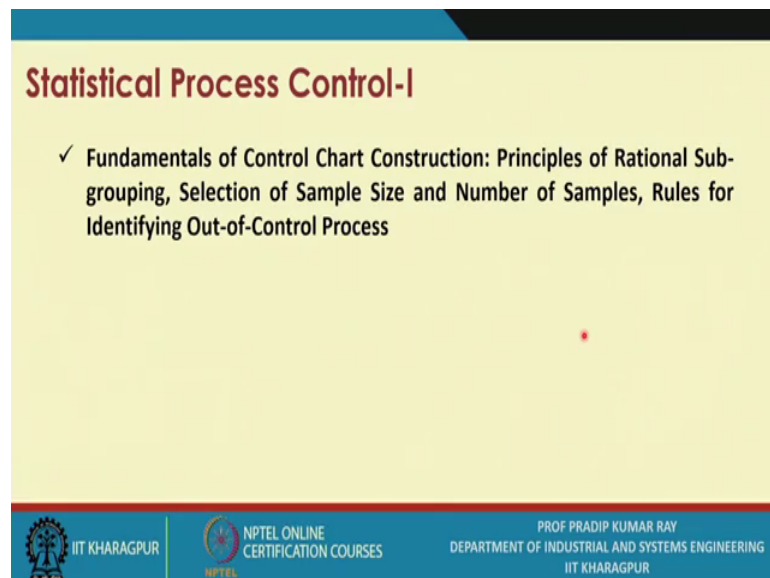


Quality Design and Control
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Lecture – 19
Statistical Process Control - I (Contd.)

So in this lecture session, I am going to discuss under statistical process control.

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Statistical Process Control-I

- ✓ Fundamentals of Control Chart Construction: Principles of Rational Sub-grouping, Selection of Sample Size and Number of Samples, Rules for Identifying Out-of-Control Process

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Some of the important issues like the Fundamentals of Control Chart Construction, the principles of Rational Sub-grouping, Selection of the Sample Size and Number of Samples, Rules for Identifying Out - of- Control Process plus you know I also want to introduce the concept of control charting for the attributes data, these are all very vital issues as far as construction of a control chart is concerned and many a time.

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Principles of Rational Sub-grouping

- Shewhart mentions the **fundamental criteria for the selection of rational subgroups, or rational samples.**
- The premise is that a **rational sample is chosen** in such a way that the variation within it is considered to be due only to common causes.
- **Samples are selected such that if special causes are present they will occur between the samples.**
- Therefore, the **differences between samples will be maximized, and differences within samples will be minimized.**

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Now, you go for interpreting interpretation of the control chart and there must be right kind of interpretation from a control chart and that you know say we actually know that what extent the process has been maintaining a state of statistical control and you know it all depends on you know whether you are you have constructed a control chart following the specific rules guidelines and principles.

Now, you know by the entire control chart you know the; depends on, how would you select the sample and from the population from the process and there are many other factors. So, as far as you know getting a sample is concerned from the population or from the process there are certain principles you have to follow and these are referred to as the principles of rational sub grouping sub grouping means essentially the sampling and you know the Shewhart who is I referred to as the father of statistical process control who introduced the concept of control charting or initially for the manufacturing processes he mentions the fundamental criteria for the selection of rational subgroups ; that means, what are the basic criteria based on which you should you know select a sample or you should create a sample.

The premise is a rational sample is chosen in such a way from the process that the variation within the sample is considered to be due to only common causes; that means, what you expect that within the sample the variation should be as minimum as possible and the samples are selected such that if special causes are present they will occur

between the samples; that means, what we try to do; that means,. So, we take a sample from at a particular point in time or at instant of time or during period of time and while you take a sample will while you form the sample from the population now, you need to take the measurements from the samples.

Now, condition should be such that these you know the measurements data which you get from a sample now when you analyze this data you will find there are the minimum differences between this data. So, that condition you have to ensure whereas, one sample is not sufficient, you must have multiple samples over the time periods. So, why do you have multiple samples that means, make sure that the differences between the samples should be as maximum as possible?

There are 2 the principles we follow while you form the sample while you select the number of samples from a process the differences between the samples should be maximized and the differences within the samples should be minimized. So, these are the 2 vital principles and these principles are referred to as principles of rational sub grouping.

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Selection of Sample Size and Number of Samples

- **Selecting sample size** - number of units in each sample – is a necessity in using control charts.
- If we can tolerate smaller changes in the process parameters, a small sample size might be sufficient.
- Alternatively, if it is important to detect slight changes in process parameters, a large sample size will be needed.
- **Sampling frequency** must be decided prior to construction of control charts. Choosing large samples very frequently is the sampling scheme that may provide maximum information.

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Now, first you select the sample and then you consider the frequency of sampling which is nothing, but the determination of the number of samples for construction of a control chart spraying. So, first you construct the control chart and then you start using it and after it is use you check whether it is performing well or not; that means, whether the

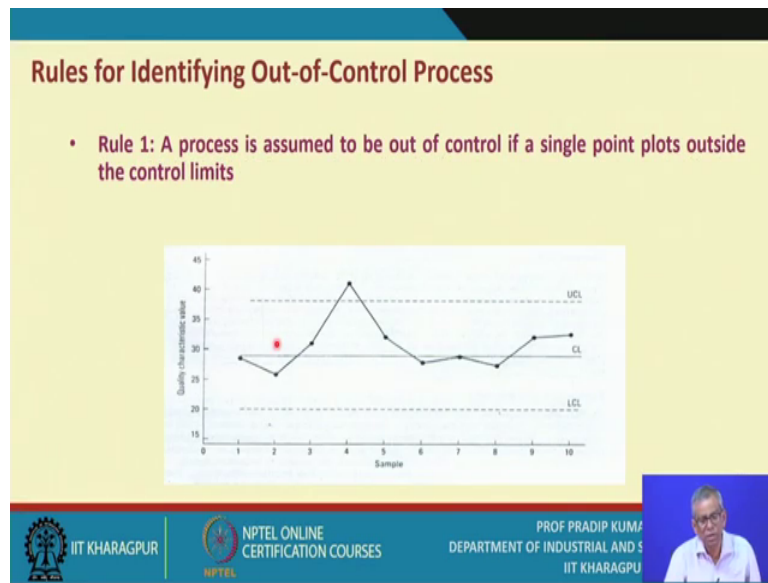
control chart is performing as for you know the objectives for which the control chart is in use. So, for that you know the 2 specific the performance measures we always use one is the operating characteristic curve or the OC curve or the control chart and the second one is the ARL curve, these the 2 performance measures have already discussed.

Now, the selecting sample size that is essentially the number of units in each sample that is to be determined is a necessity in using a control chart that is very clear otherwise you just cannot construct the control chart. If we can tolerate smaller changes in the process parameters this point is to be noted a small sample size might be sufficient is it. There are process settings like you know with respect to a machining process it could be you know like say the spindle speed or the feed rate or say you know the depth of cut with respect to a specific operation say turning and there are many other process parameters depending on the types of say the processes that we you know we deal with.

Alternatively if it is important to detect slight changes in the process parameters that is very very important like there are many sophisticated processes you deal with certain even if there is a minimum change in the process parameter setting has to be detected. So, in that case you need to have a large sample size.

Sampling frequency; that means, the number of samples to be considered for constructing a control chart must be decided prior to construction of the control charts this point I have already emphasized choosing large samples very frequently is the sampling scheme that may provide maximum information; that means, it should be as many as possible, but to just keep in mind at this stage, but there is as such there is no formula or say you know some sort of you know the formula which you may use for determining say the number of samples. So, it based on you know what kind of processes you are dealing with you must have an idea about the amount of variability in the process and accordingly you decide.

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Now as I have been telling you all the time that the main purpose of constructing a control chart main purpose of using a control chart is to identify out of control process; that means, what will assume that once you start the process our null hypothesis is that the process is in control otherwise how do you start the process it is.

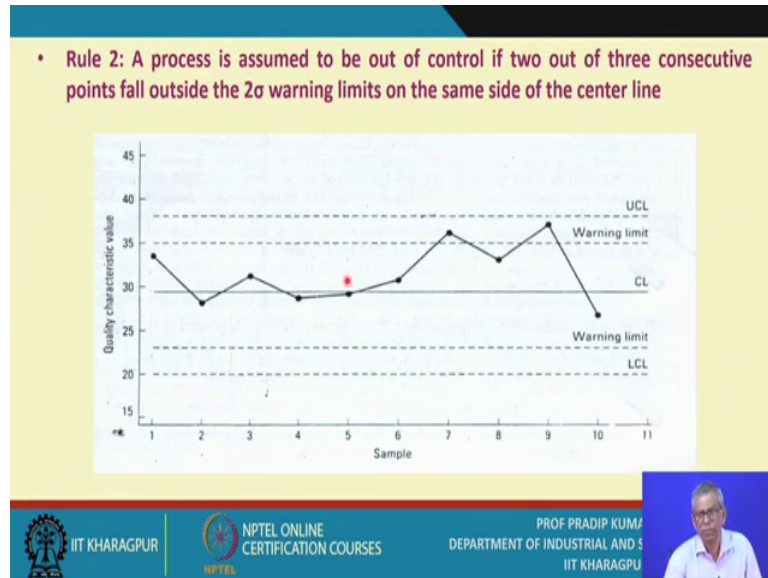
So, now what you need to do in a running process, you need to be on your toes you must check whether something has gone wrong and the process has gone out of control or not, for that this particular tool called control chart is use. Now the researchers they haven't you know the proposed the certain rules for identifying out of control process there are 5 specific rules, let me explain one by one all these rules.

The rule one is very simple the process is assumed to be out of control please note this point that is it is assumed to be out of control actually may not be out of control the or you know as you look at the process you know the kinds of well you know the data patterns you observe you find you may you are concluding at this point in time that other process may be out of control you are not definite scale. So, these interpretations may be wrong when we discussed the type1 error and type 2 error, we mentioned this point. So, please refer to these 2 concepts once again if you have any doubts.

A process is assumed to be out of control if a single point plots outside the control limits. So, here all these sample points are plotted with these upper control limit and lower control limit what you point that the fourth point is plotted outside the upper control

limit. So, immediately you conclude that at this point in time is it something has gone wrong and the process may go out of control or might have gone out of control, this is the rule 1 simple.

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Now, what is rule 2, the rule 2 is a process is assumed to be out of control if 2 out of 3 consecutive points fall outside the 2 sigma warning limits on the same side of the central line is it so; that means, what we have already mentioned in the previous lecture sessions that what is a warning limit is it. So, warning limit by conventions they are placed at plus minus 2 sigma limits from the center line.

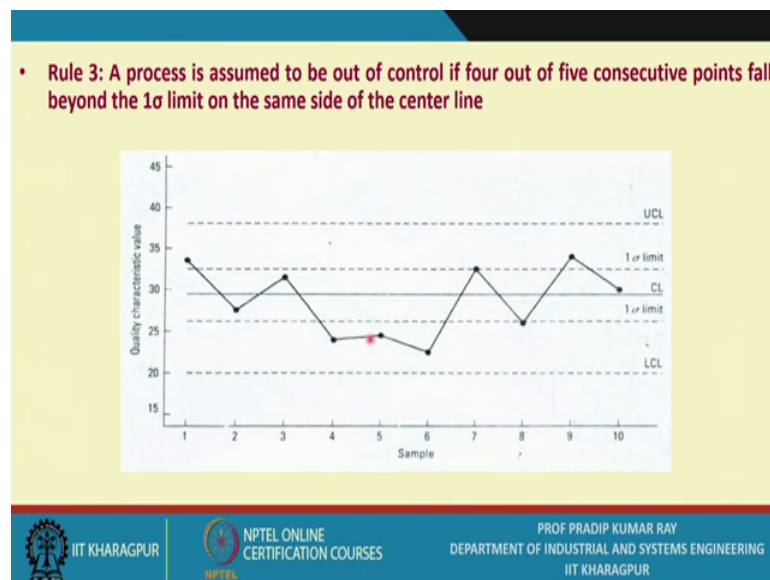
This is a warning limit, this is also warning limit on the other side of the center line now this distance between the central line and the warning limit is plus 2 sigma on this side and on the other side it is minus 2 sigma now what do you try to do; that means, as I have already pointed out that the control chart is used as in prevention based quality control; that means, as far as possible you know you use the control chart in such a way that out of control condition is avoided is it, that is your basic purpose.

Now, here, that is why you know the warning limit now here what you find that these this point and this one this point out of 3 consecutive points there they are plotted between the warning limit and the upper control limit these 2 points are not out of control point, but make sure; that means, that you can improve this situation; that means, it is all likelihood you are adding towards a situation where very soon you may cross this

upper control limit boundary and then the next point may come from and out of control state.

So, there is the time you should be very very careful right and you know that is why this rule is prescribed and you may in. So, why out of 3 consecutive points the 2 points already in this particular zone is it either the first or second third may be anywhere or the first and third like here and the second may be anywhere or you know the second and third consecutive 3 points there between the there they are plotted between the warning limit and the upper control limit whereas, the first point maybe you know anywhere which plots anywhere in this yawn, this is rule 2.

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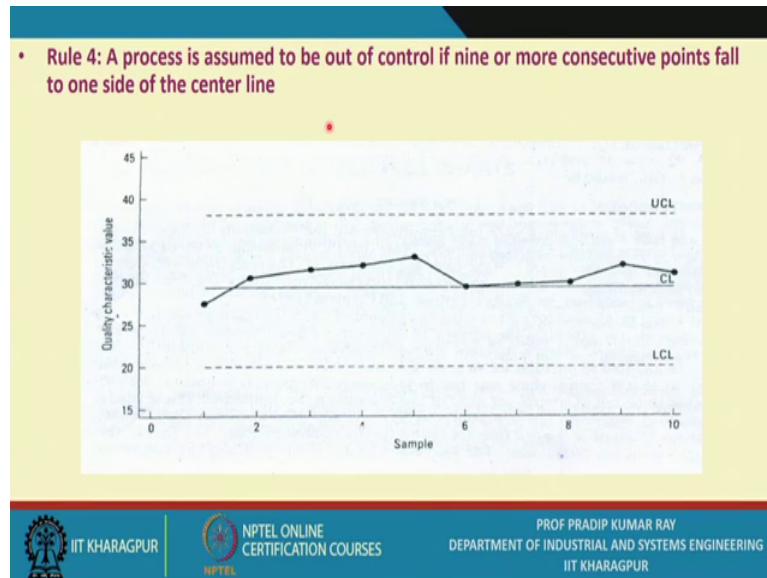


What is rule 3, rule 3 is we go one step further a process is assumed to be out of control if 4 out of 5 consecutive points fall beyond the 1 sigma limit; that means, you have a narrower band here what we do; that means, at a distance of 1 sigma from the center line on one side as well as on the other side you draw these 2 parallel lines and they are referred to as plus minus 1 sigma limit.

Then you observe that the 4 out of 5 consecutive points they fall beyond the one sigma limit is it right. So, these are the 5 points we have 1 2 3 4 5, 5 consecutive points what do you have noticed that out of 5 consecutive points 1 2 3 and the fifth one third one and the fifth one they are plotted outside of 1 sigma limit. So, you may conclude that that something has gone wrong; that means, certain pattern you observed and pattern means

even if the points are within the control limits, but if you find there is a systematic behavior systematic pattern you assume that the process is influenced by some assignable causes and those assignable causes are present in the system and that is why you know the process may be considered as out of control, this is the basic idea behind concluding that the process has gone out of control or process is in control.

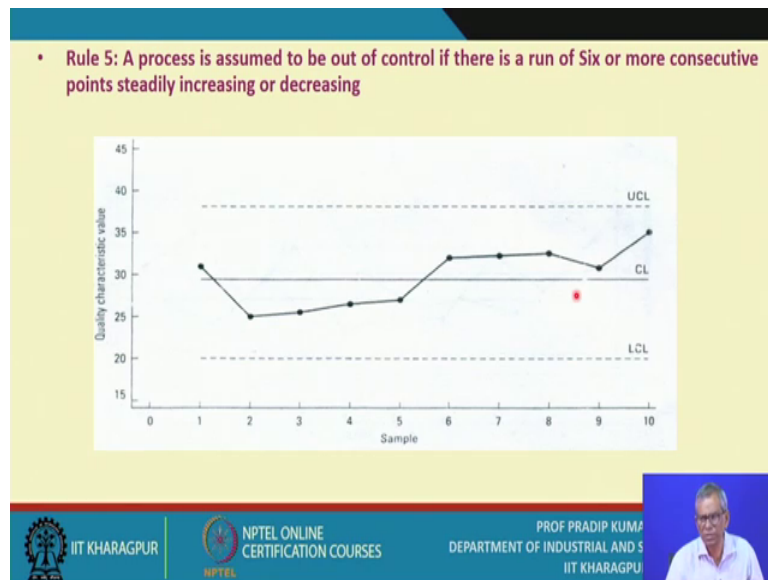
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So, this is rule 3 again we may the fourth rule is the rule 4 a process is assumed to be out of control if 9 or more consecutive points fall to one side of the center line. So, this is a very you know uncommon occurrence; that means, there is a bias here in this case you will find that 1 2 3 4 5 6 7 8 9, 9 points out of you know say the 10 points that you have plotted in consecutive the time periods what he have found that 9 out of 10 they are plotted in one side of the central line.

Actually you know when the process is in control; that means, you plot the points you get a pattern and this the pattern must be a random pattern; that means, there must not be is a nonsystematic or random pattern, but this is not random at all; that means, it is almost predictable; that means, there are some assignable causes is it. So, this rule is routinely followed that is rule 4 in many cases in many organizations for many process, they are tempted to you know use a rule 4.

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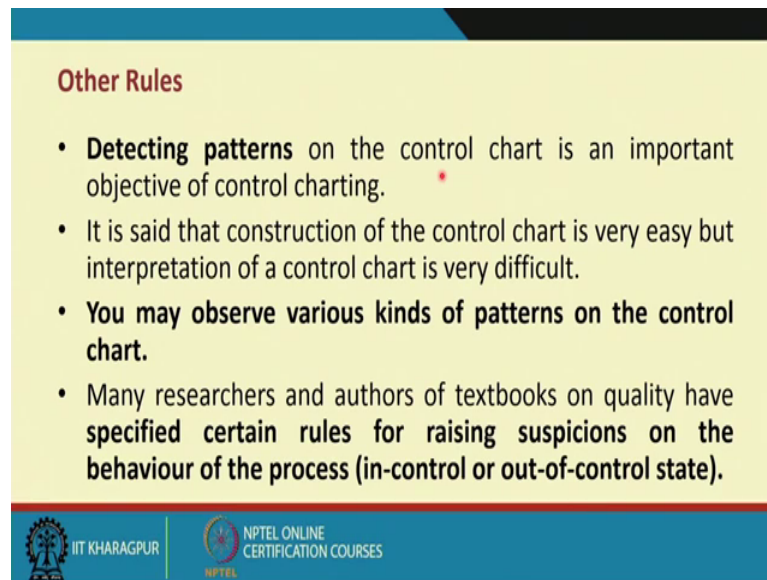


The last rule that is a rule 5 a process is assumed to be out of control if there is a run of 6 or more consecutive points steadily increasing or decreasing now what is a run, the run means actually here a sequence of you know same type of observations actually it is likely observations a sequence of where a group of observations they are of similar nature right.

So, this is basically run and whenever you say the run of sample points; that means, the total sum with the run length you also must specify; that means, here is a run of 6 or more consecutive points they are of similar nature. So, here what you find that 1 2 3 4 5 is it. So, if I run of 6 or more consecutive points; that means, here 1 2 3 4 5 6 7 8 9 that means, 6 points; that means, 1 2 constantly increasing these values getting my point or the constantly decreasing; that means, there is a trend increasing trend or decreasing trend you know in the in the set of plotted points over the time period is it ok.

So; obviously, you know you may conclude that there exist some you know some assignable causes in the process and the process behavior is influenced by those assignable causes. So, immediately you may assume that the process has gone out of control.

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Other Rules

- **Detecting patterns** on the control chart is an important objective of control charting.
- It is said that construction of the control chart is very easy but interpretation of a control chart is very difficult.
- **You may observe various kinds of patterns on the control chart.**
- Many researchers and authors of textbooks on quality have **specified certain rules for raising suspicions on the behaviour of the process (in-control or out-of-control state).**

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Now, here you might have noticed that whether it is rule 1 or rule 5 or rule 4 for all these rules all the points their existing excepting the rule one all other rules you will find that the plotted points they would they do exist within the control limits, but still they are referred to as out of control conditions why because there are 2 conditions one is the necessary condition that you have to satisfy before you say that the process has gone out of control.

The first one is first the necessary condition is for concluding that the process as is in control state that is the no points are plotted beyond the upper control limit or the beyond lower control limit this is the necessary condition, but then what is the sufficient condition, sufficient condition is even if all the points are plotted within the control limits make sure that there is a plot pattern you will observe and when you look at the plot pattern you do not find any systematic behavior; that means, it must be a nonsystematic behavior and as well as was the random pattern.

So, sometimes if you find randomness in the plotted pattern they are all existing within the control limits you say it is a natural pattern. So, what we are asking for in a process which is considered to be in control that are you getting a natural pattern is it. So, later on we will discuss about the different kinds of patterns we will study it in more depth.

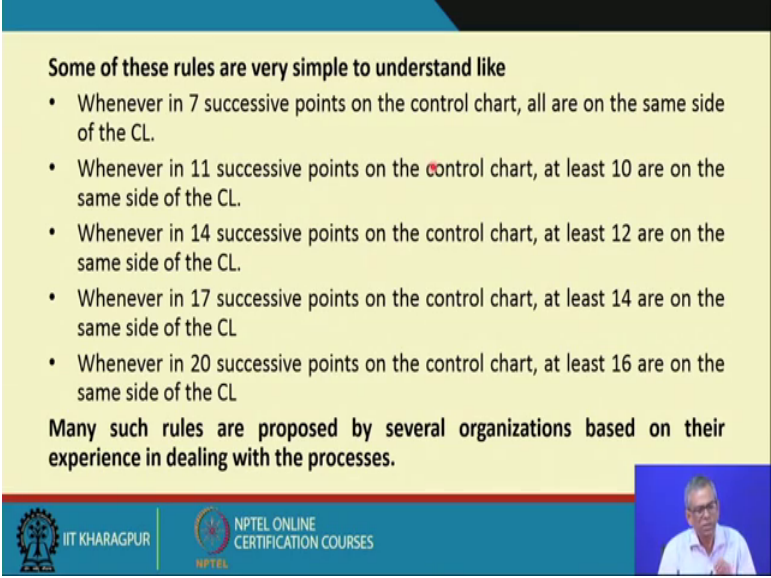
What are the other rules many companies based on their experience and they referring to their database they have proposed you know many kinds of rules. So, you should be also

be familiar with those rules is it detecting patterns on the control chart is an important objective of control charting point number one it is said in this context that construction of the control chart is very easy you follow the steps you learn the steps and you can control you can you get the data and you can propose a control chart of any type.

But interpretation of a control chart is very difficult; that means, while you interpret all these patterns you must have a processed knowledge and unless you have the processed knowledge with respect to a particular process say a machinery or a facilities is it you must have a thorough knowledge about it is behavior, it is design, it is performance you must be familiar with these past performance then only you may interpret this patterns that what are the possible causes.

You may observe various kinds of patterns on the control chart we will refer to this later on many researchers and authors of textbooks on quality have specified certain rules for raising suspicions on the behavior of the process; that means, sometimes you know you are not sure whether the process is controlled or out of control, but you by looking at the pattern you may feel that something might have gone wrong, these are referred to as the suspicions.

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Some of these rules are very simple to understand like

- Whenever in 7 successive points on the control chart, all are on the same side of the CL.
- Whenever in 11 successive points on the control chart, at least 10 are on the same side of the CL.
- Whenever in 14 successive points on the control chart, at least 12 are on the same side of the CL.
- Whenever in 17 successive points on the control chart, at least 14 are on the same side of the CL.
- Whenever in 20 successive points on the control chart, at least 16 are on the same side of the CL.

Many such rules are proposed by several organizations based on their experience in dealing with the processes.

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So, what are the suspicions some of, you are again you know many companies many such as they say that why do not you follow these rules. So, what are these rules and these rules are very simple to understand whenever inside just read them out whenever in

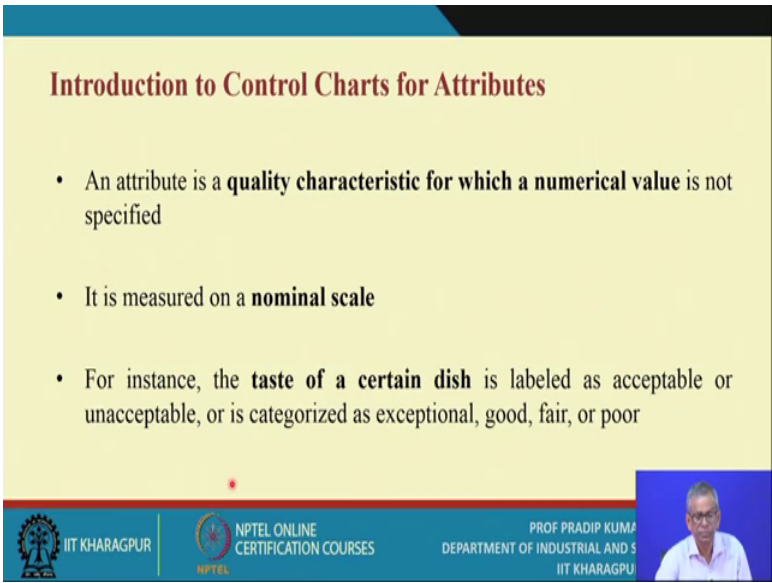
7 successive points on the control chart on a control chart. So, there are different kinds of control chart in a given situation you would just have pick up one relevant control chart and you start using it if you find in that control chart all are on the 7 side of the center, something has gone wrong it is a systemic behavior.

Next rule is whenever in 11 successive points on the control chart at least 10 are on the one side same side of the centerline is it whenever in 14 successive points in the control charts means through their empirical study these companies these organizations or these researchers they are proposing these rules very simple rule. So, out of 14 successive points plotted if you find at least 12 were plotted on the same side of the center line and you extend it.

Whenever in 17 successive points on the control chart at least 14 are on the same side of the center line and the last one whenever in 20 successive points on the control chart is it; that means, here is a situation where as you know getting 20 sample points is not that not that difficult is it, maybe while you collect the data, you may collect the data through an automated system it is not necessarily and it will be a it is a manual system.

So, out of 20 successive points on the control chart at least 16 are on the same side of the center line many such rules are proposed by several organizations based on their experience in dealing with the process.

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Introduction to Control Charts for Attributes

- An attribute is a **quality characteristic for which a numerical value** is not specified
- It is measured on a **nominal scale**
- For instance, the **taste of a certain dish** is labeled as acceptable or unacceptable, or is categorized as exceptional, good, fair, or poor

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Now, I will just you know the briefly I will tell our you know I will discuss the next topic that is you know I will make some introductory remarks related to the control charts of one type that is referred to as attributes type is it there are 2 kinds of control charts the broad classification, one set of control charts are exclusively are used when I deal with that tributes data and the other set of for the control charts with the general purpose as well as the special purpose you use when you deal with when deal with the variables data.

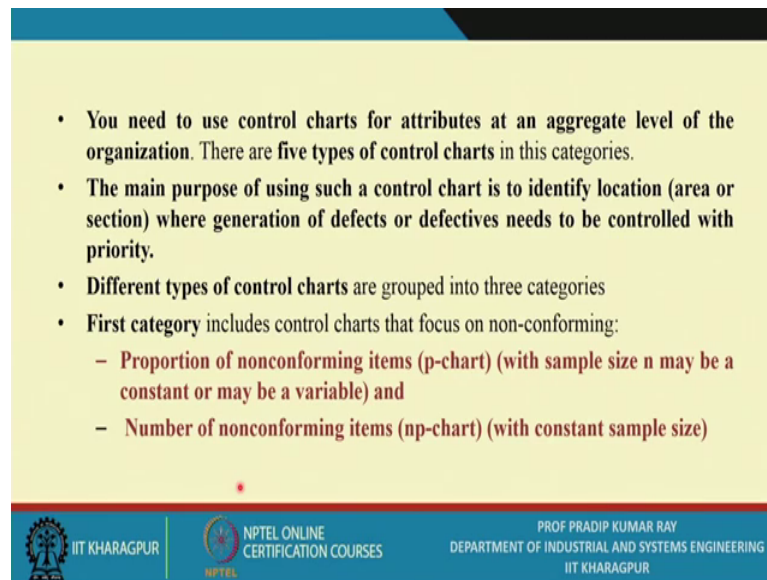
So, before I elaborate on this let me define; what is an attribute, what is an attribute any student in this particular course always you know he or she will be dealing with several kinds of data. So, the first thing he or she needs to do, one needs to understand what type of the data he or she is collecting, this is the broad classification, he or she may collect and attribute data.

So, in attribute is a quality characteristic related to a component on a system or say you know the product as a whole or a service is a quality characteristics for which a numerical value is not specified. So, it is measured on a nominal scale I have already referred to the nominal scale there are 4 types of scales already you know here.

So, it falls in the category of the nominal scale and what are the examples the taste of a certain dish can it be measured it cannot be measured is it. So, either it could be very good or very bad like this or it could be fair like this one. So, is level an acceptable or unacceptable like say you know the kinds of how the painting a you have done on the surface is it on a metal sheet?

So, the by looking at you know this that the surface, the painted surface you say you is whether it is acceptable or not acceptable. So, these are basically you know by attributes data and like you might say it is exceptional good fair or your, as I have already mentioned this is in nominal scale.

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- You need to use control charts for attributes at an aggregate level of the organization. There are five types of control charts in this categories.
- The main purpose of using such a control chart is to identify location (area or section) where generation of defects or defectives needs to be controlled with priority.
- Different types of control charts are grouped into three categories
- First category includes control charts that focus on non-conforming:
 - Proportion of nonconforming items (p-chart) (with sample size n may be a constant or may be a variable) and
 - Number of nonconforming items (np-chart) (with constant sample size)

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Now, you need to use control charts for attributes at an aggregate level of an organizations you know there are different levels you have the strategic level of the top management level in organizations in an hierarchy, you have next is the middle management level and then the third level you have the operations level and then the operations levels you have equal to have several you know the departments under each department there could be several sections under each sections there could be you know the different other work zones or work areas and in each work area there are processes.

So, the process is what we say all the time the process control and all so the question is that when do where do a particular process where do they exist; obviously, at the shop floor level and the operation level. So, now, this control chart for the attributes they are essentially used at a particular level; that means, not at the shop floor level not at the operations level; that means, at the process level, there they are used at the department level or they are used at the you know by the function level or the you know or the organization level.

There is a point to be highlighted and there are 5 types of control charts in these categories I will name them, but before I name them you also must know what is the main purpose of using such a control chart, the main purpose is to identify the location like an area work area or a section where the generation of defects or defectives needs to be controlled with priority this is very there are 2 issues involved one is you know the

location of the cause that is very very important because ultimately when you deal with a control charting you are dealing with the causes of variation. So, you first must know that here is a cause where does it locate, you have to locate is the location of the cause.

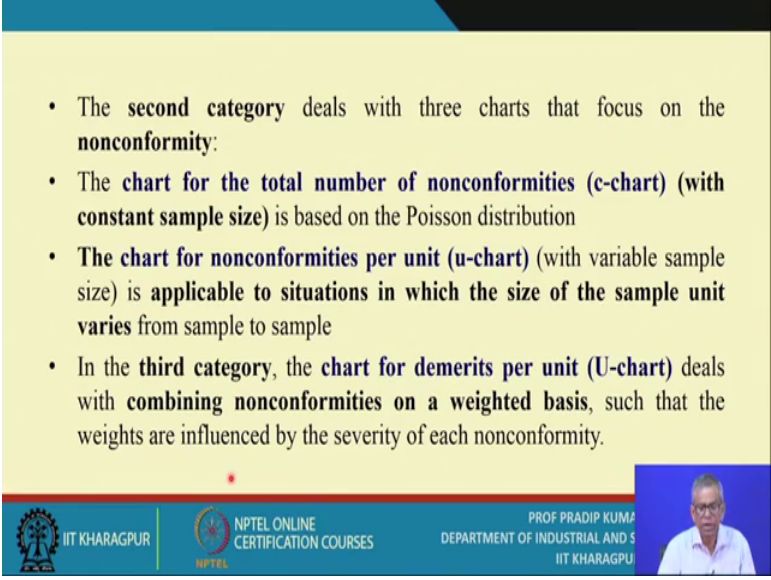
So, for that you know it is very important to use this sort of control chart attributes control charts and once the location is known then you search for the you know the exact the causes and; what are the types of causes is it and then you search for ways and means to remove them is it.

So, for that you need to use you know a different kinds of control chart that is refer to as the control chart for the variable different types of control charts are grouped into 3 categories; that means, there are 5 types they are classified under 3 categories first category includes control charts that focus on non conforming non conforming is it; that means, it is basically a defective product as already defined what is defective and the defective is a it says a general term is it any user of the product may use, but as a technical person as an engineer you say that it is non conforming.

There are 2 charts we use one is the proportion of non conforming items which refers to the proportion of non conforming items. So, it is refer to as the p chart p stands for proportion non conforming where the sample size n may be a constant or may be a variable. So, the both the cases you need to considered.

One extension of proportion non conforming is sometimes at the operational levels you may face the difficulties in calculating this proportion non conforming. So, you say that why do not we use the number of non conforming items is it you do not need to calculate the proportion non conforming of course, in this case we assume that the sample size is constant across the samples, this is referred to as np chart.

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The slide contains a list of four bullet points on a yellow background. The first bullet point states that the second category deals with three charts focusing on nonconformity. The second bullet point describes the c-chart (constant sample size) based on the Poisson distribution. The third bullet point describes the u-chart (variable sample size) applicable to varying sample unit sizes. The fourth bullet point describes the U-chart (demerits per unit) which combines nonconformities on a weighted basis. At the bottom, there are logos for IIT Kharagpur, NPTEL Online Certification Courses, and a small video inset of Prof. Pradip Kumar from the Department of Industrial and Manufacturing Engineering at IIT Kharagpur.

- The **second category** deals with three charts that focus on the **nonconformity**:
- The **chart for the total number of nonconformities (c-chart)** (with **constant sample size**) is based on the Poisson distribution
- **The chart for nonconformities per unit (u-chart)** (with variable sample size) is **applicable to situations in which the size of the sample unit varies** from sample to sample
- In the **third category**, the **chart for demerits per unit (U-chart)** deals with **combining nonconformities on a weighted basis**, such that the weights are influenced by the severity of each nonconformity.

The second category it will suite 3 types 3 charts that focus on the nonconformity or the defect on a single piece there could be many types of defects. So, for each defect you select defect which is which we may be considered to be significant a really critical one and for which you need to construct a control chart.

The chart for the total number of non conformities assuming that the concentrated sample size is constant that is a c - chart the chart for non conformities per unit; that means, when the sample size varies across the samples we need to construct the u - chart and obviously, it is a variable sample size case and this is applicable to situations in which the size of the sample unit varies from sample to sample is it that is the elaboration and in the third category the chart for demerits per unit.

We have the fifth one that is referred to as a capital U - chart deals with combining non conformities on a weighted basis; that means, there could be different types of defects or non conformities on the same item, on the same product what you need to do; that means, depending on it is impact like say the defects could be you know very very serious defects or non serious defects, major defects, minor defects and accordingly what you do each type of defects you need to consider and each type of defect is to be weighted is it.

We needs to combine these non conformities on a weighted basis such that the weights are influenced by the severity of a pinched conformity. So, this is referred to as the

demerits, is the per unit basis; that means, sample size remains constant, what it could be variable, this are the 5.

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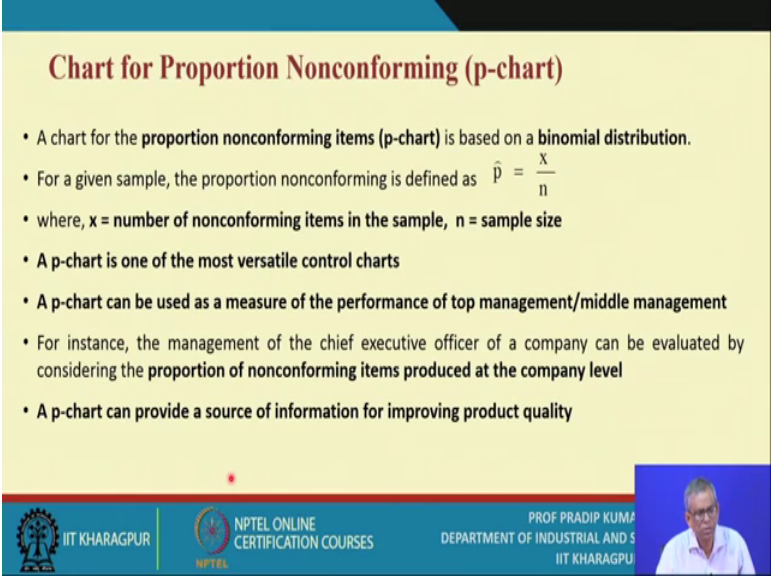


Chart for Proportion Nonconforming (p-chart)

- A chart for the **proportion nonconforming items (p-chart)** is based on a **binomial distribution**.
- For a given sample, the proportion nonconforming is defined as $\hat{p} = \frac{x}{n}$
- where, **x = number of nonconforming items in the sample**, **n = sample size**
- A **p-chart is one of the most versatile control charts**
- A **p-chart can be used as a measure of the performance of top management/middle management**
- For instance, the management of the chief executive officer of a company can be evaluated by considering the **proportion of nonconforming items produced at the company level**
- A **p-chart can provide a source of information for improving product quality**

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Now, a proportion non conforming chart I will be a proportion on p chart or the c chart or say the u chart or say capital U chart we will be discussing with examples , but there are a many issues involved. So, I will be discussing in the in the next interaction with you.

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Reference

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So, this is the reference always you can you know they refer to this textbook and also the basics we have we have discussed and then we will take up all these control charts one after another.