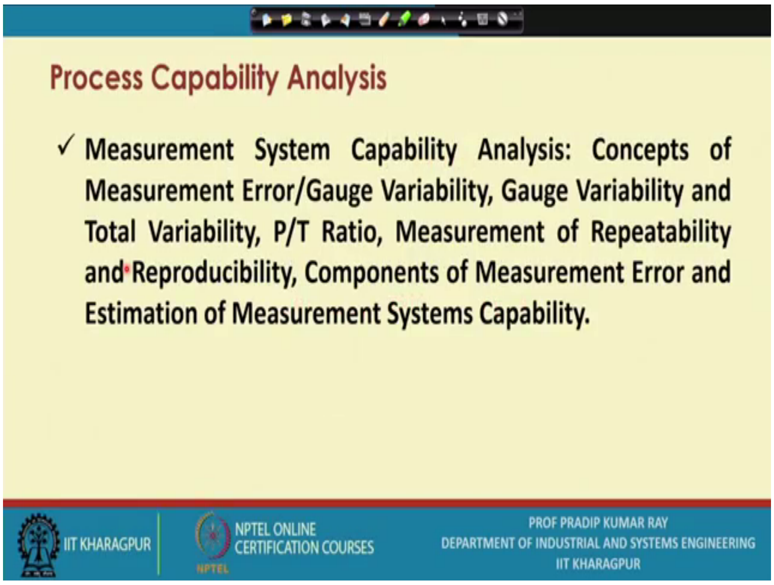
Quality Design and Control
Prof. Pradip Kumar Ray
Department of Industrial And Systems Engineering
Indian Institute of Technology, Kharagpur

Lecture – 30
Process Capability Analysis (Contd.)

Now I am going to discuss a very important topic under process capability analysis and this is the last topic I am going to discuss.

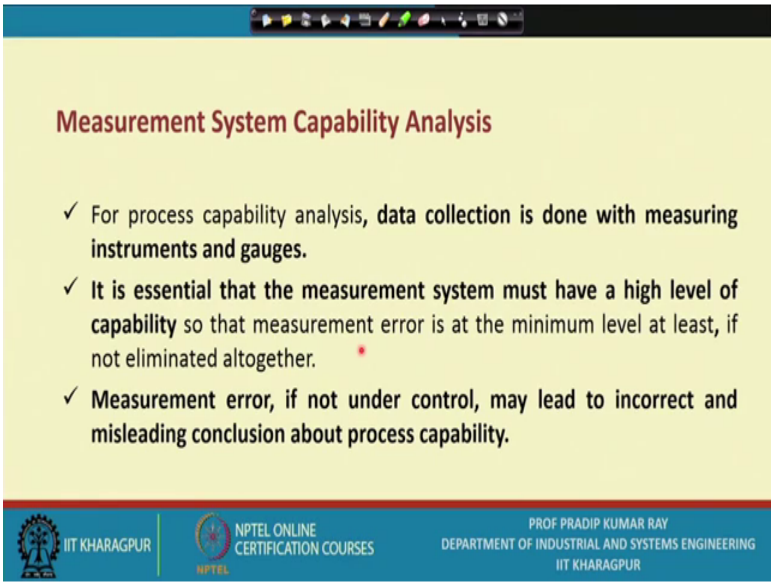
(Refer Slide Time: 00:24)



But is a very important topic and first I should mention what are the you know the specific topics I am going to discuss like related to measurement system I am going to discuss its capability and that the corresponding analysis. Concepts of measurement error or the gauge variability, I am going to discuss gauge variability and the total variability how do you relate the gauge variability or the measurement error with the total variability.

The so called precision to tolerance ratio is refer to as the P and T ratio what is it? We are going to discuss. Measurement of repeatability and reproducibility in explaining the total variability or say total variability in the gauge, also the gauge a gauge repeatability or the gauge variability, now we refer to two kinds of variability that is a repeatability due to repeatability and due to reproducibility components of the measurement error we will be focusing on and we will be definitely discussing how to estimate the measurement systems capability.

(Refer Slide Time: 01:53)



So, these are the broad issues we will be discussing. Now related to measurement system capability analysis a few points I like to highlight at this at this stage like we have been saying that for carrying out any exercise on quality you need data. And once you collect data you go for data analysis using certain tools and techniques relevance tools and techniques and then you conclude about the status of quality related to a particular process or related to a particular product or a system. Now obviously, for collecting data many a time you majority of the time, you need to use certain measuring instruments and or the gauges sometimes they are also referred to as the gauges.

So, ultimately you know the quality of data is very much dependent on the quality of the measuring instrument. So, that is why sometimes we make a statement that the quality is as good if you the recollect in the or the other lectures, I have mentioned this point that quality is as good as the measuring instrument; that means, there cannot be any situations the where you know you do not look into the quality of the measuring instruments, you have been always looking into the quality of the processes, you deal with and you expect a very good quality output is it ok.

So, when we talk about the process capability that is one part, now the process capability is definitely dependent on the measurement systems capability; now why measurement systems because through by using the measurement systems you establish your inspection system. So, if the inspection system quality is poor it is not at all depend dependable; obviously, it will have an adverse effect on the quality of the process or say you know the conclusions you draw from the process performance analysis that could be misleading conclusions too.

So, and if you have a misleading conclusions regarding the process performance, you know the implications could be very very expensive and the company may have to pay a heavy price for this one there are many such instances, these particular situations you to avoid. So, whenever you discuss a process capability analysis, this discussion remains incomplete if you not if you do not discuss the measurement systems capability. So, I think in the context in which this particular topic is being discussed is now made very very clear.

For process capability analysis, some of the important points let me go through which I have written. For process capability analysis data collection is done with measuring instruments and gauges that is point number 1. You cannot avoid using say measuring instruments; it is essential that the measurement system must have a high level of capability. Capability means in the same definition ability to produce as far the specification. So, here the definition is ability to produce as per you know as per the requirements; that means, supposing to what extents you are able to measure a particular say dimension.

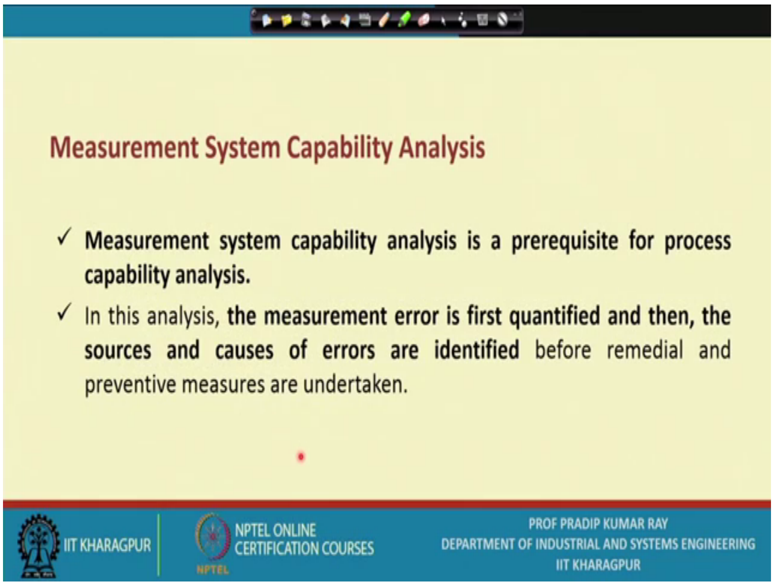
Suppose it is 20.5 the true value. Now are you getting 20.5 with your measuring instrument or you are getting 20 or 19.5 or 20.8 see is there any substantial and the difference. So, to what extent you know it is unavoidable, to what extent it is avoidable, to what extent the measurement system that the measuring instruments quality is affecting this one. Like when if you remember I have mention related to measuring instruments when you start using the measuring instruments, 3 important aspects you must be bothering about.

The first one is definitely the precision. So, you must have what is the precision level of the measuring instruments at a point in time, the second one is you know the accuracy, and the third one is the resolution. So, you just go back to these 3 types of concepts whenever you start using a particular measuring instruments. So, you must have you know your ideas you must be able to measure all these you know 3 aspects or 3 features of a measuring instruments.

So, these are the preconditions, now what do you try to do; that means, you create a measurement system in such a way, that the measurement error is it; that means, the deviation from the actual. Measurement error is at the minimum level at least minimum level, if not eliminated altogether. The ideal situation is that I have created an inspection system measurement systems in such a way that there is no measurement error, but that is your goal, but; obviously, you know the achieving that condition is very difficult maybe you will be heading towards that, but make sure at this point in time that the measurement error is held at the minimum level.

So, if the measurement error if not under control, what is these implication it may lead to incorrect and misleading conclusion about the process capability.

(Refer Slide Time: 08:00)

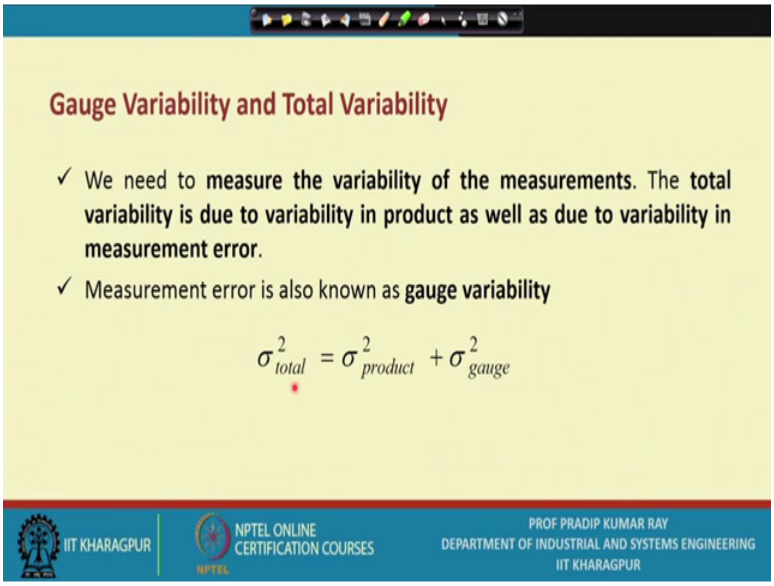


Now there are certain other the remarks I would like to make; measurement systems capability analysis is a prerequisite for process capability analysis is it. So, first you concentrate on the measurement system and then you start using the measurements is if you say that the measurement system is highly capable, you have tested the measurement systems from several relevant perspectives, and then you say now I will go for process capability analysis.

And while I carry out process capability analysis, I will collect data and while I will collect data I will be using a capable you know inspection systems or measurement systems. So, in these analysis the measurement error is first quantified, this is to be quantified and then the sources and causes of errors are identified; is it and what we believe in that the measurement error is a function of time primarily; that means, as the measuring instrument becoming say older and older; that means, getting aged. So, this is most likely most likely that the measurement error will be more.

So, first you estimate the measurement error, you identify the sources and causes of errors and then you take remedial and preventive measures is it or collective measures you take.

(Refer Slide Time: 09:34)



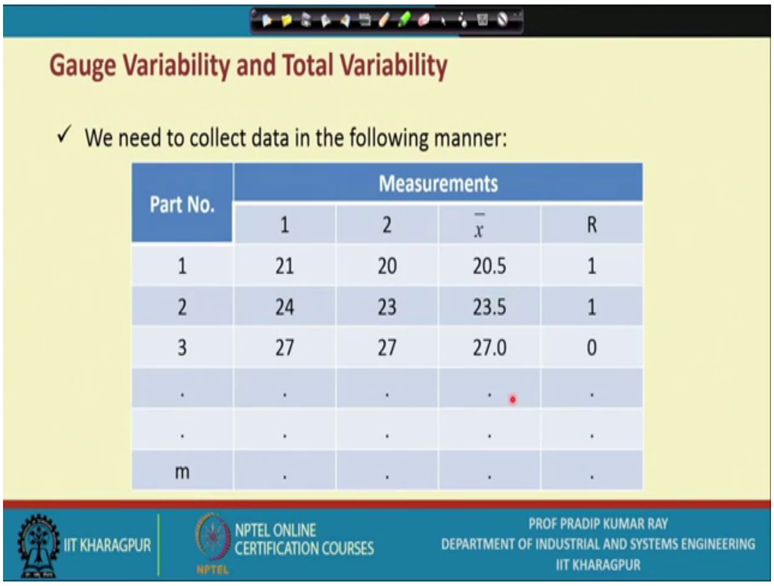
Now one important term we need to explain at this stage that is what is gauge variability and how the gauge variability is related to the total variability is it. You are getting several sorts of you know a for different parts we are getting, and suppose I start measuring the part; that means, related to a part you know what is that quality characteristics.

So, these quality characteristic now the value you are getting for the different units of the part. So, you need to measure the variability of the measurements, variability of the measurements. So, how do you do that you need to carry out a certain experimentation and certain approaches you have to follow, you have to use. The total variability first you have to define the total variability total variability is due to the variability in product that is most important. In fact, variability in the product itself is it because in in reality what you are trying to do; that means, you are producing certain dimension is it.

Now, this dimension you are producing you know the physically. So, that is basically the variability in product. As well as due to variability in the measurement error is it there is one dimension which actually it is there physically. Now you get an you measuring that while you measure the dimension, is it you use the measurement may I know measurement systems or measuring instruments, while you use the measuring instrument again there could be some error is it due to measure measurement systems is it. So, 2 kinds of, the variability you have in the total variability. So, the notation is how do you measure variability; obviously, you know that is basically a measure of dispersion, and the best possible measure of dispersion is a variance.

So, the total variance is equated with the product variance is it, and the gauge variance gauge means the measuring instrument is it. Sometimes this is referred to as the measurement error is it. So, measurement error the gauge variability is also known as the measurement error.

(Refer Slide Time: 12:08)



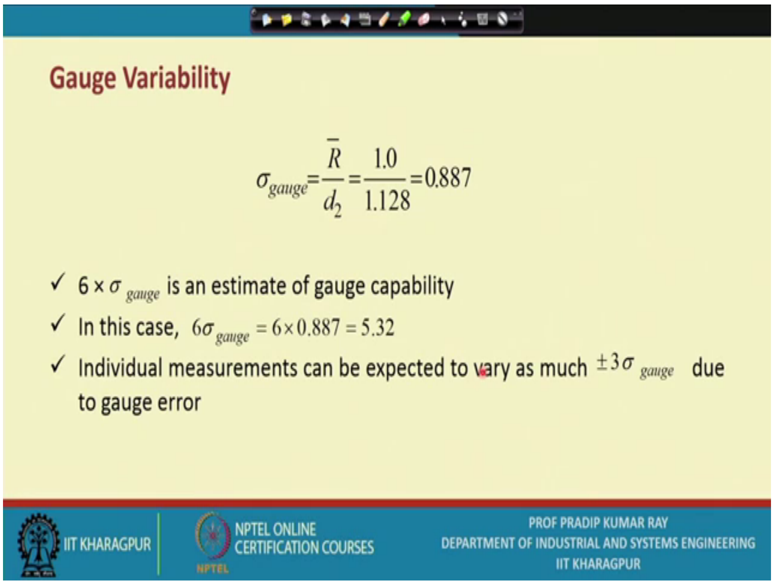
Now here is an example let me make this concept very clear to you; suppose for you know measuring the gauge variability, what you need to do or the measurement error? You have to collect data in a in a specific manner.

So, here you need to consider a number of parts say 20 parts, I will collect. So, 1 to m, m could be 20 it could be 25 a reasonable number say 30 such we have collected. Now what do you try to do; that means, you use a measuring instrument you identify the corresponding quality characteristics a length or the width or the diameter, which a related to a particular diameter or a particular say the length that is your quality characteristics of interest. What do you try to do you use the measuring instrument and you measure the same dimension twice and what we are assuming that these the 2 dimensions are independent; that means, no way you are influenced by the previous dimensions is it ok.

So, independent. So, the part number one you measure twice, in the first you get 21, whatever may be the units of measurement. The next time when you measure with the same instrument you get a value of 20 is it clear. So, what we are assuming that while you measure for the second time, you know you are not influenced by you know the way you measure for the first time is it. So, the second time measurement is no way influenced by the first time measurement. So, what is a x bar values; that means, 2 values the 21 plus 20 20.5 what is the range; that means, there is a sample size is 2, 2 values you have obtained and what the range is just one; that means, 21 minus 20.

So, similarly for all other parts you get the value is it in this manner like for the third part first time you a you measure it is 27 shown, the second time is also 27; obviously, the range will be 0 and the mean will be 27. So, this the data are collected in this manner you create such a table.

(Refer Slide Time: 14:43)



What is your objective? Objective is I am going to measure the gauge variability, then from this what you try to do; that means, the sigma gauge that is R bar by d 2 is clear.

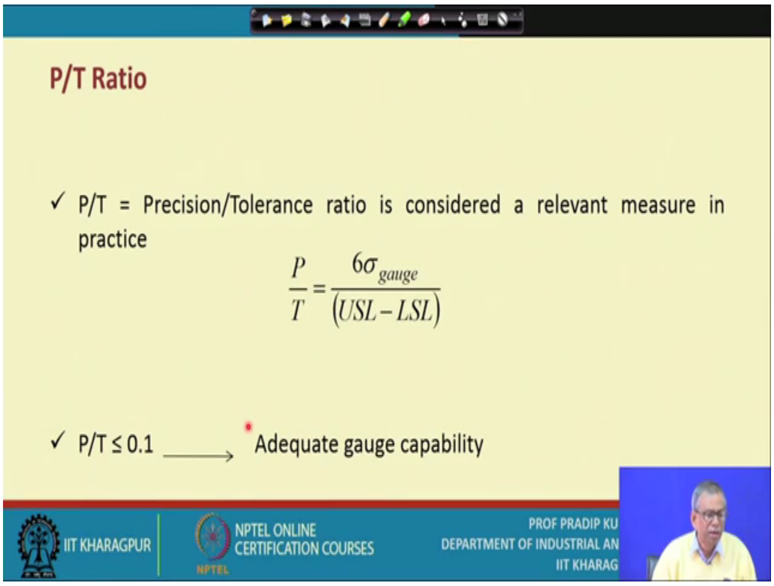
So, the R bar; that means, several R values you have considered. Now you get the value of R bar divided by d 2, these expressions already you know we have derived when we discussed say R an x bar control chart. So, the corresponding values you get and this d 2 is a function of say the sample size and here the sample size is 2, is it n equals to 2 please note that n equals to 2 and you refer to the control chart factor table.

So, against n equals to 2 you get the value of d 2 as 1.128. So, it is 0.887 now as you know that we say that the capability; that means, the NTL natural tolerance limit we calculate is it, and that is indicative of say the capability is it. So, here the gauge capability is referred to as say 3 k into sigma are you getting my point. So, already you have computed sigma gauge and this is 6 times; that means, what we are assuming that is you know the 3 sigma on one side that is UNTL, and on the other side that is a minus 3 sigma from the process the mean that is the mean value, that is LNTL low natural tolerance limit.

So, 6 into sigma gauge is an estimate of the gauge capability is clear. So, so what is this value? The value is here 0.887 you have already obtained; just I am get I am giving an example. So, 6 into 0.887 that is 5.32. So, what do you conclude? The conclusion is that individual measurements can be expected to vary as much as plus minus 3 sigma gage is it. So, the total area is UNTL minus LNTL is it, now what is the variation? Variation is across the across the mean is it ok.

So, the very plus minus 3 sigma gauge from the mean; that means, that is the area you specify the area due to gauge error. So, these simple calculations we do, but just a make a note that you have to collect data; that means, some sort of experimentations you have to do, and you have to collect data as per the given format of the table.

(Refer Slide Time: 17:46)



Now, in industries or in manufacturing organizations and the shop floor, what do you find that many a time. When you refer to this particular concepts a measurement error they might refer to one particular ratio that is called P by T ratio P stands for position and T stands for the tolerance.

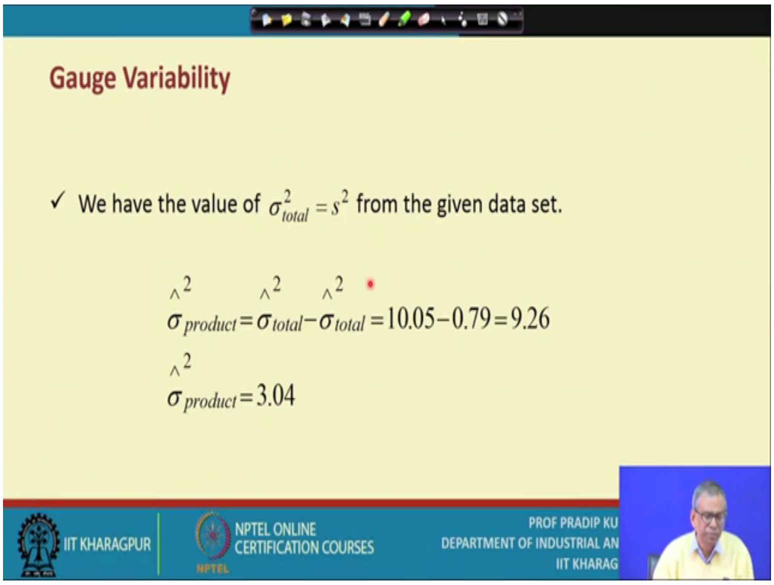
So, precision to tolerance ratio now what is this precision to tolerance ratio? Now many a time you will find at a many shop floors at a you know manufacturing plants, you will find that it is considered to be a relevant measures and that they conclude about you know the gauge variability or measurement error by referring to the value of the P and T ratio is it. So, first let me explain what is this P and T ratio; that means, the precision; that means, the repeatability that is very very important, one aspect sometimes the precision is referred to the repeatability. So, that is just a one dimensions of the precision.

So, this is referred to the 6 into sigma gauge already we have computed, and then it is what we are trying to do; that means, you are trying to measure a quality characteristics is it. So, you refer to that particular quality characteristics, suppose it is a double specification limit case; that means, you have USL we have also LSL for the given quality characteristics suppose this could be say a 60 this could be say ten. So, this is 60 minus 10 that is 50. So, USL and LSL these 2 values are prescribed by the designers they are known they are assumed to be constant for the time being.

So, you can immediately calculate P and T now there is some thumb rule existing or they are referring to sometime rule as far as P and T ratio is concerned they say that that the P and T, P by T ratio must be less than 0.1; that means, the 10 percent of that; that means, this is the total you know the variability am I lying as far as gauge is concerned, now you check whether it is the 10 percent of the of you know the required spread; that means, allowed spread this is basically the allowed spread and this is basically the actual spread is it 6 into sigma and assuming normality obviously.

So, what is the proportion? If the proportion is less than or equals to 10 percent they say is fine means the measurement error or the gauge variability is acceptable is it. Now when you refer to such data and you go for data analysis there is a possibility that instead of a referring to P by T ratio that could be better measures. So, that I am going to discuss and definitely we will be calculating P by T ratio, but sometimes know you may have a misleading conclusions.

(Refer Slide Time: 21:07)

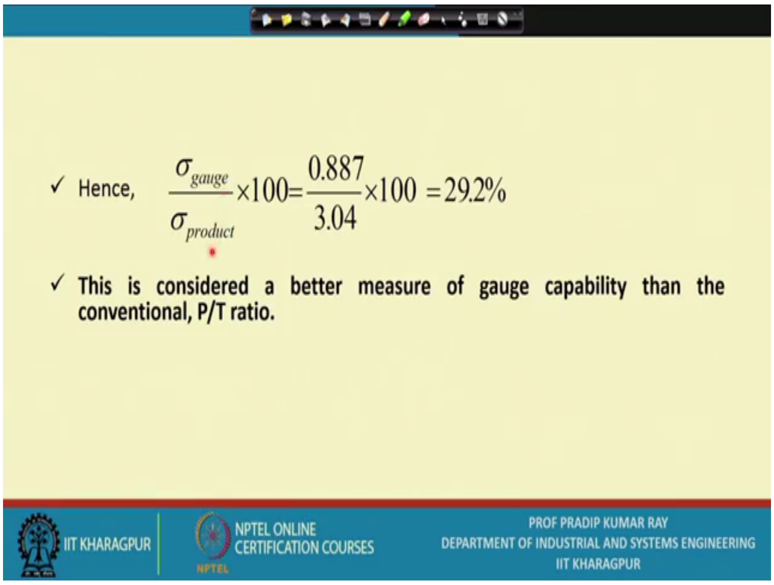


So, a better approach is always prescribed. Now what do you try to do actually from the given set of data if you refer to this particular table, what we will find that suppose there are you know say 30 say the parts you are dealing with a shell table and each part is to be measured twice. So, you have essentially suppose there are 30 parts; that means, you will be getting some 60 such measurements. So, what you try to do; that means, first you calculate the total the variance, the notation is sigma square total that is basically the sample variance.

So, you know how to compute the sample variance from the given data set is it. So, you compute this. Now what do you try to do? You have. So, if this is known; obviously, now you can calculate the sigma square product is it; that means, the variance of the due to product that is sigma square total minus sigma square, sigma square total minus sigma squared gage it is not total it should be gauge sigma square total minus sigma square gauge is it ok.

So, it is 10.05 suppose this is the data you have got and you are calculated as 10.05 that is sigma square total suppose for a given case this time setting one example and this is say suppose 0.79; that means, it is 9.26 so; obviously, the sigma square product; that means, due to the product you have these variance that is 3.04.

(Refer Slide Time: 22:53)

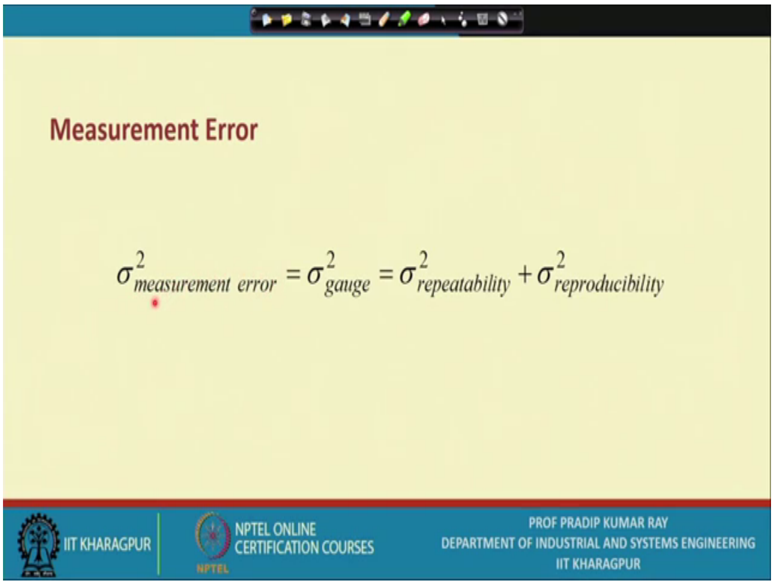


Now what you try to do; that means, you now you calculate that how much is the sigma gauge in with respect to sigma product; that means, what is the proportion of the sigma gauge is it a proportion with respect to the sigma product that is the proportion ok.

So, sigma gauge by sigma product you calculate. So, it is 0.887 divided by 3.04; that means, it is 29.2 percent; that means, one part of the variability is due to the product and one part of the variability is due to the measuring instrument. Now you check that what is the error proportion due to the gauge or the measuring instruments we service the product variance is it. So, this may be treated is a better measure than P and T ratio. So, that is the point I have made that is this is considered a better measure of gauge capability than the conventional P by T ratios.

But again you may be referring to several case studies; that means, under certain conditions maybe the P by T ratio may be recommended and you might have taken the best decision, but in southern other situations maybe you know the P by T ratio may not be valid, for first you have to prove that it is not valid and then if it is not valid then we waft for the alternative ratio that is sigma gauge by sigma product is it. So, this is an important consideration.

(Refer Slide Time: 24:33)

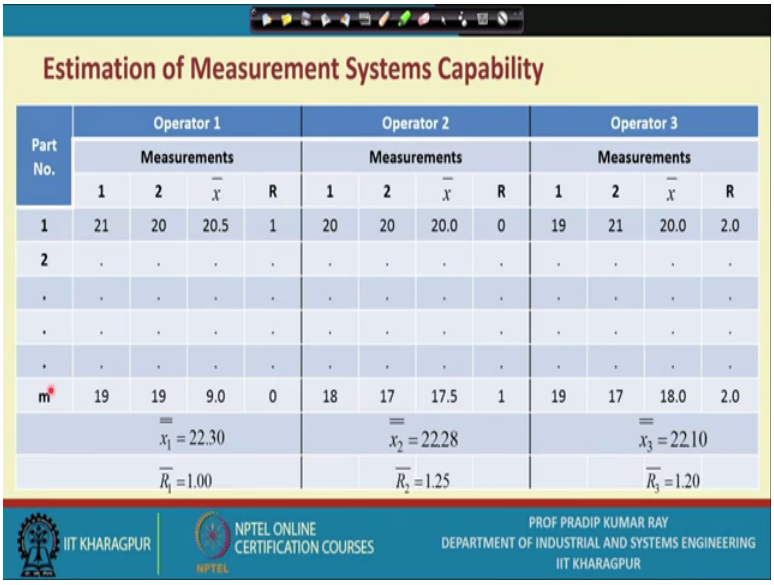


Now, then how do you explain the measurement later? So, first what you do; that means, the total variability is equated with the variability due to the product, and due to and variability due to the a measuring instrument.

So, that is referred to as measurement error now what you need to do; that means, further you need to study this measurement error. So, now, if you have this measurement error or the gauge variability, that is sigma squared measurement error that is sigma square gauge that is the notation we have been using. Now there are 2 aspects; that means, this error may be due to the repeatability or maybe reproducibility now what is the repeatability; that means, you know you will be using the measuring instrument under the same conditions you will be you will be taking measurements a number of times.

So, you are not changing the conditions or the constants. So, that is repeatability is it that is the repeatability. And the second aspect is the suppose you use a measuring instrument, but you change the conditions and in which a particular part is getting is getting measured. So, that is basically when you start you know this measurement the data under different conditions. So, again you may observe there is a variability. So, these variability is referred to as the sigma square reproducibility is it ok.

(Refer Slide Time: 26:21)


Estimation of Measurement Systems Capability

Part No.	Operator 1				Operator 2				Operator 3			
	Measurements				Measurements				Measurements			
	1	2	$\bar{x}$	R	1	2	$\bar{x}$	R	1	2	$\bar{x}$	R
1	21	20	20.5	1	20	20	20.0	0	19	21	20.0	2.0
2	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.
m	19	19	9.0	0	18	17	17.5	1	19	17	18.0	2.0
	$\bar{\bar{x}}_1 = 22.30$				$\bar{\bar{x}}_2 = 22.28$				$\bar{\bar{x}}_3 = 22.10$			
	$\bar{R}_1 = 1.00$				$\bar{R}_2 = 1.25$				$\bar{R}_3 = 1.20$			

So, these 2 aspects to be looked into right. So, for looking into these 2 aspects what you need to do; that means, again you have to collect data in a different form. So, here I am just explaining this particular this particular table like say you are dealing with say 30 parts m number of parts is it now you have engage 3 operators; that means, this is one condition, this is the separate the second condition then this is the third condition; that means, what we believe in; that means, the concept of individual difference; that means, this is the data operator in one has generated; same say with by using a measuring instrument and he measures each part twice or thrice you said twice.

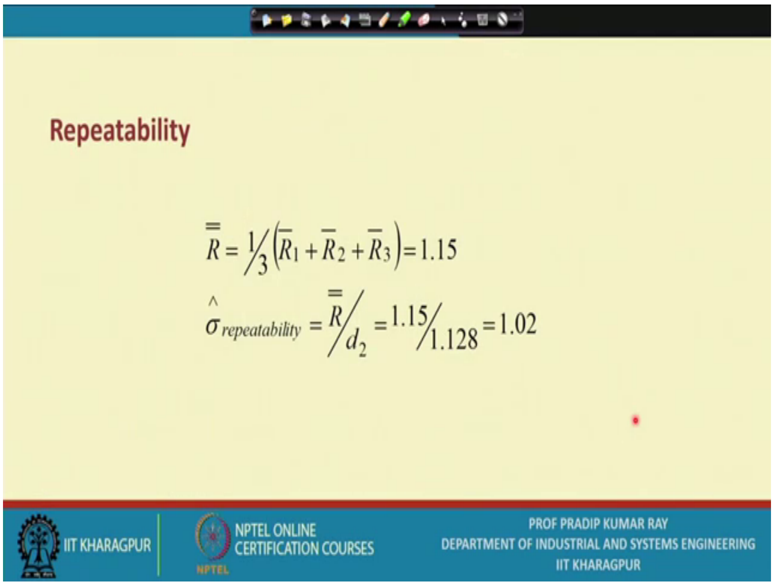
So, you have this data set is it you calculate say corresponding x bar and R individual one. So, you have for the first operator x 1double bar mean of the means that is twenty point 3 0 this is just an example and R 1 bar; that means, R 1 stands for actually for the first operator. So, this is 1.00 its clear; that means, you have all these values you add them and you take this average that is basically R 1 bar.

Similarly, you ask the second person to measure the same parts twice with the same measuring instruments is it clear? So, what do you try to you will be getting, you will be getting a different set of data these least likely that the same data set will be repeated by the second person or the third person it will never happen because the way I as an operator you know using the instrument right it has be bearing on the you know the data which you get is the quality of data which you get.

The other persons he or she will be using the same instrument in a different manner it is unavoidable it is a noise factors. These un avoidable noise factors or uncontrollable noise factors unit to unit variation uncontrollable noise factors. So, the operator to to operator variation that is emerged. So, these variation to be looked into and when you look into this variation, we this is referred to as the reproducibility. So, so the level of reproducibility is this one and when you engage the third person and ask him or her to measure these parts twice.

So, he or she you know that produces this sort of data set so; obviously, they just cannot be the same. So, the level of difference is essentially the level of reproducibility that we are going to measure. So, for the second set of data you have x 2 bar, and this is R 2 bar similarly for the third set of data you have x 3 double bar as well as R 3 bar.

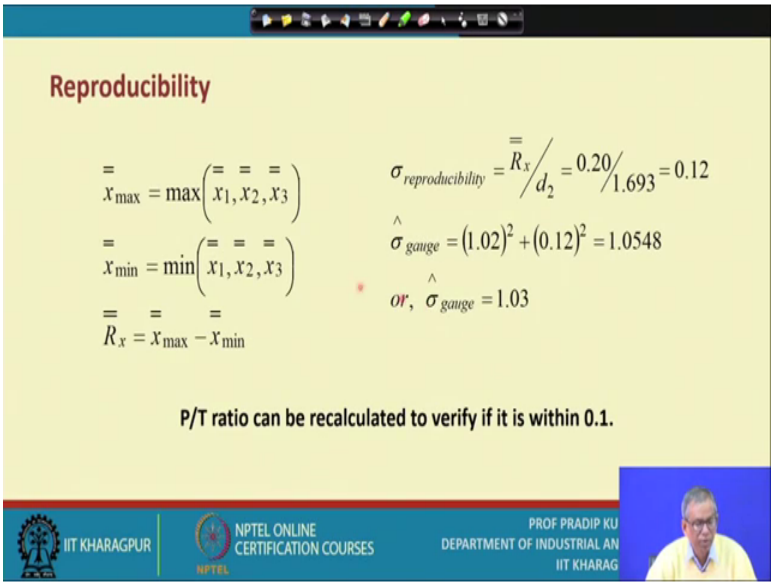
(Refer Slide Time: 29:40)



Now what you do you do some calculations; that means, as far as repeatability is concerned, you have you refer to the range data is it you have R 1 bar from the first operator R 2 bar from the second operator R 3 bar from the third operator. So, you have 3 operators ok.

So, this is the value is R double bar; that means, the mean of all these 3 ranges that is one point one five. So, what is sigma repeatability that is R double bar by d 2 the same formula we have been using that is 1.15 divided by 1.128; and this is a 1.128 when n equals to 2 is clear because the range calculation is possible only when the minimum 2 number of data points are available is it.

(Refer Slide Time: 30:36)



So, this is 1.02 and the next for computing reproducibility what do you need to know; that means, you have a 3 values x 1 double bar x 2 double bar x 3 double bar.

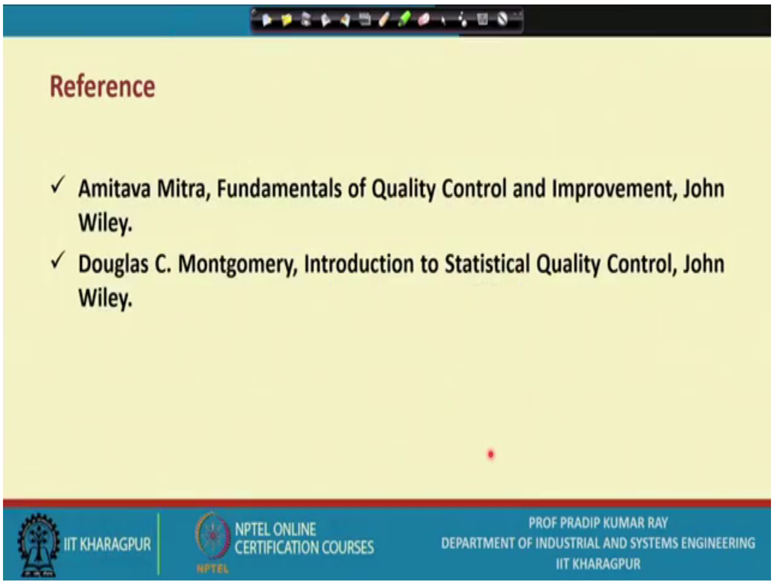
Now, out of these 3 values one will be the maximum one and one will be the minimum one. So, if you subtract the minimum one from the maximum one you get the range value is it related to x. And so, how do you define sigma reproducibility; that means, R subscript x double bar divided by d 2; now this d 2 is actually for n equals to 3 because 3 values you have taken. So, not 2.

So, it is 0.02 suppose just an example divided by 1.693. So, this value you can read from the control chart factor table. So, that is 0.12. So, now, what is the sigmas 2 square gauge; that means, 1.02 square plus 1.12; that means, 1.0548. So, sigma gauge is 1.03 its clear. So, now, you can calculate P and T ratio you will find; that means, it is 6 into sigma gauge that is 6 into 1.03 the P and T ratio can be recalculated to verify if it is within 0.10 0.1 or not; that means, the 10 percent is it clear.

So, now what we have done; that means, sigma square total we have calculated and one important are the component in sigma square total is sigma square gauge. So, that is the first level calculations we did at the second level now the sigma square gauge is equated with a sigma square repeatability as well as sigma square reproducibility. So, that is why we are engaging you know say other persons or other conditions is it, and then you get the data set you calculate both; that means, sigma square say repeatability as well as sigma square say reproducibility.

When you add them you get sigma square gauge, and then you check that what extent you know whether the P by T ratio the value of P by ratio even same or not later on you know we will be taking up you know you know examples of the numerical examples and; obviously, you know we will be we will be using these formulations for solving the numerical problems ok.

(Refer Slide Time: 33:06)



So, I conclude the lecture sessions on process capability analysis, we have dealt with several important the issues related to process capability analysis. Is it including the most important aspect that is the measurement error, and we believe in the philosophy of say the few of a quality is as good as the measuring instruments is it. So, the process capability we have dealt with as well as a measurement systems capability, sometimes the measurement systems capability is referred also as the inspection systems capability. So, I conclude the session.