Quality Design and Control
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Lecture – 29
Process Capability Analysis (Contd.)

So under process capability analysis now I am going to discuss a one important topic called setting tolerances and specifications for components and assemblies.

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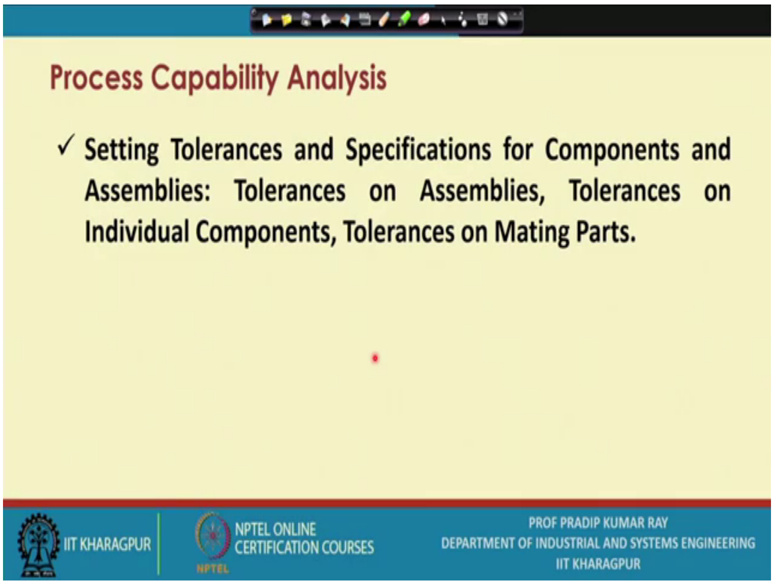

And you know there are 3 issues involved over here, like how to set on the tolerances on assemblies that is the first issue we are going to discuss this. The second important issue is how to say tolerances on individual components. Assembly consists of a number of components. So, here given the assemblies the tolerances, how to set the tolerances of the individual components.

And the last important topic in this context we are going to discuss that is that how to set tolerances on the mating parts is it. So, this will be our discussion topics.

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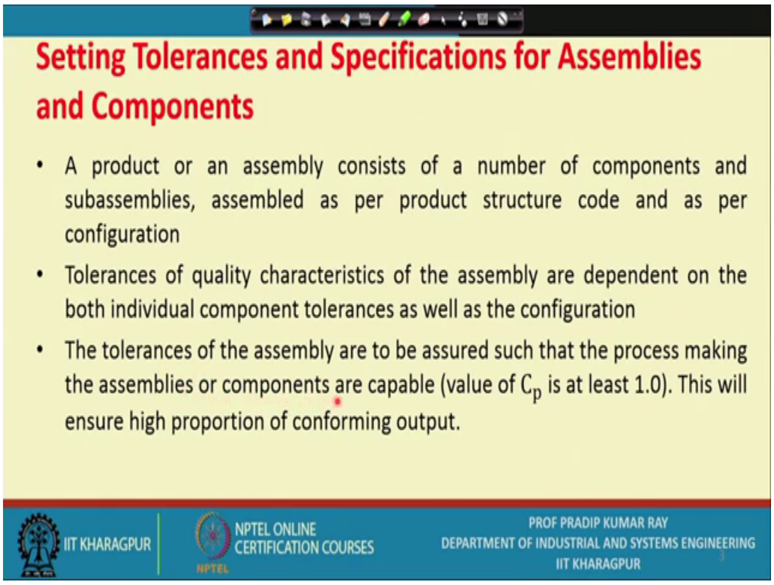

And the first let us talk about in the first issue that is the setting tolerances and specifications for assemblies and components. Now you know you are dealing with a different kinds of products. So, usually for a from a repetitive manufacturing systems or from say a batch production systems.

So, the kind of product we get this any product is nothing, but an assembly; assembly of what? Assembly of a different types of components and or the different kinds of subassemblies or the components on subassemblies, and they are assembled as per the product structure code; that means, whenever you design a product. So, you develop the bill of material. So, what is this bill of a material? Bill of material is basically a document which says are which mentions that how many and how many different types of the parts and subassembly you require to produce one unit of the product. So, that is the bill of material.

Now, this bill of material this document can be made in terms of the hierarchy; that means, from the assembly level. So, then you go to the you move to subassembly levels subassemblies to sub assemblies and ultimately you reach down the line to the component level. So, this sort of you know the structure is refer to the product structure code. Now this product structure code must be known before you go for assembly work and you also must know what is the configuration. So, the 2 important aspects must be known related to product design one is the one is your product structure code and the second one is and the second one is the configuration.

Now tolerances of quality characteristics of the assembly. Now what you need to do; that means, suppose you get the and get the assembly. So, the assembly when you look at the assembly on the product, you may find you know you know you may you can identify a several say quality characteristics of the final product. So, you just pick off one, and then you relate these quality characteristic to the component quality characteristics. Whether you know the final assembly quality characteristic and is in which way this quality characteristic is related to the individual component quality characteristics. So, this relationship must be known and only when you have the information related to the configuration of the assembly.

You may be able to they relate the final you know the assembly quality characteristics with the individual component quality characteristics. So, similarly, the tolerances of the quality characteristics of the assembly; obviously, are dependent on the both individual component tolerances as well as the configuration is it. If the configuration changes the relationship may change is it. Now the next important point to be noted that is the tolerances of the assembly are to be assured such that the process making the assemblies or the components are capable, you know what is you know under what condition we will say that the process is capable.

So, the first condition is as far as possible the process must be make centered centered of the that conditions you have to achieve and the next thing what we have to do; that means, you make sure that the value of C p process capability ratio is at least one. Now these assumptions will make and based on the assumptions, we will the determine the relationship between the component tolerances and the final assembly tolerance. This will ensure high proportion of conforming output; that means, we say that if we say that the C p process is centered and C p is 1; that means, we are concluding is the process is highly capable; that means, there will be hardly any non-conforming units you find from the process is it ok so, that condition is to be ensured.

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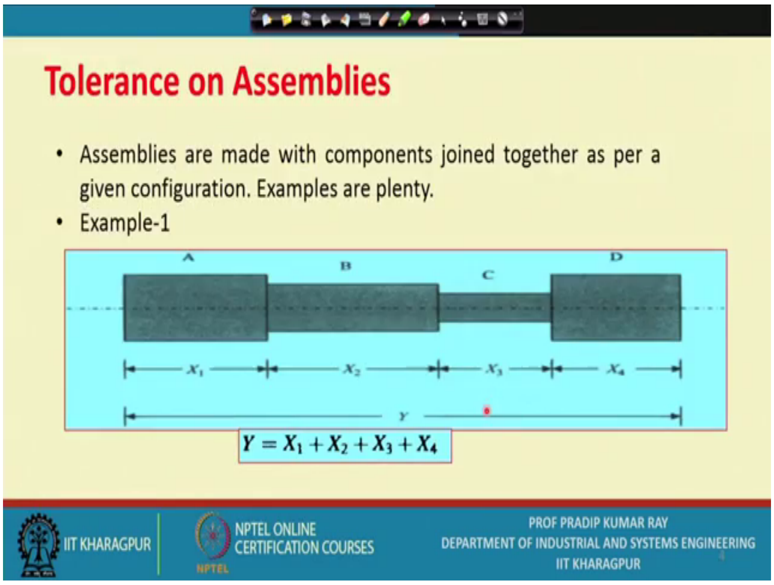

Now let us talk about the tolerances on assembly how do you determine the tolerances on the assemblies; how these then assembly tolerance is dependent on individual component tolerance. So, assemblies are made with components joined together as per a given configuration. Now how do you join there could be many kinds of you know the joining technology like say welding, and we assume that the weld quality is very good. So, that that the weld strength is acceptable to you, and the length of the weld length is it between say there 2 parts getting joined, this weld length is very very small and that is why it can be negligible.

So, these are the assumptions will make. Now there could be several examples of plenty. So, like I am just you know the mentioning 2 simple examples one example is this you have this configuration; that means, the final assembly it consists of 4 parts A B C and D right and these parts are joined means these are the weld joints and we are assuming that these 3 weld joints the weld length, the weld width is negligible is it now; obviously, you know these the components they are produced in large numbers and. So, the component the dimension is x 1 component a dimension, similarly component b dimension is x 2 for c it is x 3 and for d it is x 4 and all these components they are being produced in large numbers; obviously, the these x 1, x 2, x 3 and x 4 these are all numbers you is it ok.

Now, when you get the final assembly, you have this shape and now you have to identify that what is the most critical quality characteristics of the final assembly. So, supposing the most critical quality characteristics is y. So, how do you relate y with individual x values; obviously, y is x 1 plus x 2 plus x 3 plus x 4 simple example.

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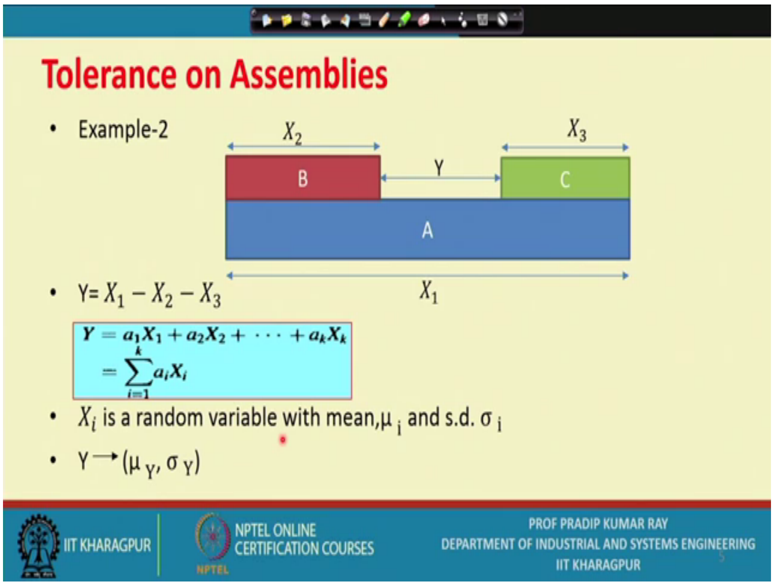

Now let us move to the second example; example two; that means, in these assembly what you have you have 3 parts and part A, part B, and part C. So, what you have that you know that this is the configuration.

So, as per the configuration what you do; that means, you weld part B with part A and similarly you weld part C with part A. Now you get this assembly this is the shape; now what is the critical quality of the final assembly you say y what is this y; that means, this is the exposed area or exposed length. So, suppose the exposed length is the critical quality characteristics of the final assembly. Now this is; that means, how this exposed length is related to the individual x values; that means, related to a the random variable is x 1, related to v x 2, and similarly related to c component c you have the random variable x 3.

So, how do you relate a y that exposed length with through the individual you know the random variables; that means, y is x 1, minus x 2, minus x 3. So, once the configuration is known. So, you have probably understood that while you deal with an assembly you know the basic the relationship will be known. Only when the configuration is known if the configuration is not known, you cannot have you cannot you do not know the relationship between the final assembly quality characteristics and individual component quality characteristics.

So, that is the first you know the conditions we have arrived at. Next is you know it is in general what do you find that y is basically suppose the y is the final assembly as a quality characteristic of interest. So, you may assume it to be linear combination of individual component dimensions. So, this x is are basically individual component dimensions. So, it is a linear combination; that means, you say that a 1 X 1 a 1 is a weightage plus a 2 X 2; that means, related to the second dimension the weightage is a 2 and similarly if you have k number of components is it for the k th component, the corresponding weightage is ak. So, it is the sigma i equals to 1 to k ai Xi.

So, it is a linear combination this is a general form now; obviously, as I have mentioned that all these individual dimensions xi s are assumed to be a random variable, because you are getting several values it is not that. Just one unit you are producing or one assembly unit you are producing you are producing multiple units, and multiple units in the sense the large numbers is a running production system. So, that sort of many a time we may assume it to be a mass production system says large number.

So, under certain. So, under these conditions X i is a random variable; that means, individual dimensions and if it is a random variable. So, you can you know what is this mean. So, the mean notation is mu i and standard deviation s d stands for the standard deviation that is sigma i is it. So, this information you must have at any point in time is it. So, what we need to do? When these individual dimensions are assumed to be a random variable, with their with their mean and standard deviation values known and they are all estimated, then you need to know that related to the final assembly quality characteristics that is y relevant quality characteristics, what could be the expressions for the final assembly quality characteristics mean and similarly the final assembly quality characteristics standard deviations. So, what are these notations that is mu y and sigma y. So, how do you relate mu y with individual mu is, and how do you relate sigma y with individual sigma is. So, that is the questions.

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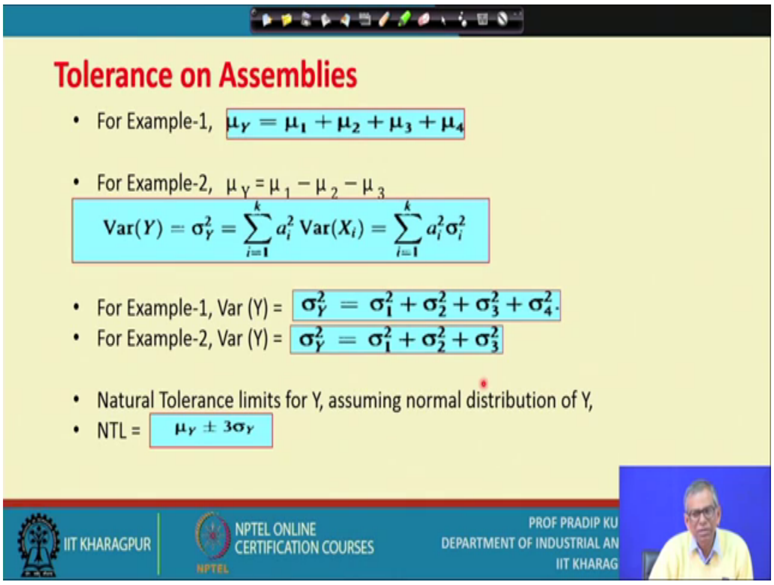

So, for example, you refer to example one that is you know the 4 components we we consider and the 4 components are joined and you get one assembly. So, if you refer to example one so; obviously, mu y is equals to mu 1 plus mu 2 plus mu 3 plus mu 4 is it; that means, what is that equation; that means, the sigma i equals to one to k mu i a i mu y is it. So, a i mu i.

So, what is a i over here? Here all these a i values will be one. So, that is why it is 1 into mu 1, plus 1into mu 2, plus 1 into mu 3 equal weightage plus 1 into mu 4 only 4 components you have considered is it. So, in general we consider k number of components for example, 2 obviously, it is mu one a mu y is equals to mu 1 minus mu 2 minus mu 3; that means, for the first component the weightage is 1, for the second component and weightage is minus 1, and for the third component weightage is minus 1 again is it ok.

As far as the variance of y is concerned, the notation is sigma y square. So, you have this generalization; that means, a i square what is a i? A i is the corresponding weightage we assume a i square variance of x i is it. So, what is the notation of variance of xi that is sigma i square because the standard deviation is sigma i. So, it is sigma i equals to 1 to k, a i square into sigma i is it. So, for both the cases; that means, for example, 1 what is the variance of y? That is sigma y square is; obviously, you add them the individual variances that is sigma 1 square plus sigma 2 square plus sigma 3 square plus sigma 4 square is it ok.

And similarly for example, 2 the variance is given by sigma 1 square plus sigma 2 square plus sigma 3 square because use a square term is it. So, the weightage is minus 1 that minus 1 square is 1 is it that is why it is plus sigma 2 square plus sigma 3 square is it I think the logic is very clear to you. Now once you have these values; that means, the sigma y is known as well as mu y is known. So, the remaining steps are very very simple to understand. So, what you try to do, you try to determine the natural tolerance limit just you know the remember one very vital point that whenever we refer to a particular process, and we try to conclude about other process mean and the process standard deviation.

So, the corresponding the tolerance limits are referred to as the natural tolerance limit is it natural tolerance limits. We add the term called natural, because actually you know you have one the design specifications, but to what extents designed specification is actually your target. Now when you get a process to achieve this target now where the process condition is varying, and you are affected with the many types of factors, but what you have mentioned that at least I have made the process controllable; that means, the process is in statistical control; that means, the process is affected by the natural causes; that means, if you refer to different kinds of patterns we have discussed; that means, you know it is a natural pattern.

So, that is why the natural you know the term may come and that is why these tolerance limits which are actually you know which you get, these are referred to as the natural tolerance limit. I think I know the logic is made very very clear. So, why it is refer to as the natural tolerance limits. So, how do you determine the natural tolerance limits for the y, that is the critical quality characteristics of the final assembly assuming normal distributions again that there is a basis in fact. So, right now we are discussing the situations, where the normality assumption holds. If suppose due to some reasons supposing you are dealing with less number of units many cases, is it like say the production loss size is very very less unique one.

So; obviously, the normality assumption may not hold is it, but those are very special cases right now we are referring to a manufacturing system, which is referred to the batch production repetitive manufacturing and in any country you will find that as far as manufacturing system is concerned 70 to 80 percent of the products, which you use which you get which you manufacture they come from a batch production system. So, our focus is on a batch production systems and so, in majority of the cases the normality assumption holds if you deal with the data you will come to these conclusions.

So, what is the natural tolerance limit; obviously, mu y plus minus 3 times sigma y.

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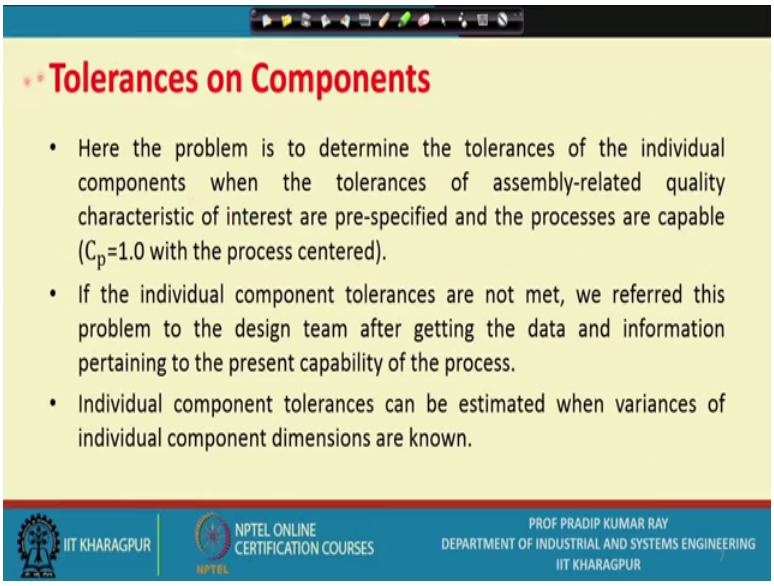

So, once this is known next one is. So, the next important issues that is the tolerances on components; that means, now what you try to do; that means, in the previous case the individual tolerances are known you are determining the tolerances of the final assembly. Here the case is just the reverse here you know assembly tolerances, assembly related quality characteristic is known corresponding tolerances are specified. Now you know the configuration of the assembly what do you need to do you need to determine the tolerances of the individual components and once you determine these tolerances then you check that whether these tolerances are acceptable to you or not; that means, why do you say it is acceptable because if those tolerances you can produce; that means, you have the corresponding process capability.

So, like we say the ability to produce as per the specifications as per the tolerances. So, basically suppose you find that the individual tolerances are not acceptable it is possible, you cannot relax the tolerances of the final assembly because ultimately the product will be known by it is assembly by the assembly quality characteristics. So, what you need to do; that means, at the component level you must look into the design aspects design improvement aspects. So, just all these points I have elaborated here, let me read it out here the problem is to determine the tolerances of the individual components, when that the tolerances of the assembly related quality characteristic of interest are pre specified. This point I have mentioned the pre specified it is already known, and the processes are capable; that means, C p is equals to 0 a C p is equals to 1.0, with the process centered.

This is all we have been saying that the process is to be made centered and then make sure that the process is capable; that means, the value of C p must be at least 1. If the individual component tolerances are not met as the point, I have already highlighted we refer this problem to the design team after getting the data and information, pertaining to the present capability of the process that must be known. Whether to that extent you know this the tolerances individual components tolerances, which are not acceptable to you to what extent they affect the process capability corresponding to that particular say the component (Refer Time: 22:37).

So, each component whenever you identify each component, you are also in a position to identify the corresponding processes. So, individual component tolerances can be estimated when the variances of the individual component dimensions are known is it.

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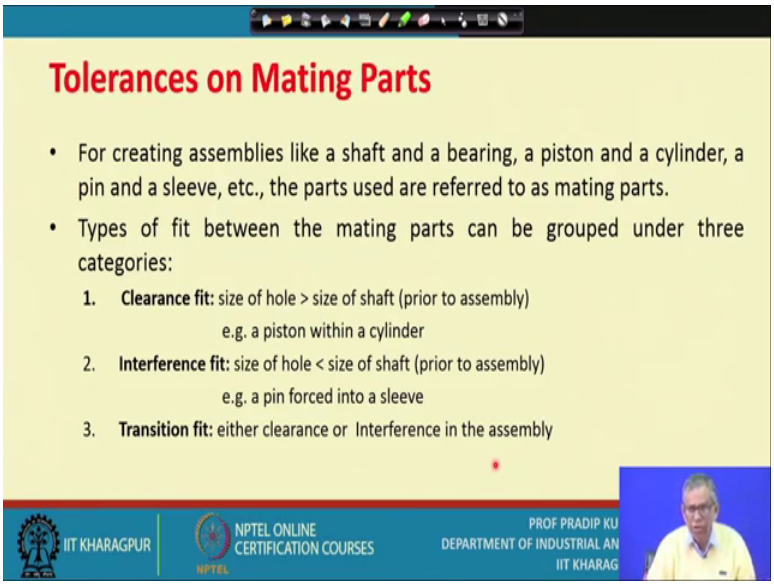

So, the same exercise we do, and we will take off a numerical problems and then you know you will come to know that how the process has been reversed and how to determine the individual component tolerances and how to decide on reach their acceptability.

Now, the third important aspects we are going to discuss and this aspect or these conditions frequently you encounter; that is how to say tolerances on the mating parts you must be aware of you know for different kinds for producing different kinds of assemblies, we need to use the mating parts. So, what are these mating parts like say a shaft and the bearing so that is one kind of assembly. So, they are all mating parts similarly a piston and a cylinder another example of mating parts for creating assembly, then a pin and a sleeve. So, there are many such examples in fact. So, whenever you know you get an such an assembly you need say the mating parts, and the parts are used are referred to as the mating parts for these kinds of assemblies this is a special kinds of assembly, and the this types of assemblies you frequently use and you design. Types of fit now here what is important is that between the mating parts when you get the assembly what kind of fit you may have.

Now, there are 3 types of a fits you may be knowing and these the fits are clearance fit, interference fit and the transition fit so; that means, the type of fit between the mating parts can be grouped under 3 categories. So, what is a clearance fit? That means, the size of the whole dimension is greater than the size of the shaft. So, prior to assembly. So, this condition; that means, in in we you create hole, you create shaft and make sure that these condition holds; that means, size of hold greater than size of the shaft prior to assembly. For example, a piston within a cylinder interference fit is just the opposite size of the hole is less than size of the shaft; that means, you have to press fit, prior to assembly these conditions you have to ensure and a pin forced into a sleeve there are many such examples in fact.

So, it will be better as a student as a learner you identify those assemblies, where the mating parts are used is it. So, when you will carry out the exercises, we will be referring to many such examples in fact. And then you have the transition fit; that means, you are unsure whether in some cases you have a production systems, you are creating the mating parts we are going for the assemblies. Now due to many reasons sometimes you produce a clearance fit, sometimes you also produce the interference fit. So, it is either clearance or interference in the assembly is it ok.

So, in such cases you will find that the majority of the components, a substantial proportion of the assemblies may be considered rejected.

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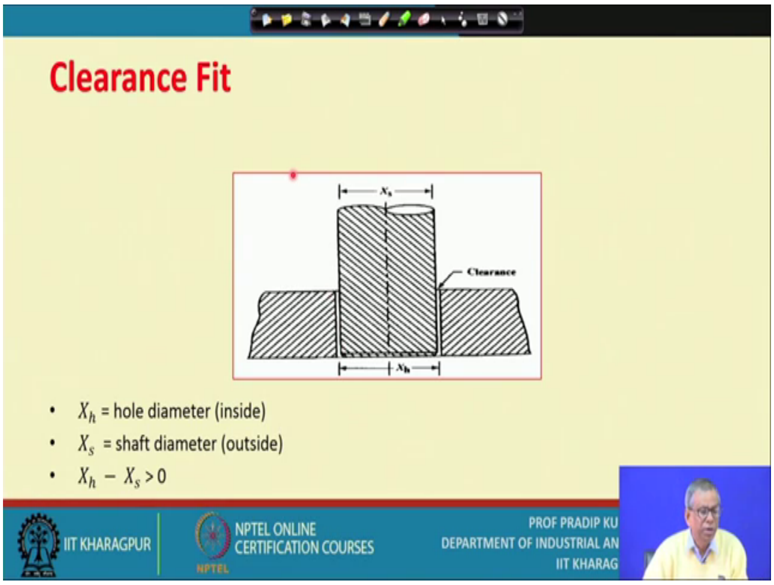

So, here is a clearance fit, it is very clearly mentioned this is basically your the bearing, this is the bearing and this is the shaft is it. So, this is the bearing dimension of the whole dimension; that means, this is the inside dia or the bearing. So, X h this is the hole diameter where we used X because you are producing a such bearings is large number so; obviously, they are considered a random variable that is why you use this notation X h.

So, hole diameter inside and what is the shaft, shaft also being produced in large numbers is it; obviously, this is also X s is also a random variable is a random variable. So, X s is a shaft diameter that is a outside diameter and for clearance fit, what is the condition you have to ensure that is X h, Xh Xh that means, inside dia must be greater than the outside dia of the shaft that is Xs this is. So, this is the conditions.

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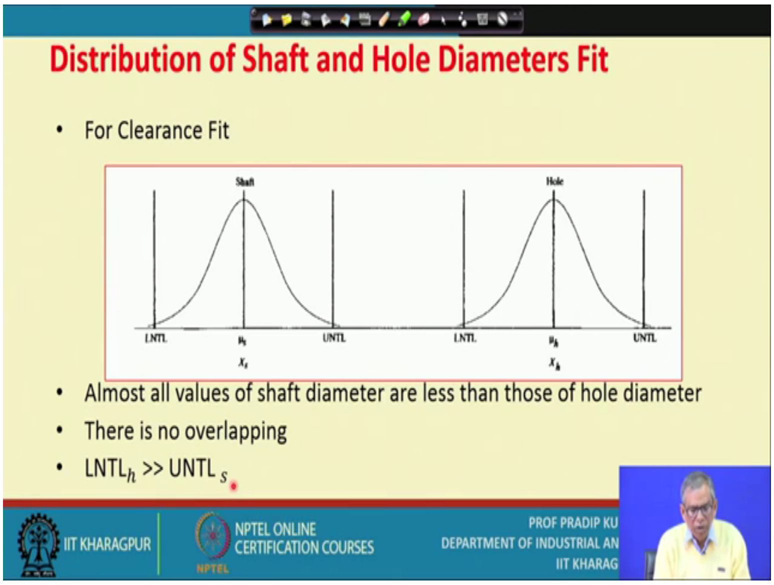

And if you consider the distribution of shaft and hole in hole diameters is it hole diameters the fit; that means, in the clearance fit easy to go on diameters.

So, for clearance fit you have these conditions is it? So, for X s is the random variable that is the shaft dimensions and this is the hole dimension. So, the hole dimension should be greater than the shaft dimension, is it corresponding shaft dimensions. Now you have this distribution; that means, several values you get and they are assumed to be normal distribution. So, you have mu h as the process mean and you have sigma h as the standard deviation of the hole dimensions, and similarly you have for the shaft dimensions you have the process mean as mu s and the standard deviation as sigma s.

Now, you have you know UNTL upper natural tolerance limit for s and lower natural tolerance limit for s is it. So, this is you know say actually the natural tolerance limit, and similarly for the hole you have UNTL h and LNTL h subscript h. So, the difference between this 2 is referred to as natural tolerance limits now almost all values of the shaft diameter are less than those of the whole diameter; that means, here the shaft dimensions are all lying over here whereas, the whole dimensions are all lying over here.

Suppose you are producing tens 10,000 such the shafts as well as the 10,000 such bearings. So, you have 10,000 values over here you have 10,000 values over here. So, this is the distribution. So, there is hardly any overlap there will be no overlap there is no overlapping and lower natural tolerance limit for the whole is significantly greater than upper natural tolerance limit of the shaft is it. So, you just referred to this diagram, you keep in mind this diagram or this figure. So, everything is made very very clear.

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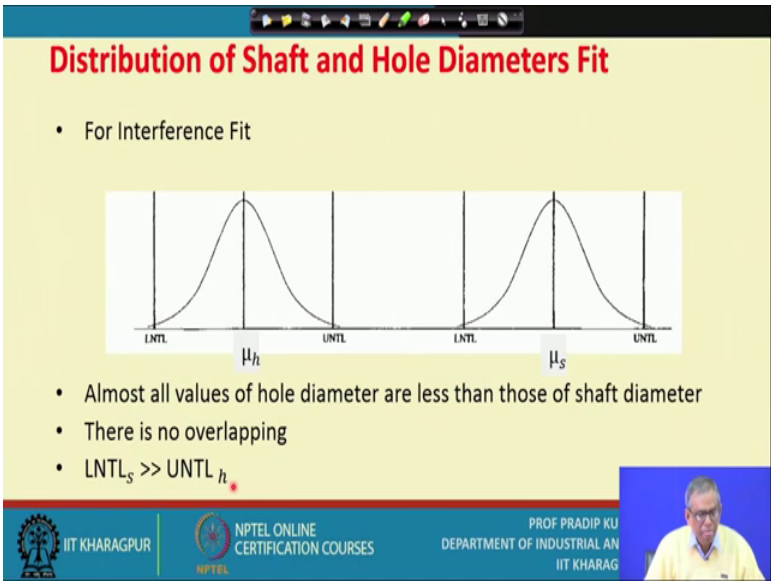

Similarly, for the interference fit is just the opposite; that means, on the left hand side you have the hole dimensions, on the right hand side you have the shaft dimensions. So, almost all values of the hole diameter are less than those of the shaft diameter is it. There could be few in the straight cases, but we ignore. So, you are not 100 percent sure because you say statistical analysis, that is why I mentioned almost I cannot say hundred percent because there could be a stray case like one point maybe somewhere here, or another point or the dimensions one value related to the shaft dimension may be somewhere here. So, that is unavoidable those are the stray cases those are refer to is the outlier.

So, you just note down the point called outliers, now here is a systems in any practical system you cannot create a system where there will be no outliers. So, there is no overlapping and LNTL s; that means, lower natural tolerance limit for the shaft dimension is substantially greater than upper natural tolerance limit for h.

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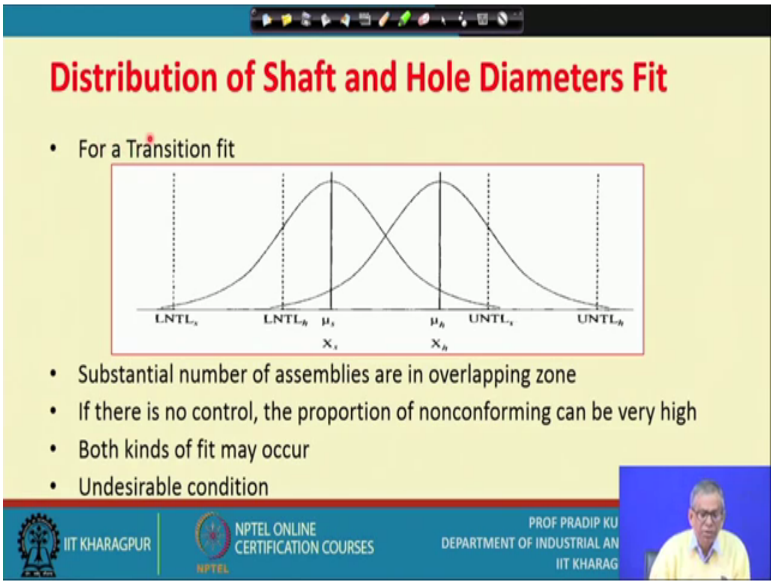

Now, for a transition fit there is overlapping is it? So, this is the distribution with the shaft dimensions and this is the distributions with the hole dimensions; obviously, this area this area is the overlap is it; that means, here is a production system it produces like this.

So, suppose you walked for a interference fit interference is desirable one, but what you produce this one so; obviously, there will be many transitions, there will be many non-conforming output. So, substantial number of assemblies are in overlapping zone this may be 25 percent of the cases or maybe even 50 percent of the cases, depending on what sort of for the data pattern you have. If there is no control the proportion of non-conforming can be very high say 25 percent 30 percent you are seeking for say transition fit or say you are seeking for the interference fit, but what do we get you know with a clearance fit say 50 percent of the cases. So, those will be the reactions both kinds of fit may occur as I have been pointing out and this is; obviously, an undesirable condition is it. So, as far as possible say a transition fit condition you should avoid.

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So, this is the reference you can for this one. So, I conclude the discussion on this