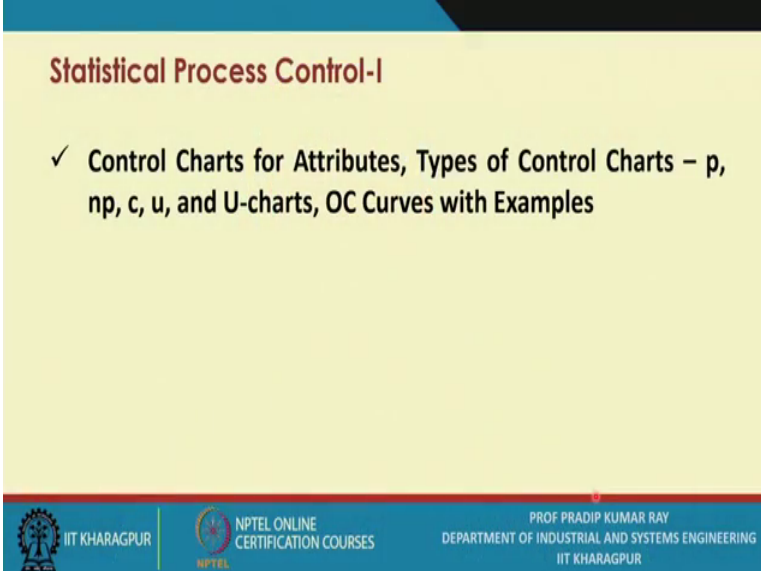


Quality Design and Control
Prof. Pradip Kumar Ray
Department of Industrial and Systems Engineering
Indian Institute of Technology, Kharagpur

Lecture – 20
Statistical Process Control-I (Contd.)

In this session, I will be discussing control charts for attributes. So, I have already or the mentioned that why do you use the attributes data and when you use attributes data related to the quality characteristics. So, you need to use this set of control charts exclusively designed for the attributes data.

(Refer Slide Time: 00:53)



The slide features a yellow background with a blue header and footer. The title 'Statistical Process Control-I' is in red. A bullet point with a checkmark lists 'Control Charts for Attributes, Types of Control Charts – p, np, c, u, and U-charts, OC Curves with Examples'. The footer contains logos for IIT Kharagpur, NPTEL, and the Department of Industrial and Systems Engineering.

Statistical Process Control-I

- ✓ Control Charts for Attributes, Types of Control Charts – p, np, c, u, and U-charts, OC Curves with Examples

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | PROF. PRADIP KUMAR RAY
DEPARTMENT OF INDUSTRIAL AND SYSTEMS ENGINEERING
IIT KHARAGPUR

So, in this particular you know say session I will be discussing about different types of the control charts in use along with now their OC curves; that means, as a performance measure and with examples.

(Refer Slide Time: 01:06)

Chart for Proportion Nonconforming (p-chart)

- A chart for the **proportion nonconforming items (p-chart)** is based on a **binomial distribution**.
- For a given sample, the proportion nonconforming is defined as $\hat{p} = \frac{x}{n}$
- where, **x = number of nonconforming items in the sample**, **n = sample size**
- A **p-chart is one of the most versatile control charts**
- A **p-chart can be used as a measure of the performance of top management/middle management**
- For instance, the management of the chief executive officer of a company can be evaluated by considering the **proportion of nonconforming items produced at the company level**
- A **p-chart can provide a source of information for improving product quality**

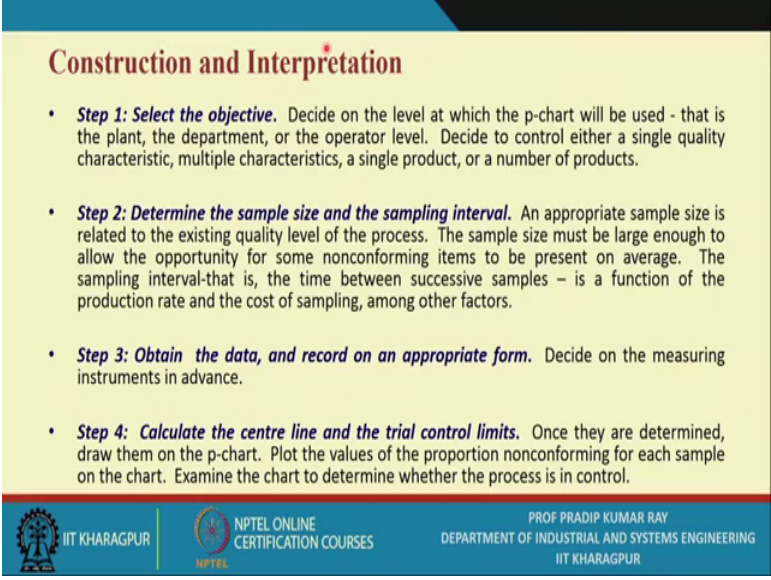
IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | PROF PRADIP KUMAR RAY
DEPARTMENT OF INDUSTRIAL AND SYSTEMS ENGINEERING
IIT KHARAGPUR

So, you know we have already mentioned that there is the p control chart, chart for the proportion nonconforming; that means, your purpose is to control the proportion nonconforming. So, how do you define this proportion nonconforming; that means, if the sample size is n and out of n you know you find x number of are the nonconforming units they are not acceptable to you then; obviously, the proportion nonconforming is p hat is equals to x by n and this the p is the proportion nonconforming is assumed to be following a binomial distribution is it.

So, here clearly mentioned x is the number of nonconforming items in the sample or the items or the units n is the sample size. So, now, in many cases we will find that whenever you deal with the attributes data many companies they feel like using A p control chart. So, as I have already mentioned that at that is a top management level you can use it, at the middle management level and even as the operations management level you can use the p control charts.

For example the management of the chief executive officer of a company can be evaluated by considering the proportion of nonconforming items produced at the company level like say you know on an average it produces 3 to 5 percent on conforming there is a company level. So, in order to are the control this proportion nonconforming of the output. So, you may feel like using p control chart. So, A p chart can provide the source of information for improving product quality.

(Refer Slide Time: 03:04)



Construction and Interpretation

- **Step 1: Select the objective.** Decide on the level at which the p-chart will be used - that is the plant, the department, or the operator level. Decide to control either a single quality characteristic, multiple characteristics, a single product, or a number of products.
- **Step 2: Determine the sample size and the sampling interval.** An appropriate sample size is related to the existing quality level of the process. The sample size must be large enough to allow the opportunity for some nonconforming items to be present on average. The sampling interval-that is, the time between successive samples - is a function of the production rate and the cost of sampling, among other factors.
- **Step 3: Obtain the data, and record on an appropriate form.** Decide on the measuring instruments in advance.
- **Step 4: Calculate the centre line and the trial control limits.** Once they are determined, draw them on the p-chart. Plot the values of the proportion nonconforming for each sample on the chart. Examine the chart to determine whether the process is in control.

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | PROF PRADIP KUMAR RAY
DEPARTMENT OF INDUSTRIAL AND SYSTEMS ENGINEERING
IIT KHARAGPUR

It is obvious how to construct a control chart there are certain steps first you select the objective what is the purpose of using the control chart. So, the details are here and at which level you will be using the p control chart, this should be made very very clear.

Now, the next important solution is that you need to determine the sample size and the sampling interval; that means, how frequently you will be collecting the sample a sampling frequency are the number of samples now as such there is no formula for determining this so, but one formula is that as you are dealing with the attributes data or some rules do not say it is a formula it is a rule that as you are dealing with attributes data; that means, the exact net in the information is missing.

So, what you need to do; that means, usually; that means, the sample size must be on the higher side is it. So, that is one say you know or the guideline and the second guideline is you know you restrict to the number of or the samples in such a way that you can capture in almost also sub situations that will occurring is it; that means, instead of collecting a data only for one day why do not you collect data at least for one month; that means, is a month is a representative and if you consider 25 days in a month or say 30 days in a month you say that within these 30 days, I am facing all sorts of situations and the process performance is influenced by all these situations.

So, this is the basis and this way you determine. In fact, these rules you follow then on the step 3 you have to collect the data and record on an application form many a time

you know you need to design these the data collection form as a step 4 calculate the central line and the trial control limits is it, for that you need to use the formula.

(Refer Slide Time: 05:24)

No Standards Given

- When **no standard or target value** of the proportion non-conforming is specified, it must be estimated from the sample information.
- The **average of these individual sample proportions nonconforming** is used as the center line (CL_p); i.e.,

$$CL_p = \bar{p} = \frac{\sum_{i=1}^g \hat{p}_i}{g} = \frac{\sum_{i=1}^g x_i}{ng} \dots(5)$$

where **g** represents the **number of samples**. Since the true value of p is not known, the value of \bar{p} given by (5) is used as an estimate. In accordance with the **concept of 3 σ limits**, the control limits are given by

$$UCL_p = \bar{p} + 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$LCL_p = \bar{p} - 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \dots(6)$$

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | PROF. PRADIP KUMAR, DEPARTMENT OF INDUSTRIAL AND MANUFACTURING ENGINEERING, IIT KHARAGPUR

And when you use this formula you know there are 2 kinds of situations you might come across, the first one is the no standards given; that means, right now you have started the process and the process has become stable and we say that first the stability condition you have to meet and once the process becomes stable then only you assume that this process may be a controllable process.

So, you may think of the concept of stability; that means, what are the features in a process that you may observe indicating of it is stable is stability. So, that is the first point to be looked into this is a very good assignment. In fact, for any kinds of process once you declare the process to be a stable one then you say whether and it might be assumed that it is a controllable process.

Now, if it is a new process is a stable process controllable process, but there may not be any standards, what you need to do you have to start from the scratch you have to collect data and in a systematic manner and in this case that means, the no standards or the target value of the proportion nonconforming is specified it is not known. So, for that you have to collect data, what you try to do, first you collect data and you calculate based on the data the average proportion nonconforming \hat{p} \bar{p} , how do you collect this that

means, suppose there are g number of samples and for each sample the average value is for the ith sample it is \hat{p}_i and the average is \bar{p} .

Now, how do you get this \hat{p}_i ; obviously, for each sample say ith sample you have x_i number of the nonconforming divided by you know the n or the sample size. So, that n is over here and n is assumed to be constant, this formula you use and this is basically the central line for the p control chart that is why $CL_{\hat{p}}$ then you determine, you know the upper control limit for the p chart as well as the lower control limit or the p chart.

So, g represents the number of samples and since the true value p is not known this is just an estimate. So, the value of \bar{p} as given by 5 is used as an estimate is it. So, it is not known; that means, standards are not known. In accordance with the concept of 3 sigma limits the control limits are given by this one; that means, you know the proportion that is p is assumed 2 binomial distribution, this is the formula you use whereas, for binomial distribution assumptions.

So, this is the standard deviation, this is you know the average you know the center line plus 3 times the standard deviation value. So, here the value of k is 3 and similarly the lower control limit on the other side is \bar{p} minus 3 times the standard deviation value. So, what is the standard deviation value root over $\bar{p}(1 - \bar{p})$ divided by n, with binomial distributional assumption you referred to equation number 6.


(Refer Slide Time: 09:07)

Standards Given


- If the target value of the proportion of nonconforming items is known or specified, the centre line is selected as that target value. In other words, the centre line is given by $CL_p = p_0$... (7)
- where p_0 represents the standard or target value. The control limits in this case are also based on the target value. Thus,

$$UCL_p = p_0 + 3 \sqrt{\frac{p_0(1-p_0)}{n}}$$


$$LCL_p = p_0 - 3 \sqrt{\frac{p_0(1-p_0)}{n}} \quad \dots (8)$$
- If the lower control limit for p turns out to be negative for (6) or (8), the lower control limit is simply counted as zero because the smallest possible value of the proportion nonconforming is zero.



NPTEL ONLINE
CERTIFICATION COURSES



PROF. PRADIP KUMAR
DEPARTMENT OF INDUSTRIAL AND
IIT KHARAGPUR



Then if the standards are given what do you need to do that you do not need to calculate \bar{p} because already you know the standard is given; that means, the target value is mentioned. So, suppose the target value based on your experience, suppose say you are running the systems for a long time is it you are aware that at the value of p_0 is it the process is around this value the process is running. So, that is referred to as a center line that is p_0 is pre specified and once the p_0 is known that is referred to as the standard; obviously, you can determine upper control limit as well as the lower control limit.

So, you use the same formula only the, you know instead of writing \bar{p} you write p_0 that is the standard is given and 3 times the standard deviation is it. So, if the lower control limit for p turns out to be negative in many a time there is a high probability that that lower control limit may be negative. So, negative as such it does not carry or does not carry any meaning. So, what do you need to do; that means, the lower control limit is simply counted as 0 is it, make it 0.

Now, my question is this is an important the question that under you know why do you make it 0 why do not make it you know let it are the; remain at minus value is it. So, this may occur not only for this kind of control chart or any kind of control chart this may occur. So, always it is a right approach that that we make it 0 convert into 0, but there are certain cases where you know the negative value has a meaning.

So, again when we refer to a special case in control charting, we will discuss this aspect.

(Refer Slide Time: 11:07)

- **Step 5: Calculate the revised control limits.** Analyze the plotted values of \bar{p} and the pattern of the plot for **out-of-control conditions**. Typically, one or a few of the rules are used concurrently. On detection of an out-of-control condition, identify the special cause, and propose remedial actions. The out-of-control point or points for which remedial actions have been taken are then deleted, and the revised process average \bar{p}_r is calculated from the remaining number of samples.
- **The revised centre line and control limits are given by**

$$CL_r = \bar{p}_r$$

$$UCL_r = \bar{p}_r + 3 \sqrt{\frac{\bar{p}_r(1-\bar{p}_r)}{n}}$$

$$LCL_r = \bar{p}_r - 3 \sqrt{\frac{\bar{p}_r(1-\bar{p}_r)}{n}} \quad \dots(9)$$
- For p-charts based on a standard p_0 , the revised limits do not change from those given by (8).
- **Step 6: Implement the chart.** Use the revised centre line and control limits of the **p-chart** for future observations as they become available. Periodically revise the chart using guidelines similar to those discussed for variable charts.



This is the standard given next we go to step 5 calculate the revised control limits. So, what do you try to do first you know control limits these are basically the preliminary control limits and then you draw the control chart and suppose you are dealing with some 25 sample points, these sample points are plotted is it.

And then you analyze the plotted values and the pattern of the plot for the out of control condition typically one or few of the rules are used concurrently on detection of an out of control condition identify the special cause and propose remedial actions; that means, you may find there suppose you are dealing with some 20 samples or 25 samples. So, all these 25 and the sample points will be plotted sample values will be plotted and you may find the 2 or 3 sample points are basically you know are plotted outside of the control limits.

So, you have to remove those 2 or 3 values from the from the list assuming that they are due to the occurrence of assignable causes assignable causes are identified and then the remedial actions of the corrective actions are taken to remove those causes that is why these points are removed. Now with the remaining points again you calculate the revised center line same formula you use only thing is this value will be different because already you have removed those points which are found as out of control points in the list and similarly upper control limit and lower control limit is it.

So, the p for p charts based on a standard p_0 the revised limits do not change from those given by 8; obviously, because you do not need to collect data is it. So, you just you construct the control chart with the standard and you start using it and then you have to implement the chart is it so implementation means.

(Refer Slide Time: 13:15)

Example: Twenty-five samples of size 50 are chosen from a plastic-injection molding machine producing small containers. The number of nonconforming containers for each sample is shown in the table.

Sample	Date	Time	Number of Inspected Items, n	Number of Nonconforming Items, s	Proportion Nonconforming, \hat{p}	Comments
1	10/6	8:30	50	4	.08	
2	10/6	9:30	50	2	.04	
3	10/6	10:00	50	5	.10	
4	10/6	10:20	50	3	.06	
5	10/7	8:40	50	2	.04	
6	10/7	9:50	50	1	.02	
7	10/7	10:10	50	3	.06	
8	10/7	10:50	50	2	.04	
9	10/8	9:10	50	5	.10	
10	10/8	9:40	50	4	.08	
11	10/8	10:40	50	3	.06	
12	10/8	11:20	50	5	.10	
13	10/9	8:20	50	5	.10	
14	10/9	9:10	50	2	.04	
15	10/9	9:50	50	3	.06	
16	10/9	10:20	50	2	.04	
17	10/10	8:40	50	4	.08	
18	10/10	9:30	50	10	.20	Drop in pressure
19	10/10	10:10	50	4	.08	
20	10/10	11:30	50	3	.06	
21	10/11	8:20	50	2	.04	
22	10/11	9:10	50	5	.10	
23	10/11	9:50	50	4	.08	
24	10/11	10:20	50	3	.06	
25	10/11	11:30	50	4	.08	
Total			1,250	90		

Now here is an example of p control chart right, the date wise collected the data and the number of inspected items is all 50; that means, here the sample size is 50 and sample size remains constant the number of nonconforming items that you note down these values against each sample. So, the total is 90 and the total number of the units inspected that is 1250. So, individual you know the sample wise you can calculate the proportion nonconforming is it and against each one you know the certain these values are very high compared to other values is it. So, you just note the points. In fact, you write down the points; that means, a drop in pressure sudden is it that is why the value is like this 0.20 is it, this may be considered as an outlier.

(Refer Slide Time: 14:14)

Solution:
The average proportion nonconforming is $\bar{p} = \frac{90}{1250} = .072$



This is the center line of the p-chart.
The trial control limits are found to be

$$CL_p = .072$$


$$UCL_p = .072 + 3\sqrt{\frac{(.072)(1-.072)}{50}} = .182$$

$$LCL_p = .072 - 3\sqrt{\frac{(.072)(1-.072)}{50}} = -.038 \rightarrow 0$$

Since the calculated value of the lower control limit is negative, it is converted to zero.

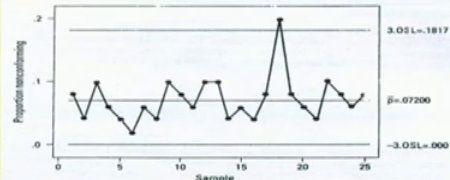



PROF PRADIP KUMAR
DEPARTMENT OF INDUSTRIAL AND S
IIT KHARAGPUR



And then you calculate the average proportion nonconforming 90 divided by 1250 then you calculate the center line you control, you calculate the upper control limit and you can you calculate the lower control limit, the lower control limit is actually it is found as negative and that is why it is converted to 0 and we are assuming that the control limits are a plus minus 3 sigma limits as we have been saying all the time that unless otherwise specified the control limits are assumed to be at the plus minus 3 sigma limits from the central lines.

(Refer Slide Time: 14:54)



Proportion-nonconforming chart for containers



- A remedial action is taken to eliminate the special cause, sample 18 is deleted, and the revised center line and control limits are then found to be

$$CL_p = \frac{90 - 10}{1200} = .067$$


$$UCL_p = .067 + 3\sqrt{\frac{(.067)(1-.067)}{50}} = .173$$

$$LCL_p = .067 - 3\sqrt{\frac{(.067)(1-.067)}{50}} = -.039 \rightarrow 0$$

- The remaining samples are now in control.

PROF PRADIP KUMAR
DEPARTMENT OF INDUSTRIAL AND S
IIT KHARAGPUR





This is this way you calculate and then you plot these data is it. So, this point is found as nonconforming is it not nonconforming out of control and that is why this point is the removed is it the sample 8 is deleted and then where; that means, the number of nonconformities nonconforming in that particular sample 18 was 10. So, the total number is 90 minus 10 divided by 1200, but; that means, it was 1250. So, one sample is removed is it? So, it is the revised value of the center line and similarly the revised value of the upper control limit as well as the lower control limit as it here computed these 2 values are computed is it and we find that we the remaining samples are now in control.

(Refer Slide Time: 15:50)

Example: Management has decided to set a standard of 3% for the proportion of non-conforming test tubes produced in a plant. Data collected from 20 samples of size 100 is shown in the table below, as is the proportion of nonconforming test tubes for each sample.

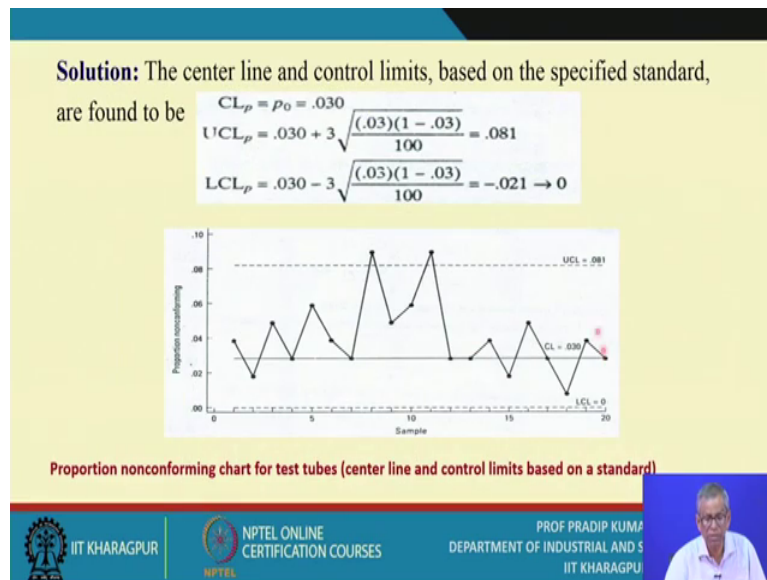
Sample	Date	Time	Number of Inspected Items, n	Number of Nonconforming Items, c	Proportion Nonconforming, \bar{p}	Comments
1	9/8	8:20	100	4	.04	
2	9/8	8:45	100	2	.02	
3	9/8	9:10	100	5	.05	
4	9/8	9:30	100	3	.03	
5	9/9	9:00	100	6	.06	
6	9/9	9:20	100	4	.04	
7	9/9	9:50	100	3	.03	
8	9/9	10:20	100	9	.09	Die not aligned
9	9/10	9:10	100	5	.05	
10	9/10	9:40	100	6	.06	
11	9/10	10:20	100	9	.09	
12	9/10	10:45	100	3	.03	
13	9/11	8:30	100	3	.03	
14	9/11	8:50	100	4	.04	
15	9/11	9:40	100	2	.02	
16	9/11	10:30	100	5	.05	
17	9/12	8:40	100	3	.03	
18	9/12	9:30	100	1	.01	
19	9/12	9:50	100	4	.04	
20	9/12	10:40	100	3	.03	
Total			2,000	84		



 PROF PRADIP KUMAR
 DEPARTMENT OF INDUSTRIAL AND S
 IIT KHARAGPU

So, that is the final control limit, if in this case what we have found that this is a second example; that means, a standard is set; that means, what is that standard. So, the value of p_0 it is 3 percent. So, please go through this example and you will find the here we have assumed that the sample size is 100 it is quite common when you deal with the attributes data and you use the control chart. So, usually it is at least 25 the sample size when whenever you deal with the control charts for the attributes.

So, this is just an example and again you know this particular sample you will find that the number of nonconforming items is significantly more and that is why you know the die not aligned there is a specific reasons as a process engineer you must be able to identify is it that is your job while you construct the control chart.

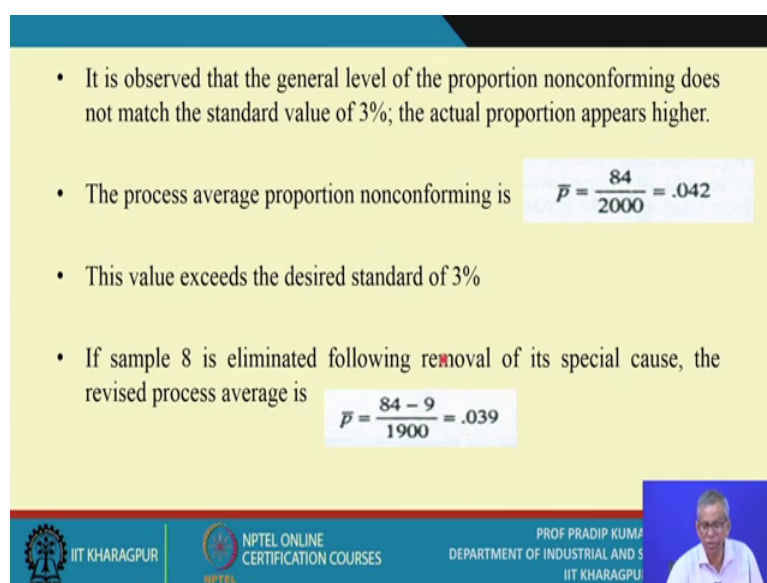
(Refer Slide Time: 16:56)



And; obviously, in many cases this may be considered as an outlier.

So, what you try to do first you calculate the p_0 is point 3 and then based on this you calculate upper control limit lower control limit and straight you start using the control chart. So, all these values are plotted is it and you find that these 2 sample points are plotted outside the upper control limit is it. So, this and then you start interpreting whether these you observe any plotted patterns or not.

(Refer Slide Time: 17:30)




Whether this behavior of the process is systematic or non systematic, this way you calculate all these details are here and the sample 8 is eliminated.


(Refer Slide Time: 17:44)


When the control limits are calculated based on this revised average, we have

$$CL_p = .039$$
$$UCL_p = .039 + 3 \sqrt{\frac{(.039)(1 - .039)}{100}} = .097$$
$$LCL_p = .039 - 3 \sqrt{\frac{(.039)(1 - .039)}{100}} = -.019 \rightarrow 0$$


If a control chart were constructed using these values, the remaining samples (including sample 11) would indicate a process in control, with the point hovering around .039, the calculated average.



 IIT KHARAGPUR

 NPTEL ONLINE CERTIFICATION COURSES

PROF. PRADIP KUMAR
DEPARTMENT OF INDUSTRIAL AND SYSTEMS ENGINEERING
IIT KHARAGPUR



So; obviously, you have this \bar{p} values calculate you recalculate at these values and then again you calculate the center line upper control limit lower control limit again you find the lower control limit is negative. So, it is converted to 0, the remaining samples including sample 11 would indicate a process in control with the point hovering around 0.039 that means, it is the processes around exist process exist; that means, it is process is performing around this value 0.039 is it, the calculated average, that is the interpretation.

(Refer Slide Time: 18:20).

Chart for the number of nonconformities (c-chart)

- A nonconformity is defined as a quality characteristic that does not meet some specification
- A nonconforming item has one or more nonconformities that make it nonfunctional
- It is also possible for a product to have one or more nonconformities and still conform to standards
- A c-chart is used to track the total number of nonconformities in samples of constant size
- The occurrence on nonconformities is assumed to follow a Poisson distribution
- This distribution is well-suited to modeling the number of events that happen over a specified amount of time, space, or volume

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | PROF PRADIP KUMAR | DEPARTMENT OF INDUSTRIAL AND S | IIT KHARAGPUR

When you refer to the c – chart, now the c - chart when you use c - chart you are trying to control nonconformity.

So, what is the nonconformity, nonconformity is nothing, but the defect is it. So, a product an item may have the several kinds of or the defects, it is your responsibility to identify these defects is it against a particular item. So, a nonconforming item; that means, a defective item may have one or more nonconformities that make it nonfunctional like. So, suppose in an automobile if the engine with the becomes the defective the entire system the entire automobile becomes defective whereas, suppose there is say certain defects in other components like say you know the seating arrangements and all those like on at the dashboard and those defects as such do not affect the performance of the product in that case the performance of the vehicle is it. In that case you know even if there are many defects a nonconformities the product as such is never considered to be nonconforming or defective.

There are many such instances we will find, a c chart is used to track the total number of nonconformities in samples of constant size; that means, sample set does not change is it and what do you assume that these number of nonconformities; obviously, in a sample it is a random variable and this number means the discrete random variable and, we assumed that it follows a Poisson distribution; that means, there will be defects, but the

occurrence of such defects is it is a probability of such occurrence is very very less is it. So, that is why you may assume it to be of Poisson distribution.

(Refer Slide Time: 20:33)

• If x represents the number of nonconformities in the sample unit and c is the mean, then Poisson distribution yields

$$p(x) = \frac{e^{-c} c^x}{x!}$$

• **No Standard Given**

• The average number of nonconformities per sample unit is found from the sample observations and is denoted by \bar{c} . The center line and control limits are

$$CL_{\bar{c}} = \bar{c} \quad \dots(18)$$

$$UCL_{\bar{c}} = \bar{c} + 3\sqrt{\bar{c}}$$

$$LCL_{\bar{c}} = \bar{c} - 3\sqrt{\bar{c}}$$

• If the lower control limit is found to be less than zero, it is converted to zero.

• **Standard Given**

• Let the specified goal for the number of nonconformities per sample unit be c_0 . The center line and control limits are then calculated from

$$CL_{c_0} = c_0$$

$$UCL_{c_0} = c_0 + 3\sqrt{c_0}$$

$$LCL_{c_0} = c_0 - 3\sqrt{c_0} \quad (19)$$

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | PROF. PRADIP KUMAR, DEPARTMENT OF INDUSTRIAL AND MANUFACTURING ENGINEERING, IIT KHARAGPUR

This is as per the Poisson distribution this is the probability that there will be x number of defects in a sample of size n in e to the power minus c , c to the power x by factorial x . So, what is c , c is the average number of nonconformities is it, here usual in any textbooks on the quality. So, we the c is this is the term always we use, that is why it is sometimes referred to as the c control chart again there could be 2 cases no standard is given in that case you have to collect data and you have to compute the average value like \bar{c} is it from different samples and then you determine upper control limit, lower control limit the same procedure you follow the procedure which you have followed for the p control chart.

So, again when the standards are given, the standard you suppose a c_0 that you say that this is the number of nonconformity is it a sample restricted to c_0 say 1 or 2 whatever it is, when the sample size is specified and accordingly directly you can calculate the upper control limit, the lower control limit as well as the center line is it.

(Refer Slide Time: 21:54)

Example: Samples of fabric from a textile mill, each 100 m² are selected and the number of occurrences of foreign matter are recorded. Data for 25 samples are shown the table below. Construct a c -chart for the number of nonconformities.

Sample	Nonconformities	Sample	Nonconformities
1	5	11	11
2	4	15	9
3	7	16	5
4	6	17	7
5	8	18	6
6	5	19	10
7	6	20	8
8	5	21	9
9	16	22	9
10	10	23	7
11	9	24	5
12	7	25	7
13	8		

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | PROF PRADIP KUMAR | DEPARTMENT OF INDUSTRIAL AND S | IIT KHARAGPU

So, this is another example like there are 25 samples and the sample size remains constant like say 100 square meter and this fabric from a textile mill. So, that is the area of the fabric is 100 square meter. Now, what you try to do it remains same for all the samples and you just count the number of nonconformities of a specific type; that means, the defects like 5 4 7 6 and so, on, you have all this data.

(Refer Slide Time: 22:29)

Solution: The average number of nonconformities based on the sample information is found as follows.

The center line is given by $\bar{c} = \frac{189}{25} = 7.560$

The control limits are given by $UCL_c = 7.560 + 3\sqrt{7.560} = 15.809$
 $LCL_c = 7.560 - 3\sqrt{7.560} = -0.689 \rightarrow 0$

c-Chart for the foreign matter data

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | PROF PRADIP KUMAR | DEPARTMENT OF INDUSTRIAL AND S | IIT KHARAGPU

Then what do you do you calculate the average nonconformities and that is c bar that is 7.560 and then you determine the upper control limit and the lower control limit for the c

control chart, again you find that the lower control limit is actually negative converted into 0 then you plot all these values 25 and you find that this the 9th sample may be his this plot it outside of the upper control limit that is 15.81.

So, what you try to do you remove it; that means, it is considered to be out of control point the assignable causes you are able to identify and when you are able to identify. So, you are also able to remove them that is why the point is removed.

(Refer Slide Time: 23:20)

• Assuming remedial action has been taken for the special causes, the center line and control limits are revised as follows (sample 9 is deleted):

$$\bar{c} = \frac{189 - 16}{24} = 7.208$$
$$UCL_c = 7.208 + 3\sqrt{7.208} = 15.262$$
$$LCL_c = 7.208 - 3\sqrt{7.208} = -0.846 \rightarrow 0$$

• After this revision, the remaining sample points are found to be within the limits

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | PROF PRADIP KUMAR | DEPARTMENT OF INDUSTRIAL AND SYSTEMS ENGINEERING | IIT KHARAGPUR

So, with this removal again you recalculate is it now there will be 24 rather samples, you calculate c bar. So, that is the center line then you calculate again the upper control limit and the lower control limit is it, this is the procedure you follow.

(Refer Slide Time: 23:39)

Chart for the number of nonconformities per unit (u-chart)

- For situations in which the sample size varies, a ***u*-chart** is used.
- For companies which **inspect all items produced** or services rendered for the presence of nonconformities, the output per production run can vary because of fluctuating supplies of labor, machinery, and raw material; consequently, the number inspected per production run changes, thus causing varying sample sizes.
- When the **sample size varies**, a u-chart is constructed to monitor the number of nonconformities per unit.
- Even though control limits change as the sample size varies, the **center line of a u-chart remains constant**, which permits meaningful comparisons between the samples.

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | PROF. PRADIP KUMAR | DEPARTMENT OF INDUSTRIAL AND MANUFACTURING ENGINEERING | IIT KHARAGPUR

Now, when you know the sample size varies and you are trying to control the nonconformities or the defects on a particular item what do you try to do; that means, you move to say u control chart and that is referred to as the number of nonconformity per unit; that means, here what we have assumed in the first case we have assumed that you know the example you have taken up that the area of the fabric is 100 square meter.

So, here what we are assuming that it may be 100 square meter, 150 square meters and all those is it. So, if you come across such situations; obviously, c - chart is not applicable what is applicable is u - chart. So, that is why it is effort is the number of nonconformity per unit.

So, there are many instances where you use u chart and though, how do you compute.


(Refer Slide Time: 24:41)

- When the sample size varies, the number of nonconformities per unit for the i th sample is given by


$$u_i = \frac{c_i}{n_i} \quad \dots(20)$$

where c_i is the number of nonconformities in the i th sample, and n_i is the size of the sample. Note that the sample size n_i need not always be an integer.

- The average number of nonconformities per unit (\bar{u}), which is also the center of a u -chart, is given by


$$\bar{u} = \frac{\sum_{i=1}^g c_i}{\sum_{i=1}^g n_i} \quad \dots(21)$$


IIT KHARAGPUR



NPTEL ONLINE
CERTIFICATION COURSES


PROF PRADIP KUMAR
DEPARTMENT OF INDUSTRIAL AND S
IIT KHARAGPUR




So, here the parameter is u_i ; that means, i sample; that means, c_i number of as a nonconformities as sample sizes n_i is it so; that means, sample size varies across the samples this is the basic assumptions. So, you have to for u_i and this u_i is also a Poisson distributed and then you calculate the \bar{u} when then the no standards are given. So, the σc_i divided by σn_i .

(Refer Slide Time: 25:16)

- The control limits are given by


$$UCL_u = \bar{u} + 3 \sqrt{\frac{\bar{u}}{n_i}} \quad \dots(22)$$
$$LCL_u = \bar{u} - 3 \sqrt{\frac{\bar{u}}{n_i}}$$


IIT KHARAGPUR



NPTEL ONLINE
CERTIFICATION COURSES

PROF PRADIP KUMAR
DEPARTMENT OF INDUSTRIAL AND S
IIT KHARAGPUR



It is very simple one and now with this u when \bar{u} is known you calculate upper control limit and the lower control limit and this is the expression for say the standard deviation.

So, 3 times standard deviations is it on one side or upper control limit and you subtract; that means, from say central line or from say the average; that means, it lies on the other side of the central line is it ok. So, this is this way you calculate a upper control limit and lower control limit.

(Refer Slide Time: 25:49)

Example: The number of nonconformities in carpet is determined for 20 samples, but the amount of carpet inspected for each sample varies. Results of the inspection are shown in the table below. Construct a control chart for the number of nonconformities per 100 m²

Sample	Amount Inspected (in m ²)	Number of Nonconformities, c_i	Sample	Amount Inspected (in m ²)	Number of Nonconformities, c_i
1	200	5	11	300	9
2	300	14	12	250	16
3	250	8	13	200	12
4	150	8	14	250	10
5	250	12	15	100	6
6	100	6	16	200	8
7	200	20	17	200	5
8	150	10	18	100	5
9	150	6	19	300	14
10	250	10	20	200	8
					Total 192

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | PROF PRADIP KUMAR | DEPARTMENT OF INDUSTRIAL AND MATERIALS ENGINEERING | IIT KHARAGPUR

So, this is an example like here you will find that the inspected area varies from one sample to another like for the sample 1 it is 200, in the sample 2 it is 300 hundred and similarly for all other samples and then you count the number of nonconformities in each sample like 5 14 8 and so, on. So, you are considering 20 such samples.

(Refer Slide Time: 26:17)

Solution: With 100 m² as a unit, the sample sizes are computed for each sample.




The center line is found as
$$\bar{n} = \frac{\sum c_i}{\sum n_i} = \frac{192}{41} = 4.683$$

The control limits are found to be

$$UCL_{nr} = 4.683 + 3 \sqrt{\frac{4.683}{2}} = 9.273$$

$$LCL_{nr} = 4.683 - 3 \sqrt{\frac{4.683}{2}} = 0.092$$

The control limits for other samples are calculated in the same manner.




PROF PRADIP KUMAR
DEPARTMENT OF INDUSTRIAL AND S
IIT KHARAGPUR

Then what you do you calculate the centerline is it sigma ci divided by sigma ni. So, that is 4.683 and immediately after that you calculate the upper control limit and the lower control limit and here interesting finding is that the lower control limit is positive is it and then when you construct the control chart. So, is the same procedure you follow the procedure which I have explained with the reference to p control chart.

(Refer Slide Time: 26:50)

Control Limits for nonconformities per unit in carpets

Sample	Sample Size, n _i	Nonconformities per 100 m ² , u _i	Upper Control Limit	Lower Control Limit
1	2	2.506	9.274	0.092
2	3	4.667	8.431	0.935
3	2.5	3.200	8.789	0.577
4	1.5	5.333	9.984	0
5	2.5	4.800	8.789	0.577
6	1	6.000	11.175	0
7	2	10.000	9.274	0.092
8	1.5	6.667	9.984	0
9	1.5	4.000	9.984	0
10	2.5	4.000	8.789	0.577
11	3	3.000	8.431	0.935
12	2.5	6.400	8.789	0.577
13	2	6.000	9.274	0.092
14	2.5	4.000	8.789	0.577
15	1	6.000	11.175	0
16	2	4.000	9.274	0.092
17	2	2.500	9.274	0.092
18	1	5.000	11.175	0
19	3	4.667	8.431	0.935
20	2	4.000	9.274	0.092
Total	41			

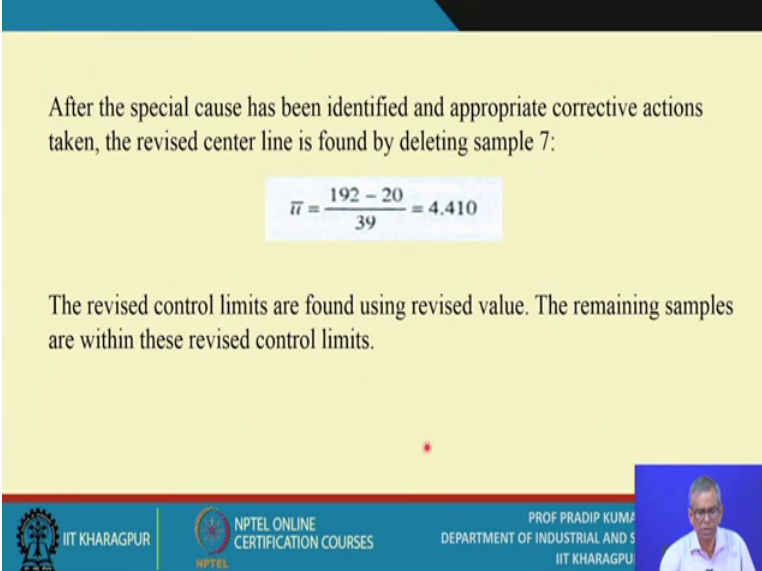




PROF PRADIP KUMAR
DEPARTMENT OF INDUSTRIAL AND S
IIT KHARAGPUR

So, this is just the example the sample size ni and then you know this is the procedure you follow please go through these examples is it whatever the formulation whatever the

steps we have discussed with respect to a particular control chart. So, those formulations we use and you follow the steps for construction of the control chart.

(Refer Slide Time: 27:12)



After the special cause has been identified and appropriate corrective actions taken, the revised center line is found by deleting sample 7:

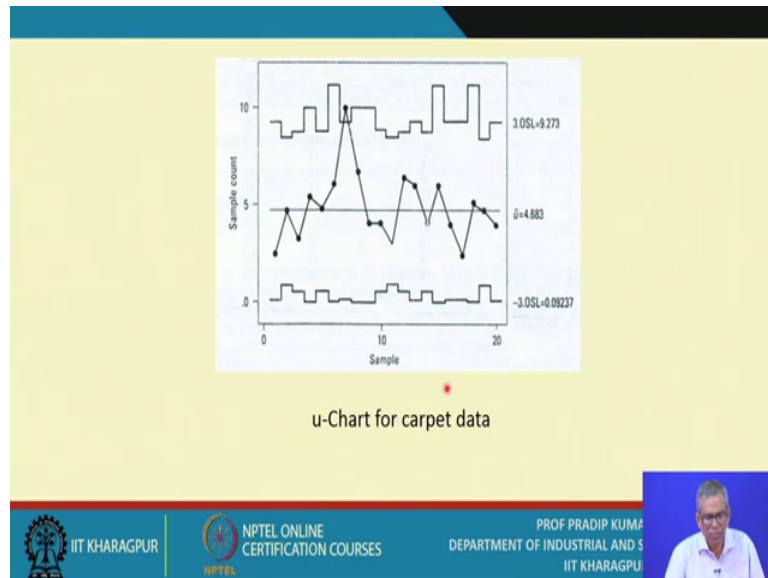
$$\bar{\bar{x}} = \frac{192 - 20}{39} = 4.410$$

The revised control limits are found using revised value. The remaining samples are within these revised control limits.

The slide features a blue header, a yellow main content area, and a blue footer. The footer contains the IIT Kharagpur logo, NPTEL Online Certification Courses logo, and the name and affiliation of Prof. Pradip Kumar. A small video inset of the professor is visible in the bottom right corner.

These details are here and always what we will try to do; that means, first you go for a preliminary control chart and then assuming that the process is in control right then you go for a determination of the revised control limits and when you determine the revised control limits assuming that I am creating and I am designing a control chart assumption is the process is in control this you know these conditions you have to ensure that is before you start constructing a control chart make sure that the process is in control is it.

(Refer Slide Time: 28:00)



So, this is the procedure this is just an example of u chart and you know this is the idea is that as the sample size varies; that means, theoretically speaking even if you assume say one average value for you, but as the sample size varies; that means, n_i is a sample size for the i th sample; obviously, the upper control limit and lower control limit may also vary.

(Refer Slide Time: 28:27)

Operating Characteristic curves for Attribute Control Charts

- An operating characteristic (OC) curve plots the probability of incorrectly concluding that a process is in control as a function of a process parameter
- For p-chart, if the process proportion nonconforming is some value p , the probability of a Type II error is

$$\beta = P(\hat{p} < UCL_p | p) - P(\hat{p} \leq LCL_p | p)$$
$$= P(X < nUCL | p) - P(X \leq nLCL | p)$$

- Where, n = sample size
- \hat{p} = sample proportion nonconforming
- X = number of nonconforming items

So, what is impact that also can be analyzed; that means, when you deal with the say variable sample size. So, how many different say the alternatives you have for

constructing the control charts. So, usually there are 4 options later on will discuss it; that means, So, the variable sample size case will be dealing it with separately particularly with respect to control chart for the variables.

Now, here just in short briefly I will just explain that how to construct the say operating characteristics curves or the OC curves as a performance measure for an attribute control chart. So, how do you do it; that means, essentially as I have already explained that OC curves in control charting means a plot of beta versus the process parameters. So, like say suppose it is a p control chart; obviously, the process parameter is p, you need to consider are the several values of p and for each value of p you need to compute beta is it.

So, suppose 10 or 15 such values of p you consider and corresponding beta values also you are computed and then you just plot them and by looking at the plot pattern you can conclude that whether the performance of the control chart is all right or not is it as you may be recollecting that beta is nothing, but the probability of making type 2 error and you must have sufficient control against type 2 error, say type 1 error is not even if you ignore type 1 error as such there is there you may not face any problem only thing is unnecessarily you may be stopping the process and you try to find out some assignable causes and those assignable causes do not exist is it.

So, you may be less productive, but as such there is other assurance that the performance of the product or the performance of the process will not be affected, but if the beta you do not control this error the type 2 error then; obviously, you know there could be a serious impact; that means, suddenly there could be a damage to the process as we took a which you may not be aware of that why it has happen and it could be a permanent damage to the process; that means, what we are assuming that if the process remains in control state the process performance will be very high and it will be a sustainable process, how to compute beta; that means, even if at a particular value of p. So, what is the probability that the point will be plotted is it within say a region defined by the upper control limit and the lower control limit.

So, this is the expression probability that the value of p is less than UCL_p minus the probability that the p is less than equals to LCL_p is clear given a value of p. So, this is a $\frac{X}{n}$ so; obviously, it is X that is the random variable number of

for nonconforming or nonconformities n into this one. Now, n is the sample size with a sample proportion nonconforming and the number of nonconforming items.

(Refer Slide Time: 32:05)

• Let r_1 and r_2 be defined as follows:

$$r_1 = [n \text{ UCL}]$$
$$r_2 = [n \text{ LCL}]$$

where $[n \text{ UCL}]$ denotes the largest integer less than or equal to $n \text{ UCL}$ and $[n \text{ LCL}]$ denotes the smallest integer greater than or equal to $n \text{ LCL}$.

The probability of a Type II error may be expressed as

$$\beta = P(X \leq r_1) - P(X \leq r_2)$$

The slide footer includes the IIT Khharagpur logo, NPTEL Online Certification Courses logo, and the name of Prof. Pradip Kumar, Department of Industrial and Manufacturing Engineering, IIT Khharagpur. A small video inset of the professor is visible in the bottom right corner.


So, we are explaining respect to p chart now n into use UCL may not be necessarily an integer similarly n into LCL may not be necessarily an integer now; obviously, the X is an integer. So, there will be some other corrections you have to make; that means, you say r_1 is n into UCL this expression denotes the largest integer less than or equal to n into UCL is it to make it an integer 1 and n into LCL denotes the smallest integer greater than or equal to n into LCL because this n into LCL is not necessarily an integer similarly n into UCL you have to make them integer. So, which rule you follow, you may have this sort of rules. So, please go through these rules will very easily you can understand.

So, what is the expression for probability of type 2 error that is beta equals to probability that X is less than equals to r_1 , r_1 is this one is it with these corrections minus probability that X is less than equals to r_2 is it.

(Refer Slide Time: 33:13)

Example: Twenty-five samples of size 50 are chosen from a plastic-injection molding machine producing small containers. The revised control limits for the p-chart are $UCL_p = 0.173$ and $LCL = 0$, with the revised center line at 0.067. The sample size is 50. Construct an OC curve as function of the process average proportion nonconforming.

Solution: The probability of a Type II error found using a binomial distribution is

$$\begin{aligned}\beta &= P\{X < 50(.173) \mid p = .10\} - P\{X \leq 50(0) \mid p = .10\} \\ &= P\{X < 8.65 \mid p = .10\} - P\{X \leq 0 \mid p = .10\} \\ &= P\{X \leq 8 \mid p = .10\} - P\{X \leq 0 \mid p = .10\} \\ &= \sum_{i=1}^8 \binom{50}{i} (.10)^i (.90)^{50-i} = .937\end{aligned}$$


IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | PROF PRADIP KUMAR | DEPARTMENT OF INDUSTRIAL AND MANUFACTURING ENGINEERING | IIT KHARAGPUR

So, this there is an example we have, UCL_p is given 0.173, LCL is 0 and the center line is 0.067. So, this is the just an application, with respect to p equals 0.10 I am trying to compute what is the value of beta.

So, probability X is less than 50 is the sample size and this is the upper control limit. So, this is 50 is a sample size and this is the lower control limit, lower control limit is 0. So, ultimately you get these expressions and you can use say here the binomial distributions is directly you can use that you know that the probability mass functions or binomial distributions and then you consider all the possible values and then you get the value of beta.

(Refer Slide Time: 34:02)




- If a Poisson approximation to the binomial is used when n is large and p is small (such that np is less than or equal to 5), we have $np = 50(0.10)$
- Next, using the Poisson cumulative probability tables, we get

$$\beta = P(X \leq 8 | np = 5) - P(X \leq 0 | np = 5)$$

$$= .932 - .007 = .925$$

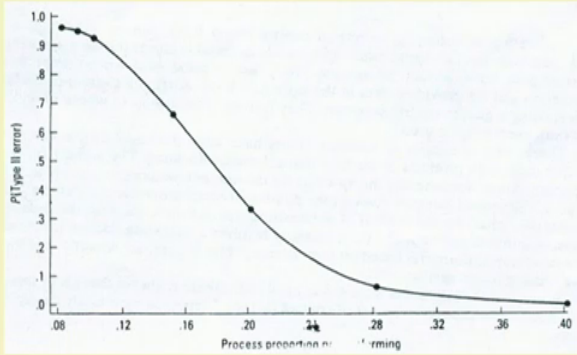
Probability of a Type II error for several values of p is given as follows:

Process Proportion Nonconforming, p	$P(X \leq 8 p)$	$P(X \leq 0 p)$	P(Type II Error), β
.08	.979	.018	.961
.09	.960	.011	.949
.10	.932	.007	.925
.15	.662	.001	.661
.20	.333	.000	.333
.28	.062	.000	.062
.40	.002	.000	.002




 IIT KHARAGPUR
  NPTEL ONLINE CERTIFICATION COURSES
 PROF PRADIP KUMAR
DEPARTMENT OF INDUSTRIAL AND SYSTEMS
IIT KHARAGPUR
 

So, you go for Poisson approximation also in many a time you go for Poisson approximation to binomial distributions. So, if you go for it, beta is computed as 0.925 that means, the probability of non detection is very very high is it. So, as you know the shift from the center line; that means the average across which the process exists. So, as the shift magnitude increases probability of making type 2 error is it is the decreases; that means, the probability have non detection the decreases, this is just an example.

(Refer Slide Time: 34:50)



Operating Characteristic Curve for p-Chart

 IIT KHARAGPUR
  NPTEL ONLINE CERTIFICATION COURSES
 PROF PRADIP KUMAR
DEPARTMENT OF INDUSTRIAL AND SYSTEMS
IIT KHARAGPUR
 

So, this sort of curve we will get; that means, in one quadrant; that means this is the probability of type 1 error the probability of non detection of the shift in the process parameters value. So, in this case if it is the p chart what is the process parameter p, if it is a c chart what is the process parameters if it is c, if it is the u control chart, what is the process parameter that is small u and so on.

(Refer Slide Time: 35:20)

Reference

- ✓ Amitava Mitra, Fundamentals of Quality Control and Improvement, John Wiley.
- ✓ Jerry Banks, Principles of Quality Control, John Wiley.

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | PROF. PRADIP KUMAR, DEPARTMENT OF INDUSTRIAL AND SYSTEMS ENGINEERING, IIT KHARAGPUR

So, this is the procedure of constructing the OC curves or attributes control chart. So, later on when you discuss the variables control charts. So, we will refer 2 other cases both you know the general purpose control charts and the variables as well as the special purpose control chart. So, in specific situations the researchers the practitioners they recommend different kinds of spatial types of control charts. So, will be discussing in another coming sessions these are the 2 references you can opt for where you know all these details will get either I suggest that as a student as an as a learner as a practitioners. So, just go through all the details and; obviously, if you have any queries and all always you can keep in touch with us.

Thank you.