Quality Design and Control
Prof. Pradip Kumar Ray
Department of Industrial and Systems Engineering
Indian Institute of Technology, Kharagpur

Lecture – 21
Statistical Process Control – II

So, in this section under statistical process control this is the second part of statistical process control that we are going to discuss.

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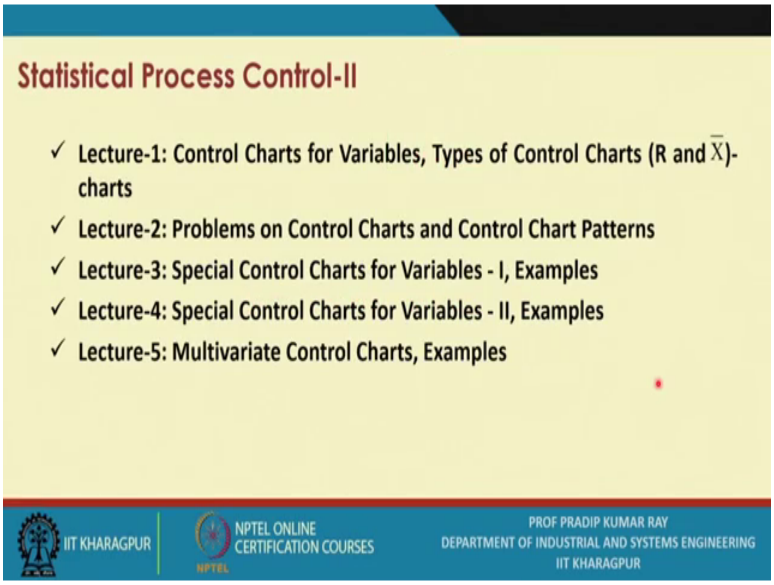

We have 5 lecture sessions. The first lecture will be on control charts for the variables. types of control charts specifically I will be discussing R and X-bar chart. In the next lecture I will discuss the problems on the control charts and the control chart patterns. Third lecture the special there are many special control charts for variables and with examples I will discuss. And in that subsequent lecture, lecture 4 then I will continue discussing special lecture, special control charts for the variables with examples. And in the last lecture session multivariate control charts will be discussed with examples. So, this is our plan as per as the second part of the statistical process control is concerned.

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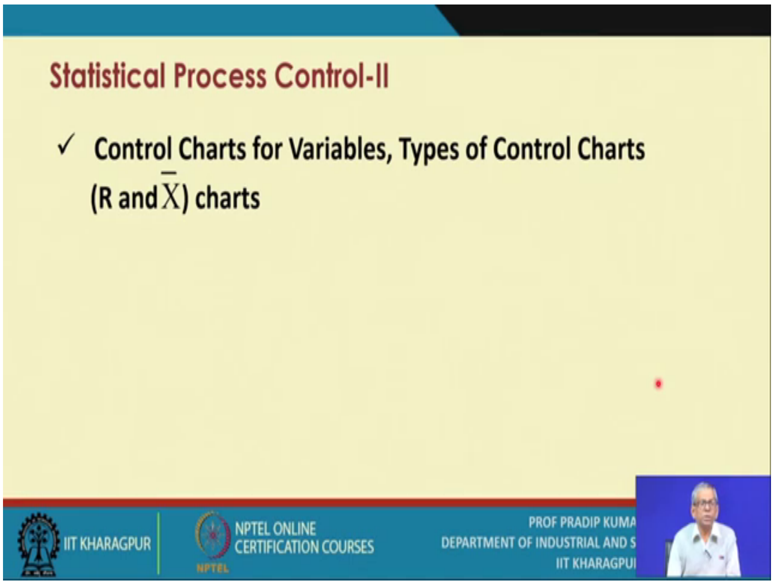

Now, let us first I discuss the sub topic in this particular lecture session that is the control charts for the variables and particularly the two important control charts I am going to discuss the first one is the R chart and the second one is the X bar chart.

So, let me first you know discuss some important aspects related to quality characteristics which are of variable type.

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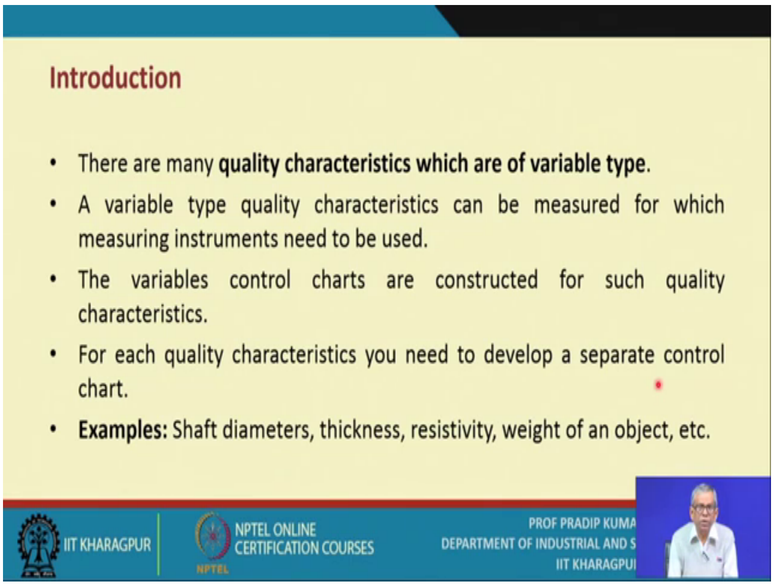

You know we have already mentioned that there are two types of the data you come across, in quality control exercises in quality control related problems. So, this the two types of data, I referred to as attribute data and the variables data. We have already discussed attributes of control charts and now, we are referring to the various control charts.

Now what is a variable type quality characteristic? Now, these quality characteristics are such that they can be measured for which measuring instruments are to be used is it certain times. Now, we also can use certain gages, but essentially you know this characteristics are such that it can be measured. The variables control charts are constructed for such quality characteristics. So, this is its domain. There are many types of the control charts under this category, some are referred to as the general purpose control charts and some are referred to as the special purpose control charts.

Now, what you need to do; that means, for each quality characteristics you need to develop a separate control chart. So, this point is to be noted. Suppose in a given situation you are dealing with some say twenty such quality characteristics which are of variables type and you have decided that you will go for control charting for this quality characteristics. So, what you need to do? That means, for each one of them you have to construct a separate control chart, is it ok. So, this point is to be noted it is not like attributes control charts right.

So, attributes control charts are referred to the proportion and conforming or the number of non conformities demerits per unit etcetera etcetera. So, those aspects are not related to a particular type of quality characteristics, are you getting my point. So, here; that means, it should be ready with you know your approach for constructing control chart for the specific variable quality characteristics, and usually in manufacturing situations when you deal with many kinds of parts components ok. So obviously, we dealing with the various types of your know the quality characteristics which are of you know the variables types. So obviously, it may not be necessary at it may not be possible to construct control chart or recover control chart for each of the quality characteristics. So, what you need to do out of many quality characteristics initially you come across you have to select a few which are considered to be very important or critical, is it ok. So, this is the point to be noted.

Now that means, we will be dealing with you a few non trivial or important quality characteristics. What are the examples of say variable quality characteristics? There are many examples like the shaft diameters, like the thickness resistivity weight of an object etcetera; that means, if you look at all these characteristics what you find that they are measurable, is it ok.

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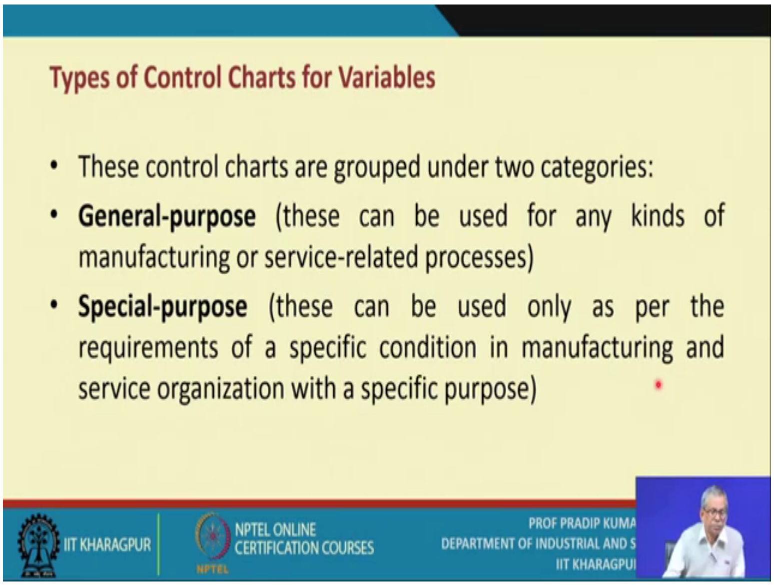

Now, what are the types of control charts for variables you come across? Now, these control charts are grouped under two categories the first one is the general purpose. Now, what do you mean by general purposes? That means, these charts can be used for any kinds of manufacturing or service related processes or the situations; that means, those are general and as you know that by using a control chart you try to you know achieve certain objective.

Now, if you use such general purpose control charts; that means, there are certain general objectives which you need to achieve through control charting and these general objectives are relevant for any kinds of manufacturing situations. That is why you know and the related to this general kinds of you know the situation and related to the process control aspects there are some general problems you have to address. So, this type of control charts are suitable for addressing those problems; obviously, related to the process control.

Now, you have you know the special purpose control charts; that means, there are certain specific objective, you have to achieve specific objective related to a particular situation. So, these on basically situation specific and there are many special purpose control charts the researchers as well as the practitioners have already developed. So, what are these the special purpose control charts? That means, these charts can be used only as per the requirements of a specific condition or the situation in manufacturing or service organization with a specific purpose its clear. Like say in a in a typical situation where you know the process is very very sophisticated and you know the slide deviation the slide deviation of the process parameter value may not be accepted to you ok. So, this is a very specific situation and what actually you need to recommend is special purpose control chart.

Now, at this point in time I will just note down that there are two specific objectives of control chart in the first objective as I have already told you, that the first objective is that you know at any point in time you must be able to conclude whether the process is in control or out of control. So, this is the first purpose. That means, first purposes is to ascertain or to conclude whether the process using control or not is it ok. In an online the control system looked at as in online and the real data or the real time control system.

Now, the second purpose is very very important. The second purposes is if there is any shift in the process process parameter value is it ok, now obviously, there is a shift you conclude that the process has gone out of control whether your control chart is able to detect this shift or not as quickly as possible. So, the first one is your know identification of the state of the system in control or out of control the second one is ability to detect the shift in the process parameter value.

Now, many a time you know what you do you recommend the special purpose control chart. So, that the second objective is fulfilled, is it ok. So, these are the two categories.

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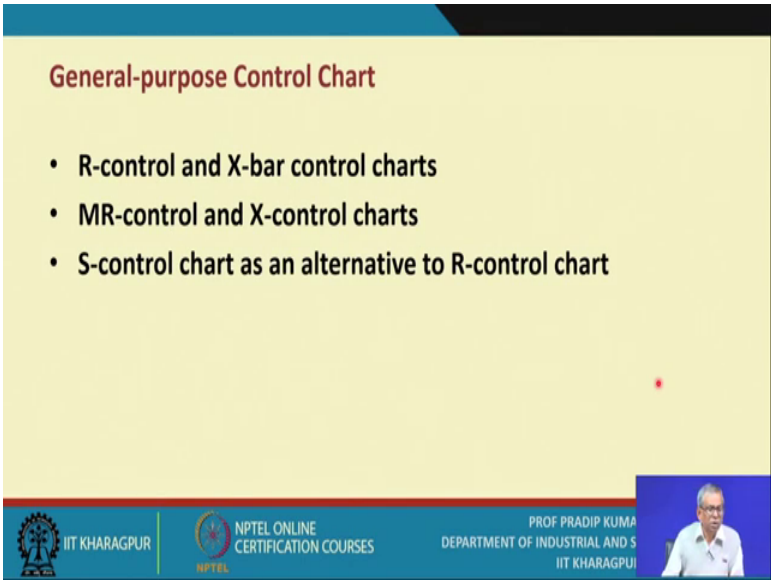

Now, let me the focus on say in this particular session first general purpose control charts. And if you go through the text books if you go through the practitioners you know the manuals related to quality control what you find that essentially there are the 5 types of control charts are recommended under general purpose category. So, what are those?

The first one is the R control chart. So the R control chart if we use the basic the basic purpose of using a R control chart is to control the variability as measured by the range is it that is why R stands for the range. So, this is also referred to as the range control chart and along with the range control chart what you need to do; that means, you must use or must construct and use X-bar control chart.

So, what us X-bar? X-bar actually it is referred to as the mean it is a general notation and X is X is a random variable and X is a variable; that means, the measurable quality characteristics, is it ok. So, X is the value. So, X-bar control chart X-bar means the mean of a sample. So, this the two control charts you use under general purpose category. The second one is your know whenever you find that the there are situation I will discuss that situation also, like in many cases in certain cases the sample size sample size may be 1, may be 1; that means, it is essentially the individual say individual units individual units forming the sample.

So, in that case obviously, the range you cannot calculate. So, you have to think or so, alternative means and this one such you know alternate the means is moving range; that means, what you need to do; that means, you define or you calculate the dispersions or the variability with moving range. So, we will explain it what is what is moving range. So, the moving range control chart you have to use and along with the moving range control chart what you need to do; that means, X control chart for the individual units that also you need to use

So, this two charts will be discussing and the last one I have mentioned that is s control chart as an alternative to R control chart in X for the standard deviation control chart, is it ok. Now, as I have already pointed out that if you measure say dispersion with range. Now, it is not that accurate. So, and as soon as you know suppose the value of n; that means, a sample size is greater than 10. Now, you may conclude that you know by the range cannot be a good measure of for dispersion it may be you know it is an inaccurate measure if n or the sample size is greater than 10 right now. So, what do you do? Obviously, you know say you opt for such cases the standard deviation control charts. So, this is point number one to be noted.

Another point you must agree that when we started constructing the control chart particularly the variables control charts as you may be knowing that this the top point to be noted there is a variable control chart any variable control chart is to be used at the operations level at the process level. That means, it with the responsibility of a developing this control chart rests with the workers or the operators dealing with a particular process for which control chart is recommended is it for process control.

So; obviously, you know some 50 years back, 60 years back the workers were not that you know qualified educational qualified to it is to be you know very difficult to ask them you know it is impossible for many cases to compute the standard deviation for sample. So, what you try to do you search for as a supervisor, as a manager the line manager what you say that I am restricting the number of say sample size to less than equals to 10, within 10 and you have to just you know the major or calculate the range; that means, you just identify that the maximum value and the minimum value and take it is that the difference and the difference is basically the range.

And today what you find that that the process has become so sophisticated and its technology intensive that these many such processes are essentially done by educated workforce, so obviously, if you have the set of conditions in today's work place. So, why do not you say recruitment the standard deviation say the control chart instead of recommending R control chart, is it ok.

So, in this discussion what we will discuss the standard deviation control chart as such will not be discussing today later on we can refer to these, are the basic norms the basic approach remain same, only thing is you are a measuring the variability or the dispersions not by range, but by standard deviation, is it ok. So, what will be discussing now, like say we will discussing R control chart along with the X-bar control charts, is it ok. And later on I will highlight the specific points as per as control charting is concerned with respect to them mr control chart and X control chart.

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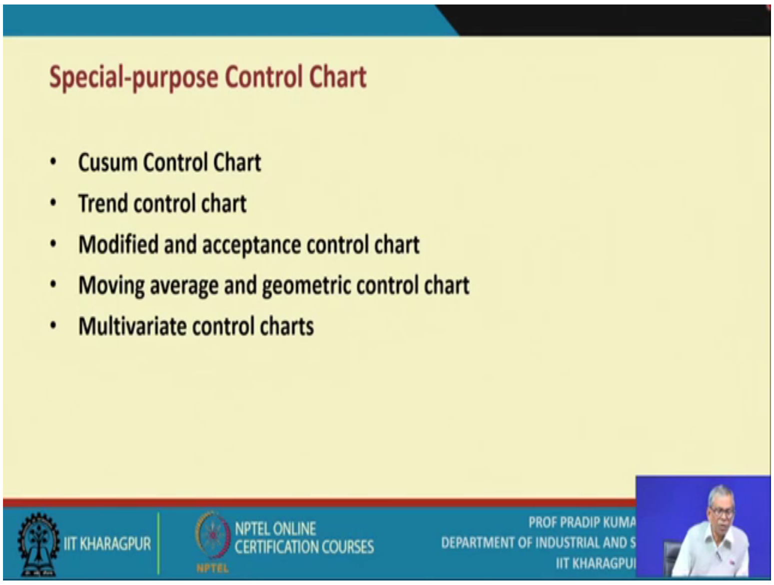

Now, what are the special purpose control charts? You saw there are many in fact, so just to the important one I have mentioned over here you can just note it down like say the cusum control charts cusum stands for cumulative summation control chart. Second one is the train control chart or sometimes referred to the regression control chart. Then we have a class of control charts known as the modified control chart along with the acceptance control charts. Why I have written together because if you used modified control chart; obviously, you have to use acceptance control charts; that means, these two control charts must be used simultaneously.

Then we have the moving average and the geometric a moving average control charts; is it geometric or exponentially weighted moving average control chart also you can use and the last one is the multivariate control charts. There are many varieties and so of the important varieties will be discussing later on.

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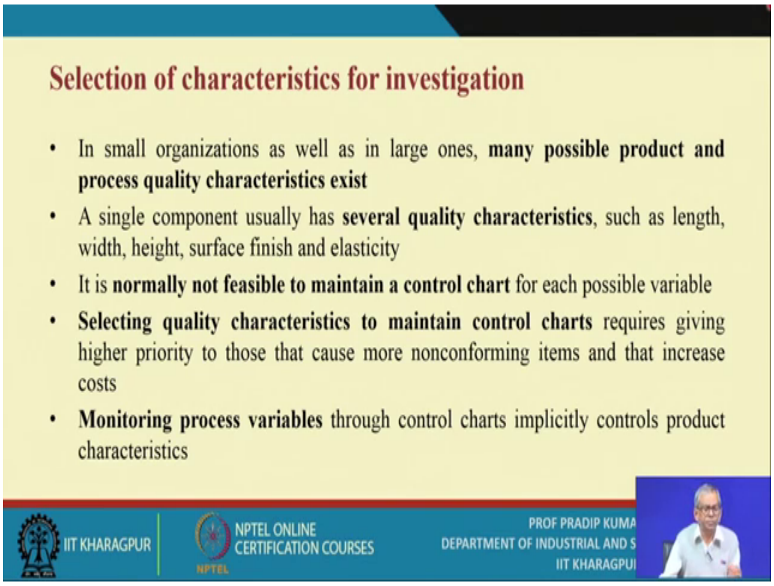

Now, what do you do? That means, selection of the characteristics for investigation. So, this is very very important. In small organizations as well as in the large ones many possible product and process quality characteristics exist. A single component usually has several quality characteristics such as length, width, height, surface finish and say elasticity. So, this is just an, just I am citing some examples it is not mention not feasible to maintain a control chart for each possible variable. I have already you know say the point pointed out, I have already whether mentioned this point. So, it is just not possible to you know maintain a control chart and it is not necessary also to maintain a control chart for each possible quality characteristics.

So, what you need to do? You have to select quality characteristics you have to very very selective to for which control charts will be maintained and this requires giving higher priority to those that cause more nonconforming items and that increase costs, is it ok. So, I will be referring to those critical quality characteristics which are measurable and if you cannot control their values or say the processes from who is this quality characteristics are generated. Suppose you lose control on this processes what will happen? Might happen that the entire production process may become very very expensive is it ok, and along with you know there is a high chance that you will be producing nonconforming output its clear.

So, these the two situations you must be able to avoid is it ok, prevent the occurrence such situations. So, you have to be you know very selective and you select the quality characteristics in such a way for control charting so that this in the critical quality characteristics you must be able to avoid. So, monitoring process variables to control charts implicitly controls product characteristics because the control there is a process is related to process when I you know referred to as a particular process then for which a control chart is to be drawn.

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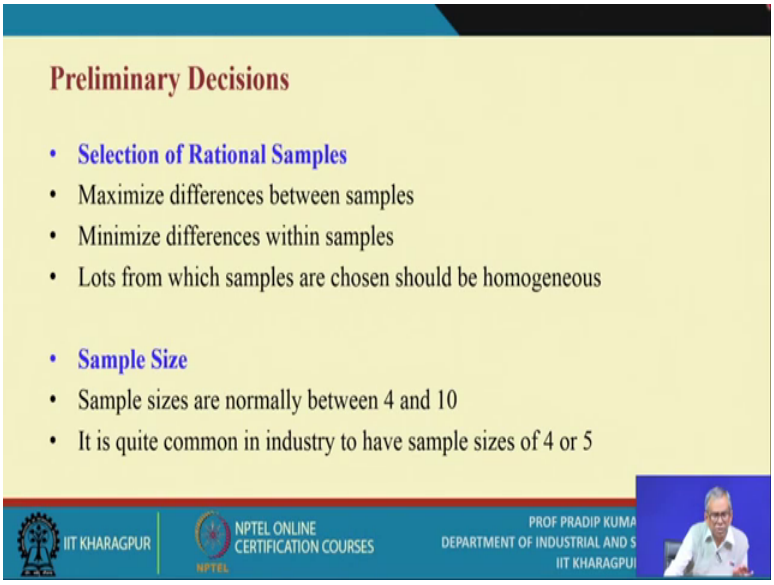

So, the control charts is to be drawn or constructed for a quality characteristics this quality characteristics is related to a particular product so obviously, directly as well as a indirectly by controlling the process you are also controlling the quality of the product, is it ok. So, this is the context.

Now, what are the preliminary decision? Selection of the rational samples. I have already mentioned that the principles of the rational sub grouping. So, you need to maximize the differences between the samples these pointed to be noted minimizes differences between the samples lots from which samples are chosen should be homogeneous, is it ok. So, homogeneous the condition homogeneity condition we have to arrive at.

Related to the sample size sample sizes are normally between 4 and 10 because you are dealing with the variables data the information contained is more and that is why what you try to do; that means, you may you may have you know the smaller sample size. So, 4 to 4, between 4 and 10 and usually now, we say many cases we will find the sample size is may be restricted to 5 it is quite common in industry to have sample sizes of 4 or 5.

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What are the preliminary decisions? That means, you have to take decisions on the frequency of the sampling; that means, the number of samples for construction of the control chart. So, what could be the number of samples to be drawn from the process? The sampling frequency depends on the cost of obtaining information compared to the cost of non detecting a non conforming items. So, this point is to be noted in fact, is it ok; that means, there is a cost for collecting the information is it ok.

There is also a cost suppose you do not collect this information so obviously, we are assuming that the cost of non detecting and non conforming item is significantly more than the cost of obtaining information are you getting my point. So, this is the condition or this is the conditions you have in your say when your process and that is why you know you feel like you know collecting data is it clear. So, this is an very important point I have highlighted choice of measuring instrument this is very very important; that means, while you select a particular measuring instrument you make sure that it has the right kind of say the accuracy then you know the precision as well as you know the resolution for the sensitivity and then you have to design a data recording form.

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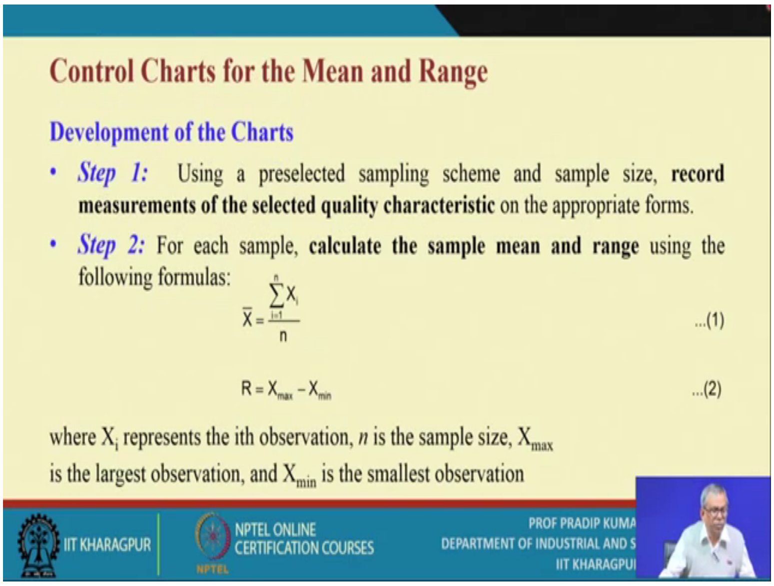

So, the control charge for the mean and range; that means, is notation is X-bar and the range notation is R. So, what you have to do? That means, in the step one what are you using a free selected sampling scheme and a sample size is it record measurements of the selected quality characteristics on the appropriate forms, is it ok. So, this is step one.

In the step two what you do; that means, for each sample calculate the sample mean and the range using the following formulas. So, X-bar you calculate is it for say X-bar is the over overall say is a sample mean that is X i values are there. So, this sample sizes is n. So, this is X-bar; that means, of sample mean. And what is R? R is basically in a particular sample. What is the maximum value? What is the minimum value? So, why I mentioned that X I represents the ith observation n is a sample size X max is the largest observation and X mean is a smallest observation.

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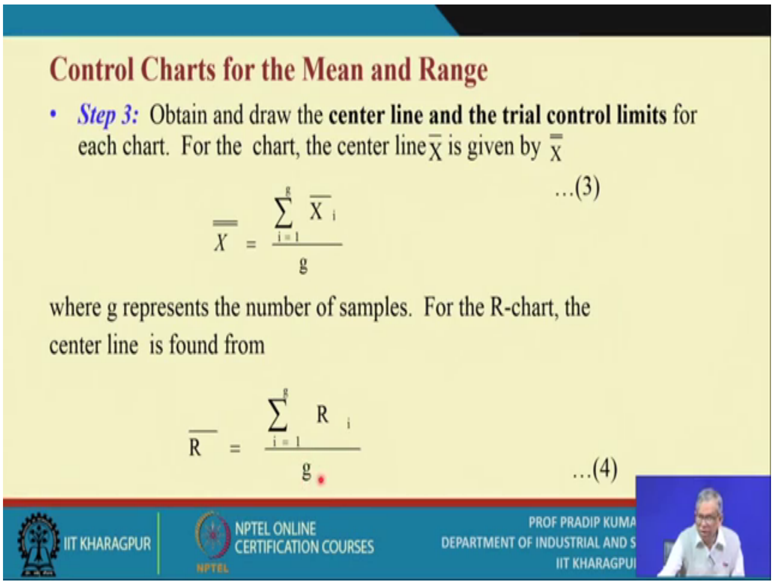

Then what you do? That means, you have drawn g number of samples that is your sampling frequency. So, when you consider all this g number of samples you calculate you know the mean of the means mean of the means that is X double bar so obviously, you have for the ith sample X i bar and when you consider all this is the g number of such a sample means and you take the average; obviously, you get you know X double bar what is the mean of the means. And similarly a for each sample you calculate R i that is the range and you are considering you know the g number of samples and you get R bar. So, that is the measure of process control process, you know in say the variability.

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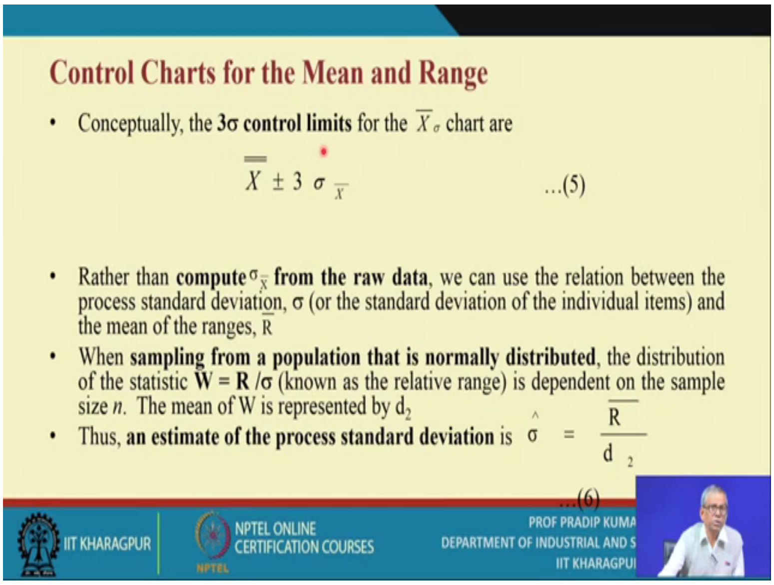

Now, what we will assume initially that the 3 for the 3 sigma control limits for the X-bar chart you have this journal formulation that is X double bar plus minus 3 times sigma X-bar, is it ok. Now, rather than computing sigma X-bar from the raw data we can use the relationships between the process standard deviation sigma this is an alternative way and it is an easier way to calculate you know this sigma X-bar, the standard deviation of the individual items that is sigma X-bar and the mean of the range, mean of the ranges that is R bar.

So, when sampling from a population that is normally distributed the distribution of the statistic we have these statistics that is you know the relative range this is relative range is defined as R divided by sigma. Now, what the researchers have found that this if you consider the statistics and its distribution. So, you will find that the mean of W is represented is dependent on the sample size. So, also the standard deviation also say now, then that depends on the R bar by d 2, is it ok.

So obviously, you know you for this statistic. So, you have the corresponding mean value and the corresponding standard deviation value. So, the mean value mean is denoted as d 2 which is the function of sample size and similarly this standard deviation is denoted by d 3 which is you know, which is also a function of the sample size.

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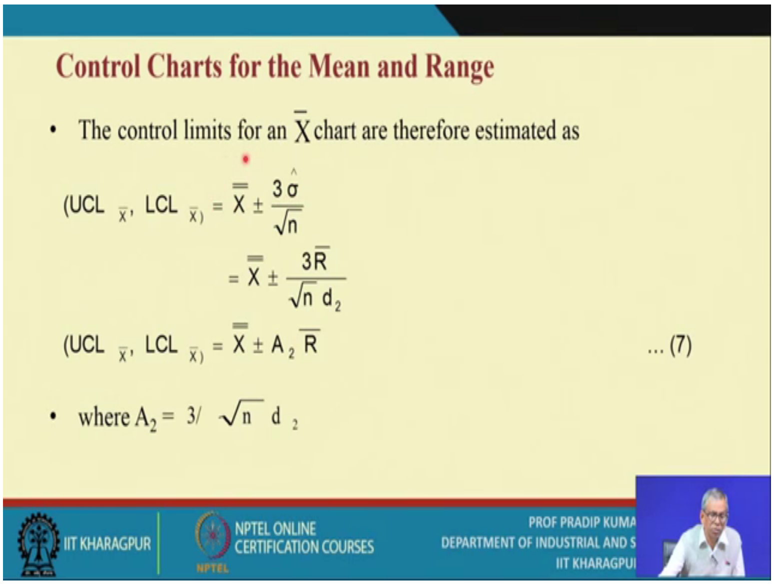

Now, we have this manipulation; that means, control limits for the X-bar chart is upper control limit for X-bar and lower control limit for X-bar is basically this is sigma and by root n.

Now, this sigma is basically R bar by d 2 this is sigma upon root n that is sigma X-bar and the sigma estimate of sigma is R bar by d 2 is it ok. And, ultimately you know this one; that means, 3 divided by root n d 2 is we have this notation that is capital A 2. So, upper control limit and the lower control limit. So, these two limits are given as X double bar plus minus A 2 R bar is it where A 2 is 3 upon root over n d 2. So, just one assumptions we have here when you use this particular you know the formulations; that means, you are assuming that the control limits are at plus minus 3 sigma limits.

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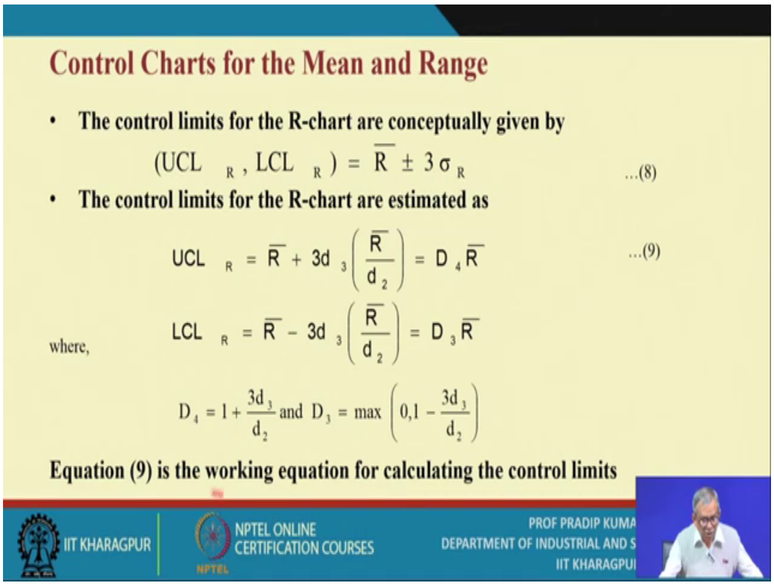

So similarly for the R chart what you do; that means, upper control limit and lower control limit for R chart is given by R bar plus minus 3 sigma, sigma R and then with this particular you know the manipulations what you have to do what mention that is a sigma R, R bar by d 2; that means, d 3; that means, d 4 R bar and this is d 4 R bar and d 3 R bar. So, where d 4 is 1 plus 2 into d 3 by d 2 and d 3 is maximum 0 and 1 minus 3 d 3 minus d 2; that means, if it is negative; that means, convert it into 0. So, equation, this equation is the working equation for calculating the control limits.

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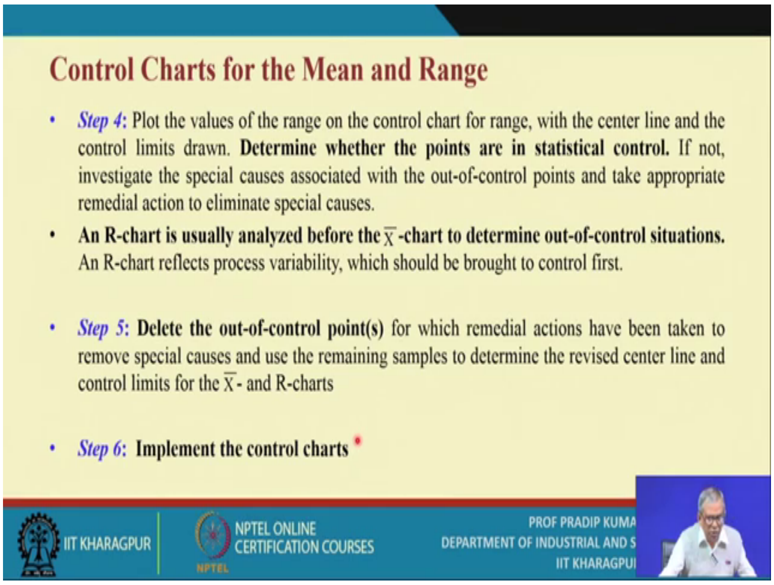

So, what do you try to do? That means, for if you want there are certain steps in constructing the control chart. So, the third step we have already covered.

Now, we go to the 4th steps to determine whether the points are in statistical control, if not investigate the special causes associated with out of control points. Suppose you start with 30 the points, so all this 30 points will be plotted is it after you determine the upper control limit and the lower control limit and check whether all these points are plotted within the control limits are not. And R chart is usually analyzed before X-bar chart to determine out of control situation and then the delete, the out of control points; obviously, and for which remedial actions have been taken and then you go for the final control chart like we follow for the p control charts. So, the same procedure you have to follow, is it ok. So, first you determine the preliminary control limits and then from the preliminary control limits you may go for the final control limits and then you take actions for implemented control chart, is it ok.

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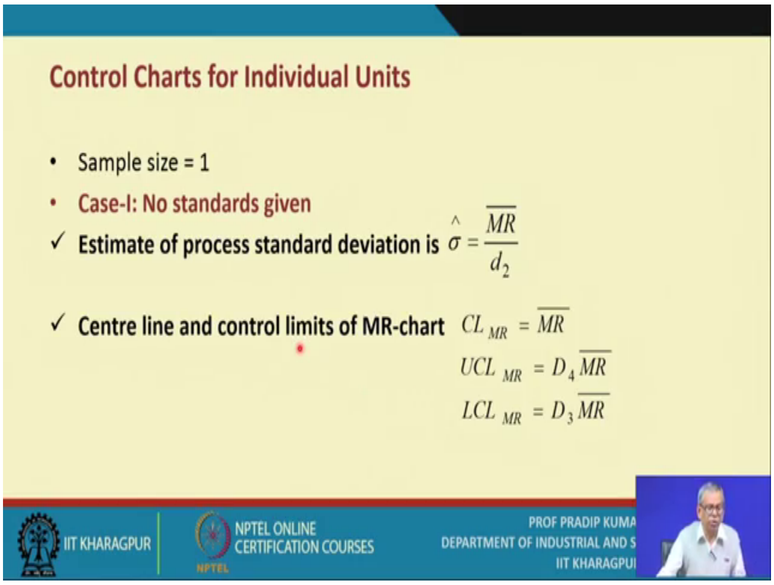

So, this is the thing. Now, what you try to do; that means, that there are two things very very important for controlling the process with the help of control chart. The first one is you try to control the variability in the process is it and this is a very important assignments hence related to any process.

So, for that you go for using the R chart and if you are successful in controlling the variability then the next stage you go for using the X-bar chart. So, that mean also, I mean of the process also can be controlled, is it ok. So, now, that means, essentially you have first step to give importance to the variability if there and you know the variability of the process is related to are the design of the process the functioning of the process functioning of the internal components of the process. So, that is a challenging assignment and usually you know the process engineer takes the remedial steps or the preventive measures to control this variability based on your interpretation of the control charts. So, that is point number one.

And number two that usually you know about the controlling mean is not that difficult and usually you will find that the workers while the process certain items on a process, so there always trying to control the process parameters, even if it is an automated systems or the semi automated system. So, what actually that trying to control; that means, a controlling the process the parameters essentially they are trying to control the process means. So, with these say now, the introductory lecture, I have highlighted a certain important issues related to control charting for the variables.

So, in the next phase or the next lecture sections I will be discussing the other types of the control charts under variables category.