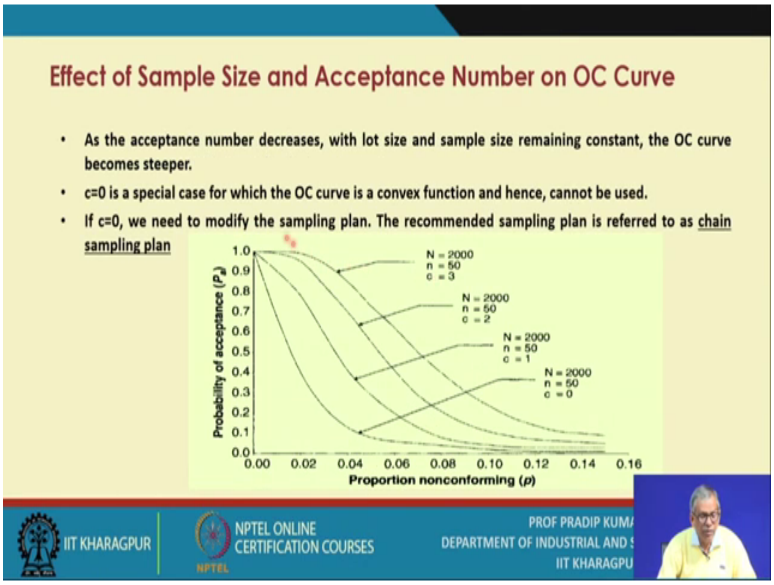

Similarly, what you can do? That means, as the acceptance number decreases, with lot size and sample size remaining constant. So here, what you find? We have considered the 4 cases and for all these 4 cases, what do you find? That the sample size as well as the lot size, they remain same. Whereas, the sample is acceptance number changes from 3 to 2 to 1 to 0. So, as you, as the acceptance number decreases with lot size and sample size remaining constant, the OC Curve becomes steeper.

Now, here c equals to 0 is a special case for which the OC curve is a convex function; this is the convex function. And hence, cannot be used. Is it ok? So, this is a very special case and usually you will find that the single sampling plan, all sorts of single sampling plans which you use, the acceptance number is always greater than 0. Is it ok?

Whereas, in certain cases acceptance number has to be 0 and for which a special purpose sampling plant called chain sampling plan you need to use. So, in, we will refer to this chain sampling plan, when we discuss the special purpose a sampling plans. If c equals to 0, we need to modify the sampling plan. The recommended sampling plan is referred to as the chain sampling plan.

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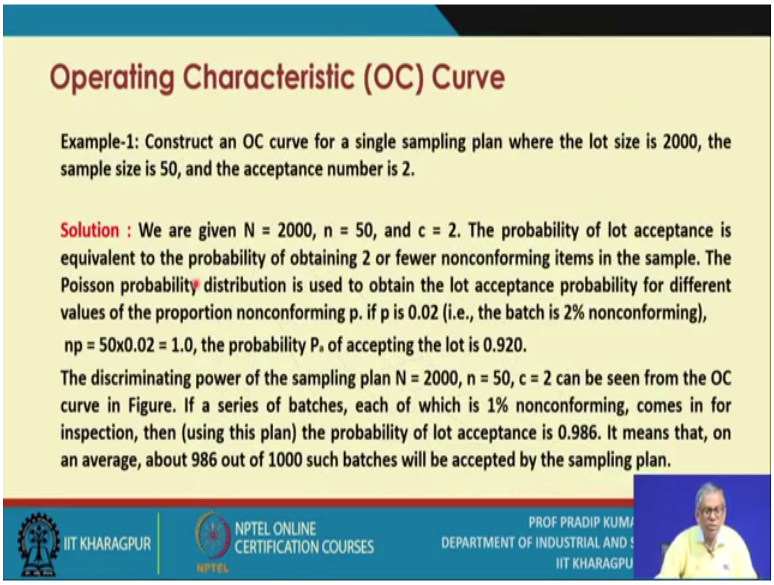

So, this is the way we analyze the OC Curve. And now, then this is an example, how to relate it to construction of an OC curve? Construct an OC curve for a single sampling plan where the lot size is 2000, the sample size is 50 and the acceptance number is 2.

So, just you follow these steps. So, what are given basically that is this N value of capital N, value of small n and value of c. The probability of lot acceptance is equivalent the probability of obtaining 2 or fewer nonconforming items in the sample. So, this is the condition you impose.

The Poisson probability distribution is used to obtain the lot acceptance probability for different values of the proportion nonconforming p; that means, even if you consider say isolated lot, a single lot or a series of lots either Hyper geometric distribution or the Binomial distribution is approximated with the Poisson distribution. And that's why Poisson approximation to Hyper geometric or the Binomial distribution is used.

So, if value of p is 0.02; that means, the batch is 2 percent nonconforming. So, np is 50 that is the sample size into 0.02 that is 1. The probability of p of accepting a lot is 0.920; that means what you try to do? You refer to cumulative Poisson distribution table against a value of the lambda is basically n p.

So, you say that what is the value of x? You refer to Poisson cumulative Poisson distribution table and the corresponding probability from the table, from the table you get as 0.920. So, these process is, these repeated several values of the p possible values you consider and against each value of p assuming Poisson approximation to binomial or hyper geometric distribution. So, you calculate or you get the values of the probability of acceptance.

So, when you get ah the several values of p.

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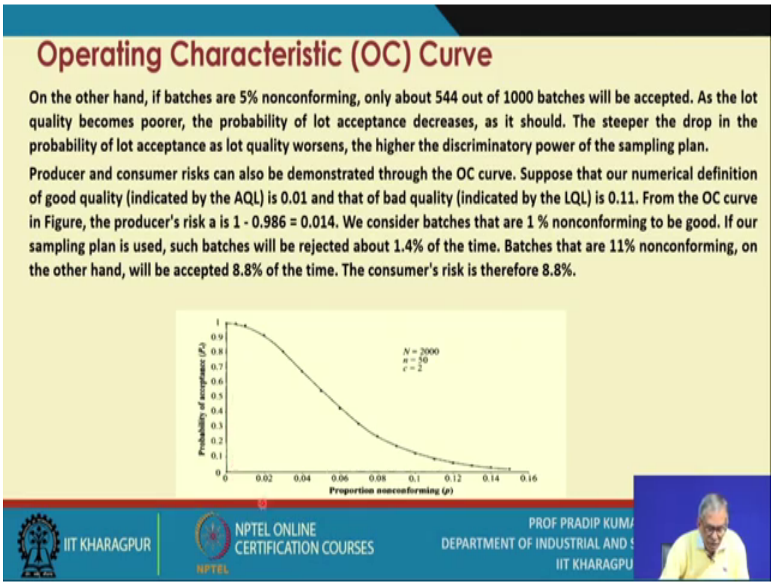

When you get several values of p, for different values of, for several values of say when you get several values of p, the corresponding probability of acceptance you calculate and then, you plot these values; values of the probability of acceptance and you get this OC Curve. Is it ok?

So, please go through all these, the steps and I am sure that you will be able to construct the OC curve by referring to cumulative Poisson distribution table. Only thing is the make sure that the pos that the values which you consider, that is the values of proportion nonconforming that you consider as incoming lot quality; these the values are acceptable to you. Is it ok?

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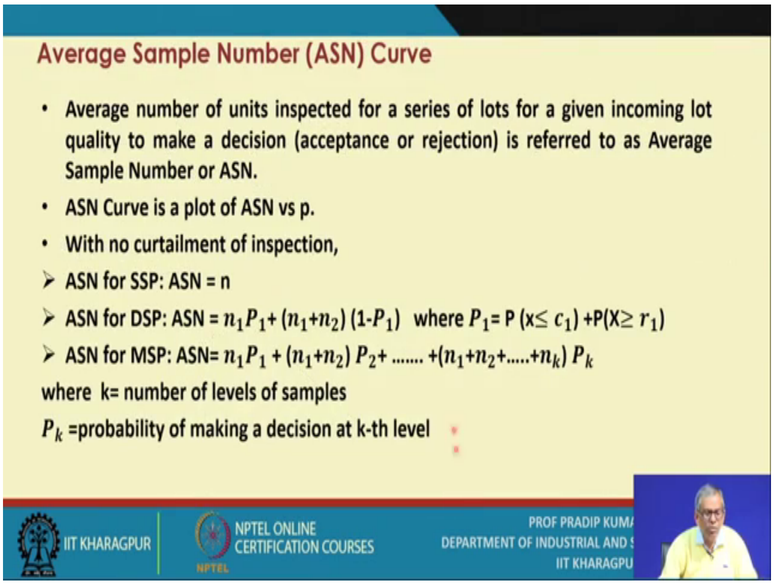

So, you have come to know that how to construct the OC curve. The next one is Average Sample Number Curve. So, what is Average Sample Number that you must know? Every sample that number actually when you define, we define average sample number like this. This is the average number of units inspected for a series of lots. What actually you are inspecting? You are inspecting the units in the sample against a particular lot.

So, the several lots you are considering, you are getting series of lots and for each lot you are the drawing 1 sample. Now when you consider the sufficient number of lots, now you check that on an average, how many units you are you are inspecting for a given incoming lot quality to make a decision? So, the decision could be either lot acceptance or lot rejection.

Now, this average number of units is referred to as Average Sample Number or ASN. Now ASN curve is a plot of ASN versus p; is it ok? That means, what you need to do against a given value of small p incoming lot quality, what how do you, you must get an expression for ASN. So, With no curtailment of inspection, this is a special case; we are saying it is a normal situation; that means, no curtailment of inspection.

ASN for single sampling plan, it simply n; is it ok? Single sampling plan is n; ASN for the double sampling plan that means, you say that the number of units we are going to inspect when there is a probability of acceptance of rejection at the first stage.

If you cannot make a decision at the first stage; then, you go to the second stage and at the second stage you are, you know the inspecting n 2 more units that's why it is n 1 plus n 2 at the second stage and what is the probability that there will be a decision taken at the second stage; obviously, it is 1 minus P 1.

So, what is P 1? That means, of probability of making a decision either acceptance or rejection; that means, a probability that x could be less than equals to c 1, the is you know the acceptance number for the first sample are or it is a probability that x is greater than equals to r 1; r 1 is the rejection number for the first sample.

So, now if it is a multiple sampling plan, what is the expression for ASN? That is n 1 into P 1 plus n 1 plus n 2 into P 2; that means, the probability of making a decision at the second stage and similarly, how many stages you consider in general; there could be k number of stages. So, what is P subscript k? That is the probability of making a decisions at the kth stage. So, the k is the number of levels of samples and capital P subscript k is probability of making a decision at the kth level.

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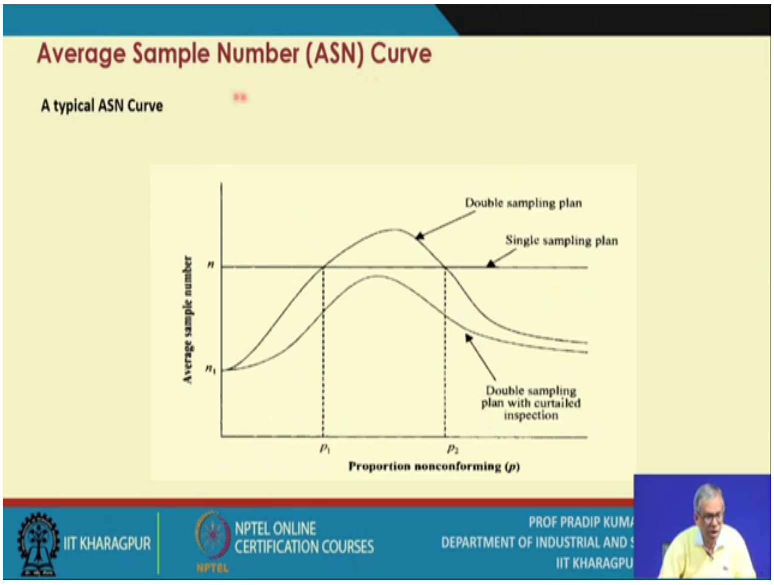

So, this is just a plot; that means, ASN values are plotted against the proportion nonconforming. So, for a single sampling plan, this is an horizontal line and this value is n and for a double sampling plan the nature of curve is like this and; obviously, the first sample size for an equivalent to double sampling plan; that means, you have a single sampling plan and if you opt for an equivalent double sampling plan, what do you find that the first sample size is less than the sample size for the single sampling plan.

So, n 1 and this is the typical nature and you have the 2 values of proportion nonconforming that is P 1 and P 2. So, these values are considered to be very good and these values beyond P 2 are considered to be very very bad. So, for both the cases, what you will find? If the incoming lot quality is extremely good or extremely poor there is a high chance that you will be, you know will be able to make a decision based on the first sample size or the first sample.

Now, this is a special case that is a double sampling plan with curtailed inspection. So, this point means whenever you go for curtailed inspection, what you find that on an average the number of units you are going to inspect for making a decision is expected to be lesser.

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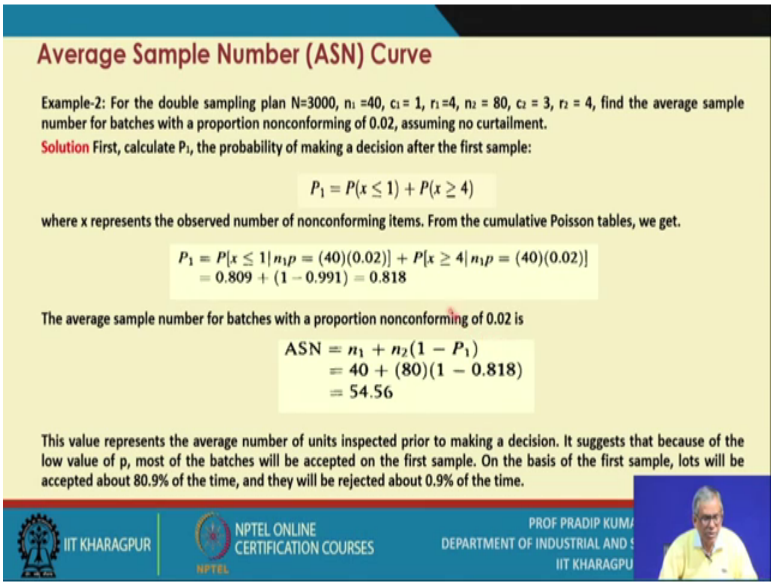

So, this is another example related to average sample number. For the double sampling plan N equals to 3000, n 1 is equals to 40, c 1 equals to 1, r 1 equals to 4, n 2 equals to 8, c 2 equals to 3, r 2 equals to 4. Find the average sample number for batches with the proportion nonconforming of 0.02 assuming no curtailment.

So, all the steps are mentioned. So, please go through all the steps. I have already explained all the steps. The required for constructing an OC curve. Here, what you try to do? That means, against a specific value of P that is p equals to 0.02, you need to calculate the ASN. So ultimately, you must calculate that, what is the probability of making a decision that is this one? The P 1 is this that is 0.818 and immediately after calculating P 1, you calculate ASN and ASN is 54.56.

Now, you go through this interpretation; that means, what you will find that 80.9 percent of the time, the decision is taken to either accept the lot or reject a lot are based on the first sample. So, this is a typical examples related to ASN.

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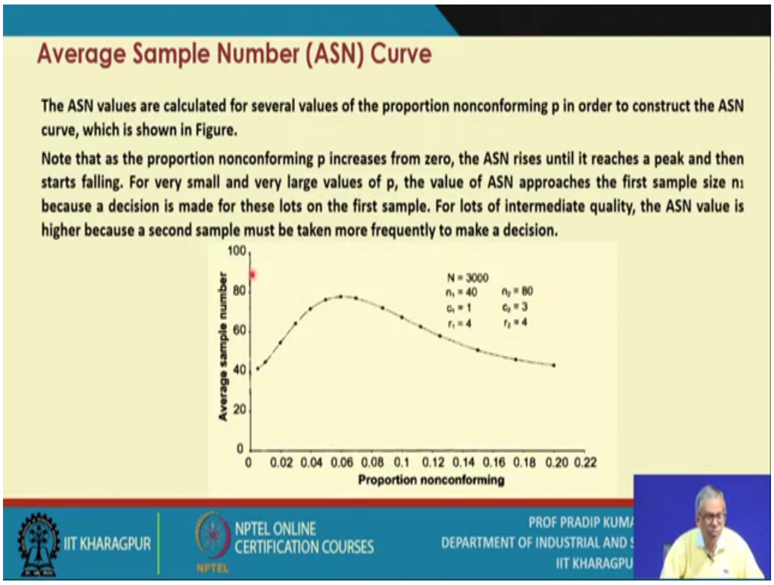

Now, the ASN values are calculated for several values of the proportion and conforming p in order to construct the ASN curve which is shown in figure. So, when you consider several values of p, you get several values of ASN; you plot these ASN values and you get the corresponding ASN curve.

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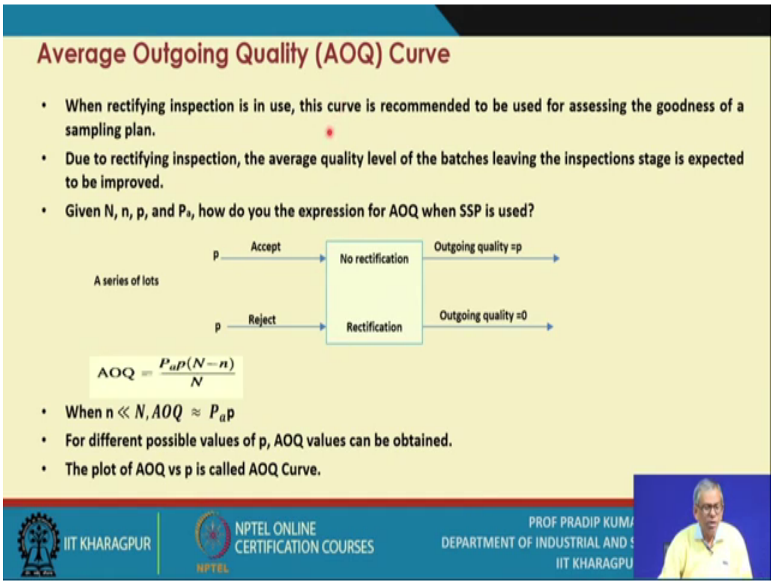

Now, next we move to Average Outgoing over Quality, AOQ Curve and you know that AOQ Curve, you draw as a performance measure; when the rectifying inspection is in vogue. So, when rectifying inspection is in use, this curve is recommended. I have already told you and this curve is used for assessing the goodness of a sampling plan like any other performance measures.

Due to rectifying inspection, the average quality level of the batches leaving the inspection stage is expected to be improved; that is the sole purpose of instituting rectifying inspection in acceptance sampling. So, the given capital N, small n, p and P a; that is the probability of acceptance.

How do you get the expression for AOQ when SSP is used? SSP means Single Sampling Plan. So, here you are getting a series of lots, a lot with incoming lot quality P, you can accept it. What is the probability of acceptance? That is P a. So, obviously, if you accept, you do not go for rectification. So, outgoing qualities also will be P; that means, if the incoming quality is P outgoing quality also will be p.

Now, supposing with the incoming lot quality of P, there is a possibility that the lot may be rejected. So, as soon as it is rejected, you go for rectifying inspection; that means, all the nonconforming units of the lot will be, you know the removed and all the nonconforming units in the lot will be replaced with conforming lots.

So, what you get ultimately when the lot is lot is returned with rectifying inspection, the lot will not have any nonconforming units. So obviously, the outgoing quality will be 0; in the sense that the proportion nonconforming we will be 0.

So hence, what will be the expression for AOQ? That is Pa into p into N minus n divided by capital N. So, when n is less than equals to significantly less than capital N and that is the case in majority of the situations; you will find that the AOQ is approximately Pa into p. But this is a very special case; make sure that the small n is significantly less than the capital [noise]; for different possible values of p AOQ values can be obtained. The plot of AOQ versus p is called AOQ Curve. So, there is a typical AOQ curve. So, the nature of a typical AOQ Curve.

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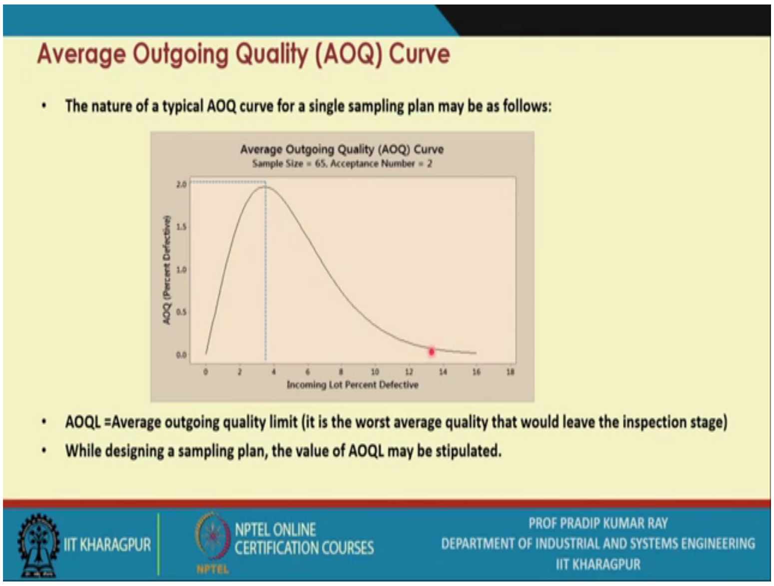

Or a single sampling plan may be as follows, like here you know as the incoming lot percent defective or so the increases, what happens that then the AOQ value increases at a certain and it increases up to a certain the level of P and thereafter, as the quality deteriorates you will find that AOQ value also the decreases.

So, this is a typical shape and against a particular value of P you get the maximum value of AOQ and this is referred to as AOQL. So, AOQL is the Average outgoing quality limit. So, it is the worst average quality that would leave the inspection stage. While designing a sampling plan, so this point is to be noted the value of AOQL may be stipulated.

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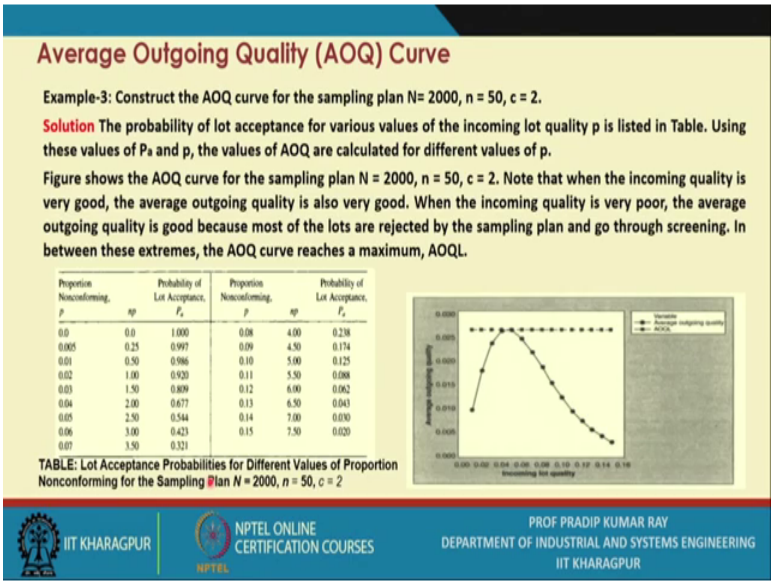

So, we will refer in to this case; now here, is an example related to Average Outgoing Quality Curve, AOQ Curve. So, construct the AOQ curve for the sampling plan N equals to 2000, small n equals to 50 and c equals to 2. So, all the steps are given; that means, against say the value of p equals to 0, n p equals to will be 0 and corresponding probability of lot acceptance, if you refer to the cumulative Poisson distribution table, you will find that this probability is 1.

Similarly, if suppose another value, you have selected that is P equals to 0.03. So, np will be 1.50 and against this value of lambda n p is nothing but the lambda, you refer to x value of 2 in the cumulative Poisson distribution table, you get the corresponding probability as 0.809. So, this process you repeat and you get, but the several values of Pa against all sorts of possible values of p. So that means, from 0 to 0.15 that is the range of the values for small p and the corresponding, the p values are like this.

Then, what you do you just plot these values probability of lot acceptance and once you have these Pa values then immediately you calculate the ATI.

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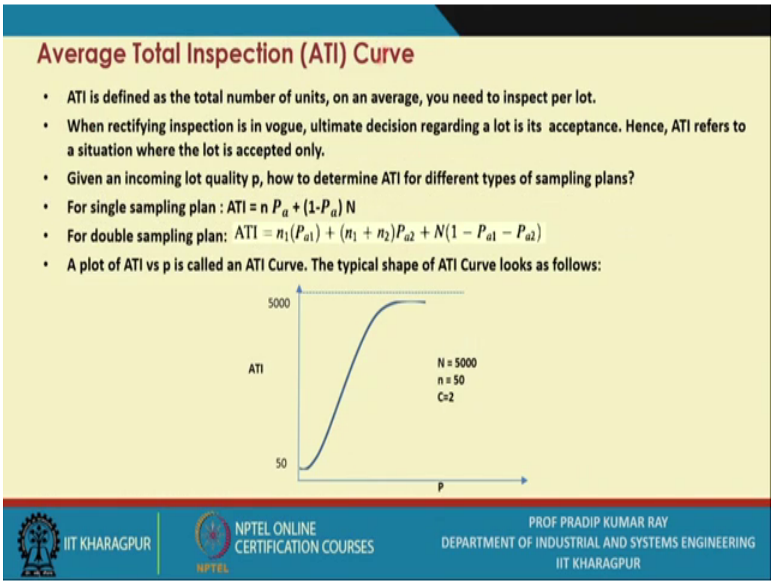

So, AOQ values by using that formula, Pa into p into capital N minus small n divided by capital N. So, against the value of p, you know what is the value of P a and against the value of P a, you also can calculate what is the value of AOQ and then, you plot the AOQ values against p. So, you get AOQ curve.

What is the Average Total Inspection Curve? ATI is defined as the total number of units on an average, you need to inspect per lot; when rectify inspection is in vogue ultimate decision regarding a lot is its acceptance. Hence ATI refers to a situation where, the lot is accepted only. Given an incoming lot quality p, how to determine ATI for different types of sampling plans? Like, we have done for say AOQO or ASN.

For single sampling plan ATI is n into Pa; that means, it is the probability of acceptance. And if you cannot accept it, what do you do? That means, you reject the lot and the rejected lot goes through 100 percent inspection that is rectifying inspection and so obviously, you know all the capital N number of units in the lot you are going to, going to inspect.

So, that is why it is 1 minus Pa into N. So, this is one event or this is another event and these 2 events are mutually exclusive. So, this is the expression for ATI for double sampling plan n 1 into Pa 1, the probability of acceptance based on the first sample n 1 plus n 2 into Pa 2, probability of acceptance based on the second sample and even you go to the second stage, you cannot accept this lot; that means, you go for rectifying inspection. So, you are inspecting capital N number of units with the corresponding probability 1 minus Pa 1 minus Pa 2.

So, a plot of ATI versus p is called an ATI Curve. The typical shape of ATI curve looks as follows. So, this is a typical ATI curve; that means here, the minimum value is 50 and maximum value could be the lot size is it and as you know the quality incoming lot quality deteriorates; that means, as the value of P increases what happens; that means, ATI value starts increasing. So, this is a typical say the plot of the ATI.

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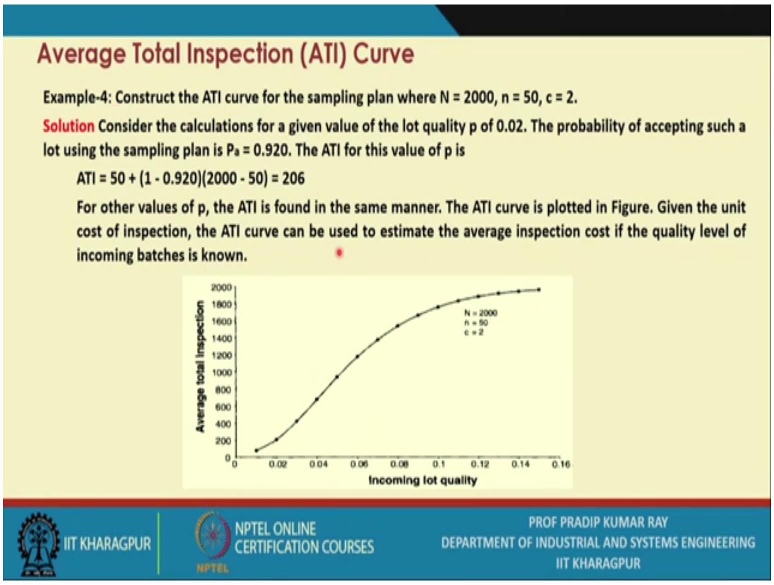

So, here is an example. So, for the given sampling single sampling plan capital N equals to 2000, small n equals to 50 and c equals to 2. So, how to against a particular value of p say p is equals to 0.02, you need to calculate the ATI. So, you just apply the formula, you get a value of 206.

So, we have already derived the expressions for ATI for the single sampling plan and this value is known. That means, you get this value of probability of acceptance by referring to the cumulative Poisson distribution table and that value is 0.920. So, it is this is 1 minus Pa into 2000 minus 50 that is 206.

So, for other values of p, the ATI is found in the same manner and ultimately for different values of incoming lot quality that is p, you get the average total inspection and these values are plotted. So, this is a typical examples of ATI Curve.

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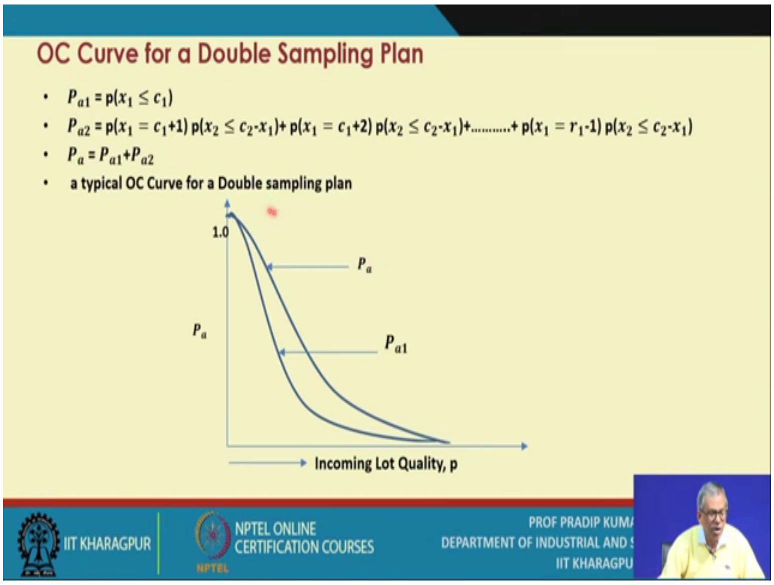

And the OC Curve for a Double Sampling Plan. So, this is the probability of acceptance based on the first sample. This is probability of acceptance based on the second sample. So, you just follow these steps, later on or we may be referring to certain numerical problems on this.

That means, it is the first stage, you go to the second stage; the condition is when at the first stage the number of nonconforming units is greater than x 1 and up to r 1 minus 1 is it x 1 and obviously, at the second stage, this should be less than equals to c 2 minus x 1.

So, everywhere that's why you know you are going to accept the lot at the second stage. So, what is the total probability of acceptance? That is Pa 1 plus Pa 2. So, for a tp for a the double sampling plan, what you try to do? That means, you compute the values of Pa 1 for different values of p and you plot them; plot those values.

Similarly you get the expressions of total probability of acceptance and against all the values of p you calculate ah the total probability of acceptance and you plot them. So, if this is. So, it is a plot of Pa versus p as well as Pa 1 versus p for a Double Sampling Plan.

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So you refer to this, the two textbooks.