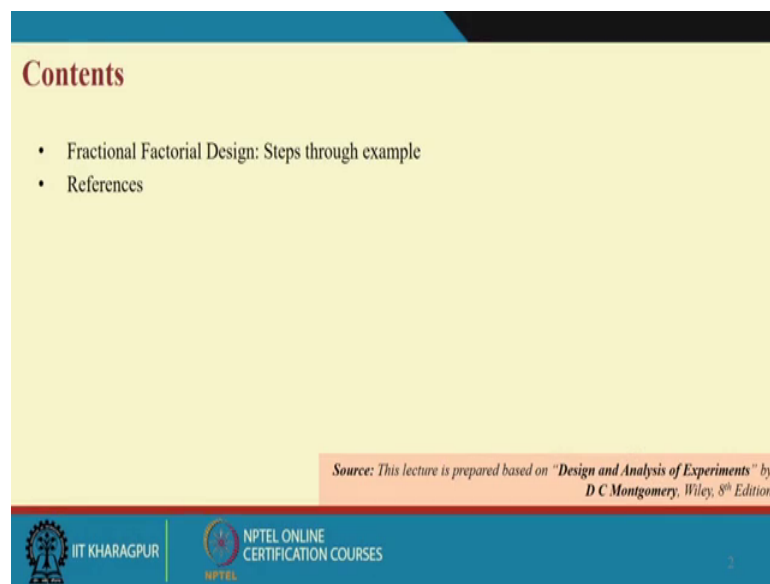


Design and Analysis of Experiments
Prof. Jhareswar Maiti
Department of Industrial and Systems Engineering
Indian Institute of Technology, Kharagpur

Lecture – 59
Fractional Factorial Design using MINITAB

Welcome all; in today's video lecture today, we are going to show you the fractional factorial design using MINITAB to begin with these things.

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Let me introduce myself. I am Sobhan Sarkar, the PhD student in the department of industrial and systems engineering, IIT, Kharagpur and also the teaching assistant of the subject of design and analysis of experiments.

To begin with these things, let me have a content in today's lecture, there is a fractional factorial design; the steps through the example and the references are given to begin with this things, we have to first start with an example.

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Example:

A chemical product is produced in a pressure vessel. A factorial experiment is carried out in the pilot plant to study the factors thought to influence the filtration rate of this product. The four factors are temperature (*A*), pressure (*B*), concentration of formaldehyde (*C*), and stirring rate (*D*). Each factor is present at two levels. The design matrix and the response data obtained from a single replicate of the 2^4 experiment are shown in **Table 1** and **Figure 1**. The 16 runs are made in random order. The process engineer is interested in maximizing the filtration rate.

Run Number	Factor A	Factor B	Factor C	Factor D	Run Label	Filtration Rate (gph)
1	-	-	-	-	(1)	65
2	+	-	-	-	a	71
3	-	+	-	-	b	68
4	+	+	-	-	ab	65
5	-	-	+	-	c	68
6	+	-	+	-	ac	60
7	-	+	+	-	bc	80
8	+	+	+	-	abc	65
9	-	-	-	+	d	63
10	+	-	-	+	ad	100
11	-	+	-	+	bd	65
12	+	+	-	+	abd	104
13	-	-	+	+	cd	75
14	+	-	+	+	acd	86
15	-	+	+	+	bcd	70
16	+	+	+	+	abcd	96

Fig. 1

This example is a chemical product is produced in a pressure vessel, a factorial experiment is carried out in the pilot plant to study the factors through to thought to influence the filtration rate of this product, the 4 factors are temperature, pressure, concentration of formaldehyde stirring rate.

So, this 4 factors are abbreviated as A, B, C and D; each factor is present at 2 levels the design matrix and the response data obtained from a single replicate of 2 to the power 4 experiment are shown in table 1 and figure 1 the 16 runs are made in random order the process engineer is interested in maximizing the filtration rate.

So, here in table 1, we can see there are 4 factors A, B, C and D is a run level 1; AB, ABC, ACBC, ABCD, ADBD, ABD, CD, ACD, BCD and ABCD and their corresponding filtration rate and this is the design. So, in this design, we have to show how to conduct the fractional factorial design.

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One half-fraction of the 2^4 design is run instead of the full factorial.

2^{4-1} design with resolution IV and I=ABCD

Table 1

Four Factor Filtration Rate Experiment

Run Number	A	B	C	D	Run Label	Filtration Rate (gph)
1	-	-	-	-	(1)	45
2	+	+	+	+	ad	100
3	-	+	-	+	ab	45
4	+	-	+	-	bc	65
5	+	+	-	-	cd	75
6	-	-	+	+	ac	60
7	-	+	-	-	bc	80
8	+	-	+	+	abd	96

The 2^{4-1} Design with the Defining Relation I = ABCD

Run	A	B	C	D = ABC	Treatment Combination	Filtration Rate
1	-	-	-	-	(1)	45
2	+	+	+	+	ad	100
3	-	+	-	+	ab	45
4	+	-	+	-	bc	65
5	+	+	-	-	cd	75
6	-	-	+	+	ac	60
7	-	+	-	-	bc	80
8	+	-	+	+	abd	96

Run	Filtration rate
1	45
2	100
3	45
4	65
5	75
6	60
7	80
8	96

Fig. 1

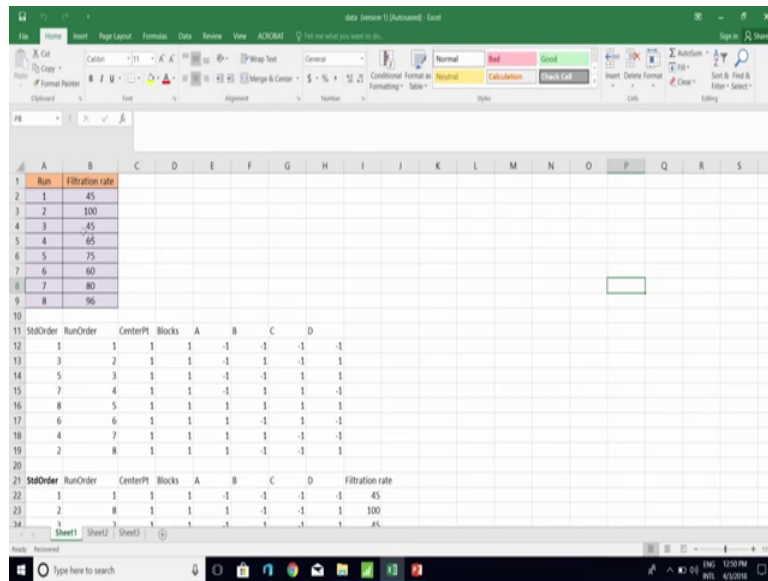
So, in that case, one of fraction of 2 to the power 4 design will have to be conducted. So, 2 to the power 4 design, if you conduct these things. So, we have to select the design 2 to the power 4 minus 1 design with resolution 4 and I equal to A, B, C, D. So, this is a data.

And this is the corresponding design from this data we have selected 8 runs and with basic designs AB and C; these are generated from ABCD, ABC. So, corresponding plus minus signs are given A for A the plus minus sign are minus plus minus plus minus plus minus plus for B minus minus plus plus minus minus plus plus for C minus minus minus minus plus plus plus plus for D, it is just product of this as you know that is A minus minus and minus the product is minus plus minus minus is plus.

Similarly, minus plus plus minus plus minus minus plus; so, treatment combination, we have got from the; from this table and their corresponding filtration rate. So, now, we are going to show you how this can be analysed through MINITAB.

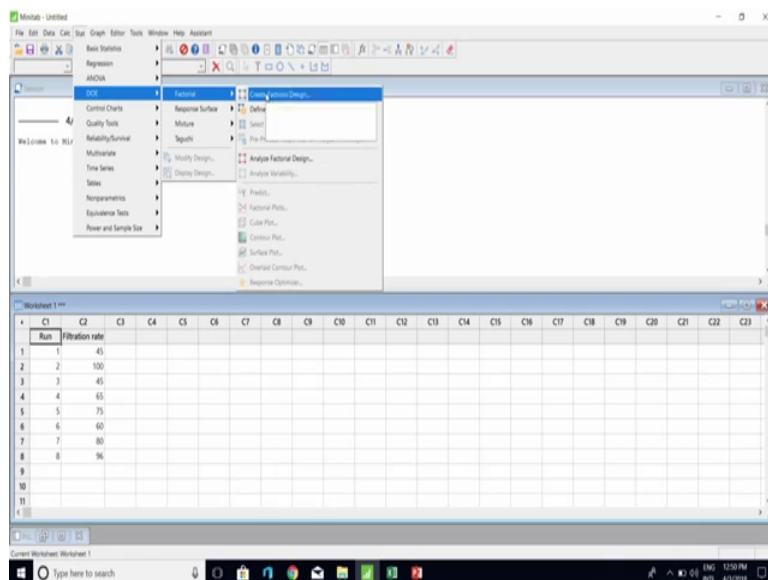
So, from this things, we have just writing down in excel file this run and the filtration rate that is 1, 2 up to 8 runs and the filtration rate up to corresponding run the filtration rate is recorded.

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So, first we open the excel file we have copied; it into this excel file for our future references. So, first open MINITAB window initially, we have to copy this thing ultimately, it will be used later or initially.

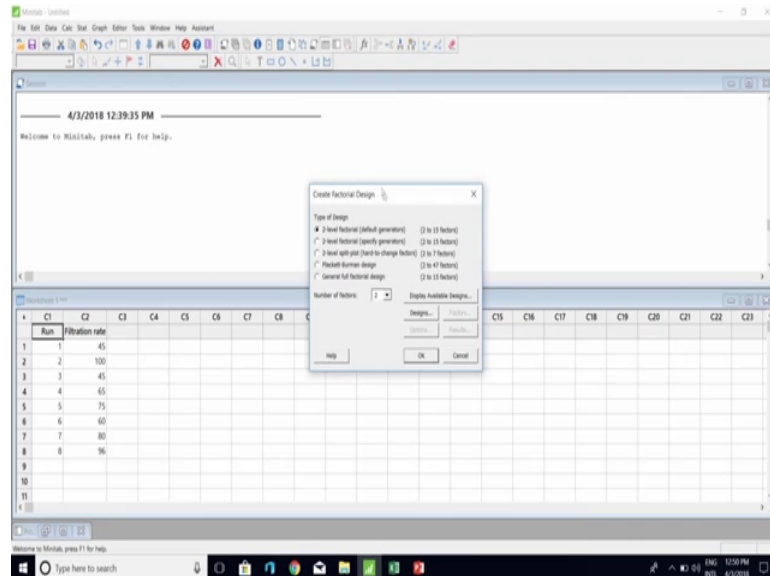
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We have to copy it. So, after copying this thing we can see the run and filtration rate as shown in the MINITAB environment. So, after that click the stat bar under the stat bar doe option is there.

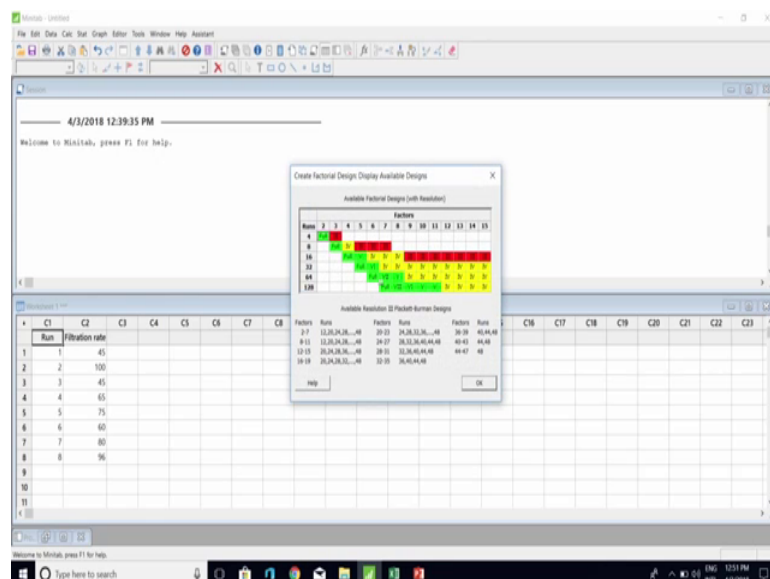
So, and then under doe factorial option is there and under factorial create factorial design option is available.

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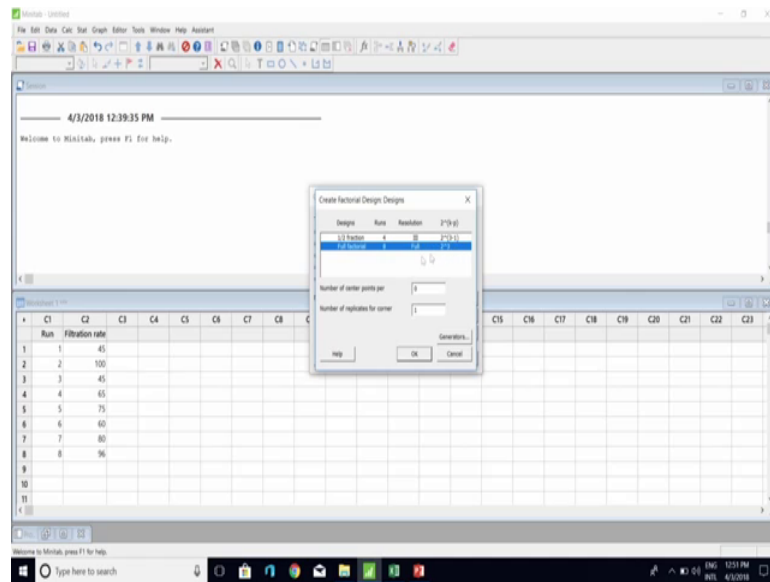
So, after clicking it, we have got a set of options under the type of design then number of factors then display available designs then designs and the factors options results will be coming 1 by 1 in that case, in our particular case, we are selecting 2 level factor factorial specify generator.

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Display design available for our case is resolution 4; maximum resolution 4, then coming to the number of factors here our design is as you can see our design is 2 to the power 4 minus 1, basically, the basic design becomes 2 to the power 3 design as here the basic design we can see A, B and C are the basic design.

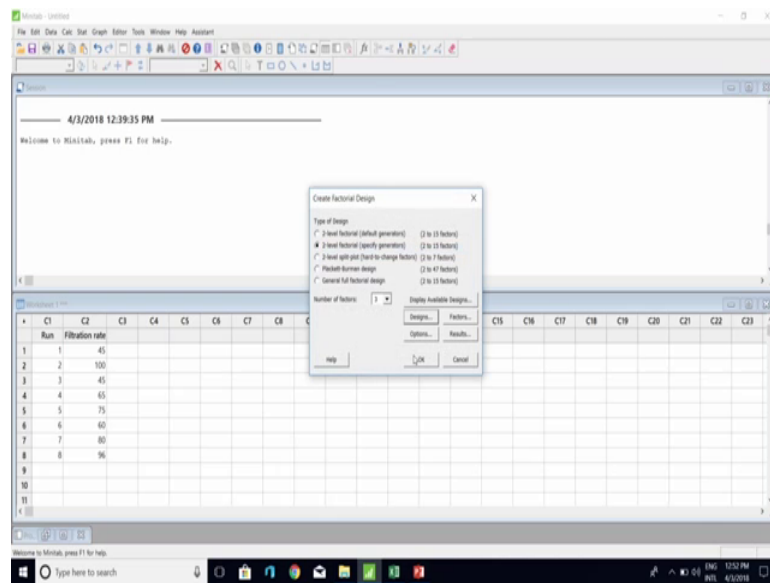
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So, here we can select 3 design, then click the designs here we will select the full factorial for the 2 to the power 3 here the design is 2 full factorial and runs is 8 resolution is full and 2 to the power 3 design generators we have to specify here the design generator if you look back into the slide we can see that design generator is D equal to A, B, C.

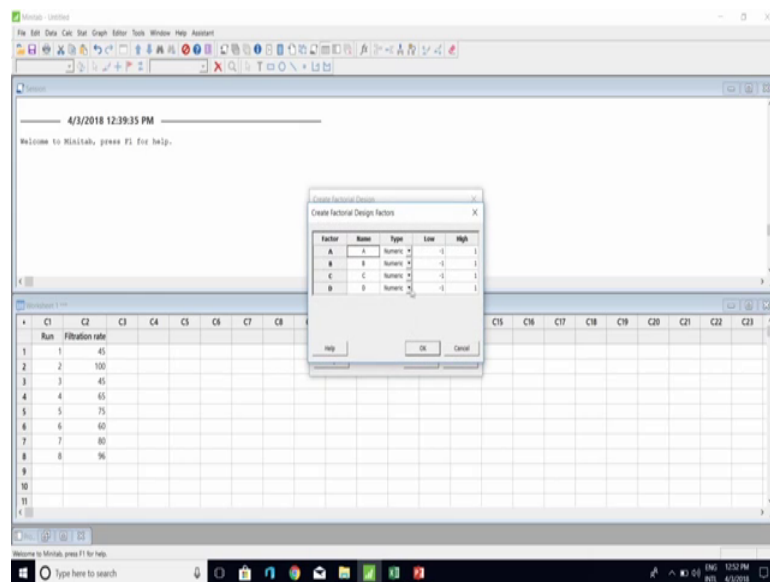
So, we will have to put it into the slide once we complete these things we will click ok.

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Then these options are available initially it was not available. Now, they are available factors options and results for clicking the factors, we can see these are all the factors options and results.

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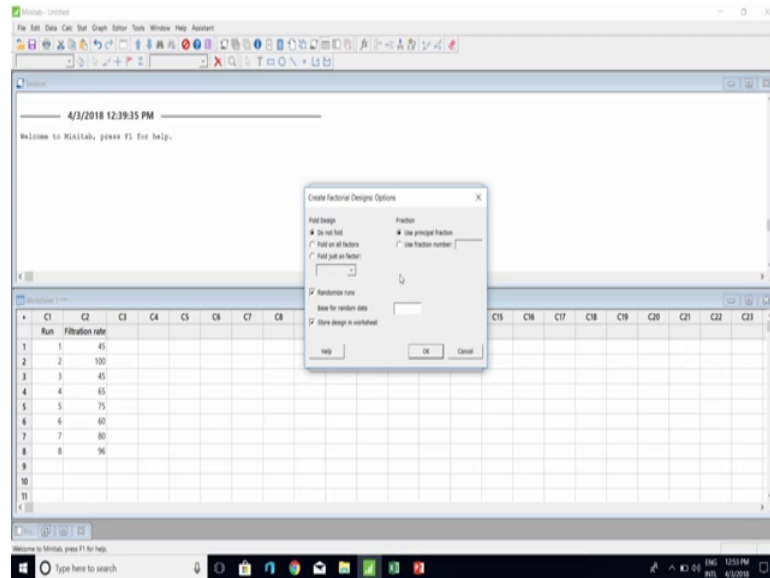


So, clicking the factors we can see these are all the factors we can see A, B, C and D.

So, here is name we can easily name them, but for the sake of the easy handling we name, we do not name this, we only keep the abbreviation as it is that is A, B, C and D, it

is a numeric type not text, it is a high level and low level abbreviated as plus 1 minus 1 respectively. So, click it then go to the options.

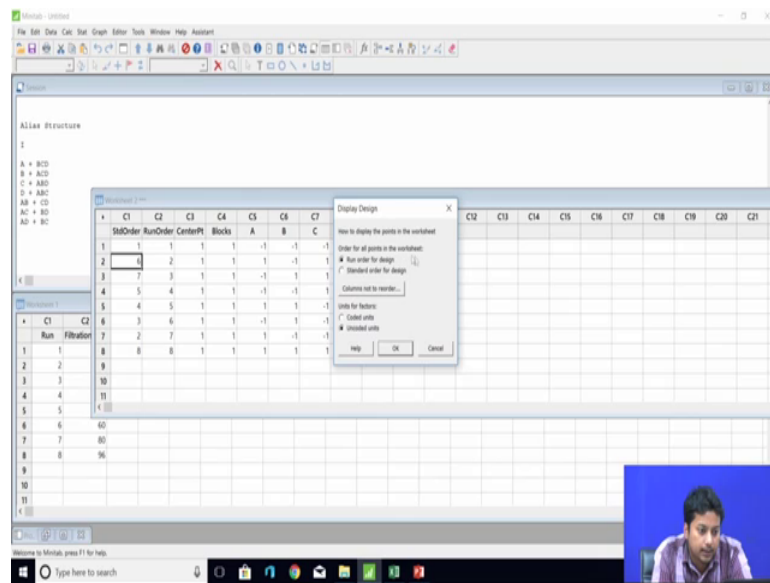
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Here in this option, we can see the fold design fraction designs. So, in fold design; we do not fold, we are not going to show you the fold over fold on all factors, it is a very simple straight forward application without folding. So, we keep it is as it is. So, here in this fraction, we will use the principal fraction that is I is equal to A, B, C, D.

So, click go to the result section we are not going to do anything with it. So, leave as it is. So, click.

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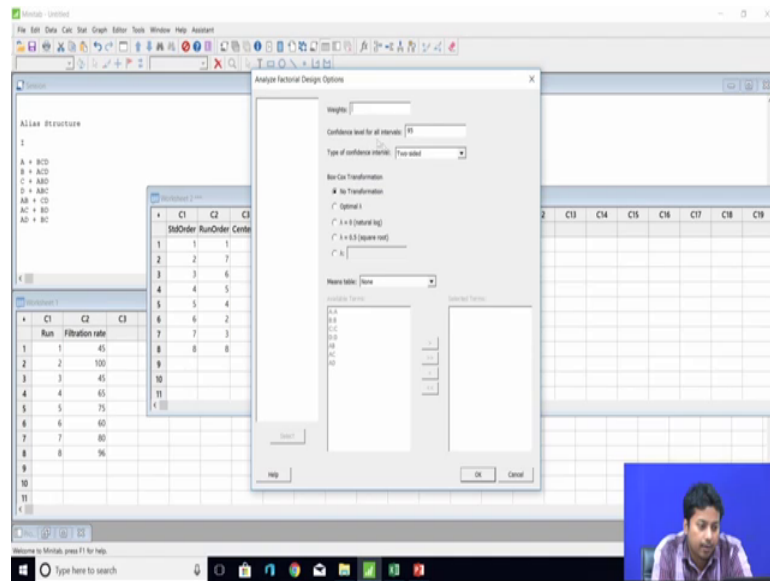
So, once we click ok, it ultimately leads you a generation of another worksheet you can see the worksheet 2, initially, it was worksheet 1, now it becomes worksheet 2 with 1, 2, 3, 4, 5, 6, 7, 8, 8 columns, initially we have to first select and make it up order the standard order column here as you can see it is placed in random 1, 6, 7, 5, 4, 3, 2 and 8. So, we have to keep it in order. So, to in order to keep it in order we have to first select stat, then coming to the doe, then click the display design here this option.

Once you click it the options would come that order for all points in the worksheet. So, we will click it the second option the standard order for the design and click it ok. Now, we can see the order has been maintained 1, 2, 3, 4 up to 8 and the corresponding run order is also given the ABCD, under the ABCD plus minus are arranged in a proper manner. Now, this is the time to put the corresponding filtration value.

So, from here to first copy and here we have to paste it after pasting these thing final analysable worksheet, worksheet looks like that standard order, run order, centre points blocks, A B C D, filtration rate.

Now, click the stat doe factorial design and then analyse factorial design here we can see the filtration rate is available.

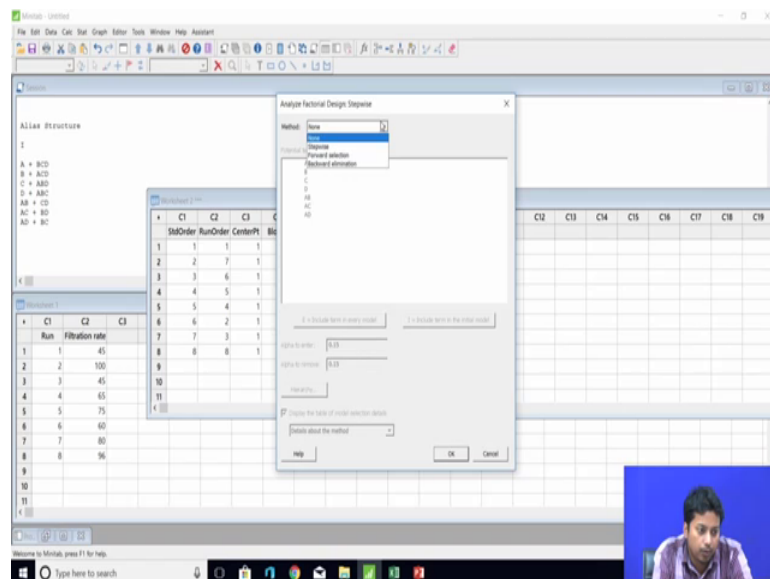
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So, selected as response variable, so, terms we are not going to do anything with it covariates options. So, many options are available 95 percent confidence level is given 2 sided test, we are going to do it is a by default setting. So, is there any requirement of transformation of the data no requirement no transformation is a by default setting

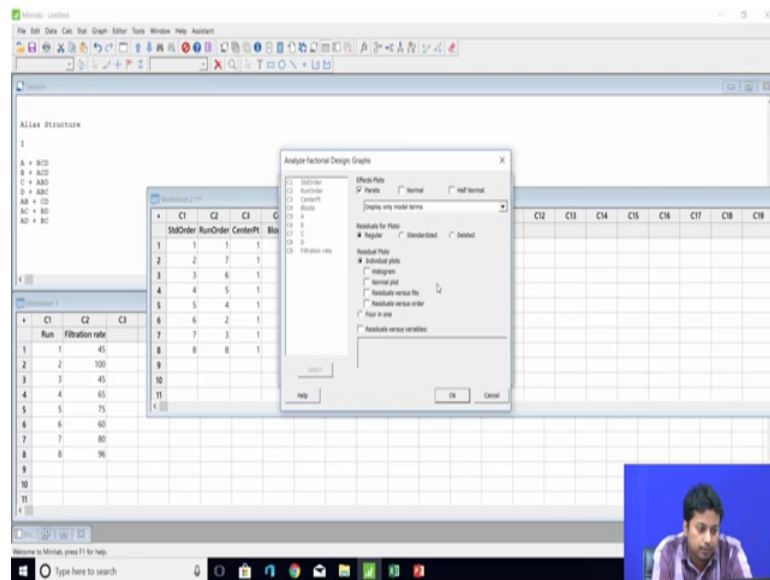
So, we are not going to alter them at all. So, we leave it as it is by clicking ok.

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Now, click the stepwise again, also we are not going to do anything in with it. So, method is nothing here, we are not going to do any stepwise forward selection and backward selection the by default selection by default method we are doing here.

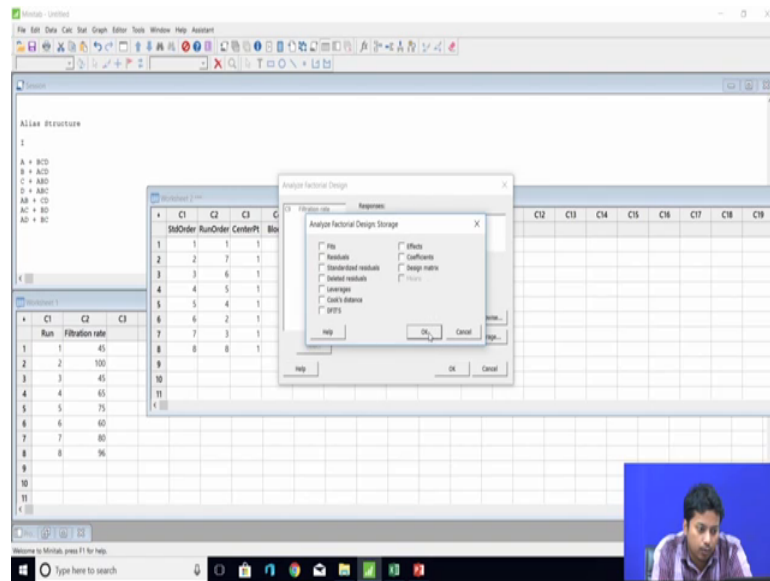
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So, cleaning the graph here so many graphs are available, we can see effects plot Pareto normal half normal if you want to get. So, we can also get half normal, the residual plot for regular standardize standardized and deleted and residual plots for individual plots for histogram normal plots residual versus fit residual versus order.

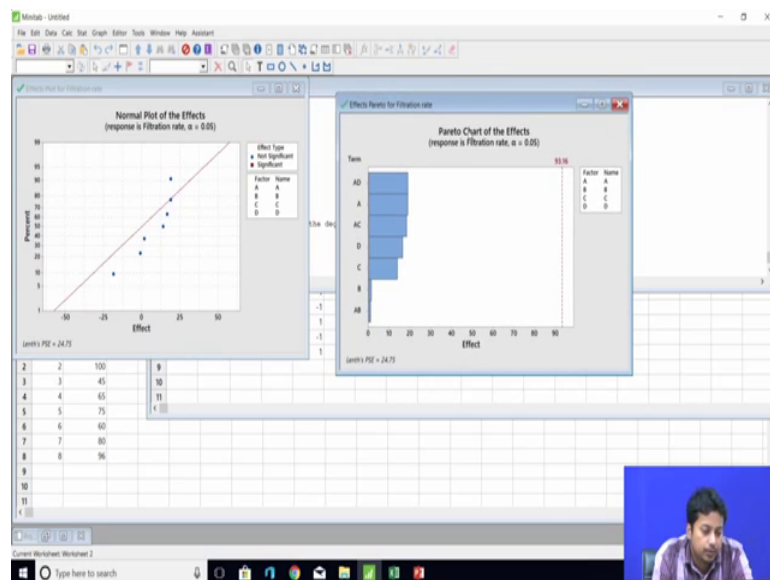
So, we are going to see these things in a same window. So, 4 in 1 after that click ok, then results leave it as it is; then storage and nothing to do with it.

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But if anyone wants to get the value of cook's distance leverages residual fits effect coefficient design matrix; one may easily afford that, but here we are not going to show you hear anything after clicking this we are getting this values.

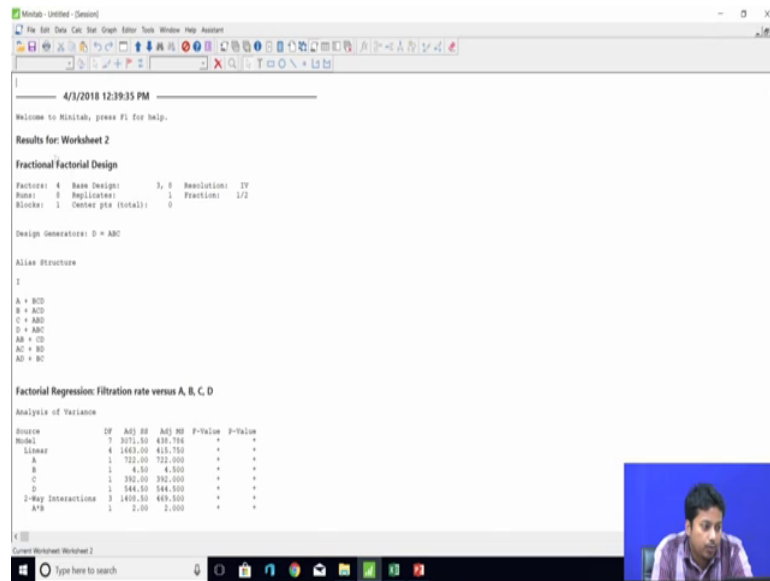
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So, as you can see in this output results the Pareto chat, we are getting here the effects that is AD A, AC, D and C seem to be very important and this a effect chart.

And based on this things, we are we can easily neglect the B and AB, this is a normal plot for the effects with alpha is equal to 0.05 and here is a results.

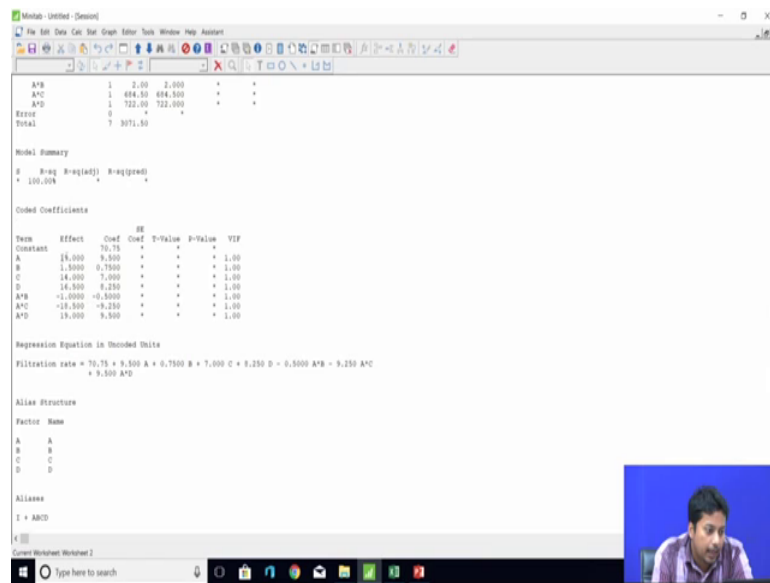
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We can see; so, coming to the starting point, we can see that it is a fractional factorial design it is a one half fraction basically. So, fact factors which were initially taken over 4 basic design history, then resolution is 4 run, we are considered is 8 because it is the 1 half fractions. So, initially it was consists of the 16 design. So, 1 half fraction will be; it means in the principal fraction consisting of only 8 runs where; it is shows in replicates is one replicates fractions one half blocks is 1 design generate D is equal to A, B, C.

So, here in the fractional re regression we can see the analysis of variance ANOVA table, we can see the A BCD, then to A interaction AB, AC, AD, we have to keep in mind that B variable is not significant that is why all. So, the AB can be neglected and A also B can also be neglected. So, ACD, AC and AD are the significant factors.

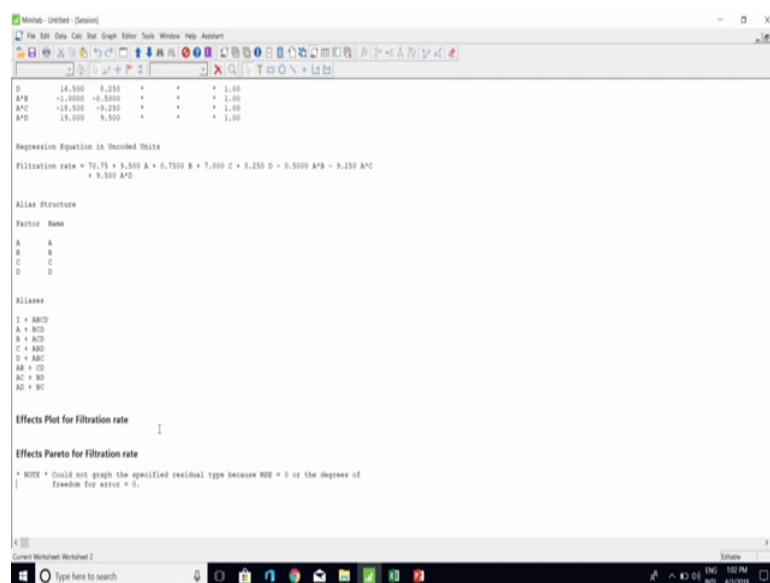
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So, ultimately we can get also the effect of each of the term ABCD, AB, AC, AD for all terms, we are getting the effects value for example, for a the effect estimate is 19 for B, it is a 1.5, C is 14, D is 16.5. Similarly for all, as A, we have already experienced from the Pareto plot that B value is very less and AB value is very less we can also see from here.

So, A, CD, holding the very high value that is a 19, 14, 16 and corresponding AC and AD are also holding the very higher values. So, corresponding coefficients are also given.

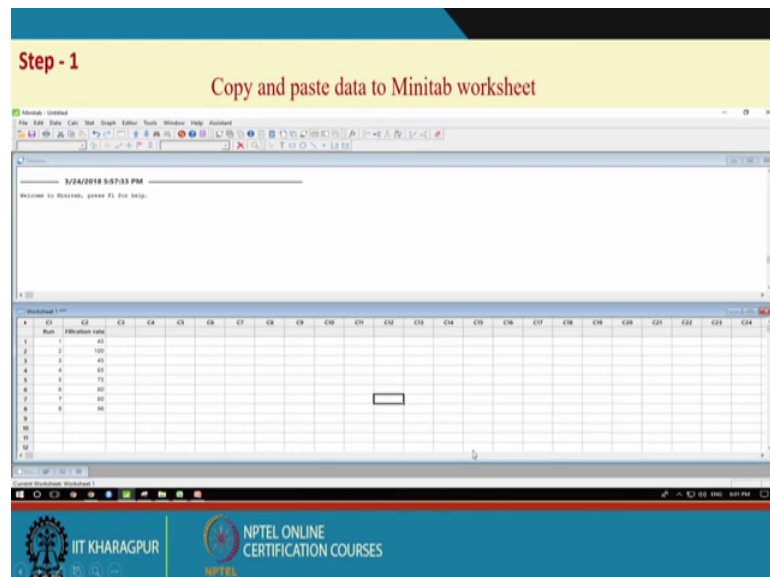
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The P value is also given and finally, we are getting the regression equation of filtration rate which has been considered as output variable. So, filtration rate is $70.75 + 9.58$, if you neglect this, then 0.75 , B should be neglected, then 7 , C 8.25 , D minus 0.5 , AB minus 9.25 AC plus 9.58 AD.

Here, if we consider if we neglect the B and AB, then left of the portion will be the final filtration rate equation regression equation for the filtration rate. So, it is alias structure I plus ABCD, A plus BCD B plus ACD C plus AB, ABD, D plus ABC AB plus AD, AC plus BD, AD plus BC.

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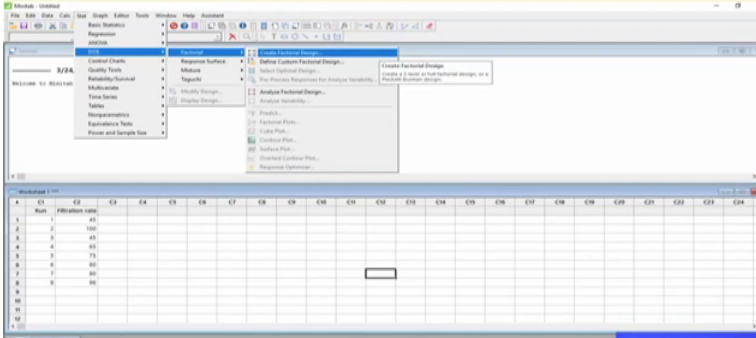


So, coming to the PPT presentation, we can show you all the steps you can follow the all the steps very clearly and practice it in your own laptop.

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Step - 2

Open "Create Factorial Design"

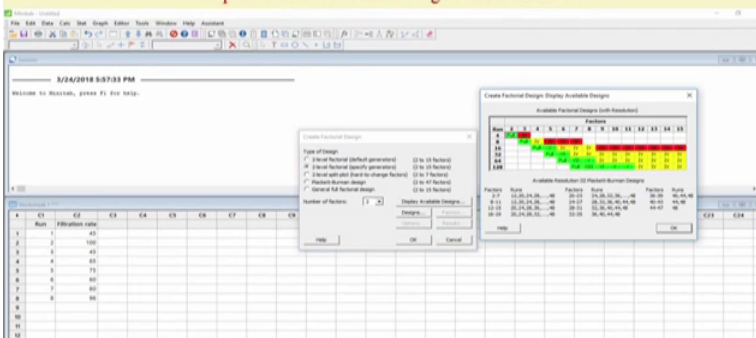


The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the 'Create Factorial Design' option is highlighted. Below the menu, a spreadsheet is visible with columns labeled C1 through C24 and rows labeled Run 1 through Run 10. The bottom of the slide features the IIT KHARAGPUR and NPTEL ONLINE CERTIFICATION COURSES logos, along with a small video inset of a presenter.

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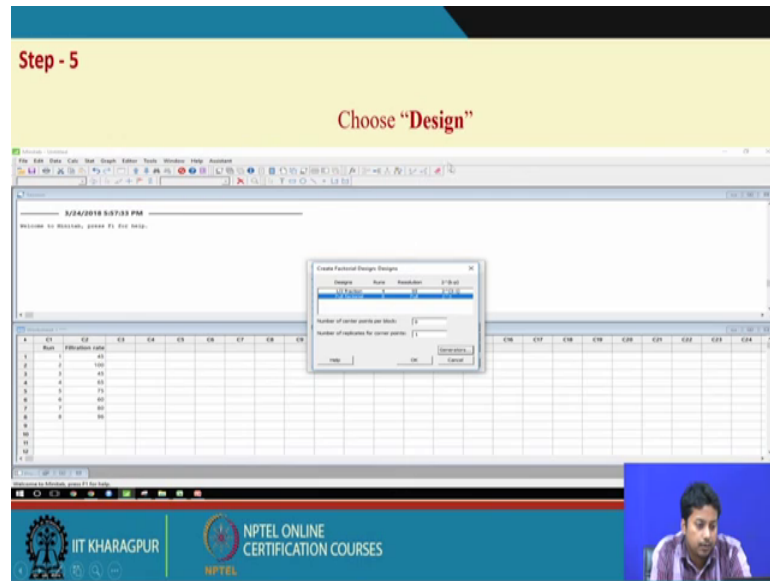
Step - 3

Open "Choose Factorial design with Resolution"

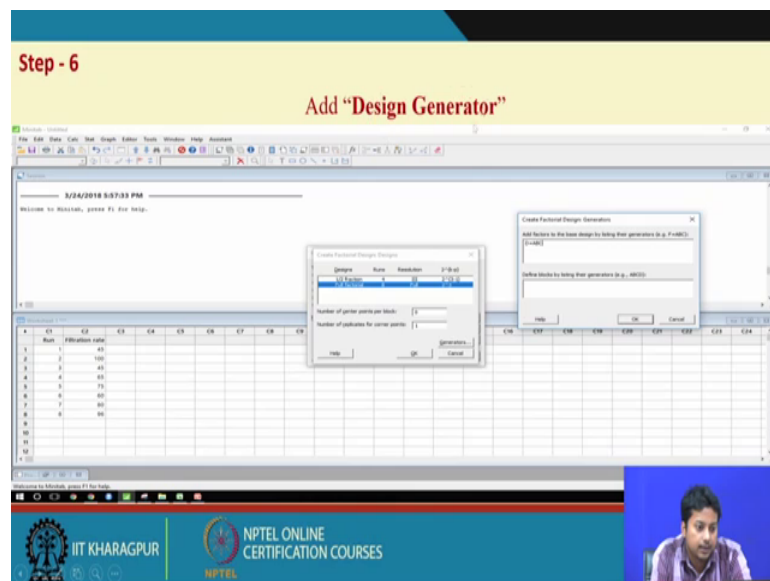


The screenshot shows the Minitab software interface with two dialog boxes open. The 'Choose Factorial Design' dialog box is in the foreground, and the 'Create Factorial Design Display Available Designs' window is in the background. The 'Choose Factorial Design' dialog box has the following options checked: '2-level Fractional (default generated)', '2-level Fractional (sparsity generated)', '2-level with one (or) more (or) change factors', and 'Fractional Run-in design'. The 'Create Factorial Design Display Available Designs' window shows a table of available fractional designs with columns for Run, Factors, and Resolution. The bottom of the slide features the IIT KHARAGPUR and NPTEL ONLINE CERTIFICATION COURSES logos, along with a small video inset of a presenter.

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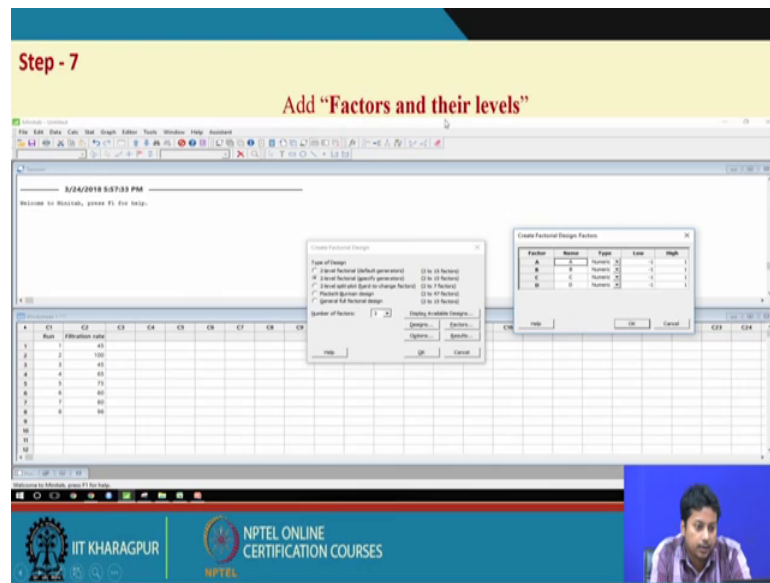


So, step 1, step 2, step 3, 4, 5, choosing the design step 6 design generator step, step 7 factors and their levels.

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Step - 7

Add "Factors and their levels"



The screenshot shows the Minitab software interface. The 'Create Factorial Design' dialog box is open, and the 'Create Factorial Design Factors' dialog box is also open. The 'Create Factorial Design Factors' dialog box shows a table of factors and levels:

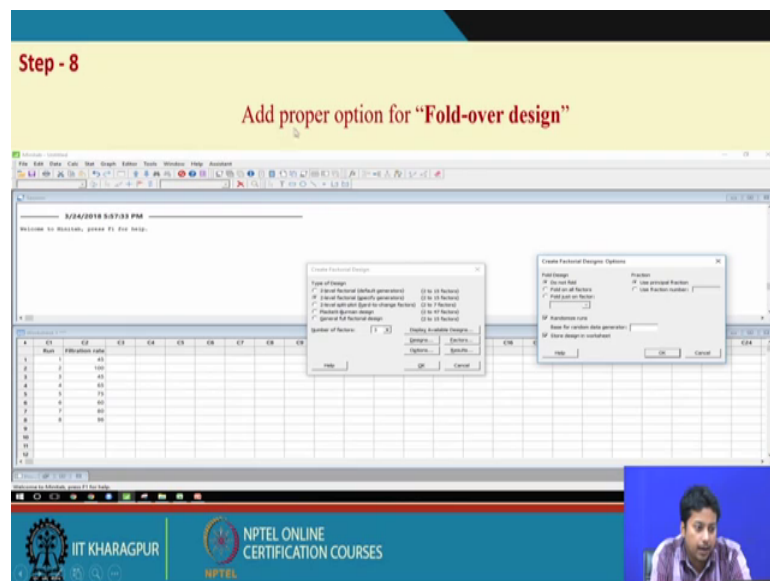
Factor	Name	Type	Low	High
A	2	Numeric	-1	1
B	3	Numeric	-1	1
C	4	Numeric	-1	1
D	5	Numeric	-1	1

The bottom of the slide features the IIT KHARAGPUR and NPTEL ONLINE CERTIFICATION COURSES logos, along with a small video feed of a presenter.

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Step - 8

Add proper option for "Fold-over design"



The screenshot shows the Minitab software interface. The 'Create Factorial Design' dialog box is open, and the 'Create Factorial Design Options' dialog box is also open. The 'Create Factorial Design Options' dialog box shows options for fold-over design:

Option	Value
Fold Design	<input type="checkbox"/> Use standard
Use standard	<input type="checkbox"/> Use standard
Use function number	<input type="checkbox"/> Use function number

The bottom of the slide features the IIT KHARAGPUR and NPTEL ONLINE CERTIFICATION COURSES logos, along with a small video feed of a presenter.

Step 8; fold over design whether we are going to choose it or not.

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Step - 9 Choose proper "Printed Results"

The screenshot shows the Minitab software interface. The main window displays a table with columns C1 through C8 and rows 1 through 8. Two dialog boxes are open: 'Create Factorial Design' and 'Create Factorial Design Results'. The 'Create Factorial Design' dialog has 'Type of Design' set to 'General Full Factorial design' and 'Number of Factors' set to 8. The 'Create Factorial Design Results' dialog has 'Printed Results' checked for 'Main', 'Sum of Squares', 'ANOVA table', and 'Residual Sum of Squares'. The bottom of the slide features the IIT KHARAGPUR and NPTEL ONLINE CERTIFICATION COURSES logos, along with a small video feed of a presenter.

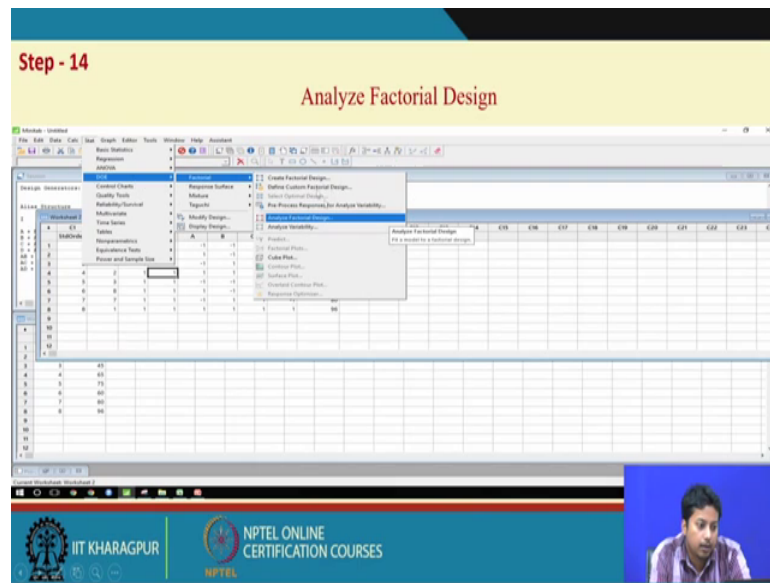
Then printed results; what exactly we are going to get the results.

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Step - 10 Generation of data to be analysed

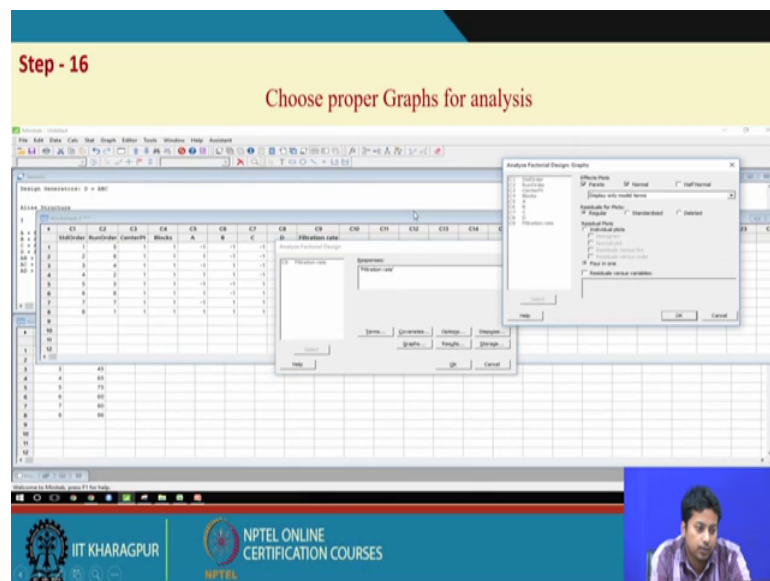
The screenshot shows the Minitab software interface. The main window displays the 'Design' window with 'Alias Structure' and a 'New form of data' table. The 'New form of data' table has columns C1 through C8 and rows 1 through 8. The bottom of the slide features the IIT KHARAGPUR and NPTEL ONLINE CERTIFICATION COURSES logos, along with a small video feed of a presenter.

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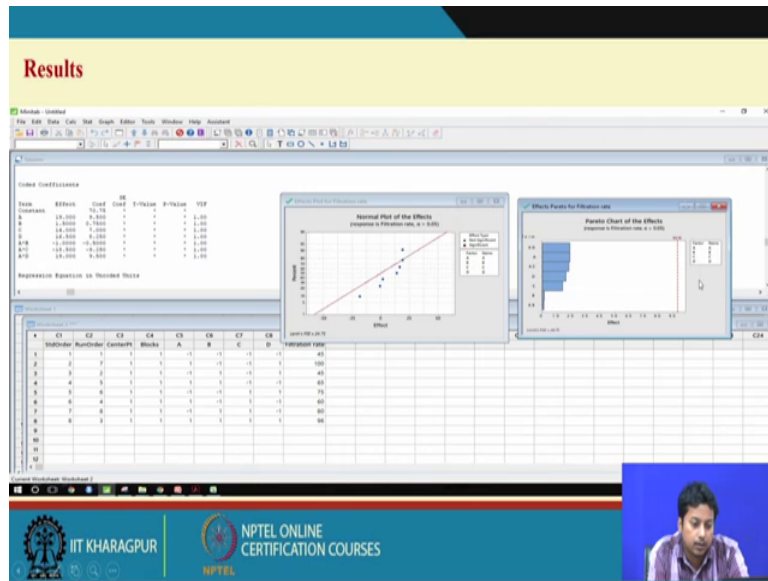
And finally, the new data as a worksheet was generated.

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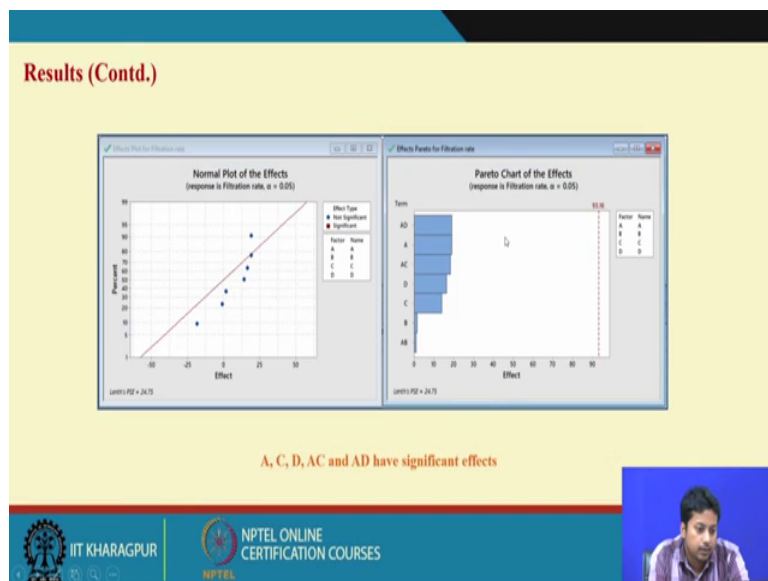
Then filtration rate was inserted using these things, then you can go further analysis the factorial variable factorial design then filtration is selected as response.

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This Pareto chart is generated.

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Results (Contd.)

ANOVA Table with other results including prediction equation for filtration rate

Results for Worksheet 2				Factorial Regression: Filtration rate versus A, B, C, D				Filtration rate = 10.19 + 8.50 A + 6.700 B + 7.000 C + 1.250 D - 0.500 AB - 9.250 AC + 9.500 AD			
Fractional Factorial Design				Analysis of Variance				Alias Structure			
Partials:	4	Best Design:	1, 0	Replications:	3	Factorial:	IV				
Block:	1	Center pts (total):	0	Factorial:	IV						
Design Generation:	D = ABC										
Alias Structure											
I											
A + BC											
B + AC											
C + AB											
D + ABC											
AB + CD											
AC + BD											
AD + BC											
Factorial Regression: Filtration rate versus A, B, C, D											
				Model Summary							
				R Squared = 0.999							
				Adjusted R Squared = 0.998							
				S = 0.125							
				Total = 0.0015625							
				Error = 0.00015625							
				Total = 0.00171875							
				Error = 0.00015625							
				Total = 0.001875							
				Error = 0.0001875							
				Total = 0.00203125							
				Error = 0.000203125							
				Total = 0.0021875							
				Error = 0.00021875							
				Total = 0.00234375							
				Error = 0.000234375							
				Total = 0.0025							
				Error = 0.00025							
				Total = 0.00265625							
				Error = 0.000265625							
				Total = 0.0028125							
				Error = 0.00028125							
				Total = 0.00296875							
				Error = 0.000296875							
				Total = 0.003125							
				Error = 0.0003125							
				Total = 0.00328125							
				Error = 0.000328125							
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				Error = 0.00034375							
				Total = 0.00359375							
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				Total = 0.0046875							
				Error = 0.00046875							
				Total = 0.00484375							
				Error = 0.000484375							
				Total = 0.005							
				Error = 0.0005							



And ACDAC and AD are found to be very significant factors and this ANOVA table.

Thank you.