Quality Design and Control
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Lecture – 25
Statistical Process Control – II (Contd.)

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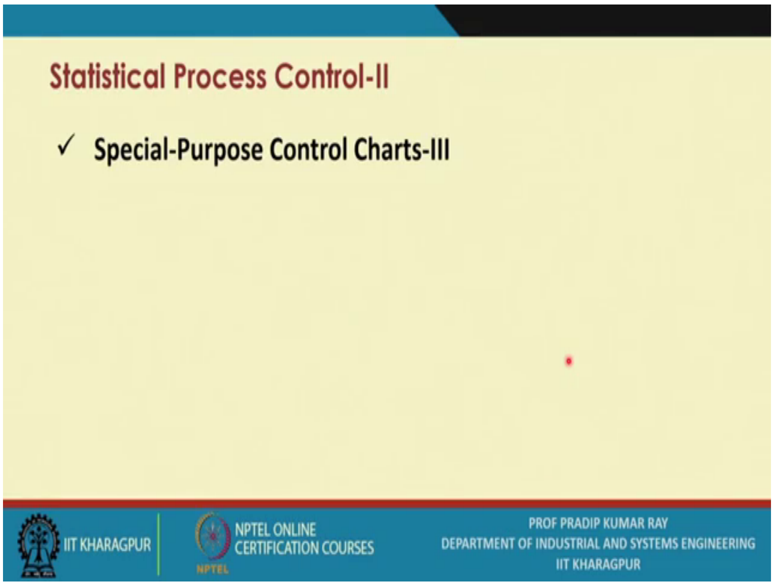

So this session I will be continuing with the cusum control charts.

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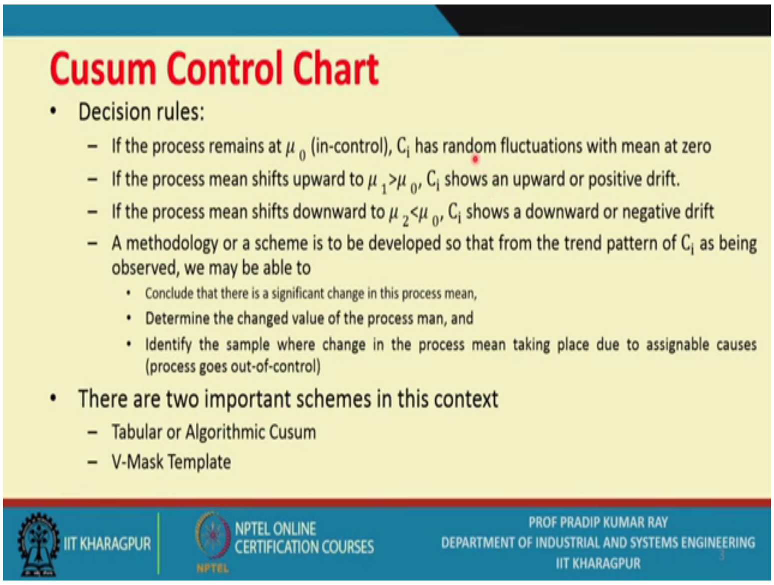

As I have already mentioned that you know when you use a cusum control chart you need to use certain decision rules. So, what are these decisions rules? If suppose the target value of the process mean is mu 0 and if you find that the C i that is a cumulative sum at the ith sample point the way you have computed has a random fluctuations with mean at 0. So, this will be your observation when the process is in control at mu 0 if the process has gone out of control and there is an upward value upward trend and reaches the mu 1 then C i must show an upward or positive drift.

Similarly, if there is you know the process mean shifts downward; that means, the C i must show a downward or a negative drift is it. Now, there are two important schemes you have already have before two what are these two schemes someone because the first one is referred to the tabular or algorithm cusum and the second one is the v mask template.

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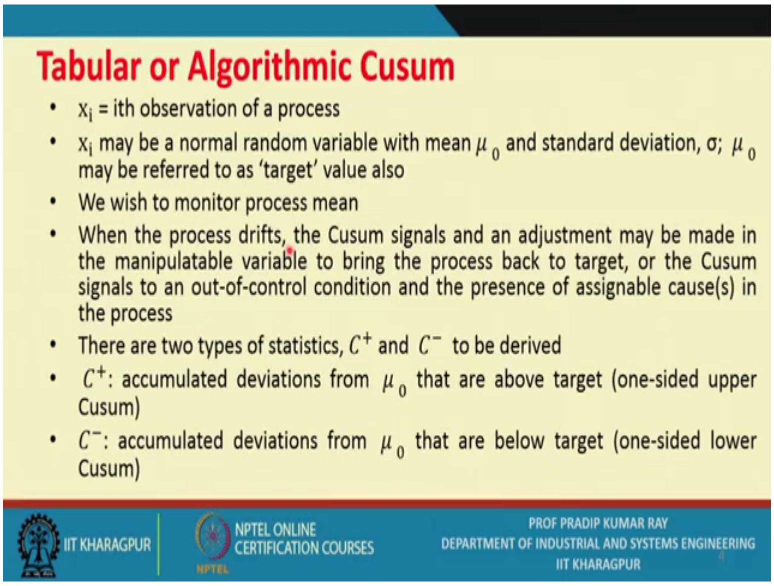

Now, let me explain the details of the tabular or algorithmic cusum, is it clear. So, this is widely used this particular the scheme you can use. So, let me explain it what are the steps involved. There are certain steps involved in order to use this particular scheme.

Supposing x i is the ith observations of a process; obviously, with respect to a particular quality characteristic, x i may be a normal random variable is it in many cases you know that there is a physical basis of assuming that the output of a particular process is normal. So, supposing x is output for a particular process, output are the response variable. So, you may assume it to be normal random variable with mean mu 0 and the standard deviation sigma, is it clear. So, whenever you have the data with you. So, you can have an estimate is it of mu 0 and an estimate of sigma mu 0 may be the referred to as the target value is it. So, the mu 0 is your target value that is the mean.

We use to monitor the process mean that is all right; that means, you need to use a control chart for controlling the process mean. So, that is a very important assignment; that means, whenever you start using a control chart you must specify the specific you know the process parameter is it ok. So, in this case is the process mean.

When the process drifts; that means, when the process drifts you assume that the we do many reasons there could be you know the change in the process mean values the cusum signals and an adjustment may be made; that means, what is the signaling; that means, if you use the cusum control chart; that means, and at a certain point in time suppose the process mean there is a change in the process mean. Now, this cusum will be sending a signal and looking at the signal you may assume that the process has gone out of control.

Now, what you do that means, you want to run the process had a particular value that is mu 0. So, these is (Refer Time: 04:36) going to you. So, what do you need to do; that means, an adjustment may be made in the manipulatable variable is it ok, it is the typical u know the chemical industries is it ok. So, there is a manipulative variable to bring the process back to target; that means, you have a thorough knowledge about the process. That means, what are the factors affecting the value of the output variable is it ok.

So, and then what you try to do if suppose you do not get the value of the output is it ok, at the target value has a target. So, you must be able to identify the manipulatable variable is it ok and you change you try to change the this manipulatable variable that is the input variables in such a way that the process is brought back to the target value or. The cusum signals to an out of control condition and the press presence of assignable causes in the process, is it clear.

So, this as you observe that when you start plotting the C i values over the samples. So, you will have this sort of observations. Now, what do you try to do; that means, while you compute C i now there could be positive deviations there could be negative deviations from the target is it depending on what is the actual value that you are getting in a sample. So, accordingly you know you can form two types of statistics the notation is C plus superscript and C minus superscript.

So, this is basically the positive deviations it indicate a positive deviation and this is the negative deviations. So, what is C plus? Accumulated deviations from mu 0 that are above target, sometimes this is referred to as one sided apart cusum its clear. So, what is C plus this is the one side of our cusum. What is it basically? Accumulated deviations from mu 0 that are above target is clear. What is C minus? Super super script minus accumulated deviations from mu 0 that are below target; that means, some of the values they will be greater than mu 0 and some other values in x is it they will be less than mu 0 is it ok.

So, you create C plus statistic one of these conditions this is referred to as one sided upper cusum and again you create you have this C minus statistic which is referred to as one sided lower cusum is it. So, it is written down into 2.

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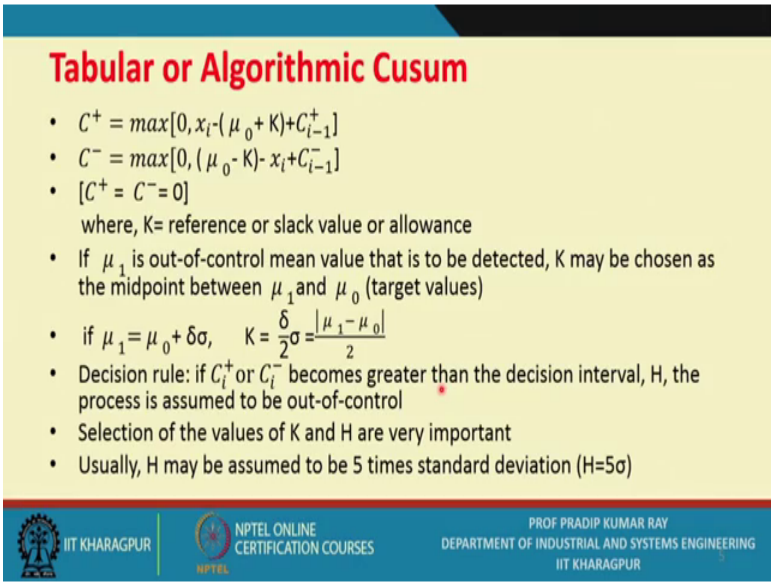

Now, next what do you do; that means, you develop a scheme. So, you define C plus as maximum 0 x i is the actual value minus the mu 0; that means, x is greater than mu 0 and what is k; that means, you add these allowance is it. So, that is very very important right. That means, before you know; that means, some allowances is to be given. So, that the process is remains under control it is it may be heading towards an out of control situation, but before you reach there you must be able to take some preventive measures of corrective measures is it because once it crosses the beyond the target value; that means, already you know it has gone out of control is it. So, you cannot take any corrective measures is it ok.

So, as we have been telling you that this is an online a real time control is it ok. So obviously, there must be some allowances. So, this key is essentially the allowance that you have to specify and this is for the, this is for the ith one is it. So, this is for the i C plus i and this is related to C i minus 1. So, this is subscript will be C i is it. So, you write it here i and similarly the negative deviation; that means, C i minus is it which is related to the previous one that is C i minus 1, is it clear. So, this is 0; that means, if it is negative suppose for some reasons it is negative obviously, it is made 0.

Similarly, here; that means, first mu 0 minus K that allows you subtract and then x i value is always be expected is less than mu 0; that means, those values of x i which are less than mu 0. So, that those values form a different set is it set of values. So, this is C i minus equals to maximum 0 of this one; that means, this is if this is positive if this is negative it is converted into 0 its clear. So, either 0 or positive value that is to be ensured is it ok.

So, please go through these equations I am sure that you will be able to understand this one; that means, ith value is related to i minus 1th value similarly i minus 1th value is related to i minus 2eth value is it ok. So, this way all these you know the individual values individual sample values are related is it ok.

Now, what is K? I have already mentioned K is the reference or the slag value or the allowances. So, 3 names are given; that means, K is refer to the reference value or it is sometimes referred to is the slag value or it is referred to that allowances. If mu i mu 1, I have already I know while I explaining the decision rules I refer to say the mu 1.

So, mu 1 is out of control mean value; that means, you have specified. Suppose the mean value is originally when the process is controlled suppose the mean is 10 and suppose the sigma is one sigma is 1, now what we are assuming. Now, that the sigma remains same suppose we are assuming that the sigma does not change, but there is a possibility that the mu may change. So, suppose the mu changes to from 10 to 11 is it ok. So, that is the value of mu 1. And you say that if it changes to 11 the process is assumed to be out of control is it there is a upward trend. And similarly you say that if the value of mu changes to 9 and this is considered to be a significant change the sigma remains same, but I will say that even mu equals to 9 is not acceptable, but this is a the downward you know our the drift in the process that is why it has become 9 and that may be another value of say the mean 2, is it ok.

So, mu 1 is the out of control mean value that is to be detected. So, you must have a thorough knowledge about that about the data that you generate. K may be chosen as the midpoint between mu 1 and mu 0 that is the norms unless otherwise specified. So, it is a mid value that is considered to the allowance. That means, as soon as the value you know where the crosses the midpoint the process is in control, but you be careful about the process condition and you have to you must know immediately that why it has crossed this midpoint is it is there any is it heading towards the value called mu 1 and then the processes we will be going to out of control condition and then; obviously, you know there could be you know unexpected bad performance from the process. So, that is to be avoided.

So, the target value is mu 0 and out of control mean value is mu 1. So, mu 1 is equals to mu 0 plus delta into sigma is it and this is the deviation in terms of sigma I have already mentioned that the deviation is always expressed in terms of sigma. So, what is the value of K? Obviously, delta by 2 into sigma; that means, mu 1 minus mu 0 that is the difference between mu 1 and mu 0 the absolute value and half of that is it.

So, what is a decision rule? That means, with the given set of data you need to compute C i plus or C i minus. What is i? i is the sample one; that means, suppose you have some 20s of sample points or 30 sub sample points and say you have to start from i equals to 1, you have to calculate C i plus C i minus both by using this the two formulae and then you continue this exercise is it; that means, from the first period you go to the next period the time wise; that means, in each period you get one sample that is the assumption. So, the time wise you do.

So, what is the decision rule? If C i plus or C i minus becomes greater than the decision interval each this is very very important; that means, there is you are controlling the cumulative sum of the deviations from the target at a particular point in time is it and it is continuing is it ok. So, as you are taking the cumulative deviation community values so; obviously, even if there is a small shift now at a certain as it is a cumulative calculation.

So, that is why you know even if this was a small shift maybe you know maybe signaled as a prominent one, as a prominent one at a particular value of i or a sample point.

So, this is the basic idea and then you check at a certain point in time or at a certain for a certain sample whether this the C i plus or C i minus is greater than H and if you find that this condition holds then immediately you are shown that the process is assumed to be out of control, is it clear. So, with the given data set what do you need to do you need to calculate for any sample i, C i plus as well as C i minus and there must be some initialization condition and you assume that the C i plus and the C i; that means, this is C say 0 plus and C 0 minus will be will give 0.

So, this is C 0 plus is equals to C 0 minus will be 0 is it; that means, our starting is 0 or you know you may have some initialization condition if it is known so that you can add is it ok, that value we may consider not nearly 0, but usually it is 0. So, at certain value of i is a particular sample point you use these conditions. So, poses is assumed to be out of control otherwise you say that the process remains in control selection of the values of K and H and very important it is obvious in fact. That means, as such there is no mathematical formula for determination of K and H, but the certain you know, based on your experience if you refer to several case studies on this. So, the researchers they have prescribed certain value is it certain expressions for K certain expression for H, is it ok.

So, at the initial stage why did not you assume those values, for K and H right see usually H may be assumed to be 5 times standard deviation. So, this is you know this value; that means, H may be assumed 5 times sigma is it look accumulative right. So, if this is the norms. So, there must be some basis; that means, researchers based on that experience based on their investigation they say that this is these assumptions you know may be assumed to be valid in most of the processes. So, H is equals to 5 into sigma. So, what we are assuming at this stage that the sigma does not change the sigma remains constant; that means, you have taken the steps adequate steps for controlling the variability and that is the precondition always you said.

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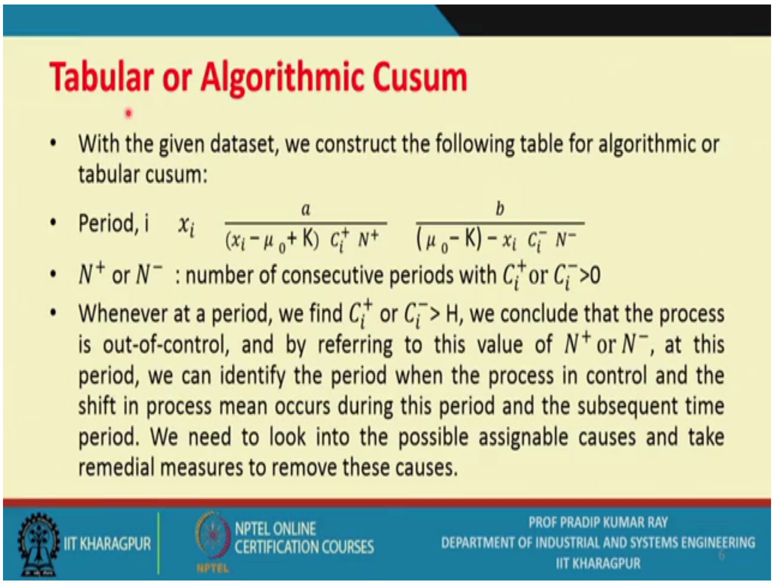

So, just make a note that whenever we carry out such exercises related to control charting. So, the first conditions you have to meet the whether you are you are able to control the variability. As I have been always telling you that any exercises or quality is essentially is a quality on ready; in an exercise on variability and exercise on consistency is it clear.

So, once this you know the necessary condition is met; obviously, you know you will be you are in a position to use such a scheme called algorithmic cusum. So, with the given data set, so every data set what is the data set; that means, related to a particular sample or related to a particular time period you have all these x values. And what is this x? x refers to a particular quality characteristics how why you are referring to a quality characteristics because you know we are referring to a process and this process is essentially you use to produce this quality characteristics. So, quality of statistics is; obviously, inbuilt in the component or the part. So, so that part processing or the component processing you do using that particular process and for that particular process you are going to create these this cusum control chart.

With a given data set we construct the following table these are actually the table format I am not showing the table later on when we take up for the numerical problems is it we will you we will refer to the table format. So, in the table what you have you have are the several columns. So, for using say algorithmic or tabular cusum what you have to do you have to create this table. So, in the first column you refer to the period that is i. In the next column that is the second column you have the corresponding value of the quality characteristics; that means, for the ith sample or the ith period is it what is the value of x. So, x is a variable.

Now, you have you know there are what you need to do; that means, against x i you need to calculate you know the C i plus as well as C i minus.

So, against a C i plus you have 3 columns is it. So, this is part small a and against C against you know say C i minus is it that also you are going to compute against x i you have 3 columns we have is it and this is a subset of the columns they refer to as the b; that means, you have one section a and another section b. So, under section a what you compute actually? Mu 0 will be known x i value is known, so against a particular period for a given period you can calculate x i minus mu 0 and K will be specified how do you specify the k; that means, you have defined mu 1 is it mu 0 is known, so is the midpoint between mu 1 and mu 0, unless otherwise specified. So, that is the value of K and the key is that allowance at the slack value right. So, this is the first column you have is it against each value of x i have to compute this when K is known when mu 0 is known.

Then C i plus; that means, what is the cumulative value. So, suppose the first value is 0.5 and the next value is say you calculate say 0.5 again. So, at the first stage; that means, when I equals to 1 it will be 0.5 and the next day it will be 0.5 plus 0.5; that means, n plus means; that means, it indicates it is basically a counter, counter of what; that means, counter of the positive deviations.

So, suppose the deviation is positive; that means, it will be 1, now the next day suppose the deviation is again positive so it will be 2, at the third stage suppose it becomes 0 is it suppose the third stage you get the value of x i such a value that you know it becomes negative and negative is converted to 0, so obviously, again you know this the counter; that means, cumulative will go; that means, third point it will be 0 this is it. So, again from the fourth the sample point all over four onwards of the fourth time period onwards you need to the count the n plus values.

So, this is the first part and similarly when you need to compute the C i minus. So, C i minus mu 0 is known; that means, it is the negative deviation and the negative deviation is converted into to the positive one like this. So, x 1 x i is on likelihood is the less than mu 0. So, you subtract minus k; that means, suppose this is 0.5 this is say say say the 10. So, 10 minus 0.5 that will be 9.5 minus the actual value is it whereas, if it will be ok. So, the actual value minus say 10.5, so the actual value will be will be you know we will subtract 10.5 from x i whereas, here the actual value will be subtracted from 9.5 and cumulatively you do and similarly you will have this cumulative value that is C i minus is it ok, and then the number of counters. So, number of the times you will have the positive values is it in the deviation 0.

So, this particular table you have to construct is it again I am telling you that when we take up a numerical examples. So, this table will be constructed with respect to a given problem numerical problem. So, what is n plus or n minus? Number of consecutive periods with C i plus or C i minus greater than 0; that means, the positive values. Are you getting my point? So, as soon as it becomes negative it you know at that point in time it becomes 0 this is converted into 0 the way we use the formula we had derived the formula we use that formula and then again you know you start from 0; that means, the chain is broken.

Whenever at a period now at this point you please go through the first thing is that whenever at a period say time t is it or say you know say the ith time period ith sample we find C i plus or C i minus like C i plus and C i minus greater than H. What is H? H is 5 times sigma. What is sigma? Sigma is the process standard deviation. Now, if these conditions at a particular you know the sample at a particular time period you observed this condition holds we conclude that the process is out of control. So, that is the norm we follow.

And by referring to this value of n plus or n minus; that means, how many you know consecutive time periods you are observing positive deviation; are getting my point. So, that is very very important how many consecutive time periods that is n plus or n minus. Now, when you refer to this value say 5 or 6 if it is 5; that means, it means that 5 consecutive time periods you have positive deviations and as you are dealing with the cumulative sum; obviously, these over the time periods for which you get the positive values these values will be increasing.

Now, at a certain point it starts increasing and if it crosses the value of H immediately you conclude at that point in time that the process has gone out of control. So, that time you conclude, but the question is that the train has already occurred; that means, it is not necessarily that when you observe or when you conclude that the process has gone out of control at that point in time or that sample you know you are observing it is not necessarily mean that out of control condition has occurred at the same the sample point.

So, already you know the out of control condition has occurred some you know you know the previous samples. So, you need to identify that particular sample or that particular time period need to identify where possibly the out of control condition has occurred and then they do existence of some assignable causes. So, that particular sample point you have to identify. So, by referring to this value of n plus or n minus we can identify the period when the process is in control. So, this is the point to be noted you have to identify. It is like you know you are referring to the plus data and you just did not check that at a particular sample or at a particular time period something has occurred in the process behind the same and you have to identify those possible in other causes.

So, the process is in control and the shift in process mean occurs during this period and the subsequent time period is it; that means, what has happened suppose you find that on say 20th sample point is it ok, this is the value of C i plus has crossed say each value. Now, you check that n plus you find that the value of n plus is 5; that means, what do you conclude that 20 minus 5; that means, a 15 point. Possibly in the 15 point that something has gone wrong and this you know this the process has gone out of control and is continuing is it. So, between, the 14th and the 15th samples up to 14 there is it is an out of control point, but the 15th sample point onwards you may assume that the process at 15th point onwards the process is out of control.

So, something has happened between say the 14th point and the 15th point. So, you have to refer to the past data and you have to find out as a process engineer as a process control engineer say are executed that what has gone wrong is it between the 14th period and the 15th period. So, we need to look into the possible assignable causes and take remedial measures to remove these causes.

So, this is the reason and what we have observed that the tabular algorithmic cusum is a very reliable tool and you know the way and the scheme is developed you know you will come to know 3 things whether the process is in control or not. Second thing is that even if the shift magnitude is very very small and that normally this happens for a sophisticated process very quickly will be able to detect that that is the second you know. So, the aspect we must look into and the third one is, that you will observe at this point in time that the process has gone out of control.

But at what point in time actually there is out of control condition has set in. So, you will be able to identify that particular you know the time period as well as the sample point so that you know the kinds of actions you take kinds of you know the remedial measures or sometimes the preventive measures you take to eliminate or the assignable causes in the process. So, that will be very very you know the specific and the solutions will be getting that is highly reliable is it because exactness will be there in your solutions approach.

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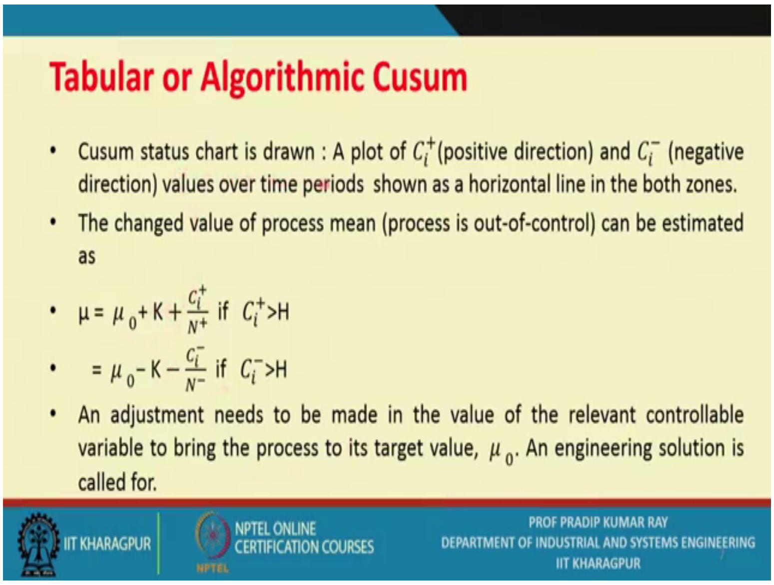

So, cusum status chart is normally is plotted; that means, a plot of C i plus and C i minus, this is the positive deviation and this is the negative deviation on the positive direction and the negative direction values over time period shown as a horizontal line in both the zones; that means, you have a horizontal line that is a 0. So, positive deviations are shown, but with C i plus and the negative deviation that is C i minus over the time period. So, this is basically refer to as the cusum status chart.

So, the changed value of the process mean can be estimated like, so this is the change value. So, it was mu 0 then the allowances C i plus divided by n plus. So, if C i plus is greater than H and mu 0 minus k; that means, on the other side is it ok. That means, this reflects the upward drift and these reflects the downward drift is it. So, in both the cases you will find that the C i minus or C i plus is greater than H and adjustment needs to be made in the value of the relevant controllable variable to bring the process to its target value musical.

So, whatever may be the current value mu 1 it is not or mu 2 it is not acceptable to you, it you must be able to bring it back to the original value on the target value that is the mu 0. So, in many cases once you refer to the case study you will find that to bring back the value to mu 0 you need so the engineering aspects you look into; that means, an engineering solution is called for.

So, with this I conclude my discussions on algorithmic cusum. So, the other charts we will be discussing in subsequent sessions.