Quality Design and Control

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Lecture – 23
Statistical Process Control – II (Contd.)

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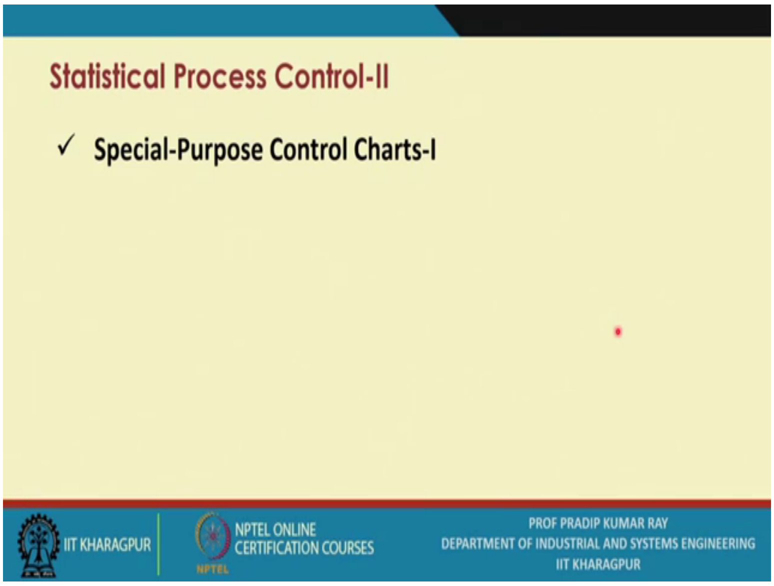

In this session, I am going to discuss under statistical process control special purpose control charts, there are many varieties of a special purpose control charts.

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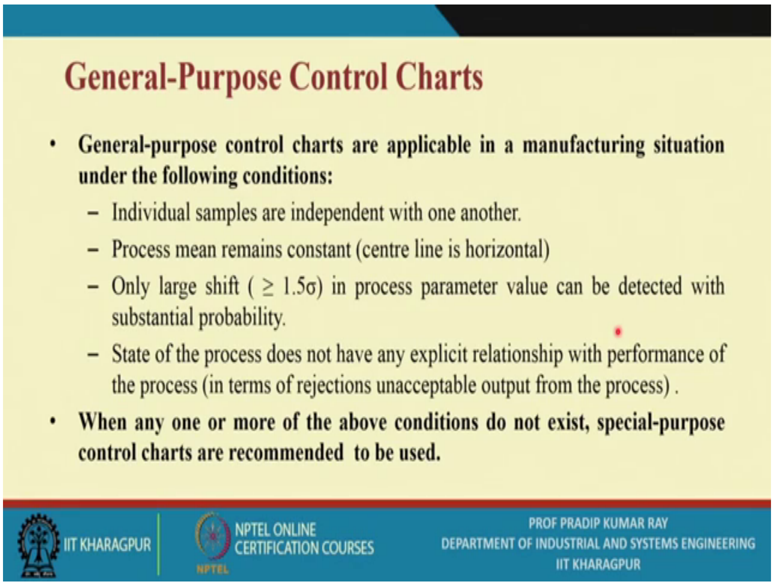

And before I discuss in detail as some of the important special purpose control charts, let me highlight some of the important aspects while we use general purpose control charts.

You are already aware of that what are the general purpose control charts under variables category and in fact, all these general purpose control charts are applicable in a manufacturing situation under the following conditions. So, if you please note then down like here for whenever you use a general purpose control chart we assume that the individual sample points are independent with one another.

In other words, we assume that the samples which you are where required to draw in order of you know maintaining the time order over the time periods, they are all independent with the one another; that means, the value you get in a particular sample is a no way it affects the values of values of values in the subsequent samples or the previous samples process mean remains constant central line is horizontal.

This is another important assumptions we make the third one is only the last shift and you know if the process standard deviation is assumed to be sigma; that means, the last shift, sometimes, we defined as a shift with magnitude of 1.5 sigma or greater than that maybe 2 sigma may be 3 sigma. So, always the shift in the process parameter he is defined in terms of the sigma.

So, only last shift in process parameter value can be detected with substantial probability as I have already mentioned that there are the 2 specific purpose of control charting the first one is with the help of the control chart a specific control chart, you will come to know at any given point in time whether the process has gone out of control this is this is your first purposes and the second purposes is if there is a shift in the process parameter value the control chart must be able to detected, it as early as possible as quickly as possible.

Now, in many cases, if you use the general purposes con troll charts or this we called shear control charts we will find that this is it is performance is very good as far as meeting the first objective is concerned, but in respect of meeting the second objective it is you know its performance may not be that acceptable.

So, the state of the process does not have any explicit relationship with the performance of the process; that means, what we try to do in many a time we will using the control chart, but then again you know in majority of the cases we say that the performance of a process is acceptable only when you are able to control the rejections in the process output to substantial or to a reasonable level if we are able to control.

Now, whenever you use control chart as such there is no relationship explicit relationship of the state of the process; that means, whether it is a in control state or out of control state with the performance of the process in other words you we may assume that the process is in control it does not necessarily mean that we will not be producing on acceptable output when any one or more of the above conditions do not exist special purpose control charts are recommended to be used.

So, I think it is made very very clear that if someone ask you that under what conditions special purpose control chart is to be used. So, we will able to highlight the special conditions.

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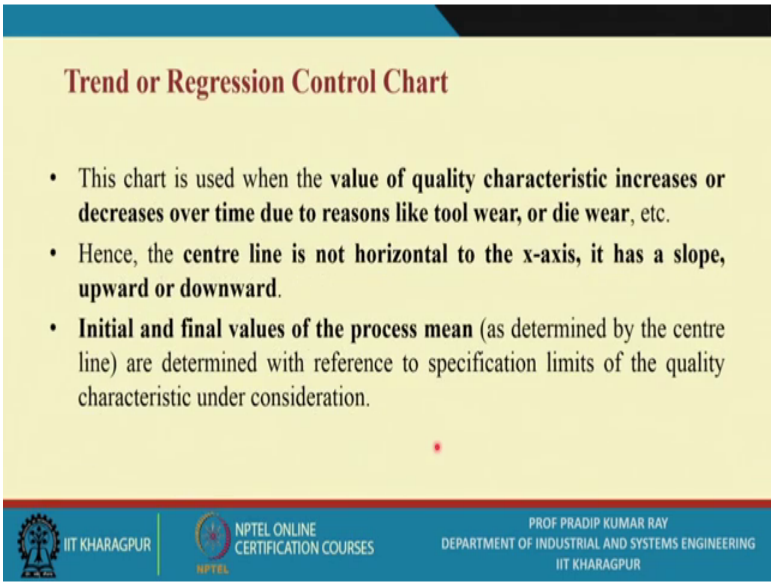

Now, given that this the conditions and which are the general purpose, control chart in the control chart to be used. Now suppose in these conditions, you are unable to make it is not under your control. So, what to do and you also you assume that the control chart is a viable tool is an important tool for process control under a special conditions.

So, what you need to do; that means, you have to judge a special condition and as per the special condition, you come across, you need to use as a type of control chart which is referred to as special purpose control chart. Now there are the researcher and the practitioners, they have already developed a number of such special purpose control charts; I am going to discuss a few of them.

The first one; I am going to discuss that is referred to as the trend or the regression control chart is it. So, first thing you must know that under what conditions these; this type of control chart is recommended to be used this chart is used when the value of the quality characteristics increases or decreases over time due to reasons like tool wear die wear, etcetera ok.

So, there are many cases. So, till now, we have assumed that the central line is a horizontal like, but in case, there is a tool wear or in case there is a die wear, what do if you assume that the central line is horizontal as for your assumptions may not be correct. Hence, the central line is not horizontal the x axis, it has a slope upward or downward is it. Now if you if you want to incorporate this aspect in the control chart. So, what you need to do; that means, you recommend a special purposes control chart called trend or the regression control chart.

Now, initial and final value of the process means it together there is a starting value there is an ending value as determined by the central line now these values are determined with refer reference to the specification limits of the quality characteristics under consideration; that means, whenever you draw a control charts a control chart is drawn with respect to a specific quality characteristics.

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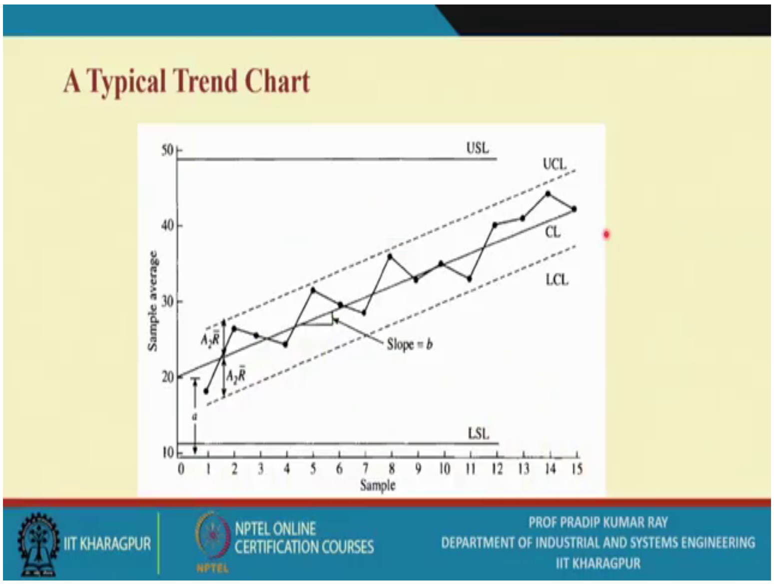

So, initial value and the final value of the process mean you need to specify you must know. So, this is a typical the trend chart let me explain it; that means, here the horizontal line no this is the central line it is not horizontal. So, it has got a slope the slope is small v is it is the slope and with the horizontal; obviously, and it has got an intercept and intercept denoted as small a is it; now what you have in a typical these 3 sigma control limits and so, what you have you have the upper control limit and the lower control limit.

So, the upper control limit is placed at 3 sigma plus 3 sigma from the central line and similarly, the lower control limit is placed at minus 3 sigma from say the central line and you are already aware of that in R control chart is it or in x bar control chart you are always expose 2 and if you remember in the x bar control charts we have the upper control limit that is placed that from the central line a 2 into R bar distance is it or the one side and the other side you know the LCL is placed from the central line at a distance of A2R bar.

So, here it is not a horizontal line it is the central line is having a slope of b and interest of a with the y axis that is sample average by representing the sample average values and. So, the upper control limit and the lower control limit you can easily you can determine is it when this distance is known A2R bar or A2R bar on the other side. Now you must specify for the given quality characteristics say what is its upper specification limit and what is its lower specification limit that this is a chart where a control chart where you know the upper control limit lower control limit the central line ok.

So, this 3 are compared actually with upper specification limit as well as the lower specification limit is it?

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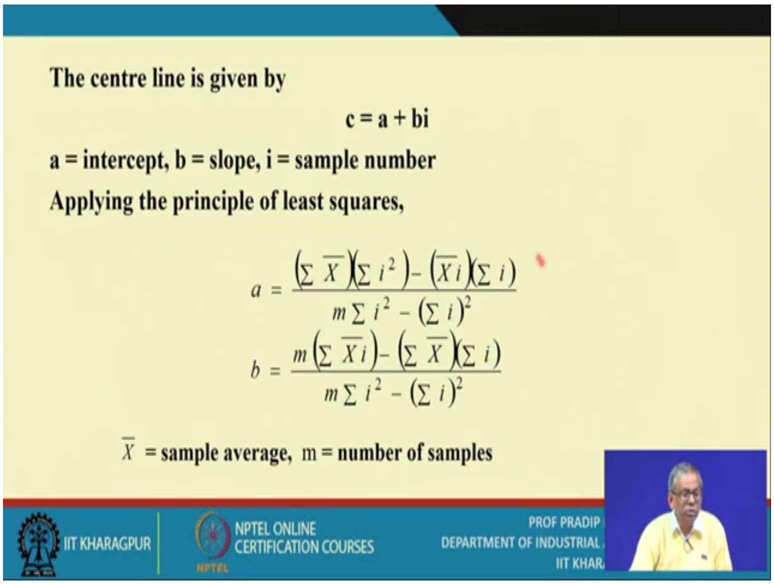

So, you must know the location of upper specification limit as well as the lower specification limit; now this is the central line is given by this equation c equals to a plus b i where a is equals to intercept is very simple b is the slope and i is the sample number ok.

So, now you are aware of how to apply the principle of this square; that means, that means fitted value you determine in such a way that that the deviation from the mean all the square of the deviation the mean from the fitted value is it is a minimum. So, this is the principle of least squares. So, this is the expressions where you can derive it I do not need to derive this expression this comes from applying the principle of least squares. So, the intercept a is given by this by this formula and the slope the b is given by this formula is it ok.

So, I suggest that you refer to typical textbooks on the regression on regression analysis linear regression. So, you will get this expressions for a as well as for b x bar is the sample average m is the number of samples. So, whenever you use a sigma; that means, sigma from i equals to say m i equals to m to i equals to i from i equals to one to m is it ok.

So, x i bar actually this is x i bar x i x x bar an i all these values will be proper notations we have.

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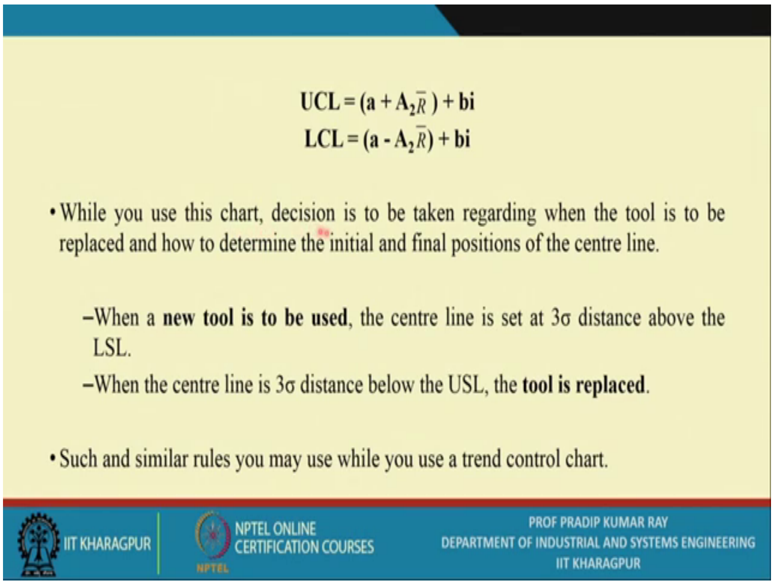

So, this is the expression of; so, what will be the upper control limit and the lower control limit for the regression control chart that is a plus A2R bar plus b i and a minus A2R bar plus b i if you refer to this figure, we will come to know the location of the upper control limit and the lower control limit is it ok.

So, is upper control limit and lower control limit, there L parallel to the the central line and the central line is having a slope of b where you use this chart decision is to take is to be taken regarding when the tool is to be replaced and how to determine the initial and final positions of the central line. So, there is a slope so; obviously, you know what is happening actually you knows d 2 tool wear or the die wear; obviously, you know the horizontal there cannot be horizontal line is always you know the central line you have a slope that there is a starting point there is an ending point; that means, it depends on what is the life of the tool; that means, when you start with a new tool. So, you get the initial value and when you replace the tool. So, you will get the final value of the central line.

So, when a new to use to be used the central line is set at 3 sigma distance above the LSL is it. So, so there must be certain norms there must be certain rules. So, what is that rule; that means, whenever say the central. So, the point this is the central line starts; that means, referring to the first sample. So, whenever you get the first sample the, you are using the new tool fresh. So, the new tool is to be used when the central line is set at 3 sigma distance above the LSL.

So, this one; we follow and when the central line is 3 sigma distance below the USL the tool is replaced is it. So, this rule we follow please refer to this particular control chart and we will find that this is made very very clear certain similar rules you may use while you use a trend control chart ok.

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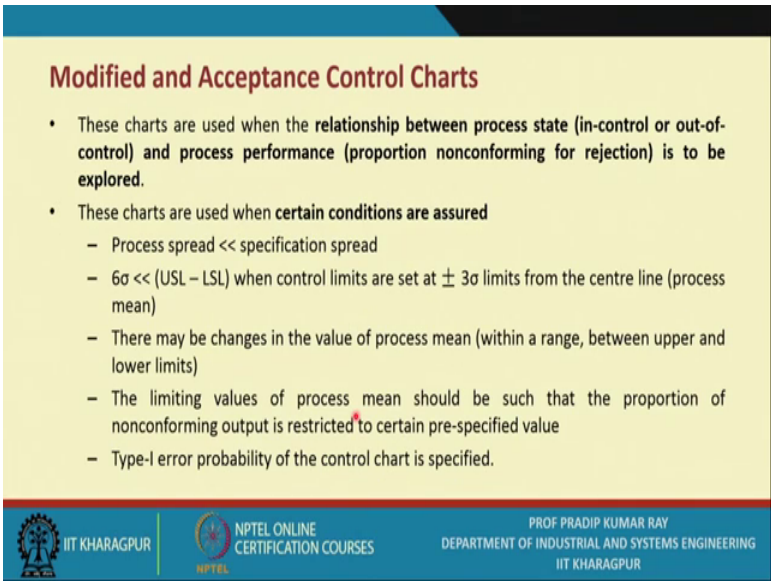

So, this is in short say the specific features of a trend control chart this is also referred to as a regression control chart and whenever you assume that the central line cannot be we say horizontal line horizontal tool the x axis and due to tool wear or the die wear and it is a very common phenomenal in manufacturing situations.

So; obviously, you have to use a regression chart or the derivation control chart or the trend control chart is it clear and so, I will said the numerical problems later on right and; obviously, when the concept is known when the control chart you know related to a particular control chart when the formulations of say upper control limit lower control limit at the central line these are known; so, with respect to a typical. So, the numerical problems will be again discussing this type of control chart.

Now, the next set of say control charts, I will be using under the special purposes category that is modified and acceptance control charts till you know till now you might have observed that while you use a control chart there is no reference to the specification limits or you know in the say related to quality characteristics you know the identifications or the reference to the specification limit is very very important because we say the process is in control or process as gone out of control this is one aspects and we hope that if the process is in control; that means, the process behavior the process performance will be very very consistent and we will and in course of time will have enough control substantial control on the process parameter value and if you have say substantial control on the process parameter value what is the expectation. Expectation is the process performance will be excellent; that means, the number of say non conforming say the units will produced as per the specifications the number will be very very less.

So, but here in this case what you try to do is; that means, first you use a modified control chart and you check that whether your perform performance of the control chart is such that the expected number of non conforming say the units is within control that is the first condition you must arrive at and when you arrive these conditions next you check that because of some reasons suppose there is some say the access the number of non conforming units from the process which we are not going to accept whether the control chart is able to detect that as an out of control condition or you say that it must have say substantial or say you know substantial value of beta ok.

So, as a first we used the modified control chart and then you referred to that certain control charts and. In fact, both this control charts must be used simultaneously now certain remarks, I have made related to this 2 types of control chart the first point is that this charts are used when the relationship between the process state you know; what is it either in control state or out of control state is it all the process condition.

So, when the relationship between the process states and the process performance how do you define process performance process proportion nonconforming or rejection. So, this is to be explored is it to what extend the state of the process is responsible for producing for producing say nonconforming say the output this charts are used when certain conditions are assured is it; it is not that at any point in time the process is newly installed and suddenly you start using this modified and accepted control charts.

So, it may not be a good decision and so, before you start using this modified and acceptance control charts as certain conditions have to be made what are these conditions the process spread should be substantially less than the specifications spread. So, what is the process spread process spread is basically the distance between the upper natural tolerance limit and lower natural tolerance limit; that means, if you refer to many a time you assume that upper natural tolerance limit for a given process is at a distance of plus 3 sigma from the mean of the process and similarly the lower natural tolerance limit is at a distance of minus sigma from the mean of the process is it ok.

Now, the now the distance between the upper natural tolerance limit and lower natural tolerance limit. So, in most cases you know we assume that if you know it is upper natural tolerance limit you may assume it to be at plus 3 sigma from the mu and the lower natural tolerance limit is a is at minus 3 sigma from the mu so; obviously, you know the process various you know the 3 plus to sigma on 1 side and plus 3 sigma on the other side. So, total is 6 sigma is it 6 into sigma and what is the specification spread.

Specification spread depends on the type of quality characteristics. So, if it is quality characteristics quality characteristics and where you know the double specifications limits are valid then it is the distance between the upper specification limit and the lower specification limit and if you deal with the physical characteristics. So, it may be upper tolerance limit minus the lower tolerance limit; is it ok.

So, the process spread should be substantially less than the specification spread second condition is I have already explained what is this 6 into sigma is it should be you know substantially less than USL minus LSL when the control limit are set at plus minus 3 sigma limits from the central line; that means, the process mean central line actually indicates the process mean.

So, this part already I have explained there may be changes in the value of the process mean within which is another way within a range between upper and lower limits; that means, suppose the mean is hundred. So, the process mean is assumed to be hundred that is a mu where mu itself may be a random variable; that means, depending on the condition of the process or if there is the drifting in the process. So, you say that the process mean I will allow in order to have an acceptable performance from the process. So, between say 80 to 60, 60 or say between 80 to 120 and on an average on I will assume it to be around 100 is it ok.

So, you have you set the upper limit you set the lower limit of the process mean is it. So, this is you have to specify this values; that means, your knowledge about the process should be substantial and a given a particular quality characteristics given a process you must be able to you know the specify the upper the value of the process mean as well as the lower value.

Now, the type one error probability of the control chart is specified that is type one error I have already mentioned; that means, the process is assumed to be say under is gone out of control, but actually it is in control. So, that is basically the type one error; now what is this type one error probability that is alpha.

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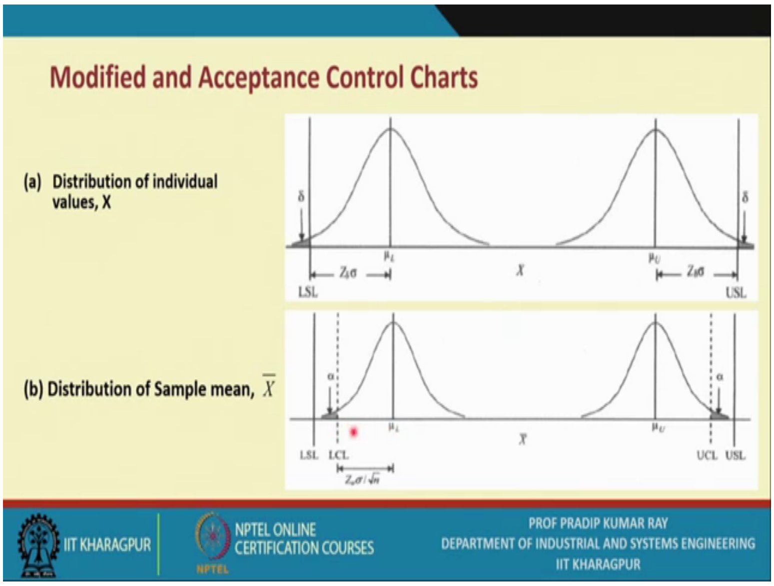

So, you know the distribution of individual values this is here distribution of the individual values is it ok.

I am assumed I am assuming that that individual the x values is normally distributed and that is the case in a running production system in a spectra of say particular quality characteristics. So, what do you find here; that means, X will this value; that means, there will be mean somewhere here is it. So, the mean value is a lowest value you assume somewhere here and the highest value of mean is the upper mu U and mu L is it ok.

Now, there is a shifting now what how do you specify the upper limit as well as the lower limit of the process mean such that that the proportion nonconforming is restricted to delta getting my point. So, suppose you have say the process, right, now over here like say suppose upward drift is it. So, ultimately you know it will be all these values will be changing to the higher sides these values and whenever you know it reaches this values say it becomes mu u, you will find that these proportions is it beyond upper specification limit this is the proportion one conforming.

Now, you determine the value of mu U in such a way that the proportion one conforming is restricted to delta and similarly suppose there is a downward drift; that means, on this side the mean value may move and ultimately also the lower the lowest value you determine that is mu L in such a way that the proportion one conforming; that means, area under the curve beyond LSL they restricted to delta.

So, delta you specified and you need to determine under this conditions what could be my expressions for upper control limit and the lower control limit so; obviously, you know you must know the what is the distribution of the sample mean that is x bar is it. So, sorry try to do now you have the lower limit for mu that is mu L you have the upper limit is it for the mu that is mu U and then you consider the distribution of x bar is it. So, here what we are assuming when you consider the individual value x. So, this is you know the standard deviation is sigma whereas, here the standard deviation will be sigma upon root n as per central limit theorem is it ok.

So; obviously, you know you specify this is the lower control limit and you specify this as the upper control limit is it now whenever you draw a control chart there as to be you know there will be always on the probability of making type say one error. So, this probability is alpha. So, here this area under this curve this is referred to as alpha and similarly this area under this curve; that means, when you know the mean value is at mu L; obviously, this area is alpha, it is beyond LCL. Now what is this distance; that means, and the distance is z alpha z alpha is it into sigma upon root n.

Similarly, this distance will be z alpha into sigma upon root n is it ok.

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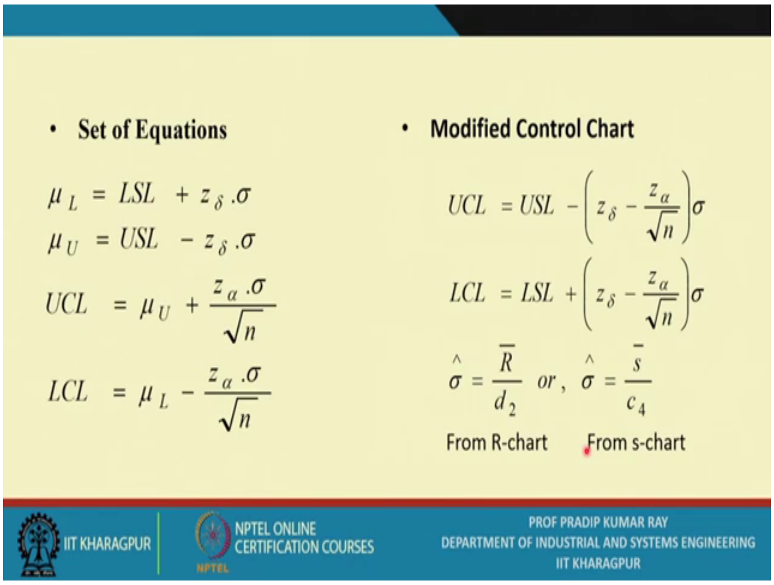

So, now what you try to do; that means, you write down the set of equation. So, if you just refer to this or the 2 figures you will find that mu value is LSL plus z delta into sigma. Similarly mu U is USL minus z delta into sigma is it and so, the UCL is ultimately mu U plus z alpha into sigma by root n and LCL is mu L minus z alpha sigma upon root n.

So, you have ultimately when you consider the both these sets of equation. So, you have the upper control limit is equals to upper specification limit minus these expression into sigma and the lower control limit is equals to LSL plus z delta minus z alpha divided by root n is the sample size into sigma now this sigma is basically the process standard deviation. So, if you already use R control chart R control chart so; obviously, R bar will be known d 2 is also known d 2 is a function of n. So, you can have an estimate of sigma R.

So, if you use R bar R control chart instead suppose you use s control chart; obviously, you know s bar will be known is it and the c four that value is again use a function of say n of the sample size. So, you get the value of c four and you get an estimate of sigma r. So, from this is this you get from the R chart and this you get from the s chart. Now before I close this section. So, just I want to highlight one important points; that means, this modified control chart you use only when it is assured that the process is in statistical control.

So, this is to be assured and how do you know that the process is in control as I have already mentioned that if the process spread is less than substantially less than the specification spread and the process is centered that is very important point. So, please note it down that sometimes you know we seek a condition called centering of the process what is centering of the process; that means, the process mean is coinciding with nominal value or the target value of the quality characteristics is it; that means, you say that mu is approximately equal to is almost same as m; m is the nominal value nominal value means desired value.

So, suppose you have a 2 sided specification in this case so; obviously, you know m is assumed to be the midpoint between the USL and LSL in other words USL plus LSL divided by 2. So, this centering the process this condition is to be arrived at and then you check that when the process spread is substantially less than the specification spread. So, if these 2 conditions are assured and then yes you can use the modified control chart one of the most important the special purpose control charts we use is it. So, other control charts like say acceptance control charts and. So, the Cusum control charts. So, we will be use in the subsequent section.