Quality Design and Control

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Lecture – 24
Statistical Process Control – II (Contd.)

Continuing our discussion on the special purpose control charts.

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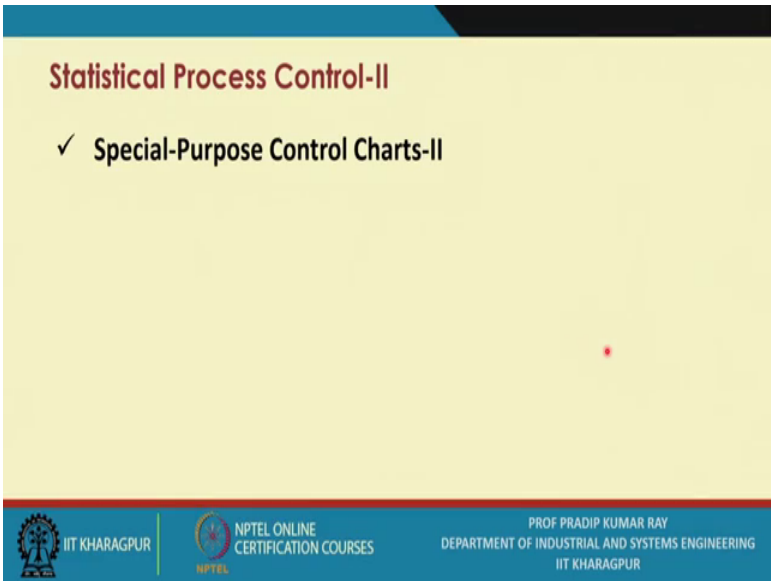

We have already refer to modified control chart if you recollect.

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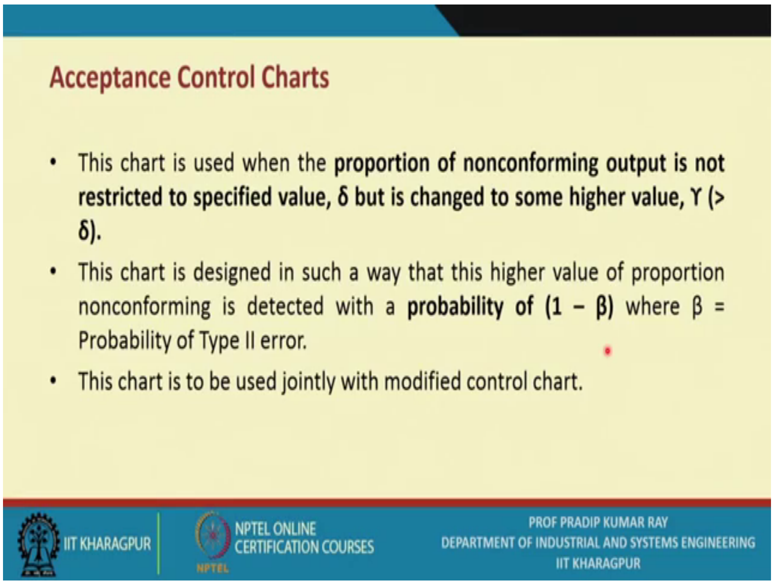

And what we have mentioned that the modified control chart is to be used along with the acceptance control charts. Like, we say that r control chart, and x bar control chart must be use simultaneously. And similarly, we also say that the modified control chart, and acceptance control charts. These 2 chart must be used simultaneously. There is a specific reason for this one. And we will come to know what is the, that reason when I discuss the acceptance control charts.

Now, there are if you just mention that there is there are ah, there are 2 conditions to be satisfied. The first condition is you need to determine the upper limit as well as the lower limit of the process plane in such a way that the proportion and confirm is restricted to delta; that is, the first condition we have to satisfy. And for that you need to use the modified control chart. Now the second condition that you frequent may frequently come across; that is, these the proportion and conforming which we have set as a standard. Say, 1 percent, 0.5 percent, is it ok? So, that is the set; that is, and the value of delta. There is no guarantee that in an actual process at that point in time there could be take many reasons, that this actual proportion nonconforming may be greater than, substantially greater than the delta, which is to be specified.

So, what do you do? Obviously, you need to use another control chart, which will help you in detecting this you know unacceptable or the proportional conforming this amount as quickly as possible. So, initially you design the modified control chart with respect to the proportional conforming delta; which is acceptable to you. Next what you will do? That means, if the delta changes to gamma; that means, say 1 percent, it was it is delta is 1 percent. Suppose because of some reason, you are getting a value of say 3 percent proportion non-conforming.

So, whether you know you are able to detect this change in the proportional conforming as quickly as possible. So, for this you need to use acceptance control charts. So, I think it is clear that you know, what is the specification, that is why the acceptance for these acceptance control chart is used. Now this so, this point I will elaborated, when I say that this chart is used when the proportion non-conform proportion of non-conforming output is not restricted to specified value delta.

If you remember, that when we design when we design the modified control chart the modified control chart is designed with respect to a specified value delta, is it ok? But because of some reason suppose the delta is changed to some higher value, say gamma. Gamma is substantially greater than delta. So, the delta is 1, say 1 percent and the gamma maybe 3 percent. Now these chart is designed in such a way that these higher value of proportion nonconforming is detected with a probability of 1 minus beta. Beta as we know these are probability of making type 2 error; that means, essentially probability of non-detection.

Obviously 1 minus beta is probability of detection. So, that is to be assured; that means, a high value of probability of detection is to be assured, is it? These chart is to be used jointly with a modified control chart. These points already are elaborated.

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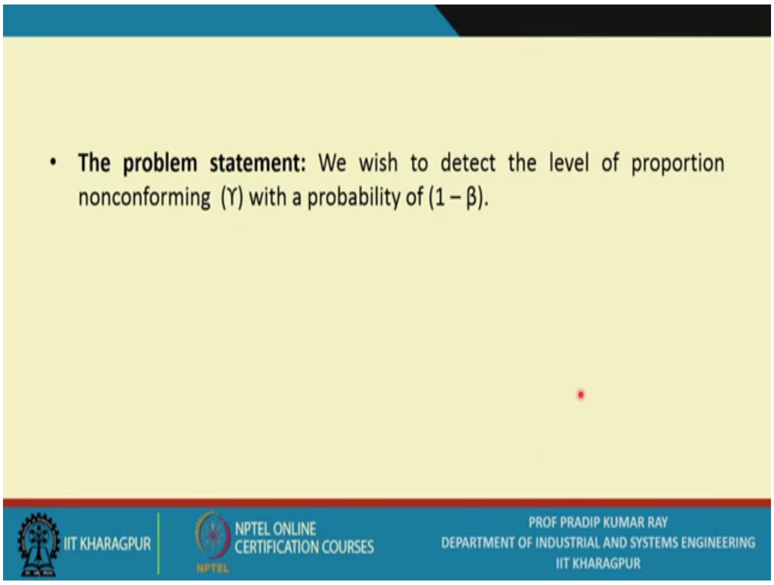

So, what is the problem statement? Problem statement is very simple. We use to detect the level of proportion nonconforming gamma with probability probability of 1 minus beta, is it ok? So, if the detection probability and make sure that whenever the professional nonconforming becomes gamma, ok? It should be detected as quickly as possible; that means, make sure that the average (Refer Time: 05:26) if you recollect we have use the term we have defined what is a r l; that means, on an average how many sample points you are going to plot before and out of control condition is reached, is it ok is detected.

Now; obviously, if the process has already gone out of control so, that error should be as minimum as possible, say 3 to 4 or say 2 to 3 sample points as in quikly as possible. Whether of suppose you know the process remains in control. So, the (Refer Time: 06:04) should be you know, as early as possible (Refer Time: 06:10). So, here in this case has you are defining the out of control condition with respect to the proportion of non-conforming. And with a specific value of proportion of non-conforming gamma, we say that the process has gone out of control. So obviously, the a r l value should be as minimum as possible?

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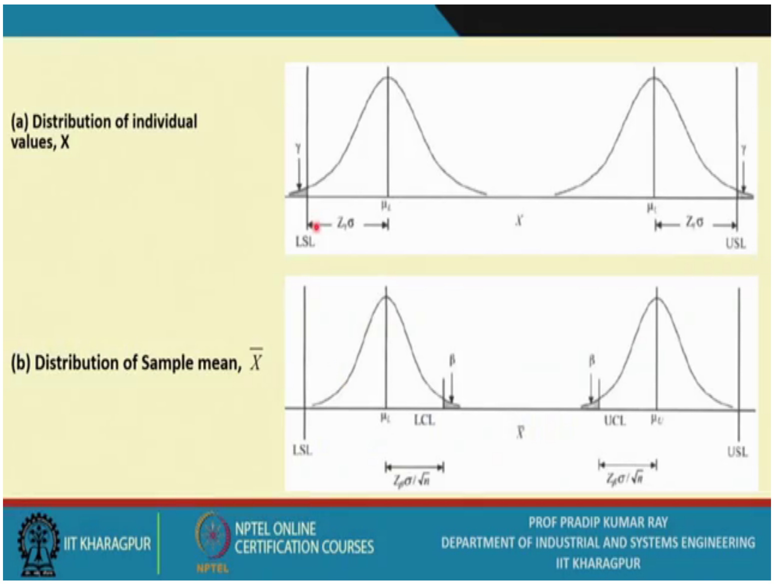

Now, again we referring to the say distribution of individual values, that is x. So, again what we are assuming now that already the mu l, and mu u. These 2 values are specified against the process mean, is it ok? And your this is the upper value. This is the lower value, is it ok? Now you know this is the distribution of x and so, the corresponding value of the standard deviation, suppose standard deviation is sigma and if the proportion of non-conforming gamma, which is not acceptable to your. So, this (Refer Time: 07:19) distance is z gamma into sigma, is it ok?

So, this is the distance from mu l to USL. USL is already specified for the given quality characteristics for which already you have drawn, you have been using the modified control chart. And similarly, on the other side this is the area under the curve of beyond LSL that is gamma, is it ok? And this is a standardized distance; that is, z gamma into sigma ok. Distance of mu l from LSL is, it is ok. So, we are referring to the distribution of individual values, and we are assuming that the central limit theorem holds, that is why the distribution of x is assumed to be normal.

When you refer to the distribution of the sample mean that is x bar; that means, around mu l around mu u; obviously, you know as per central limit theorem. So, this is the standard deviation, we will be sigma upon root n that man’s sigma z bar is equals to sigma upon root n. And this we refer to the distribution of x bar. So now, you have the say l LCL is here. And UCL is here. So, even if you know the value is at mu l, what do you find that there is an area, which is within this LCL within the UCL this zone.

So, obviously, this is a proportion of say this is the proportion indicative of the probability of making type 2 error; that means, the process has gone out of control, but actually it is shown as in control, is it ok? So, this is a type 2 error. Similarly, when the upper value is mu u ok, and the process mean, now the area which is within the upper control limit that is beta, is it ok? So, again this is when you reach at this at this point. So, you have this area that is this indicative of probability of making type 2 error.

So; obviously, the standard size distance from UCL to mu u is z 2 beta z beta into sigma upon root n. And similarly, the standardized distance of LCL from mu l is z beta z beta into sigma upon root n, is it clear? So, we are referring to distribution of the individual values x. And then we are referring to the distribution of the sample mean x bar, is it clear?

So, we are defined what is USL, what is LSL. We have defined what is gamma. We have defined what is beta; we have you know defined what is UCL and what is LCL. Now what you try to do; that means, ultimately in terms of all this parameters ok, you need to have an expression for UCL in terms of USL. And you must have an expression of LCL in terms of LSL; that means what you are trying to do, we are trying to explore the relationship between the control limits as and the specification limits, is it ok?

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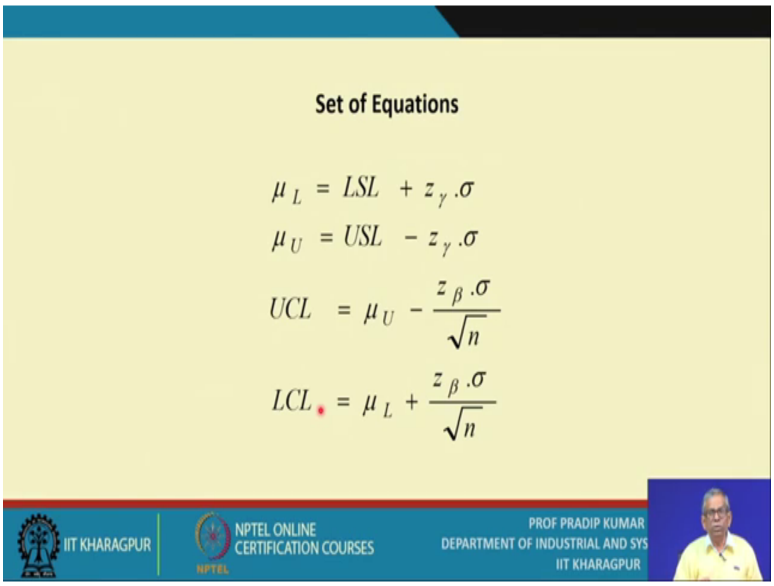

So, and then you know; obviously, you know whenever you are referring to say the delta or gamma, the proportion of non-conforming. So, these are you know this proportion nonconforming is always defined with respect to in reference to other specification limits. And so, we are going one step further; that means, till now we have not refer to the specification limits, is it ok in the control chart.

Now, here we are determining the control charts in such a way that that the relationship between the control limits and the specification limits are so, given the problem. Another important aspect we must look into; that is the specification limit is always you know applicable to individual values. Whereas, the control limits not necessarily accepting, when you draw the control chart for the individual units, that is x control chart. And along with the x control chart if you remember, we have the m r or moving range control chart, is it ok?

So, accepting you know those that that particular case, otherwise always we will find that the sample size is greater than 1, and in the sample is represented by it is statistics. So, x bar is it ok? So, so, whenever you refer to the distribution of say the process the parameter, say the process mean; that means, the sample size is greater than one.

So, we refer to the distribution of x bar, is it clear? Whereas, whenever you refer to the specification limit, we are referring to the distribution of x of the individual values. Now what are the set of equations, we if we refer to this figure, what you will find that the mu l is equals to LSL plus z gamma into sigma. We refer to the first figure; that means the institution of x for the individual values. And similarly, you get mu u is equals to USL minus z gamma into sigma, is it ok?

Now, we must have an expression for mu l, and mu u from the second figure; that means, you refer to the next figure, we will find the upper control limit is mu u minus z beta, what is beta? Beta is the probability of making type 2 errors into sigma divided by root n. What is n? N is sample size. Similarly, LCL you refer to the same figure you will find that the LCL is equals to mu l plus z beta into sigma divided by root over n. How do you get this sigma upon root n is basically sigma x bar? And as for the central limit theorem, you can say that sigma x bar is nothing but sigma upon root n.

So, these 2 equations, we are referring to the distribution of individual values. Whereas, these 2 equations, for this equation, we are referring to the distribution of x bar is it?

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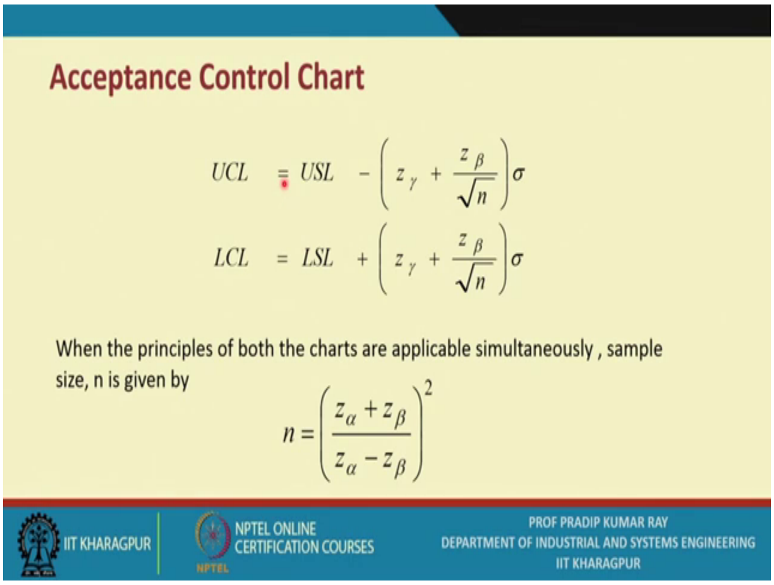

So, whenever. So, related to the acceptance control charts. So, what is the upper control limit? is equals to upper specification limit minus these expression into; that means, z gamma plus z beta upon root n into sigma, is it ok?

So, sigma is essentially the process standard deviation. Similarly, the LCL is equals to LSL plus z gamma plus z beta divided by root over n into sigma. Now again how do you determine sigma? So, that would be good estimate, acceptable estimate; that is, you referred to r bar control r control chart or s control chart. And you say that the sigma, if you refer to the r r control chart, is it ok? So, you can have an estimate of sigma, and that is sigma hat is equals to r bar by d 2. And similarly, if you instead of constructing that instead of using r chart suppose you use s chart.

So, similarly. So, related to say the s chart, you have you know the value of c 4 and as well as the s bar. So, another you know the expression for sigma, it is an estimate of sigma is say s bar divided by c 4, is it ok? So, so, whenever you declare a process to be in control; that means we say that the first I try to control the process with respect to it is variability. So, if the variability is under control, then I use the control chart, another control chart in such a way, that I say that the mean or the process mean is under control, is it ok?

So, so, the so; obviously, you know when you assume that the process is in statistical control, as in all likelihood the value of sigma; that is, estimate of sigma is made available. Now when has a we have been saying that are modified control chart must be used along with the acceptance control chart. So, when the principles of both the charts are applicable simultaneously, reading my point? So, in the first case, we know that, what is the mu u, what is the value of mu l; we know what is the distribution of x.

We also know what is the distribution of x bar. We know given the quality characteristics USL and LSL. And then we say that the given control chart there is a probability of making type one error, is it ok? And then there is a probability of making type 2 error; when the proportion of non-conforming changes say any value from say the delta, is it ok? So, then we say that under these 4 verses 6 conditions or 4 or 5 conditions, we have to meet all these conditions simultaneously.

So; obviously, the modified control chart must be used along with the acceptance contract chart. So, that is the norm. So, when so, under in this context we say the principles of both the charts are applicable simultaneously, is it clear? I think it is clear. And simple in sample size n it given by this one is it clear? That means what you try to do; that means, UCL for the acceptance control chart the expression for UCL is equated weight the expression for UCL for the modified control chart, is it ok?

So, either you equate both these expressions. And get the expressions for n, is it ok? Sample size or you can say that I will equate LCL expression of acceptance control charts with the LCL expressions for the modified control chart. So, again you will be getting an expression for n. So, what is the expression for say sample size when you combine both these charts? That is z alpha plus z beta, divided by; this is z 1, this is not z alpha by a beta, there is some problem.

So, this is z alpha plus z beta into say you have z delta z delta gamma minus z delta. So, it is z delta z gamma minus z delta, is it clear ok? So, so we will we will make this changes later one.

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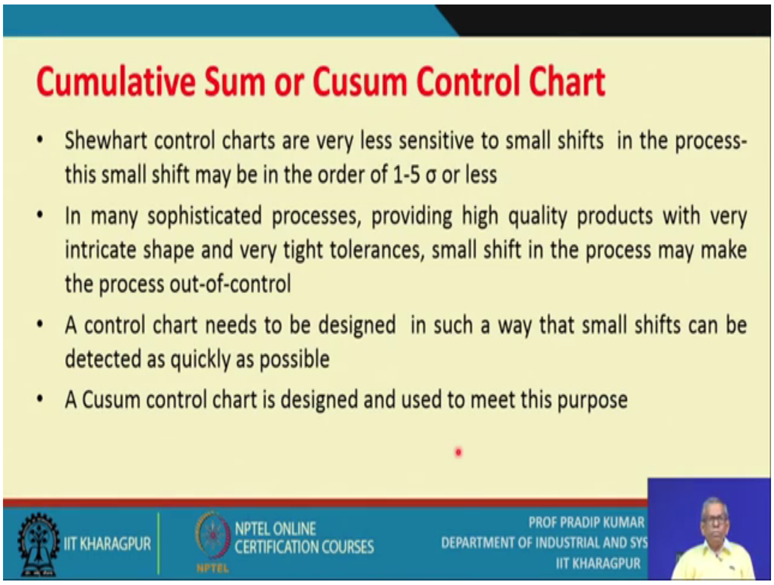

Now so, this is the special purpose say we are discussing the special purpose control chart, right. Now as I have already told you, there are there are variety of special purpose control charts So, so the 3 specific control charts we are used till now.

First one is the regression control chart, or the trend control chart that is the first one. The second one that we have discussed that is the modified control chart and the third one is the acceptance control charts. Now the next important control chart. We are going to use that is the cumulative sum or cusum control chart, is it ok? Now I will before I discuss in detail, the cumulative sum of control chart. let me say highlight some of the important points related to control charting.

As you are aware that are the basic theory behind control charting was introduced many years back in 1920's, and by Walter (Refer Time: 21:14). So, that is why I know many of these control charts are named after him. And their effect has Shewhart control chart. So, the Shewhart control charts are very less sensitive to small shift in the process. These small shift maybe, in the order of 1 to 1 0.5 sigma or less, is it ok? It is not one one to it is 1.5 sigma or less, is it ok? That means, when the sigma of the process is wrong, that is the first exercise you carry out before you start using a control chart; that means, given a quality characteristics given a process, is it ok? And it is running process. So, you must know with respect to the quality characteristics, what is the value of sigma? And this sigma with respect to the quality characteristics may be referred to as the process standard deviation, and estimate of the process standard deviation.

Now, what we are trying to do; that means, we are using control chart that is the first objective, what extends we are able to control the variability. So, the variability; that means, the sigma. That is also random variable. So, and so, if the process mean changes; suppose you need to control the process mean. So, the change in the process mean; that means, shift magnitude many a time, it is it is expressed in terms of sigma; that means, what you are assuming that in a given process, both mu and sigma may change. And their essentially, they are changing simultaneously ok.

Now, we need to define the small shift as well as the large shift, is it ok? Large shift are basically you know more than 1.5 sigma. That is the norm, or say more than 2 sigma, is it ok? That is the norms. So, what the researchers have found, that the Shewhart control chart is very, very sensitive to a small to large shift, is it ok? Whereas, it may not be that sensitive ok, when the shift magnitude is very, very small; that means, less than 1.5 sigma.

In many sophisticated processes, providing high quality products with very intricate shapes, and very tight tolerances like say you know automobile parts or the aircraft parts, is it ok? So, small shift in the process may make the process out of control, is it ok? So, if there is a small shift in the process parameter in the process; I mean process parameters ah, you say that the process has gone out of control. Because very tight tolerances you not be able to achieve.

So, the control chart is special purpose control charts need to be designed in such a case; that means, you design the control chart, you re design the control chart existing control chart in such a way. That even we have the this the small shift in the process parameter value can be detected quickly ok. So, a control chart needs to be designed in such a way that the small shifts can be detected as quickly as possible ok.

So, the cusum control chart is designed and used to make this purposes, is it ok? Now so, these are very special purpose control chart. And I will just explain. What are the steps to be followed ok. And what is the rational behind following the steps ok, in developing a cusum control chart, is it ok?

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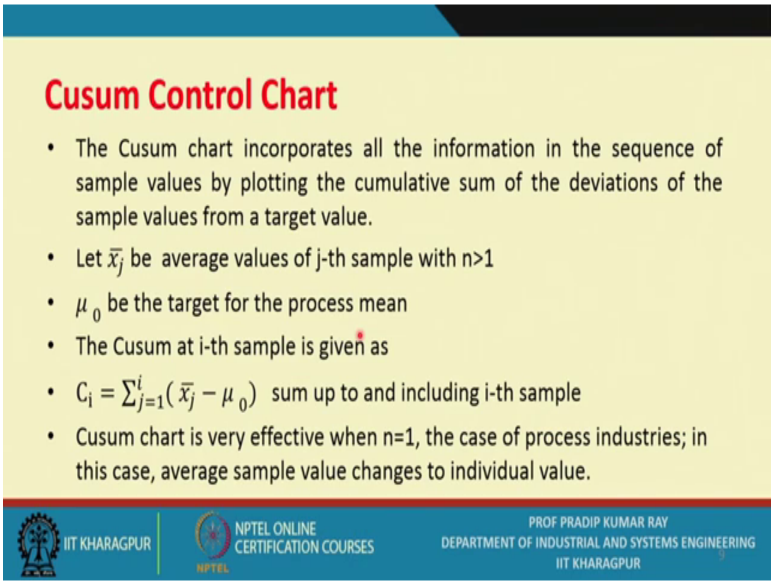

Now what do you try to do in the in the previous in previous cases? Like, when you use the Shewhart control charts, what do you assume that all the samples they are independent with one other; that means the sample values are not dependent, is it ok? And at any point in time at a particular when you draw a sample and at a particular point in time you are you know you are using a sample statistics. And you plot the value of the sample statistic, is it ok? That means, you know the value you plot is indicative of a condition at that point in time, at that during that period of time, is it ok?.

So, as got a no relationship with the previous conditions, getting my points now, what we are trying to do; that means, you try to incorporate all the information in the sequence of sample values by plotting the cumulative sum of the deviations of the sample values from a target value; that means, here is here is a process I have designed. Now with respect to the process parameter ok, given process parameter, I must know what is this target value?

Now, the actual value when you get at a particular sample. So, you know that, what is this you know the deviation from the actual value? So, that is a particular sample, what you need to do when you arrive at a particular sample, now you need to consider for all the previous samples the deviations from the targets. And you have to you know the add them together at a particular point in time. So, the cusum charts incorporates, all the information in the sequence of sample values; that means, (Refer Time: 27:57) is to be maintained that I have already told you, that when you construct a control chart make sure that the there the samples are drawn ok, as per you know the time sequence.

By plotting the cumulative sum of the deviations of the sample values from a target value; that means, at a particular sample ok, when I when I define the deviation, now this deviation incorporates all the deviations that that have already occurred in the previous samples, is it ok? From the first sample, just I will explain it means how do you arrive this community value. Let x j bar be the average value average value of the jth sample with n equals to n greater than 1.

So, this is the average value, not values ok. So, just one value; that means, you have n number of values in a particular sample jth sample you compute the average value. Mu 0 be the target for the process mean as already mentioned. Now how do you compute the cusum? So, the cusum at the ith sample; that means, how do you arrive at the ith sample; that means, you have started from the first sample second sample third sample like this you proceed, and ultimately you reach the ith sample, is it clear?

So, the I could be 1, I could be 2, I could be 3, is it ok? So, I is the variable. So, at the ith sample, what is how do you compute the cumulative the sum; that is, from j equals to 1 to i up to i, including i you compute the x j bar minus mu 0; that means, j equals to 1 to i, is it ok? That means, up to I including I th point. Some up to I mean including ith sample, is it clear? I think it is clear, when is this is you are the cumulative sum of the deviations sum of the deviations from the target value up to the ith sample point and including the ith sample, is it ok?

So, the cusum chart is very effective when n equals to 1, as you when you will be referring to that the control chart for the individual units; that means, x chart ok, you know what are the for under which conditions you have no other alternative, but to go for you know, using x chart is it ok. So, the sample size is very, very less ok. So, . So, that is one. So, the so, in many cases we will find, like if you deal with say the process industries, is it ok? The case of n equals to 1 is very, very common.

So, in this case average sample values changes to the individual value; that means, there will be just x j not x j bar, is it ok? So, what you know this cumulative sum control chart maybe recommended to be used for say the process industries, and whenever say is a chemical plant or the chemical industries ok, as an example. So, you will find that you need to assume that n equals to 1, is it ok? And accordingly you define the cumulative sum, is it ok?

So, the first thing you have to do that is cumulative say the sum you have to defined with respect to the given the process parameter, is it ok? And for which target value is to be specified.

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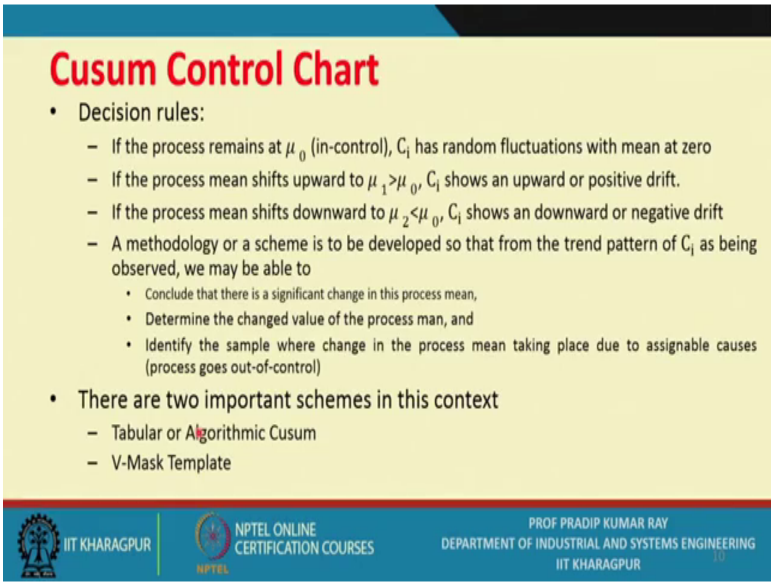

Now so, just what I am telling you certain decision rules, let me just highlight, and then in the next sections, I will be discussing the other aspects of cusum chart.

So, what is the decision rules that is to be known. That is if the process remains at mu 0; that means, again control states that is the target value. C i has random fluctuations with mean at 0. Always you know the x i values is there will be randomness; that means, there is randomness only when the process is in control. So, obviously, the mean at 0, is it ok? If the process mean shifts upward to say mu 1 which is greater than mu 0 substantially greater than mu 0 c I shows an upward positive drift and similarly if the process mean shifts downward to mu 2, which is substantially less than mu 0, c i shows a downward a downward or negative drift, is it ?

A methodology or a scheme is to be developed. So, that from the trend pattern of c i as being observed we may be able to conclude that there is a significant change in the process mean determine the changed value of the process mean, is it ok? That you have to do and identify the sample, where the change in the process mean taking place due to assignable causes; that means, at what point in time for which sample the process has gone out of control. So, that particular the sample you have to detect . And you refer back to that particular sample, is it ok? And you just like check the process conditions, and whether you are able to identify you know the assignable causes related to out of control conditions.

Now, here whenever you talk about cusum control charts, the researchers have developed 2 approaches, is it ok? In fact, there could be other approaches, but this 2 approaches are always refer to. So, you must be aware of in. In fact, on the cusum chart till lots of research going on, is it ok? So, the from. So, in perfect the systems you may be going to a perfect system, is it ok? But the 2 important schemes they refer to the scheme or the approach for the cusum control chart, you must be you must be discussing. And that is first one is the tabular or algorithmic cusum, this is the scheme you need to use. And the next one is the v mask template. So, in the next sections, I am going to discuss these 2 schemes.