Quality Design and Control
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Lecture – 27
Process Capability Analysis (Contd.)

In this the lecture session I will be referring to the process capability ratios and their measurement.

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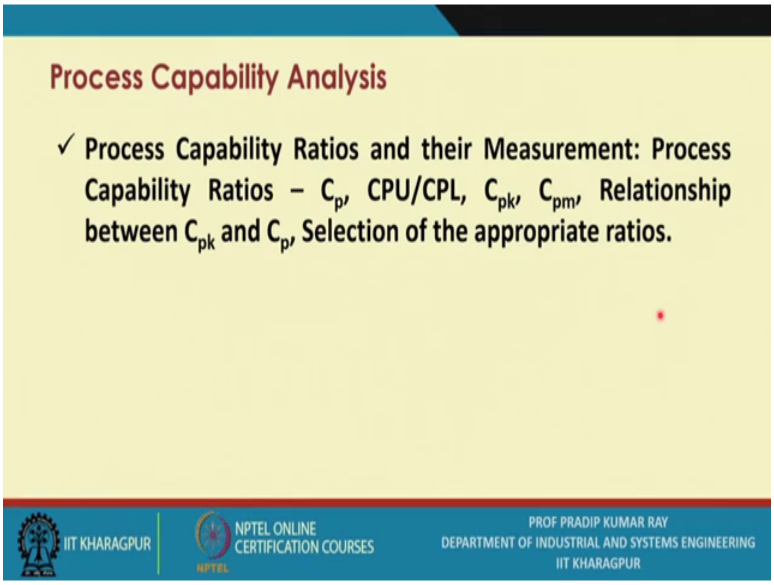

Now as I have already mentioned that there are different types of process capability ratios and they are known as Cp, CPU or CPL, Cpk, Cpm. So, if you go through the literature if you go through the text books on quality are related to process capability analysis the researchers, the practitioners, there they have recommended all these ratios for measuring process capability under different conditions.

So, we will explaining those conditions thoroughly and we will also explore the relationship between Cpk and Cp essentially you know with Cp we try to assess the potential of a process and if the potential is assured then you will go for measuring the process capability is it. So, what is could be the relationship between Cpk and Cp we explore and what is the basis scientific basis of selecting right kind of process capability ratio this is an important issue will also discuss this aspect.

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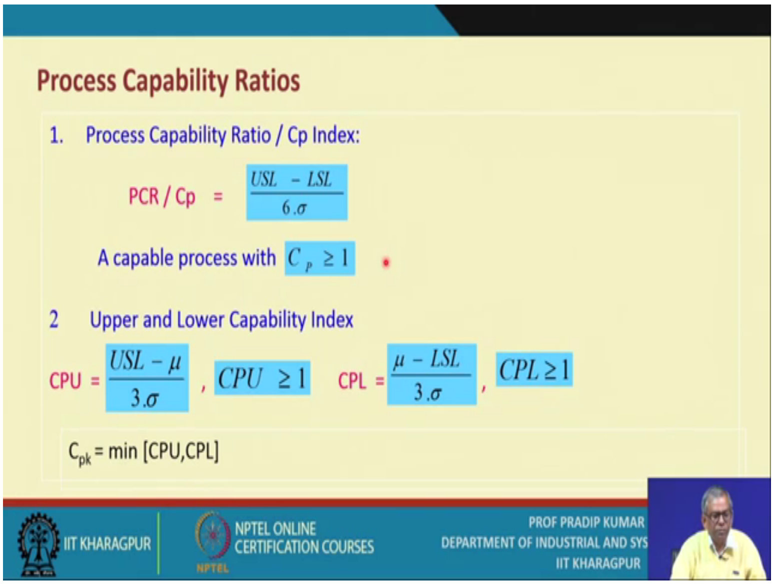

Now let us discuss these ratios one by one and the first the term we use that is called process capability ratio that is known as PCR and PCR sometimes it is referred to as a Cp index. In fact, sometimes this process capability ratios are also referred to as a process capability indices is it and as there is in the ratio form that is why you know I prefer this terminology called ratio.

Now, how do you define PCR or the Cp index right now here what do you find that on the numerator you have USL minus LSL is it. So, what is USL minus LSL for a you know the double specification limit case; that means, with respect to a particular quality characteristics; that means, the first thing you must do that you have to select a process for the given the process or the select process. Now you must identify that which particular component you are going to produce; obviously, you must know that related to the process what kind of operation you will be getting reservice that component.

What the component is known; that means, you look at the component the drying and the definitely we will find that a particular quality characteristic you will be processing with the help of this process. So, with respect to that particular quality characteristics you must know what is USL and what is LSL, now actually the you know these are the 2 values or the specifications limits are prescribed by the designer. So, you must have these details when you look at the engineering drawing you will find you know that the limits are given specification limits are given and you will also come to know what is the nominal dimension and it is a double specification limit case.

So, if it is a single specification limit case either upper or the lower is it; that means, upper specification limit means; that means, a quality characteristic is such that the value of the quality the value of the quality characteristics must not cross some upper limit like. So, the temperature of the oven must not cross a 200 degree Celsius. So, any value which is technically feasible less than 1200 degree Celsius is acceptable to you is it that is the upper specification limit case whereas, the lower specification limit case like say the tensile strength of a particular say the material must be at least 3000 kg right.

So, if kg per say whatever may be the unit like the square this said like centimeter square centimeter or the square millimeter whatever it is. So, that is the minimum value is specified any value greater than of this minimum 1 is acceptable to you. So, that is a lower specification limit case now. So, here USL minus LSL is basically the specification spread we have already defined and then what is this 6 into sigma; that means, the sigma is the process standard deviation and what we have assumed actually 6 into sigma means the distance between say UNTL upper natural tolerance limit minus lower natural tolerance limit; that means, what actually you observed and this is actually referred to as the actual spread.

So, in the denominator when you write 6 into sigma where assuming that the distribution of x is normal and UNTL is placed at plus 3 sigma from the main and similarly LNTL is placed at minus 3 sigma from the mean. So, this is nothing, but the actual spread; that means, it is allowed allow allowed spread versus the actual spread so; obviously, if the process needs to be capable and when the process is centered what do you assume that Cp must be greater than equals to 1 is it like you refer to case 1 this is basically where referring to case 1 and what we are assuming that the process is centered is clear; that means, here while you while you propose this particular ratio know where you know we mention the location of mu we are only referring to the sigma right.

So, basically we say is a capable process, but more specifically means the actual wording should be that if you find that the Cp, Cp is greater than 1; that means, the process has got the potential to be capable. So, it is had adequate potential to be capable is it. So, this is the first condition you have to the necessary conditions you have to satisfy is it clear.

Now, the next one is upper and lower capability index now upward capability index we proposed when the quality characteristics is of upper specification limit case is it or the single specification limit upper and where and in case you deal with one a single specification limit with lower is it the lower limit mentioned then we will be referring to the lower capability index how do you define CPU; that means, just USL minus mu is it.

So, you have just you know from the mu to USL that is basically you know that is the actual or the allowed spread you allow and on that side what is basically the distance from the mu to the UNTL that is 3 into sigma. So, what is the desirable value; that means, the CPU must be greater than or equals to 1. So, it should be at least 1, but if you deal with you know single specification limit case with the lower limit to be to be checked then you refer to the CPL so, here mu minus LSL; that means, mu is always greater than LSL. So, that is the allowed spread and divided by 3 into sigma.

You just in the refer to the one side of the mu is it and for the process to be capable you say the CPL must be greater than or equal to 1 right the same logic you have, now here the actual capability index that is Cpk this is you just note it down that the Cpk is actually the process capability index and what is Cpk actually one way you say that this is the minimum of CPU, CPL suppose you say that the CPU I compute as say 1.2 and the CPL is a 0.85 so; obviously, the Cpk will be 0.85 is a minimum of CPU and CPL.

You will have other expressions for Cpk will refer to this and you will have you know the clear cut understanding of the factors affecting the Cpk. So, from the Cp we move to Cpk.

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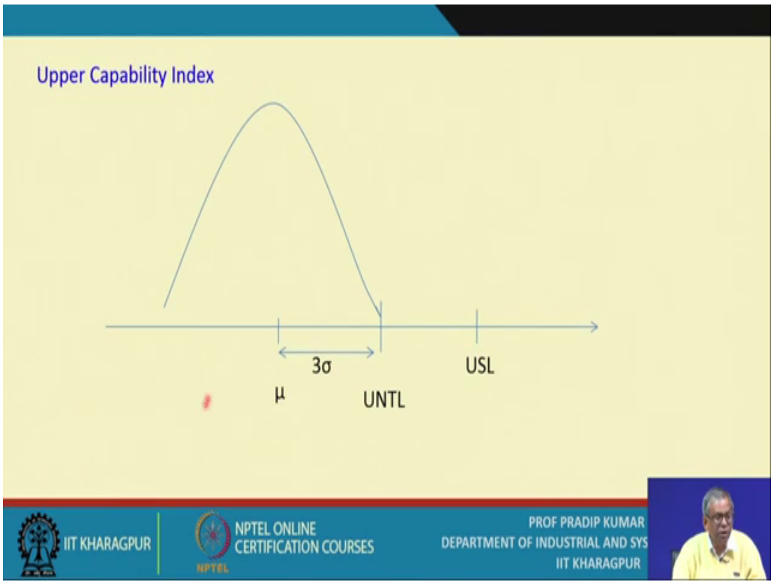

Now, this is just elaboration on say CPU; that means, here this is mu you are considering just you know this side of the distribution you have some weary USL. So, actually you can this value you know any value between mu and USL is acceptable. So, USL minus mu is actually the allowed spread and, but what do you have that UNTL and mu; that means, the process should be restricted within this limit is it. So, that is why it is when we compute upward capability index we refer to this particular diagram figure.

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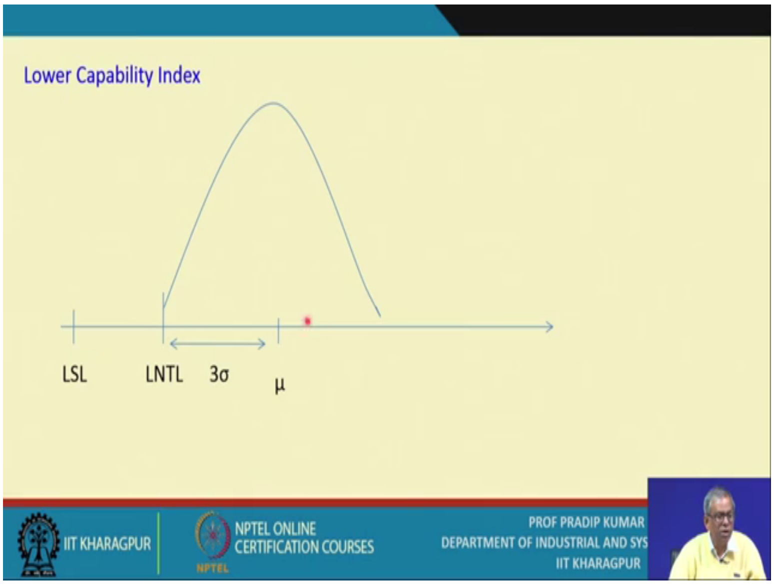

Similarly, when you refer to the lower capability index you have the LSL over here and the mu now this any value between this and this is acceptable to you is it, but if the process is like this; that means, LNTL will be somewhere here, at 3 sigma away from mu is it. So, you have to identify this one and the then this point; obviously, this is the actual one and this is the allowed one. So, allowed divided by the actual will be the capability index is it. So, lower capability index this way define.

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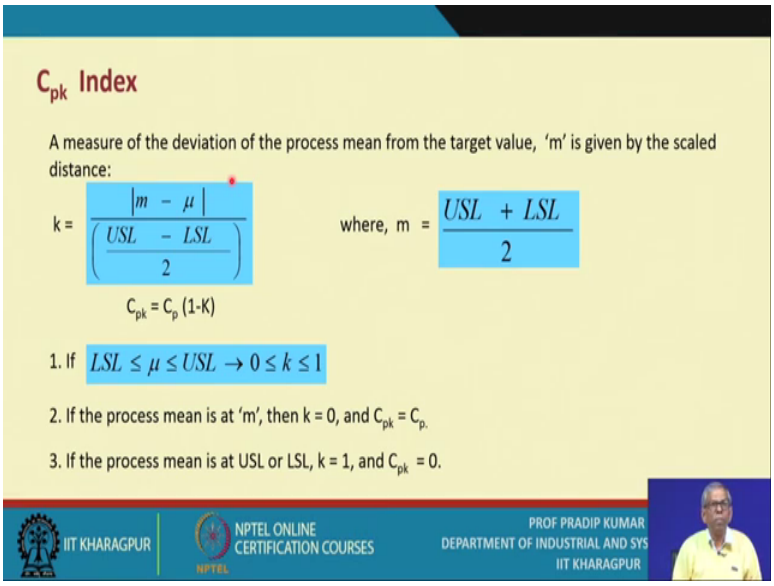

Now, let us elaborate on Cpk index is it, you have to define what is k, now a measure of the deviation of the process mean from the target value, target value is m. So, what is m unless otherwise specified it is basically the midpoint between USL and LSL. So, we are considering a 2 sided or double specification limit case for illustration and the m or the nominal value or the desired value is USL plus LSL divide by 2.

So, now there is no guarantee that the process mean will be coinciding with say the target value or the nominal value of a given quality characteristics. So, there could be some difference and these differences may be significant. So, what we try to do, you try to measure these difference is it. So, this is m minus mu absolute; that means, the difference either on the positive side either m could be say the mu could be greater than m or mu could be less than m, m is constant whereas, mu is a variable mu is a is a function of time.

So, with reference to m mu may be greater than m or may be less than m. So, what you try to do; that means, you get the absolute value; that means, the difference. Now this difference must be you know must be made dimensionless. So, what you try to do; that means, you define k as a proportion of this one so; that means, this is the distance, what is the distance, USL minus LSL divided by 2 is it. So, that is the distance, as a proportion of half of the distance between USL LSL you define k. So, as a proportion; that means, this k becomes dimensionless is it. So, as a proportion as a proportion of the distance between USL and LSL half of that right.

So, this way if you define k and we have defined what is m, then you get the relationship of Cp k is equals to Cp is to 1 minus. So, small k 1 minus small k is it. So, now there could be 3 conditions it may. So, happen that suppose the value of k becomes 1 right, whatever may be the value of Cp suppose it is greater than 1 suppose it is 1.51, 0.67 there is ultimately the process capability has defined by Cpk will become 0. So, there are many cases it may be there could be cases that the process has got the potential, but the way you handle it the way you try to you know the control it is parameters, but these wrong so; obviously, you know you are unable to control the process parameter settings.

So, you may find the situation; that means, even if the process is absolutely all right from the design point of view from it is potential point of view, but you know you cannot control the process and ultimately, you know the majority of the outputs or the units you produce they may be out of tolerance. So, that is why Cp could be very very very less is it. So, there are cases in fact, now first case is LSL less than equals to mu less than equals to USL what you must ensure that suppose you said that the mu is a variable.

Now, mu can assume any value in this range is it like say mu at extreme case mu may be equals to USL or mu may be you know exactly equal to LSL is it; that means, mu falls on LSL or the mu falls on USL at that point in time. So, what do you find that if it falls on USL or the LSL, you will find that the value of k could be 1 or the value of k could be 0. So, if it is 0; that means, the process is centered; that means, m is equals to mu or mu is equals to m is it. So, k becomes 0; that means, if Cp is 1.5 or. So, 1.67 Cpk is also 1.67. So, that condition you must be able to reach and what is these conditions, we are referring to this condition is referred to the centering of the process; that means, the first exercise you try to do you try to reduce the distance between m and mu is it.

So, you cannot do anything on m that is I know the pre specified the location is known, but the mu is location is a variable. So, you try to create a condition the process settings in such a way that majority of the times mu is you know coinciding or it is as almost same as m is it; that means, there is no another significant difference between the value of mu and the value of m is it clear. So, this is if the process mean is at m then k equals to 0 and if k equals to 0 Cpk is equals to Cp whereas, if the process mean is at USL or a LSL I have mentioned the LSL less than equals to mu less than equals to USL. So, this condition may arise; that means, if the process mean is at USL or LSL k is equals to 1 you just compute this 1 you will find that the value of k is 1 and if k is 1; obviously, it is 1 minus 10, Cp into 0 equal to 0. So, Cpk is to 0.

Now the question comes that suppose there is a drifting of the process and that the value of the mu is it and you is suppose you have lost control and you find that mu is greater than USL all the mu is less than LSL. So, if it is greater than USL; that means, you said is upward drift uncontrollable is it or suppose a mu is less than LSL like; that means, there is a downward drift. So, what will happen to the Cpk value; obviously, is such cases you will find that the value of Cpk will be negative.

So, there are you know that 3 zones you have for mu first is a mu may be anywhere between USL and LSL, the next one is mu may be in a zone is defined as that is a greater than USL or it could be in the third zone; that means, mu male lie anywhere you know any it may assume any point less than LSL. So, in the first case you will have some value; that means, in ideal condition when m equals to mu at the center mu; that means, the centering of the process you get the most desirable situation and, but other cases you will find that the value of Cpk will be less than 1 and in certain cases the extreme cases the Cpk value could be negative also there is the most undesirable situation.

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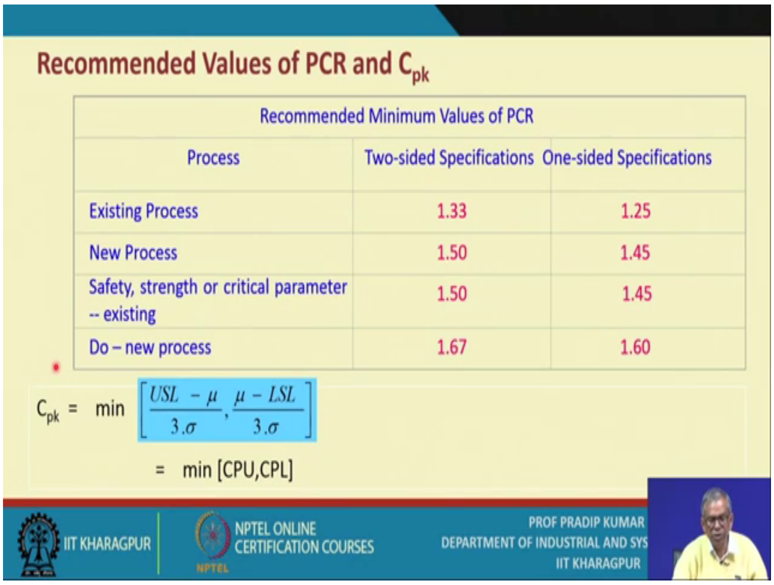

Now, you know the researchers practitioners they have recommended values of PCR that is Cp and Cpk. So, if the process is an existing one; that means, the process has been running for the last say 5 years, 10 years and all as it successfully it is being run. So, existing process if we deal with the two - sided specifications related to quality characteristics. So, you say that the recommended value of PCR could be 1.33 it is clear and if you deal with the one - sided specifications then it could be 1.25.

So, these are based on the experience the researchers, the practitioners, investigators there recommended this one is it, if it is a new process; obviously, you know, but it performance is expected to be higher. So, you say for two - sided specifications the value you said as 1.50 that is the recommended minimum value for PCR and one - sided specification slightly less than 1.50 that is 1.45 if it is related to a particular quality characteristics like the safety strength like the tensile strength or some critical parameter existing one is it; that means, the existing process right you are dealing with the existing process, but now you concentrate on a particular say another quality characteristics or so parameters.

So, then you say if you deal with the two - sided specifications it will be 1.50 and one -sided specifications 1.45 and if you deal with all these parameters or the or the parameters like safety or the strength, but you are dealing with a new process we just installed you set a value of 1.67 for PCR this is the minimum value when you deal with two - sided specification similarly for one - sided specification it is 1.60. Now, we have already mentioned that the Cpk, you just you refer to that what is Cpk, that is a minimum of CPU, CPL.

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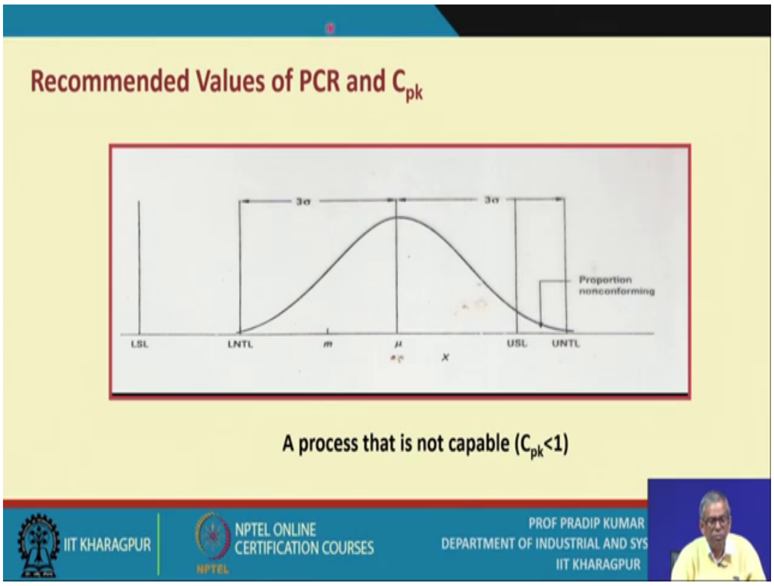

Now, the recommended values and a process that is not capable if you look at this diagram why I am saying that this process is not capable here you find that LSL is here, USL is here and any value you know you get you know the above USL or less than LSL is it. So, that value is considered to be nonconforming right and we are dealing with the distribution of x.

So; obviously, there is hardly any value less than LSL is it there could be some you know certain cases there could be someone value isolated case, but usually there is hardly any value which is less than LSL, but there is this these are the, this is the area; that means, there could be many values is it, when your production rate is very high; that means, there could be many values you may expect right which these values are greater than USL.

So, if they are greater than USL they are referred to as nonconforming. So, suppose this is a 10 percent or 15 percent is it, proportion nonconforming what do you conclude about the process performance, you say the process performance is not at all acceptable is it. If you control it within 3 percent even in many cases 3 percent nonconforming you know it is cost implication could be a very very serious very high. So, you may not be acceptable to you is it. So, we say to the process that is not capable and Cpk is less than equals to 1 why because here you find that there is a substantial distance between m and mu; that means, the process is not centered if you can you know by reduce the value of mu is it reduce the value of mu that you have not done yet.

Then; obviously, if it coincides with m; obviously, that is the best possible situation that is referred to as the centering of the process so, but till now you will find by looking at this diagram you will find that there is a significant you know is the difference between m and mu. So, whenever the process is not centered there is a high probability the process may be considered as incapable the Cpk the probability that the Cpk less than 1 this probability will be very very high.

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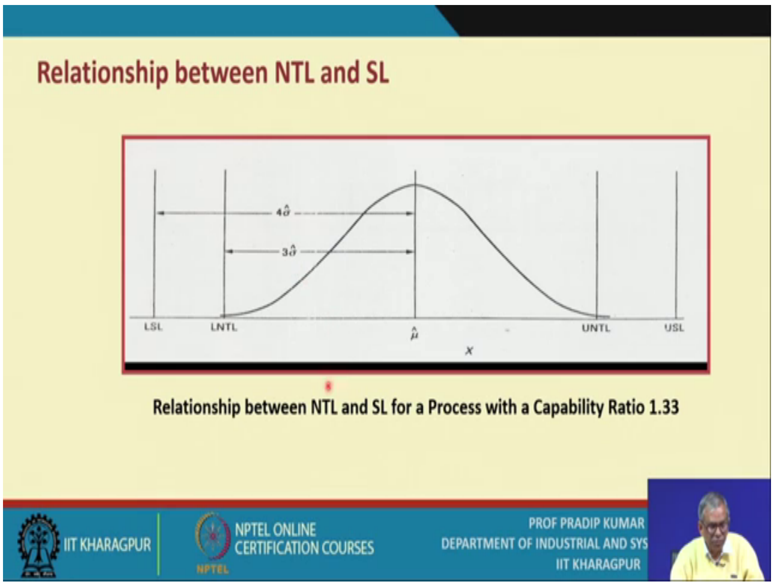

Now, relationship between NTL and a specification limit for a process with a capability ratio of 1.33. So, how do you do that; that means, here is mu and this is LNTL and what do you find that the distance between say LNTL and mu is 3 sigma whereas, distance between LSL and mu is 4 sigma is it. So, this is with respect to say you know the lower capability index, capability index lower you find that this is basically force 4 by 3; that means, 1.33. So; that means, always you remember one particular ratio; that means, allowed spread divided by the actual spread is it.

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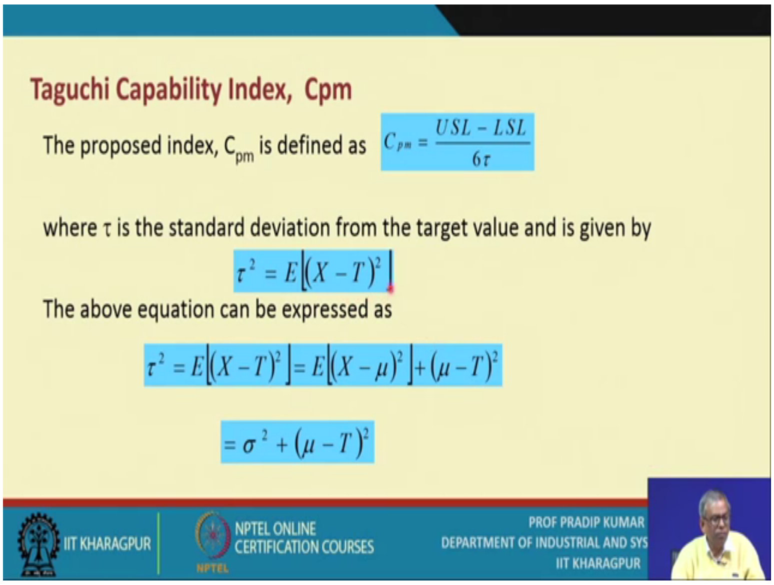

Now, the last capability index I am going to discuss that is proposed by Taguchi and as you know that the Taguchi says that you need to produce as per the target is it, each and every value; that means, each and every x you ensure that these x is you know this value is that you get that is the target value. So, that is your goal.

So, the proposed index is the notation is C subscript p m is defined as USL minus LSL for a two- sided specification limit divided by 6 into tau it is not sigma it is clear. Now what is this tau; that means, tau is the standard deviation we calculate standard deviation, but with respect to the target value is it and how do you get it; that means, it is the general expressions you are already aware of that; that means, expected value of X minus T square, what is T, T is the target value for the given quality characteristics is it, not necessarily the nominal dimension.

In majority of the cases it will be nominal dimension, but when you believe when you apply the concept of Taguchi you say that I will not say it is nominal value, I will say it is the target value. So, this variance you need to calculate with respect to the target not with respect to the mean is it. So, that is why we use a separate rotation that is the tau square and then when you go for manipulation of these equations you have these expressions; that means, you say that it is expected value of X minus mu square plus mu minus T square is it; obviously, we assume that there is you know the product term is absent; that means, the dependency conditions we will not assume, but we say that there is no dependency conditions. So, under these conditions this is basically the sigma square that is the variance and this is mu minus T square is it all right.

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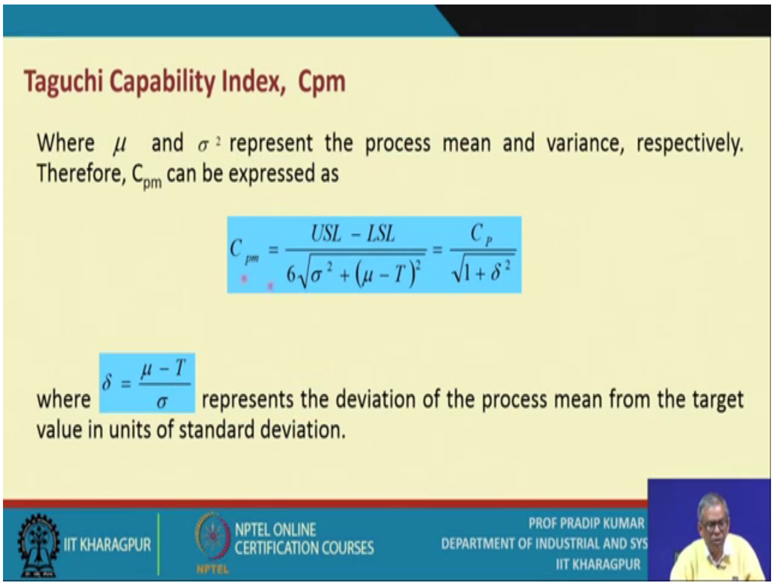

So, this is the simplified expression for tau square and when you substitute this tau in the Cpm expression substitute this expression for tau in the Cpm expression. So, you get USL minus LSL 6 times the square root of this is it. So, this is basically tau, tau square and this is 1 plus delta square where delta is mu minus T by sigma; that means, get see the mu is the process mean and this is the target.

So, what is the difference between this, and this difference is expressed in the standard deviation units and this is referred to as the delta. So, this represents delta represents small delta represents the deviation of the process mean from the target value even in it is substandard deviation. So, ultimately the Cpm; that means, USL minus LSL divided by 6 into sigma into 1 plus mu minus T divided by sigma square; that means, the delta is mu minus T by sigma; that means, root over 1 plus delta square and you have USL minus LSL divided by 6 into sigma that is the Cpk, but original Cpr.

So, the Cpm has got a relationship with the Cp into 1 upon root over 1 plus delta square is it. So, this is, whenever you apply the Taguchi method and in the process you apply the Taguchi method you believe in the concept of the target value for the given quality characteristics and you are asked to carry out the process capability analysis. So, which particular index you should apply, that is the Taguchi capability index and the basis is this one.

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So, I conclude this one. So, all sorts of you know the capability indices we have discussed and the later on in the next lecture will be discussing the other aspects of process probability analysis.