**Quality Design and Control**
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**Lecture – 22**
**Statistical Process Control – II (Contd.)**

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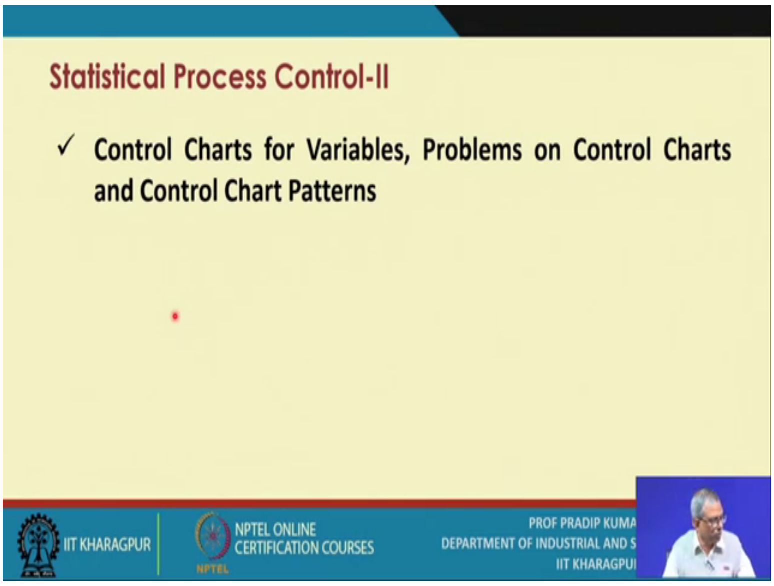


So, during this lecture session under statistical process control, I will be again referring to the control charts for the variables certain problems on one control charts. Particularly, I will take up one numerical problem related to R and X bar charts I will discuss it thoroughly and then, obviously, you know an important issue we must discuss that is the control chart patterns. So, this three issues we will be discussing this one.

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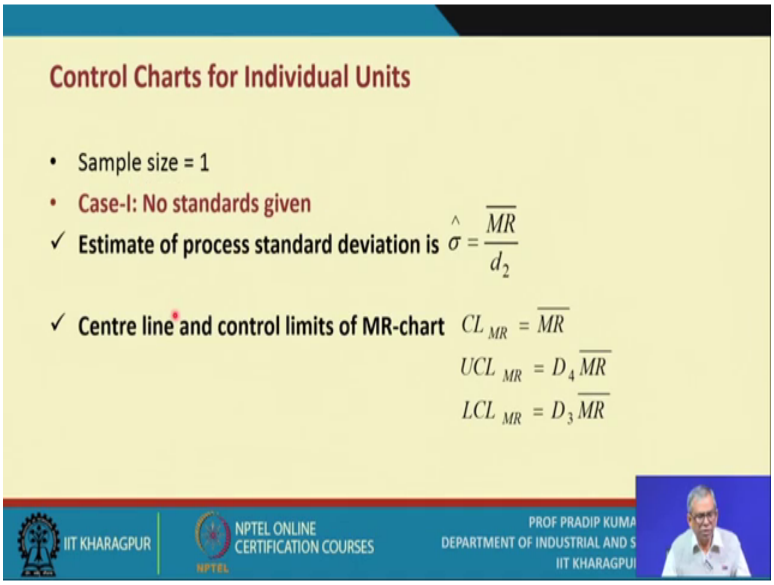


Now, continuing our discussion and control chart for the variables, I have already mentioned that whenever the sample size is 1 and what you try to do, that means, you have no other way, but to construct a control chart for the individual units. Now, you must be wondering that why the sample size is 1.

Now, there are many reasons some of the reasons you must be aware of that is you know in many cases when you go for inspecting the item essentially, what you are doing you know getting a sample and then individual units you are inspecting. Sometimes you have to adopt destructive testing; that means, in order to get the data you have to either destroy the unit. So, the destructive testing is very common in many cases for measuring the values of the quality characteristics like say tensile strength many cases. So, here what you try to do, that means, you say that the sample size should be as many on as possible and ideal size is 1.

The second reason could be like, say the production rate is very low; that means, in one shift you just get 1 unit. So, it all depends on what is your the manufacturing process time. So, that may be the reason that why the sample size may be 1. So, when a sample size is 1, now as usual there could be two cases and you are advised to construct the control chart you are advised to use that control chart. So, in the case one like in the previous case that no standards are given.

So, if no standards are given then what is the estimate of the process standard deviation the process standard deviation is given by MR bar divided by d 2. d 2 I have already explained, what is MR? That means, what you try to do; that means, suppose the 20 such the data points you have collected, now, in each sample you have just one data point. So, obviously, in each sample you cannot calculate say the dispersion. So, as soon as you get the second sample; that means, the next data point you get now you compare between this two so that is referred to as the moving range.

So, if you have 20 data points or the 20 samples now, the moving range we will have 19 moving range and what we are assuming that, you know the second data point is maybe the first moving range second moving range third moving range up to say n minus one at moving range they are essentially dependent because there are some common the data points everywhere. But, we are assuming at the initial stage that this you know this correlation between the successive moving ranger is this correlation is not that significant is it ok and as you know that this MR bar by d 2 are these expression may be valid under some distributional assumption.

So, once this is valid then center line and control limits for the MR chart you can immediately you can determine like the central line for MR or the moving range control chart is a MR bar that always you can calculate and then upper control limit for a MR is D4 MR bar it was like in the in the previous case when I you construct the R control chart and similarly the lower control limit for a MR, D3 that is D3 into MR bar.

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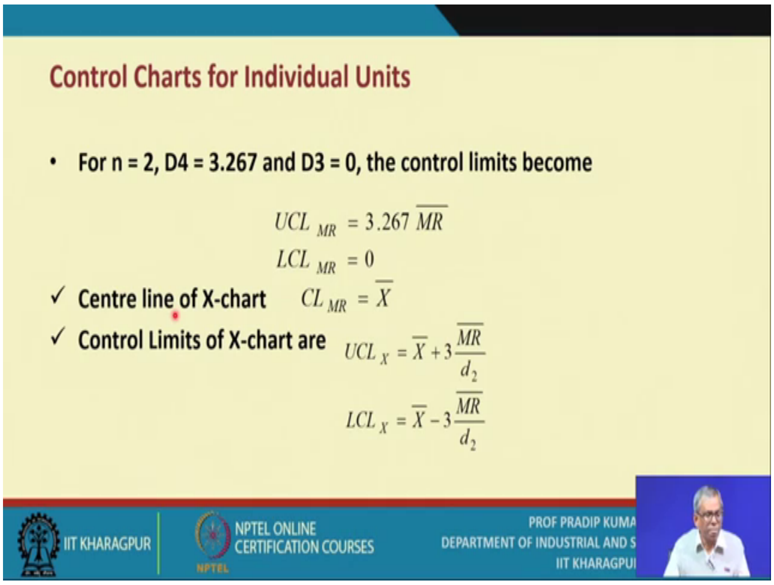


Now, when n equals to 2; that means, D4 is 3.267; that means, MR to compute MR. What is the sample size? That means, you must have the two data points. So, for n equals to 2 what is the value of D4? D4 is 3.267. In fact, there is I know if you refer to the control chart factors tables. So, you get these values 3.267 and D3 will be 0. So, the control limits becomes upper control limit it is 3.267 into MR bar and the lower control limit is 0.

Center line for X – chart; obviously, this is center line for X is X bar it is not a MR. So, this is X and that is X bar control limits for X – chart are obviously, upper control limit and lower control limits we determine and we assume that the control limits other plus minus 3 sigma limits. So, X bar plus 3 times MR bar by d 2, that is, upper control limit for X – chart and similarly for the lower control limit X – chart is X bar minus 3 times MR bar by d 2. So, later on we will take of examples on this and obviously, you know these the formulations we will be using.

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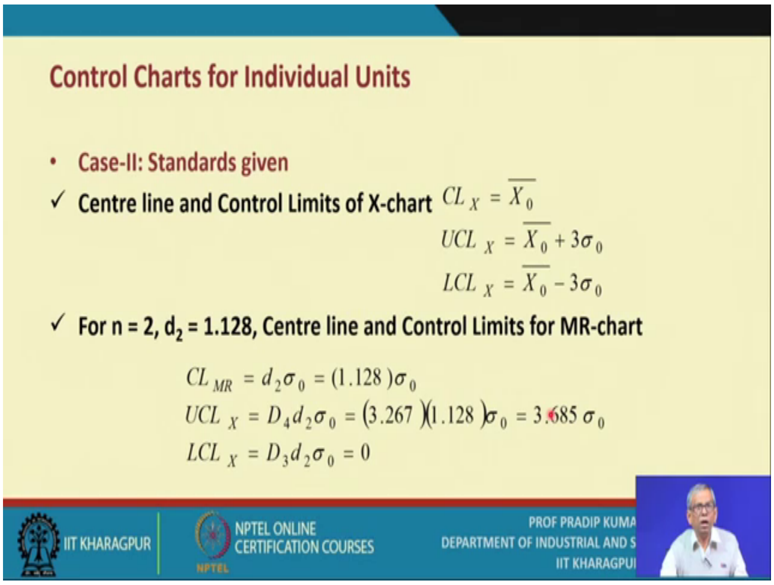


In the case II, when the standards are given; that means, you do not need to collect data. So, you assume that the standard is given as you know X o bar, so, upper control limit and the lower control limit for the X – chart you determine as 3 into sigma 0.

So, what are the standards given? Standards for the mean in X 0 bar and the standards for you know dispersion here sigma 0. So, this is straight you know the formulation and when n equals 2 to d 2 is 1.128 and the corresponding expressions for the center line, upper control limit and the lower control limit will be this one, so, d 2 sigma 0. So, this is this way you determine. You know the control limits for both MR the control chart as well as the X control charts. So, later on I will be taking up the numerical exercises or numerical problems.

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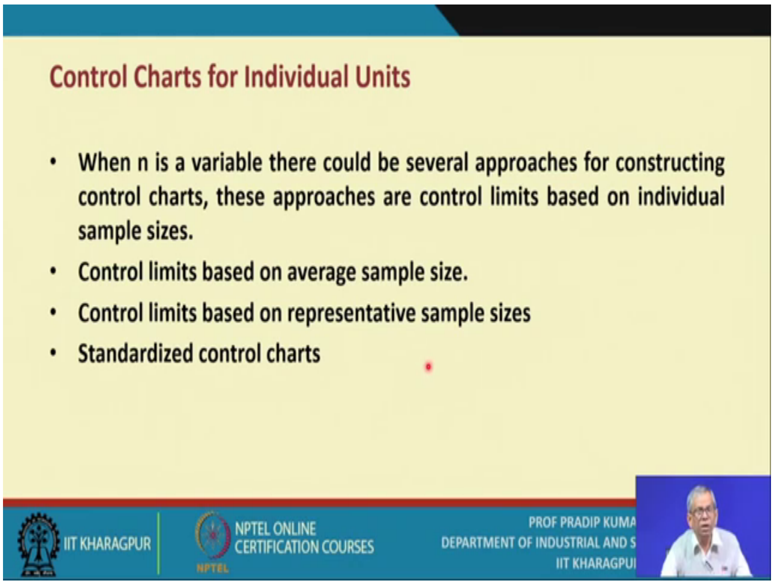


Now, when n is a variable there could be several approaches for constructing control charts and these approaches are control limits based on individual sample sizes. Control levels based on average sample size. Control limits based on representative sample sizes. Standardized control charts, that means, here, main issue is that control charts when you try to draw, if the sample size may be variable is it. So, when n is a variable; that means, over the samples n is not constant, there could be many situations like you know you say that my production rate values over the time periods and what I try to do, that means, from each say the production volume that is population I just draw the 10 percent of the population, the units as the sample units 10 percent. So, obviously, if the population size changes as a production rate changes, the sample size also might change. So, this is a very common occurrence.

Now, what do you try to do; that means, when n is a variable you have to say you can modify your formulations or you must know how to deal with this particular situation. So, there are 4 approaches in the first approach what you do? You determine control limits based on the individual sample size that you can do. So, this is very simple, but there are certain disadvantages you have. The second one is you determine the average sample size and then based on the average sample size you determine the upper control limit and the lower control limit as well as the centerline.

Now, the third I know the alternative could be that there could be out of many sample sizes you come across some may be considered as a representative sample sizes; that means, they are you know the frequency of occurrence is high. So, you select two or three such the representative sample sizes and for each one you determine the control limit. So, this is the third alternative and the fourth alternative is the standardized control chart. So, all these you know the four approaches we will we will discuss it in the later on.

So, at this point in time you just note, the possible approaches that you have to adopt or you have to use when n is a variable.

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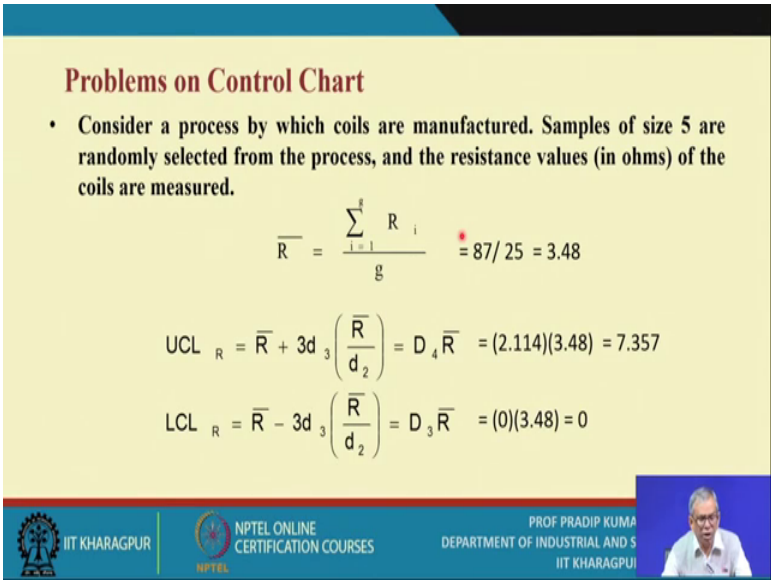


Now, here is a problem on say R control chart as well as X bar control charts. So, let me and explain it like consider a process by which coils are manufactured is it. So, this is you must know that while you select a process for control charting, what does it produce? Samples of size 5 are randomly selected from the process and as soon as you find that the sample size is less, immediately you may assume that you are dealing with a quality characteristic which is of variable type.

So, from each sample of size 5 you get the data; that means, this is the resistance values in ohms of the coils. So, this data you collect, then what do you do, you calculate the R bar; that means, there is the g number of samples g number of samples and here you know for each sample you calculate say the range and if you add them for 25 samples you get a value of 87, so, obviously, R bar is 3.48. So, how do you determine the upper control limit and the lower control limit? So, you apply the formula and when you apply this formula that is D 4 R bar and D 3 R bar we are assuming that the control limits are at plus minus 3 sigma limit.

If you have some other you know control limits like in certain cases we go for say plus minus 2.5 sigma limits, obviously, you cannot use this formula; you have to use this formula. So, 3 d 3 and the d 2 these values you have to consider while you compute say the control limit and instead of having 3 you have 2.5. So, this is a case, but usually unless otherwise stated. The control limits are set at plus minus 3 sigma limits. So, this way you calculate the upper control limit and the lower control limit. So, d 3 is 0, so, the lower control limit is 0.

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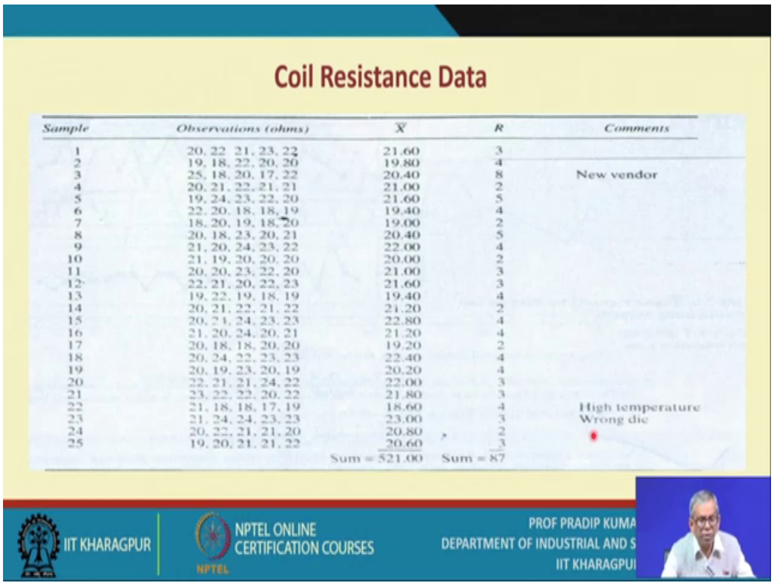


| Sample | Observations (ohms) | $\overline{X}$ | R | Comments |
|---|---|---|---|---|
| 1 | 20, 22, 21, 23, 22 | 21.60 | 3 | |
| 2 | 19, 18, 22, 20, 20 | 19.80 | 4 | |
| 3 | 25, 18, 20, 17, 22 | 20.40 | 8 | New vendor |
| 4 | 20, 21, 22, 21, 21 | 21.00 | 2 | |
| 5 | 19, 24, 23, 22, 20 | 21.60 | 5 | |
| 6 | 22, 20, 18, 18, 19 | 19.40 | 4 | |
| 7 | 18, 20, 19, 18, 20 | 19.00 | 2 | |
| 8 | 20, 18, 23, 20, 21 | 20.40 | 5 | |
| 9 | 21, 20, 24, 23, 22 | 22.00 | 4 | |
| 10 | 21, 19, 20, 20, 20 | 20.00 | 2 | |
| 11 | 20, 20, 23, 22, 20 | 21.00 | 3 | |
| 12 | 22, 21, 20, 22, 23 | 21.60 | 3 | |
| 13 | 19, 22, 19, 18, 19 | 19.40 | 4 | |
| 14 | 20, 21, 22, 21, 22 | 21.20 | 2 | |
| 15 | 20, 21, 24, 23, 23 | 22.80 | 4 | |
| 16 | 21, 20, 24, 20, 21 | 21.20 | 4 | |
| 17 | 20, 18, 18, 20, 20 | 19.20 | 2 | |
| 18 | 20, 24, 22, 23, 23 | 22.40 | 4 | |
| 19 | 20, 19, 23, 20, 19 | 20.20 | 4 | |
| 20 | 22, 21, 21, 24, 22 | 22.00 | 3 | |
| 21 | 23, 22, 22, 20, 22 | 21.80 | 3 | |
| 22 | 21, 18, 18, 17, 19 | 18.60 | 4 | High temperature |
| 23 | 21, 24, 24, 23, 23 | 23.00 | 3 | Wrong die |
| 24 | 20, 22, 21, 21, 20 | 20.80 | 2 | |
| 25 | 19, 20, 21, 21, 22 | 20.60 | 3 | |
| | | Sum = 521.00 | Sum = 87 | |



So, these are the data points. So, you have 25 samples and this is the individual observations again each sample. You just look at these values there are 5 values in each sample. So, sample size is 5 and then you calculate X bar for each sample so, but the sum is this one and you calculate the range in each sample and total sum is 87. So, you note you have certain comments to make like for a particular sample 3, suddenly the range is more. So, you say it is an inexperienced vendor and for this sample it is four, so, high temperature and when it is 3 you say the reason could be the wrong dye. So, there must be comments column.

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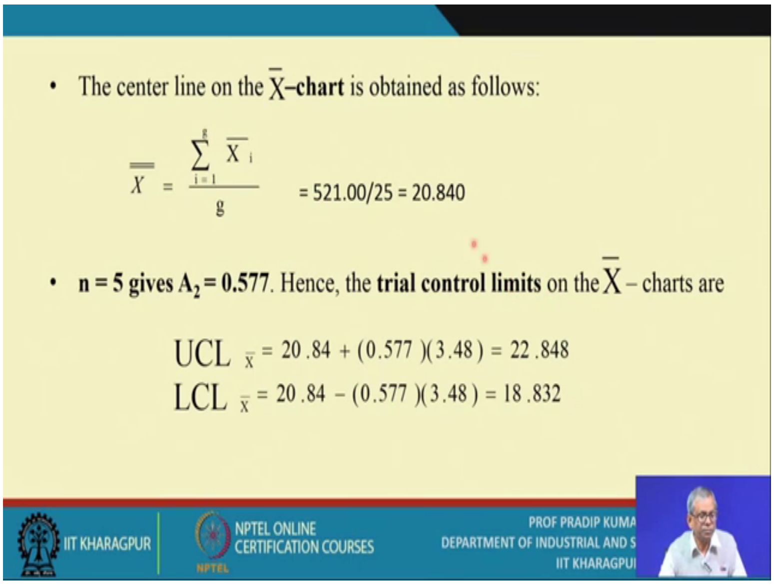


So, how do you calculate the center line; that means, the center line you calculate as X i bar sigma divided by g; that means, total is 521 by 25, 20.840 and n equals to 5 gives A 2 is equals to 0.577. So, you refer to say and the control chart factor factors table and directly you can read this value. So, what is the trial control limit? So, trial control limit you calculate. So, that is UCL x bar LCL x bar that is 22.848 and 18.832.

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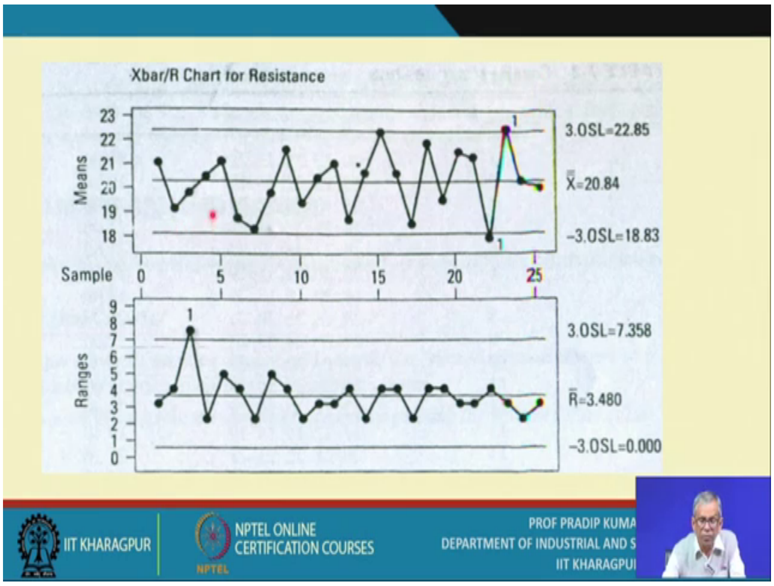


Now, what do you do now? So, first you consider say the range chart, this is your range chart you have. So, you find that particularly this sample has gone out of control. All other points are not plotted outside of the control zone. So, what do you need to do, you have to remove this point and when you are removing the sample point from the data set it means that you have identified the reasons, assignable causes and you have taken steps to remove those assignable causes.

So, and similarly this point to be removed and here is in this case when you go for plotting the individual X bar values in the X bar you know the control chart you will find. Possibly, these points and these points as well as these points they are plotted outside of the control limits. So, you have to remove those values or on those the sample values you can remove from the data set and you have to recalculate the mean as well as the control limits for both R chart as well as the X bar chart. So, and whenever you are removing certain sample points it means that you could identify the assignable causes or the special causes and you have taken you know the appropriate steps to remove those causes, so, this is the implied meaning.

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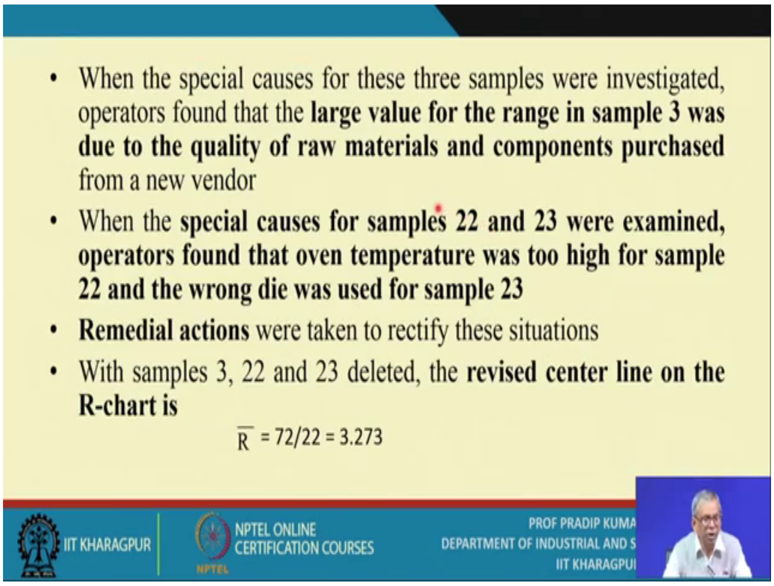


When the special causes for these three samples were investigated operators found this is the case in fact, found that the large value for the range in sample 3 was due to the quality of raw materials and components purchased from a new vendor. The point we have already mentioned. When the special causes for samples 22 and 23 were examined operators found that oven temperature was too high for sample 22 and the wrong die was used for sample 23, that means, constantly what you are trying to do; that means, you are monitoring the process is it and you are your objective is to control how the process and you are aware of what could be the possible reasons of getting an abnormally high value or abnormally low value.

So, the remedial actions were taken to rectify this situation this is obvious. With samples 3, 22 and 23 deleted, the revised center line on the R – chart is R bar is equal to 72 by 22, that means, 3 samples are gone that is 3.273.

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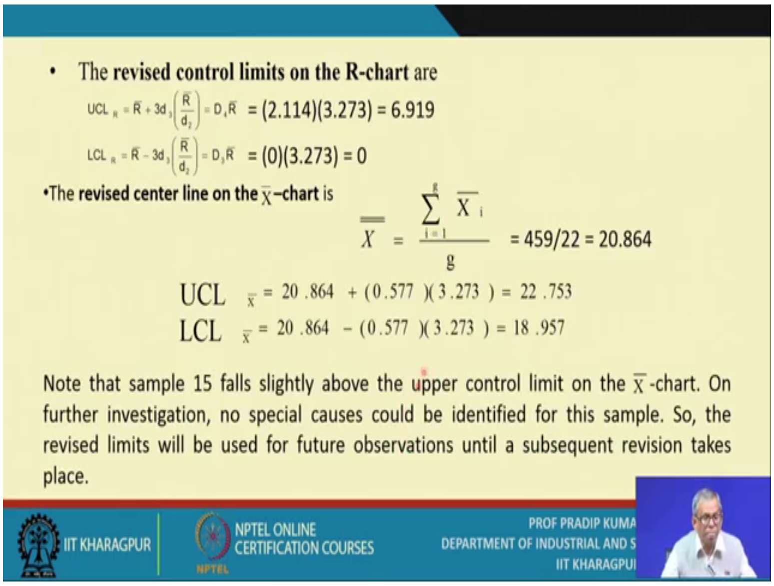


And, then the revised control limits are calculated like say upper control limit the lower control limit, the same formula you use and these are the two values you have and as far as X – bar chart is concerned you calculate X double bar and these the three points already removed so; obviously, you know. So, you do not consider those 3 samples for computing you know the mean as well as the upper control limit and lower control limit for X bar charts. So, that is why you will find this is 22 over here.

So, this X double bar is 20.864 and similarly, UCL is 22.753 you know the formula and LCL is this. Note that the sample 15 falls slightly above the upper control limit on the X bar chart. So, this is your observation. On further investigation no special causes could be identified for this sample. So, the revised limits will be used for future observations until a subsequent revision takes place. So, these are the noting or these are the comments you need to make.

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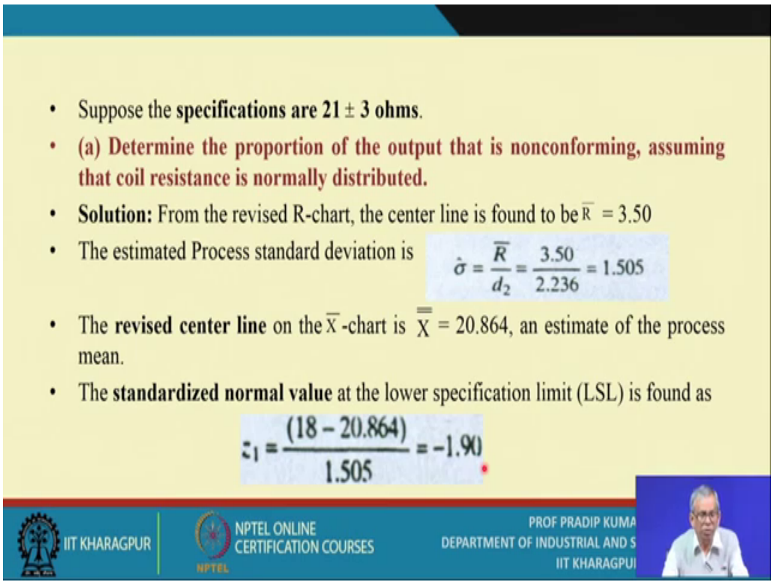


So, now, suppose the specifications are 21 plus minus 3 ohms, you are dealing with a register and this is the specifications. Determine the proportion of the output that is non conforming assuming that the coil resistance is normally distributed. So, this is a typical problem. Now, what do you find that the from the revised R chart already we have constructed the R chart the center line is found to be the center line notation is R bar that is 3.50. So, the estimated process standard deviation is R bar by d 2 that is sigma hat; that means, this is an estimate that is 1.505. So, the reverse central line on the X bar chart is X double bar is 20.864, an estimate of the process mean.

So, the standardized normal value at the lower specification limit LSL; LSL is 18, 21 minus 3 that is the LSL. So, 18 minus; that means, the mu is basically 20.864. So, the difference from the mu and expressed in the standard deviation units. So, what is the standard deviation that is 1.505. So, that is why it is 1.505. So, z value z 1 is minus 1.90 is it clear? I think it is very clear.

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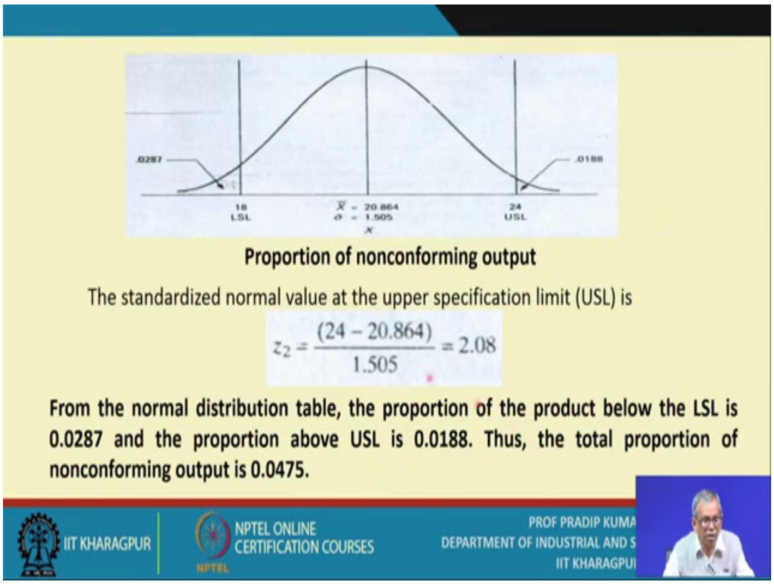


So, what do you try to do, that means, here it is we calculate. Similarly, we calculate z 2; that means, what is the upper specification limit that is 24. So, 24 minus 20.864 divided by sigma that is 1.505, that is, 2.08 it is clear.

So, from the normal distribution table the proportion of the product below the LSL is 0.0287 that is this one 0.0287; that means, this is the LSL that is 18. This area we are trying to you know determine and similarly, this area this with up means is the beyond the upper specification limit that is 24. So, this area is 0.0188. So, if you add them, you get a value of 0.0475 it means that 4.75 percent of the units you produce, they are non conforming.

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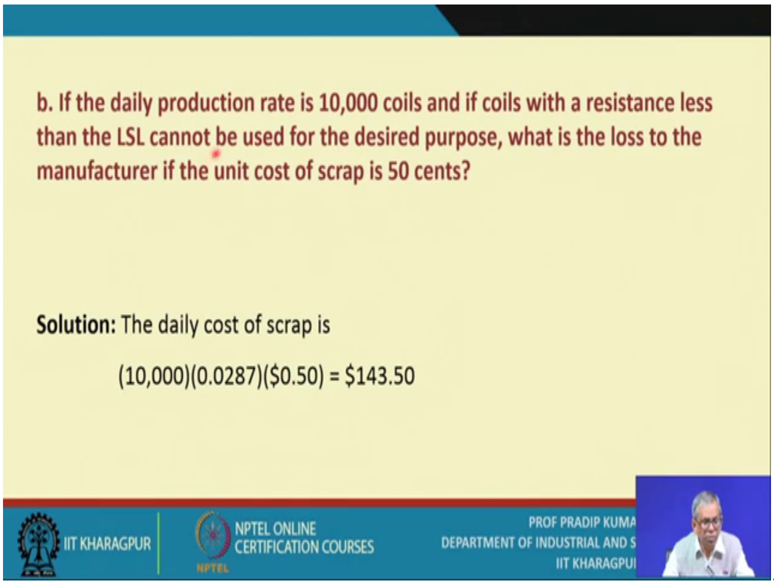


So, if the daily production rate is 10000 coils and if the coils with a resistance less than the LSL cannot be used for the desired purpose, what is the loss to the manufacturer if the unit cost of scrap is 50 cents? So, the daily cost of the scrap 10000, 0.0287. So, that is the proportion the below LSL and for each one you have 0.5 the dollars; that means, 143.50 or say rupees, whatever it is. So, in terms of say a particular say the currency you can calculate that what the total daily cost of scrap is.

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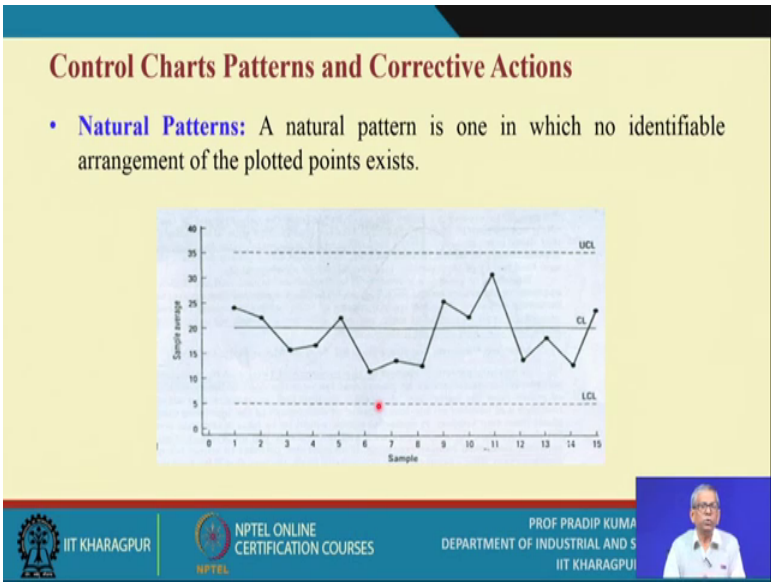

Because, here whatever the proportion you get and the units which are less than LSL, you cannot rectifying them, reworking is not possible. So, there is declared as the scrap is it whereas, if it is, the more than say 24 resistances may be you know you can take some reworking say or say, you can rectify. So, these are the typical examples, later on we will take up many such numerical problems so that, if you go through the numerical problems, your understanding will be better. So, these are the two typical problems we have taken up.

So, many such problems we can take it up, but we will have those exercises in separate sessions. Now, you know the next important topic I am going to discuss before I conclude this session that is the control chart patterns and the corrective actions. So, as we have been saying all the time, whenever you discuss these particular topics a control charting we always make a statement and what is that statement we say that the construction of the control chart is very easy; whereas, the interpretation of the control chart, look at the control chart. So, you will find that there are several you know the points, the plotted on a particular control chart. Looking at these plots you have to interpret and when you look at these plots these plots may be are or say you know you can identify certain patterns within these plots.

So, when you get these patterns, the patterns are reflecting certain kinds of conditions whether is an in control state or the out of control state. So, looking into the patterns and based on your knowledge the processed knowledge, you have to conclude whether the process is in control or process has gone out of control, is it clear. The pattern interpretation is not that easy. So, in that context we say the interpretation of the control chart is not that easy. You need sufficient process knowledge and experience and then only we will be able to interpret those patterns, when you deal with several kinds of situations several kinds of processes, you may define several kinds of patterns.

So, in these particular discussions, I will try to classify these patterns is it these and each pattern each type is effort to use a kind of pure pattern and what we will find. In fact, in this specific scenario suppose you know you have collected data you have been using your control chart for say 1 months or 1 quarter now, so, many kinds of you know the plot patterns you may come across. So, it is a combination, but first you must know what are the different types of patterns, you may come across.

So, the first one is the natural pattern. So, what is the natural pattern? Natural pattern is one in which no identifier arrangement of the plotted points exists; that means, this is a may be assumed to be a natural pattern, the all the points are plotted within the control limits. The necessary condition for an in control state is achieved, but what is the sufficient condition? When you look at this pattern is it a natural pattern? This, can you say that it is a random pattern? If you say it is a natural pattern. So, you say that the process is in control. You do not find any particular you know the systematic pattern is it.

So, first one is the natural patterns and if you find the natural pattern you say that process is in control.

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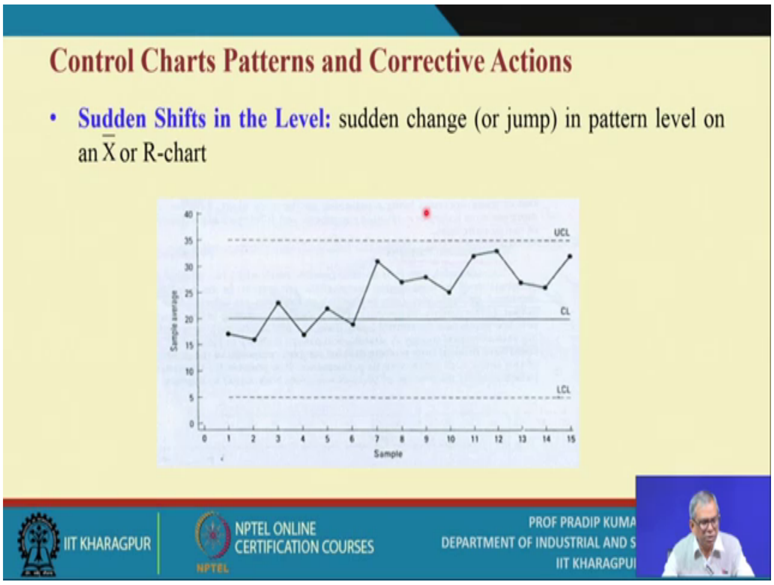


Sudden shift in the levels, sudden change at the jump in pattern level on an expert or R chart; like say these are the points you have suddenly you will find that it moves to this higher value and then almost it is continuing is it the process is continuing around the higher values; that means, this is say one level and it moves to whatever may be the cause it has moved to this the next level, from level 1 to level 2.

Now, this is essentially the sudden one; that means, in electrical control say the terminology this is referred to as a state function, in the control theory is almost a state function So, this is obviously, there must be some reasons there solving assignable causes. So, you have to find it out and just as an assignment, while I can suggest that suppose, you are you find this sort of pattern in an R chart what is the possible result.

Same sort of say you know the pattern you observe in X bar chart what could be the possible reasons.

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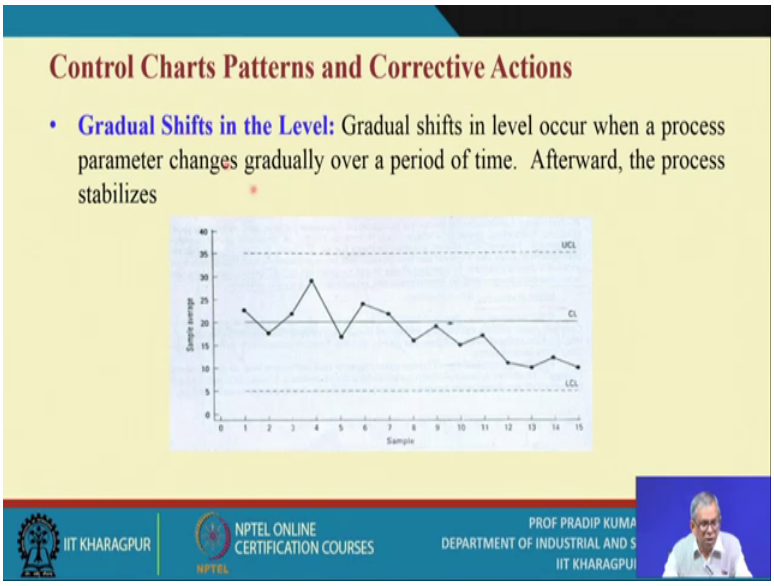


So, this is next one. Now, next one is the gradual shifts in the levels. You have a level like this, what do you find from this point onwards? This is a shift and then it took you know few samples before you gets stabilized to the next level; that means, initially it was at this level around this points process, around this suddenly something as something has happened at this point and then it is continuing and then the process gets stabilized at the 12 sample and then it reaches a different level, the next level and around that level the process continues. So, this is called gradual shifts in the levels; that means, gradual shifts in level occur when a process parameter changes gradually over a period of time.

So, this might happen like say the raw materials. Suddenly, you know you say that I will be using a different kind of raw materials; already the old raw materials are in the working process. So, it takes time for you to remove you know the old raw materials with was substitute with the new raw materials, it takes time. So, over few samples you get the next state. So, this is gradual shifts or gradual changes in the levels.

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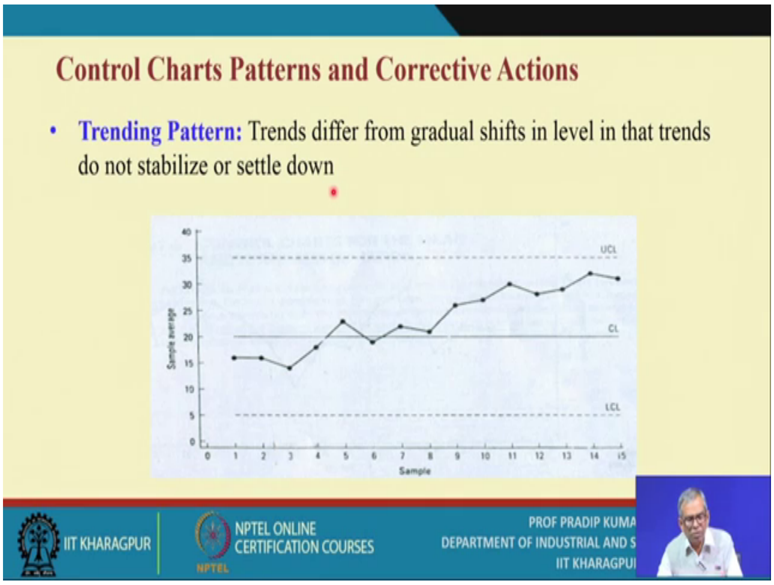


What is the trending pattern? The trending pattern in the previous two cases what you find, that from one level you are moving to will say the next level. But, in each level there is a condition of stability whereas, in the trending pattern what you find that the process has not yet become stable; that means, it is continuing; that means, you know either there could be upward trend or there could be downward trend. So, here in this case what do you find, there is an upward trend and the process has not yet become I know the stable. So, it is continuing. So, the trends differed from the gradual shifts in level and that the trends do not stabilize or settle down, that means, initially it might happen that is you have installed a process and that the process has not yet become stable and suppose you start using a control chart, so the control chart patterns may be of this type is it clears.

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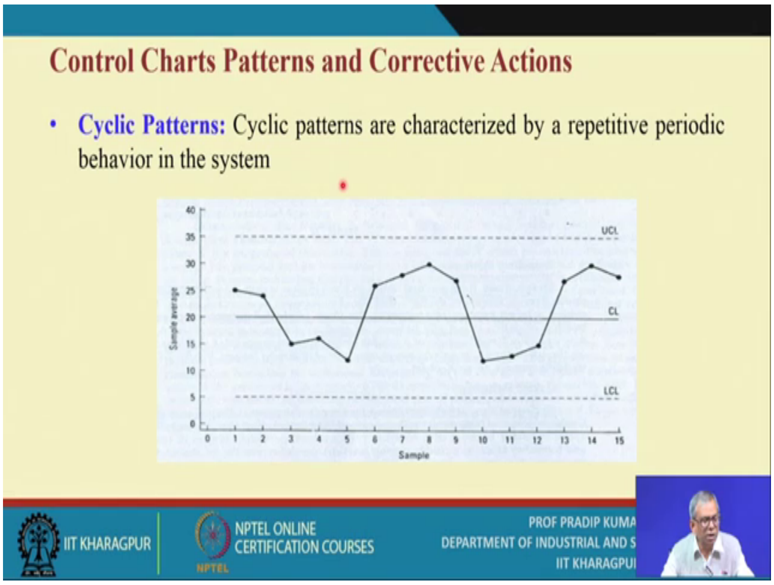


Then, cyclic patterns; so, this is a cyclic pattern; that means, which repeats then the cyclic patterns are characterized by the repetitive periodic behavior in the system; that means, these are the values you get in one condition and when you change the condition you get different higher values than this one and again you bring in the you know the original condition and again you get the same values the previous values and again the condition changes over the time period. So, this is the typical cyclic patterns that mean, they are assignable causes. So, these are the cycles you get over the other side over the samples.

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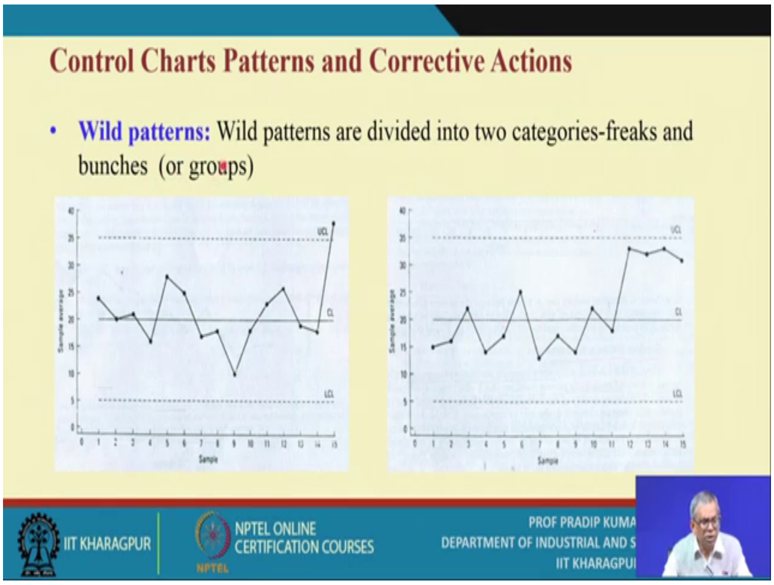

Next one, is the wild patterns is it wild patterns are divided into 2 correct categories the freaks and the bunches you know the freaks are basically you know this is a freak means is this one, suddenly you will find that one particular value sample value is very high they are referred to as the king Kong very large value or very low value somewhere here. So, you have 2 search for the causes, what has gone wrong? What is the specific reason? So, that means, this could be a Lilliput so, either it could be a king Kong or it could be a Lilliput or what do you say that this freak patterns or the freaks may continue. So, say 1, 2, 3, 4 these are all higher values have very high values compared to the other ones or very low values. So, this is referred to as the bunches or the grooves. So, these are referred to as the wild patterns.

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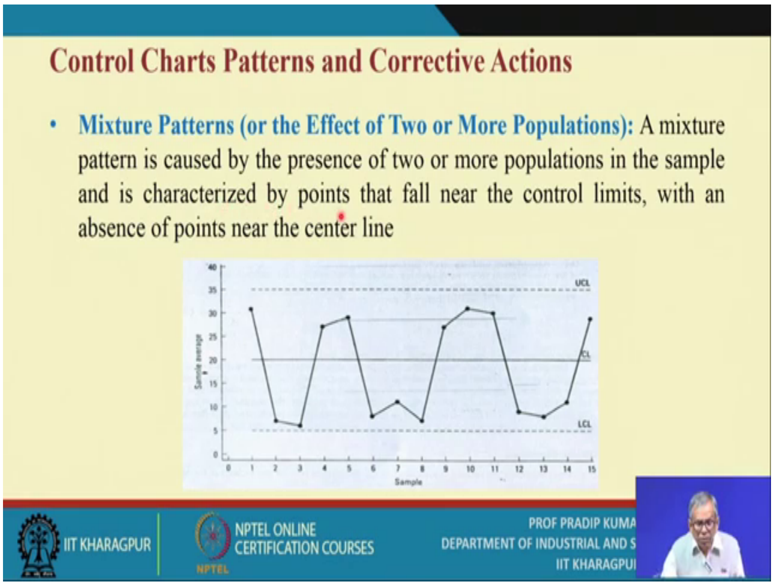


Then you have the mixture pattern; that means the effect of two or more populations that means, I have already mentioned that while you create the sample; that means, you have to follow the principles of rational sub grouping. Now, suppose you find you say that all the low values I will get and at a particular during a particular time period and I will form a sample and in the next time period you get all the high values and you form another sample and they again next to next you get the small values and next to next one you get high values.

So, what did; that means, this is very high value, this is low value, next one is also low value, next what do you do; that means, this is you say that this is almost it may be touching the upper control limit. So, you say you go for under control then it almost reaching the lower control limit then you say you go for over control and again. So, it is a phenomenon called under controlling as well as the over controlling is all you are always on pose when you are trying to control this. So, from one extreme you go to the other extreme and this process continues. So, either of this effect of two or more populations.

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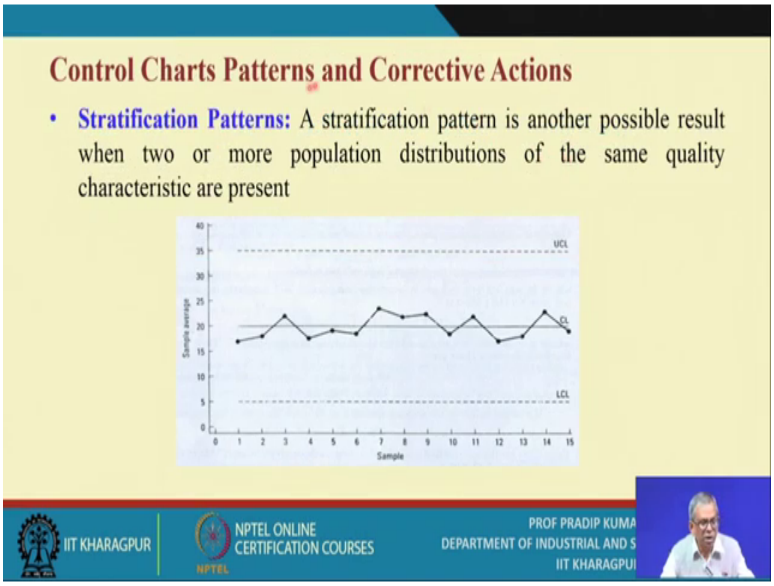


Then, you have the stratification patterns; that means, when you mix up in a sample low values with the high values. So, what is you expect; that means, the average value will be almost you know will be around the center line. So, sometimes you get confused, you might conclude that the process is highly in control, but actually it may not be. So, getting my point; that means, you are not following the principles of rational sub groupings appropriately.

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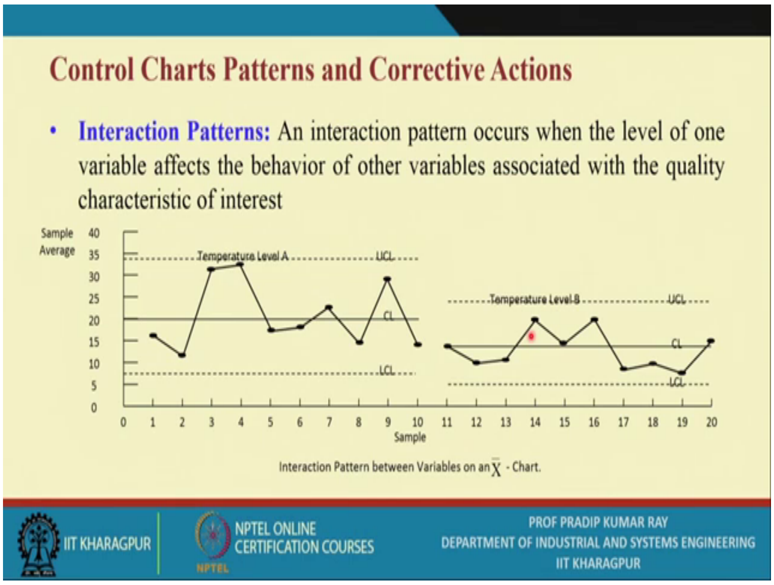


So, this is called stratification pattern and the last one that I am going to discuss that is the interaction pattern. An interaction pattern occurs when the level of one variable affects the behavior of other variables associated with the quality characteristics of interest. These are the temperatures. So, if the temperatures you change this is the level A. So, this is you know the values you will find, if the temperature level you know gets reduced you will find different kinds of values over here; that means, the output this is clear. So, the effect of temperature on the output and it is very you know significant in presence of say the pressure.

So, this is also we will discuss when we will take up you know the problem suited problems. So, these are the another control trade patterns and whenever you find one particular pattern you conclude that the process has gone out of control and the possible reasons of out of control or the assignable causes you must be able to identify and the corrective actions are to be taken.