Quality Design and Control
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Lecture – 32
Acceptance Sampling - I (Contd.)

So, during this lecture session on acceptance sampling I am going to discuss a number of issues and concepts related to acceptance sampling.

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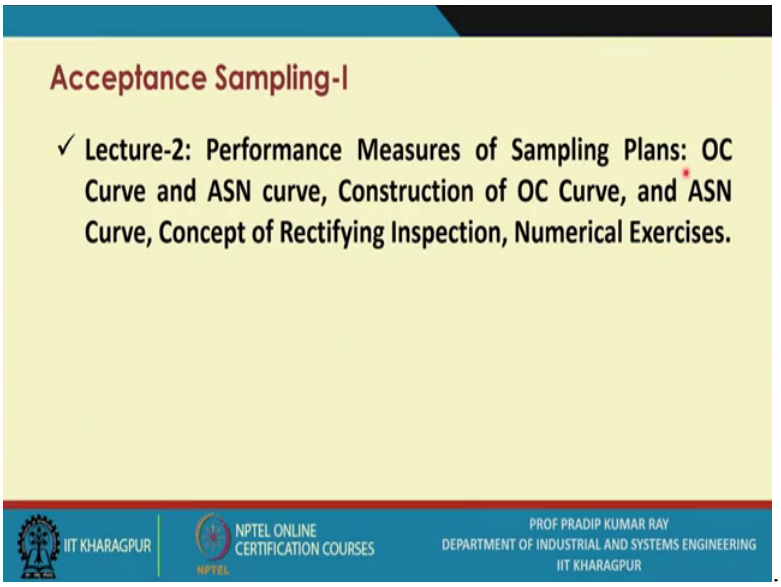

The first one I am going to discuss that is the performance measures of the sampling plans you come across different types of sampling plans and when you start using them how do you come to know that whether the given sampling plan is performing well or not. So, those are the issues we must be dealing with and here in these context we propose two types of curves or the two types of performance measures the first one is the OC curve or the operating characteristics curve that we are going to discuss and we also can propose the ASN curve ASN stands for average sample number.

So, once we discuss these measures then you also must know how to construct an OC curve and how to construct an ASN curve concept of rectifying inspection this concept is widely used and there could be few numerical exercises will be referring to now.

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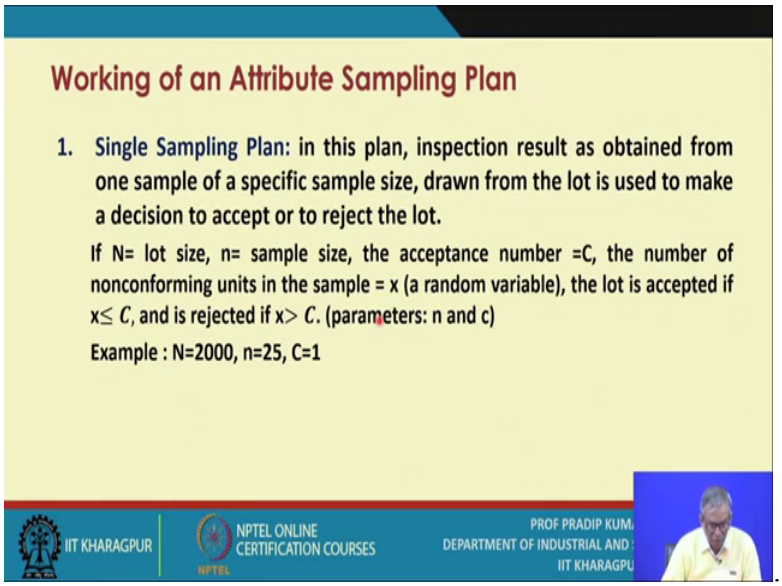

You have already mentioned that depending on the type of data you are referring to you select a particular sampling plan. So, that is this classification based on the type of data you collect or you use. So, whenever you collect attributes data and you need to use the attribute sampling plans; that means, under this category you have a host of sampling plans and one or more such sampling plans you have to use.

So, the first thing you must know that, what is the working of such a sampling plan. So, under attribute sampling plans there are on the varieties of sampling plans, we may opt for single sampling plan, we may opt for double sampling plan and we also may opt for multiple sampling plan.

Now, what is the working of such plans, if you deal with the single sampling plan what do you do; that means, in this plan inspection result as obtained from one sample of a specific sample size drawn from the lot is used to make a decision to accept or to reject the lot is it; that means, the permissible number of for the sample are to be drawn from the lot is it is just one. So, that is why it is referred to the single sampling plan.

So, if capital N is the lot size or the batch size the small n is equals to the sample size the acceptance number is equals to small c the number of nonconforming units in the sample is x a random variable; that means, when you get a sample from the from the lot you start a inspecting each and every unit in the sample; that means, 100 percent inspection of the of the sampling units.

Now you just count that how many nonconforming units occurring in the sample. So, this is a variable so; obviously, this is specified as x and x is a random variable it could be 0, it could be 1, it could be 2, or it could be 3 also any number. So, that is why it is referred to a random variable and for this particular case when you deal with attribute sampling plan; obviously, this random variable is a discrete random variable that means; only you get the integer values.

Now, this lot is accepted if x is less than equals to c; that means, what is the c, c is the acceptance number this is one of the parameters of a sampling plan it is acceptance number and the lot is rejected if the actual number of nonconforming say the units in the sample is greater than the small c. So, what are the parameters of such a sampling plan called single sampling plan that is n and c given, capital N that is the lot size.

So, what is an example, capital N is 2000, small n is just 25; that means, the remaining 1975 units you have do not need to inspect only 25 units you inspect and suppose the acceptance number is equals to 1; obviously, later on when you discuss the design of the sampling plan he will come to know say under a given condition under a given you know this stipulated condition, how to determine, how do you select the values of small n and select the value of small c given a lot size is it capital N.

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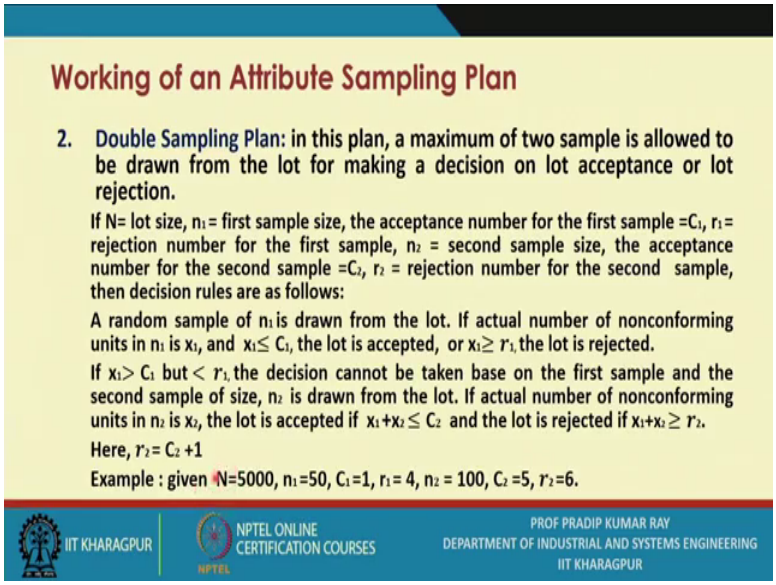

You may opt for a double sampling plan. So, what is the double sampling plan; that means, in the double sampling plan if you opt for the maximum number of say the

samples you are allowed to draw from the lot before you take a decision regarding lot acceptance or lot rejection is true that is why it is referred to as a double sampling plan. So, in this plan a maximum of 2 samples is allowed to be drawn from the lot while making a decisions on lot acceptance or lot rejection the point I have mentioned if capital N is the lot size, n 1 is the first sample size; that means, there will be 2 samples and for each sample you have to specify the sample size.

So, n 1 is the first sample size the acceptance number for the first sample is small c 1, r 1 is equal through the rejection number for the first sample n 2 is the second sample size the acceptance number for the second sample is c 2, r 2 is the rejection number for the second sample then the decision rules are as follows.

A random sample of n 1 is drawn from the lot that is the first sample of size n 1, n 1 could be 25 if actual number of nonconforming units in n 1 is x 1. So, x 1 use random variable and if x 1 is less than equal to c 1 the lot is accepted; that means, you need you do not say need to draw the second sample.

So, from the results you obtained from the first sample you can take a decision some lot acceptance or suppose this random variable that is x 1; that means actual number of nonconforming units in the first sample is greater than or equals to r 1 that is the rejection number on the first sample the lot is rejected. So, they you do not need to go to the second stage, but if x 1 is greater than c 1, but less than r 1; that means, it lies between safe c 1 and r 1 suppose c 1 is 1 and r 1 is 4.

So, if you get the value of x 1 as 2 or x 1 as 3, you cannot take a decision. So, for taking a decisions now you are allowed to draw the second sample and why you draw the second sample, the second sample size is n 2 is it clear and then what you do then again you try to find out how many nonconforming units in the second sample if x 1 is greater is greater than c 1, but less than r 1 the decision cannot be taken based on the first sample and the second sample of size n 2 is drawn from the lot..

So, this point I have already mentioned if the actual number of nonconforming units in into; that means, if the second sample is x 2. So, that is also a random variable it varies from one sample to another the lot is accepted if x 1 plus x 2, x 1 plus x 2 is less than equals to c 2; that means, what is c 2, c 2 is basically the acceptance number for the combined sample and the lot is rejected if x 1 plus x 2 is greater than equals to r 2; that

means, what is r 2 is basically the rejection number for the combined sample. So, what is the combined number samples that means, x 1 is basically the actual number of nonconforming units in the first samples you cannot take a decision based on the number which you get in the first sample.

So, that is why you have drawn the second sample, again in the second sample you start counting the number of nonconforming units and that value is x 2 now you add x 1 plus x 2 if you find that x 1 plus x 2 is greater than or equals to r 2 then that is the rejection number for the combined sample you are going to reject the lot, but if you find x 1 plus x 2 is less than equals to c 2 then you are going to say except the lot at the second stage; obviously, you know you cannot remain indecisive at the second stage. So, that is why either you have to accept or you have to reject when you go to the second stage. So, that is why the r 2 is equals c 2 plus 1 is it.

Example like say given capital N that is the lot size is 5000, n 1 is 50; that means, the 50 units are drawn from say the lot and then you start counting the number of nonconforming units, if the nonconforming units is less than equals to 1 then you are going to accept the lot if the number of nonconforming units in the first sample is greater than or equals to 4 then you are going to reject the lot.

Supposing the value you get; that means, the number of nonconforming units in the lot in the first sample is either 2 or 3 then; obviously, cannot take a decision. So, you have to draw the second sample suppose the second sample stage is 100 then again you what you do; that means, now again out of 100 units in the second sample how many are nonconforming so that you count, that is x 2.

Then you consider the number of non conferring over here and the number of nonconforming over here; that means, you add them and then you will total number of nonconforming in the combined sample you get n 1 plus n 2 and then if this total number is less than or equals to 5 then you are going to accept the lot, but if it is greater than 5 that is 6 or more then you are going to reject the lot.

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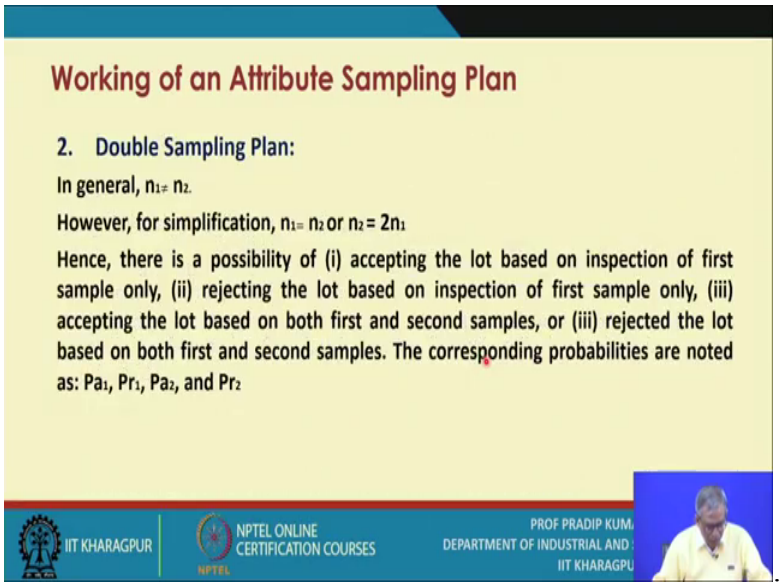

So, this is the working of a double sampling plan. So, in general what do you find when you design a double sampling plan, what do you find that usually the first sample size is not equal to the second sample size that. So, I have written n 1 is not equals to n 2 is it when you design the sampling plan.

But; however, for simplification what you try to do in many cases we will find when you refer to the strain of the standardized sampling plans. So, what you need to do any design exercise you go forward as the standardization, you go for simplification. So, here also the same norms we follow and that is why in many cases we hold n 1 equal to n 2, n 1 could be is equal to n 2 or sometimes the second sample size is twice of the first sample size to n 1.

Hence there is a possibility of accepting the lot based on inspection of the first sample only is it clear. So, there is a possibility or rejecting the lord based on the inspection of the first sample only accepting the lot based on both first and the second samples is it or rejecting the lot based on both first and the second samples.

So, these are the 4 possibilities we have and while we explain the working of a double sampling plan we have highlighted all these 4 possibilities, the corresponding probabilities are noted as Pa1 p subscript, a 1 p; that means, this is the probability of accepting the lot based on the first sample, p subscript r 1 it means the probability of rejecting the lot based on the first sample, p subscript a 2; that means, the probability of

acceptance based on the first and the second sample you have to go to the second stage and similarly p r p subscript r 2 means the probability of rejecting the lot based on the second sample combined sample.

So, these are the 4 notation. So, the; obviously, you know when you add all these 4 probabilities. So, that the sum total will be 1 and later on you will come to know in a given case how to compute the value of all these probabilities is it and only when against a particular say the lot input quality you are able to say compute this probabilities possibly you can determine or you can measure the performance same the performance measure for a given sampling plan.

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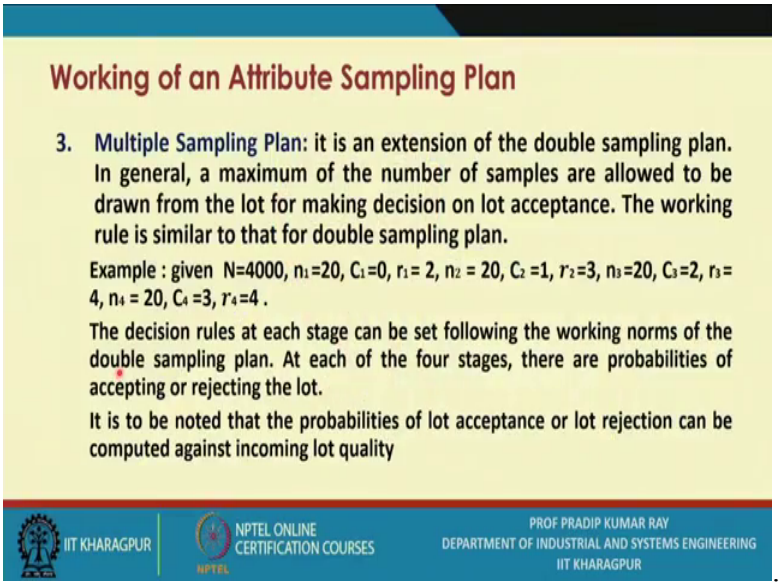

Now, in many cases you know you there could be extension of see the double sampling plan and whenever the maximum number of samples you are allowed to draw from the lot for making a decisions regarding lot acceptance or lot rejection these maximum number of the permissible the number is more than 2 then we refer to such a sampling plan is a multiple sampling plan.

So, how do you explain the working of a multiple sampling plan, it is an extension of the double sampling plan the point already have mentioned in general a maximum of the number of samples a maximum of the number of samples the 2 a maximum of say a 2

number of samples or say more than 2 number of samples are allowed to be drawn from the lot for making a decision on lot acceptance.

So, here a maximum of more than 2 number of samples the working rule is similar to that of double sampling plan. So, here is an example given the lot size of 4000 units in the lot, n 1 equals 20, c 1 equals to 0, r 2 equals to 2, n 2 equals to 20, c 2 is equals to 1, r 2 equals to 3, n 3 is equals to 20, c 3 is equals to 2, and r 3 is equals to 4, n 4 is equals to 20, c 4 is equals to 3, and r 4 equals to 4.

So, how do you explain the working of such a sampling plan; that means, the from the lot of size 4 thousand first you draw a sample of size 20 and then you check what is the number of nonconforming in the first sample, if it is 0 we are going to accept the lot you no need to go to the second stage or third stage and the fourth stage and if you find that the number of nonconforming in the first sample is greater than equals to 2 then you are going to reject in order to go to the second stage if you find it is 1; obviously, you cannot take a decision based on the first sample information say you do not need to draw the second sample, the second sample is 20; that means, now you start counting the number of nonconformity nonconforming units in 20.

So, already you have this data with the first sample of 20 units. So, the total number of nonconforming units in the combined sample first and second if it is less than or equals to 1 then you are going to accept the lot if it is greater than or equal to 3 we are going to reject the lot, but if it is 2 then you cannot take a decision. So, you need to draw the third sample, the third sample is again 20 units you draw from the lot and you start counting the number of nonconforming units.

So, now, you have the first sample second sample and the third sample. So, the total number of nonconforming units in the 3 samples will be known and if this number is less than or equals to 2 you are going to accept the lot at the third stage if this number is greater than equals to 4 you are going to reject the lot at the third stage, but if it is 3 say between 2 and 4 you cannot take a decisions. So, you have to draw the 4th sample and 4th sample size is again you specified as 20. So, again you count the number of nonconforming units in the 4th sample

So, now you have all the 4 samples and the total number of nonconforming units in all the 4 samples that you come to know if this total number of nonconforming units in all

the 4 samples is less than or equals to 3 you are going to accept the lot otherwise if it is 4 or more is it you are going to reject the lot, this is the procedure.

So, here what we have assumed that the maximum number of as the samples you are allowed to draw from the lot for making a decisions regarding lot acceptance lot rejection is 4. So, this is just an example the decision rules at each stage can be set following the working norms of the double sampling plan is it this already I have explained at each of the 4 stages there are probabilities of accepting or rejecting the lot like in the previous case is it and you need to use the notations of such probabilities accordingly it is to be noted that the probabilities of lot acceptance or lot rejections can be computed against incoming lot quality.

The incoming lot quality; that means, whenever you get a lot now you must be able to specify it is quality like, suppose when you deal with the tributes case; that means, the there could be several possibilities several values of are the possible values of incoming lot quality and this for the attributes data when you deal with these incoming lot quality is specified as proportion nonconforming.

Now, there is a concept of equivalence among sampling plans like we say you know in a given situation you may opt for a single sampling plan or you may opt for a double sampling plan or you may opt for a multiple sampling plan choice is yours, now the question is that, how do you say that a single sampling plan is equivalent to a double sampling plan and also equivalent through a multiple sampling plan.

That means, the choice is yours you may out for said that single sampling plan, double sampling plan or multiple sampling plans , but the make sure that whatever the sampling plan you use are this impact that you felt that you have is it in terms of the performance you get there is acceptable to you and almost the same level of performance you get.

So, then only you say that all these 3 types of sampling plans are equivalent to one another.

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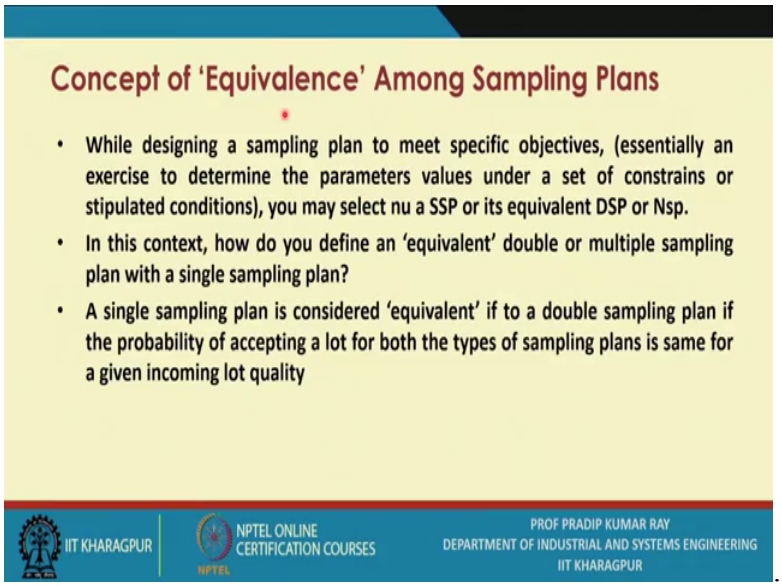

While designing a sampling plan to meet specific objectives essentially an exercise to determine the parameter values under a set of constraints or stipulated conditions is it. So, under the set of constraints or the stipulated conditions so, that we do when we discuss the design of a sampling plan you may select an SSP, then a single sampling plan or it is equivalent double sampling plan or it is equivalent multiple sampling plan is it like Nsp in this context how do you define an equivalent double or multiple sampling plan with a single sampling plan.

So, this is the key questions we have now a single sampling plan is considered equivalent if to a double sampling plan the probability of accepting a lot for both types of sampling plans is same for a given incoming lot quality; that means, what we are saying that the single plan is considered equivalent to a double sampling plan if the probability of accepting a lot for both the types of sampling plans is same for a given incoming lot quality is it clear. I think this is the definition of equivalence between the 2 sampling plans or among 3 or more sampling plans that we use.

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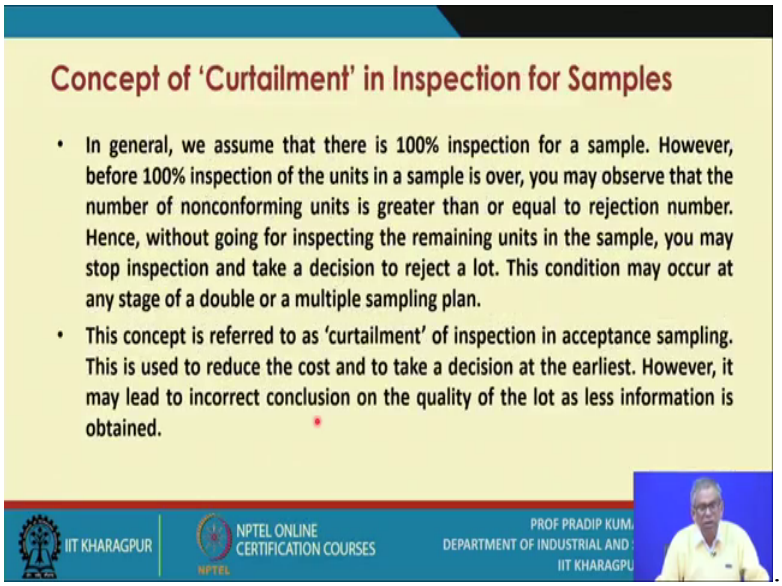

There is another concept of say the curtailment in inspection for or say samples in general we assume that there is 100 percent inspection for example, this we have been saying; that means, suppose a capital N is the lot size and the sample sizes are small n and you are inspecting unless otherwise stated that the you need to inspect each and every item in the sample before you take a decision.

So, this is 100 percent inspection for a sample usually the case; however, before 100 percent inspection of the units in a sample is over you start inspecting each and every item and you know what is acceptance number or what is the rejection number, you may observe that the number of nonconforming units is greater than or equal to the rejection number; that means, suppose the sample size is 25 you start inspecting say all the units one by one.

So, the first unit, second unit, third unit, fourth unit and so on, what do you find that suppose your inspection number is say suppose your acceptance number is 2 when you when you say inspect that the 10th unit you find that, that acceptance number has a the total number of units that you have come across is it, that is more than the acceptance number and suppose a single sampling plan what you do; that means, you do not need to the inspect the remaining 15 units.

So, immediately you can take a decision that while I inspect to the 10th unit only the 10 units I have inspected and then and I have seen that, that 10th unit is the third you know

the nonconforming unit whereas, the acceptance number is 2. So, immediately I am going to reject this I do not need to you know they inspect the remaining 50 units, if you follow this rule if you follow this norm, this is referred to as the curtailment of inspection. So, hence without going for inspecting the remaining units in the sample in this case the 15 remaining units we are not going to inspect you may stop inspection and take a decision to reject the lot this condition may occur at any stage of a double or multiple sampling plan also I am getting my point.

So, this concept is referred to as curtailment of inspection in acceptance sampling. So, in many cases in a practical this is a situation or in a real say world situations many a time the management may prefer to opt for curtailment of inspection because you have to take a decision immediately, if there is an urgency this is used to reduce the cost and took a decision at the earliest so; however, it may lead to incorrect conclusion on the quality of the lot as less information is obtained that is obvious in fact, but that is the main not the main purpose; that means, here is a case where you have to take a decisions immediately.

So, for those cases you may opt for curtailment of inspection what is rectifying inspection, rectifying inspection is also you should be aware of and if you adopt rectifying inspection in your system then certain the performance measures you need to need to prescribe is it.

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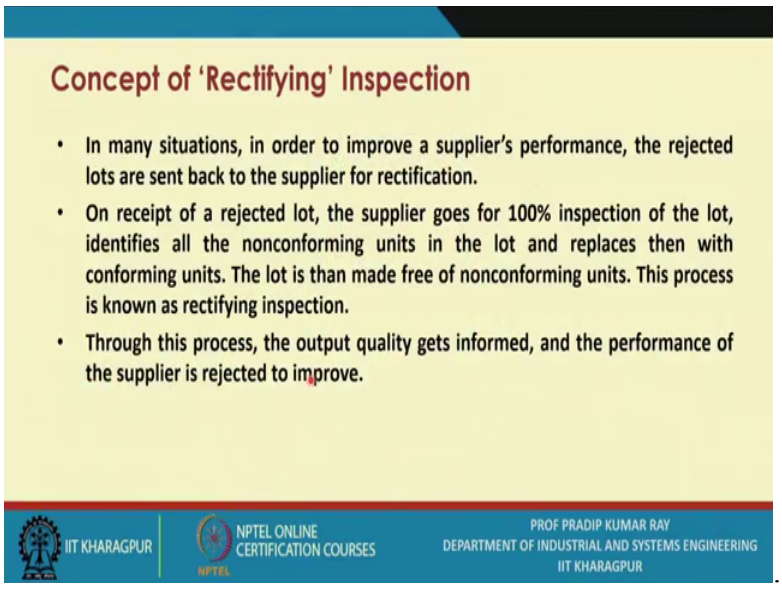

So, what is this rectifying inspection, let me elaborate in many situations in order to improve the suppliers performance the rejected lots are sent back to the supplier for rectification is it. So, decisions could be rejection. So, what you said that the rejected lot is sent back to the suppliers. So, on receipt of a rejected lot; that means, it is referred to as the returned supplier and this is a very you know the serious case in most of the organizations if the if you are you know the orders are your supplies are returned to you because of some quality problems.

So, immediately it becomes an emergency situation. So, what do you do on the shift of a rejected lot the supplier goes for 100 percent inspection. So, now, he is not going to take any risk. So, he goes for 100 percent inspection of the lot identifies all the non conforming units in the lot is it and replaces them with conforming units the lot is then made free of nonconforming units this process is known as the rectifying inspection.

So, many a time what you need to do, when you deal with the stream of lots or series of lots in order to improve the output quality you may opt for your say rectifying inspection through this process the output quality gets improved and the performance of the supplier is expected to improve is it.

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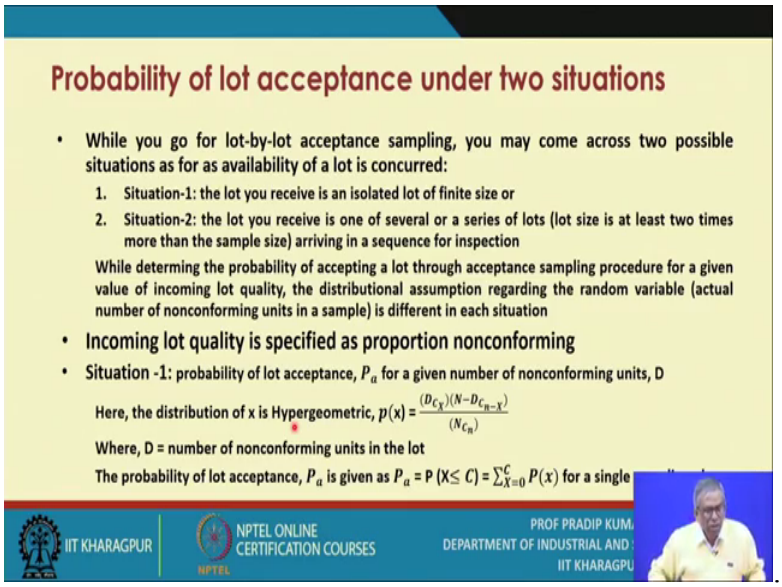

While you go forward lot by lot acceptance sampling you may come across 2 possible situations as far as availability of a lot is concerned, situation 1: the lot you receive is an isolated lot of finite size; that means, just a onetime occurrence one lot, situation 2: the

lot you receive is one of several or a series of lot as a stream of lots size is at least you know by the 10 times more than the sample size arriving in a sequence for inspection.

So, this is the condition; that means, the lot size is at least 10 times more than the sample size while determining the probability of accepting a lot through acceptance sampling procedure for a given value of incoming lot quality the distributional assumptions regarding the random variable; that means, the actual number of nonconforming units in a sample as we have already pointed out that is referred to as x 1 or x or x 1 x 2 for the double sampling plans for the x it is referred to as you know for the single sampling plan. So, there could be different kinds of description assumptions regarding x.

So, in coming lot quality is specified as proportion nonconforming. So, in the situation one if you come across; that means, isolated single say the single lot case probability of lot acceptance for a given number nonconforming units D, here what do you assume that the distribution of x that is the number of nonconforming is can be assumed to be hyper geometric function.

And this is the probability mass function for the hyper geometric distribution that is D combination x, N minus D combination n minus small x divided by N Cn, where D is the number of nonconforming units in the lot is it actual number not the proportion. The probability of lot acceptance Pa is given as; that means, C is the acceptance number that; that means, sigma x equals to 0 to C Px for a single sampling plan.

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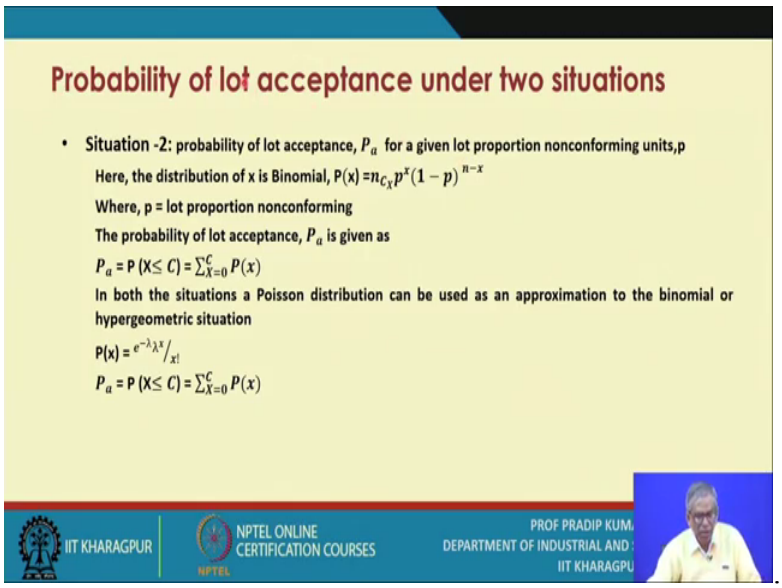

Similarly, in the situation 2; that means, how do you compute the probability of lot acceptance for a given lot proportion and confirming units P; that means, the distribution of x is assumed to binomial and this is you know the probability mass function or the binomial distribution; that means, ncx p to be power x into 1 minus p to the power n minus x, where p is the lot proportion nonconforming, the probability of lot acceptance P is similarly you calculate; that means, the probability of this in both the situations a poisson distribution may be used as an approximation to the binomial or the hyper geometric distribution.

So, this is the probability mass functions for the poisson distribution that is e to the power minus lambda; lambda to the power x by factorial x and the same the formulation you use to compute probability of acceptance.

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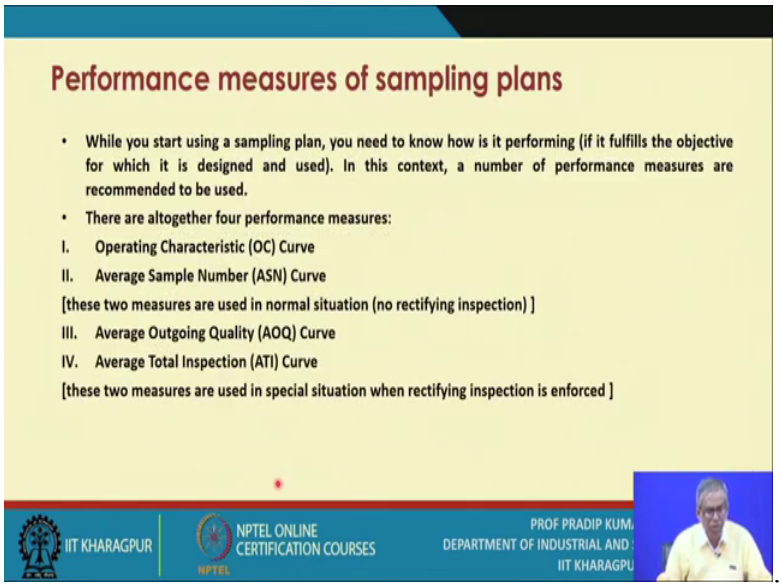

Now, here what do you try to do; that means, we will be referring to the performance measures of the sampling plans and there could be two types of situations like say in a normal situation or there could be a situation where the rectifying inspection is used.

So, what do you mean by the normal situation? That means, where there is no rectifying inspection and what is and if you have a special situation; that means, rectifying inspection is enforced then what could be the performance measures. So, I just conclude this lecture session by referring to four types of performance measures. So, four

performance measures now these operating characteristics curves and average sample number curves.

So, these two performance measures you suggest when there is no rectifying inspection we have already explained what is a rectifying inspection, whether when the rectifying inspection is enforced the these two performance measures you opt for; that means, average outgoing quality curve that you have to construct and similarly average total inspection curve. So, these two measures are used in special situations where rectifying inspection is enforced. So, I conclude these sessions in the subsequent lecture sessions we will be referring to will be discussing all these performance measures one by one.