Design and Analysis of Experiments
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Lecture - 52
Response Surface Methodology (RSM) – First Order Model (Contd.)

Welcome we will continue with Response Surface Methodology in this half an hour to 40 minutes of lecture I will explain first order model with an example.

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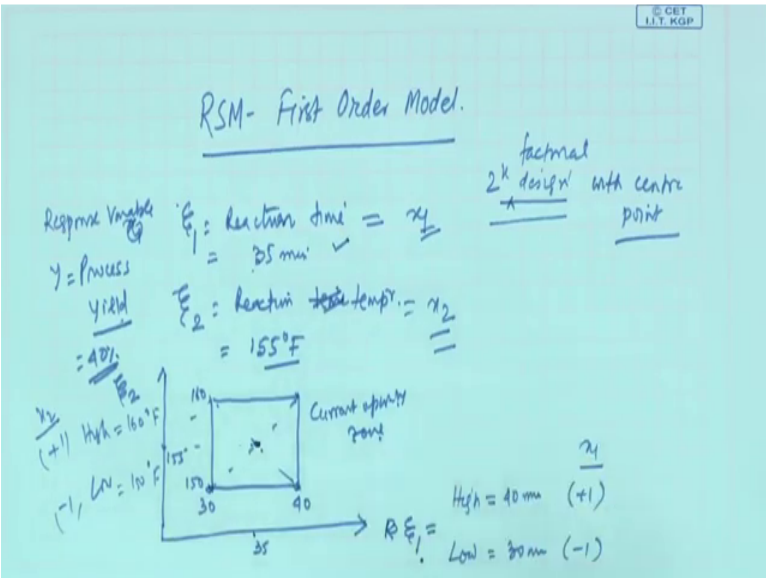

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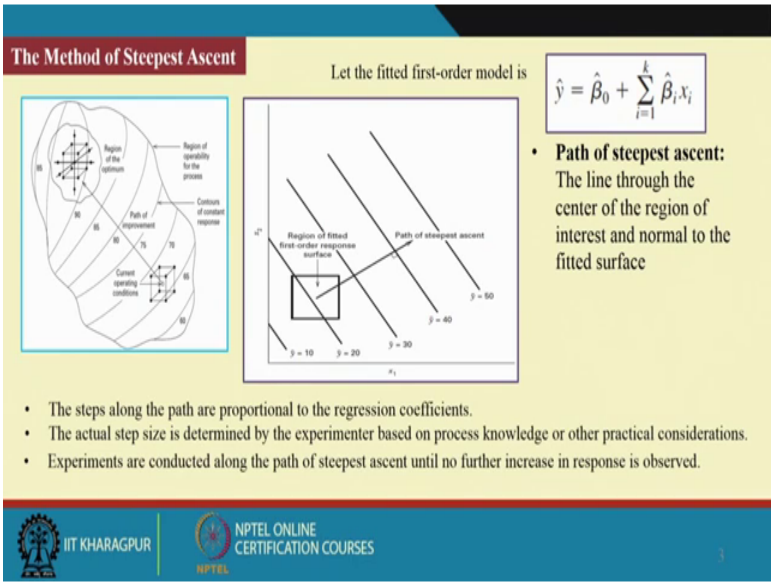

So, if you recall my last lecture on response surface methodology there I have introduce these two figures as well as this first order model. Also I told you that what is the path of steepest ascent and the purpose of conducting first order model is to find out the path of improvement ok.

Means if you are operating here then here you conduct experiment and then obtain a first order response surface and then using that first order model find out the direction of improvement so, that you can con you and you conduct sequential experimentation. And finally, reached to the regional optimum I also discuss that when you go for first order response surface model the contour plot for the responses will be parallel lines.

And if you are operating here then you start with the origin of this operating zone and then follow the path normal to the fitted surface. Also I told you that in order to conduct the sequential experimentation, what you required to do you required to know the step size means where you will go for the next experiment.

So, there you will be guided through the process variables and their corresponding regression coefficient values. You start with one of the process variables and the chosen process variable is 1 where you have maximum knowledge about that variable or that variable has the maximum amount of contribution towards the response values ok.

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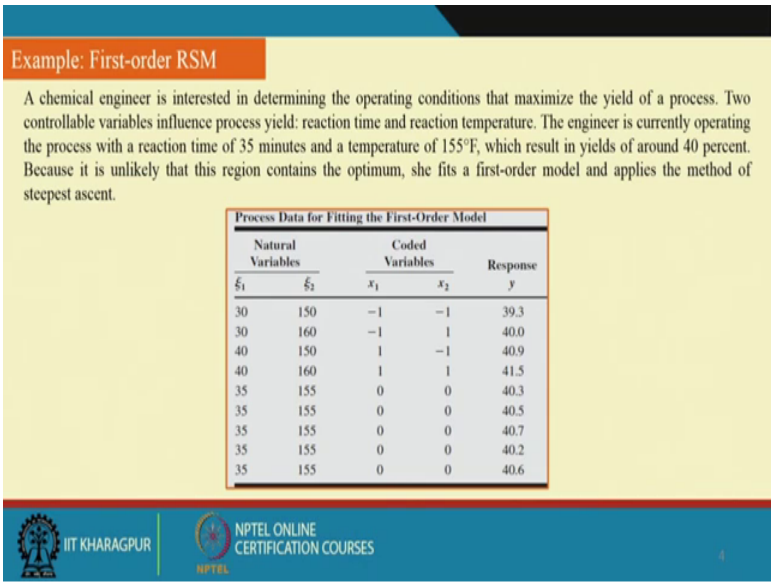

So, there with this background where we will start this example where there are two variable process variables; one process variable is x 1 is. So, for the xi 1 is reaction time and your xi 2 is your reaction temperature. And the response variable is response variable is process yield variable Y is process yield ok. Suppose you are operating with reaction time maybe at 35 minute and this with 155 degree Fahrenheit; what does it mean?

It mean that you are working with two process variable one is your reaction time that is xi 1 and your one is reaction time and here second one is sorry reaction temperature. So, reaction temperature xi 2 and the mean and your operating maybe in this zone where reaction time the center point in this zone is this one equal to I told you that 35 and this one equal to 155.

So, that mean this maybe we can say this is 30 degree minutes 40 minutes then this is maybe 150 Fahrenheit and 160 Fahrenheit this is the current operating current operating zone. And your average yield on an average yield here is 40 percent which is very less.

So, you want to it will improve it find out the where it is the maximum. So, in order to do; so, you have to start experimenting here. So, what kind of experimental design you adopt here. So, you all know that 2 to the power k design can be adopted here. So, 2 to the power k factorial design is very much known to you; suppose if we create for reaction time high and low for reaction temperature also high and low. And if we assume that reaction time high is 40 minute and low is 30 minute and here low means 150 and high means 160 degree Fahrenheit.

Then you got the 4 factorial points in addition because if you if you have two factors in their interactions and all those things we have seen earlier also that if you go for 4 you have to go for applications to get the estimates of the effects. In addition your seen if you have the center point then at the corners you may not the factorial point you may not do replication, but at the center point if you do more replic more replication what happen using center point which is divide of basically which is the maybe the current operating which is the current operating zone, where if you conduct more number of experiment you will be able to get the aero terms to be estimated even if you go for one experiment is as a single replicated the corner points that is what you are seen in 2 to the power k factorial design.

So; that means, here it is basically that factorial design with center points. So, 2 to the power k factorial design with center points with center point ok. So, what more you required to know this will be converted to coded variable x 1 and this will be converted to coded variable x 2. So, from the from; that means, high will be your plus 1 low will be minus 1 similarly here high will be plus 1 low will be minus 1. So, this is related to x 2 and this is related to x 1. So, we have discussed this issue how to convert the original variable to coded and coded two original variables; so, I am not going to discuss further.

Now, suppose under the situation the design metrics is like this x 1 x 2 minus 1 minus 1 minus 1 plus 1 plus 1 these are the factorial points and the center point 0 because you know that plus 1 minus 1 so; obviously, this point will be 0 0; x and y risk. So, the central point is 0 and if you have conducted experiment and laid the responses un like this. So, what is you are your aim you want to fitted first order model.

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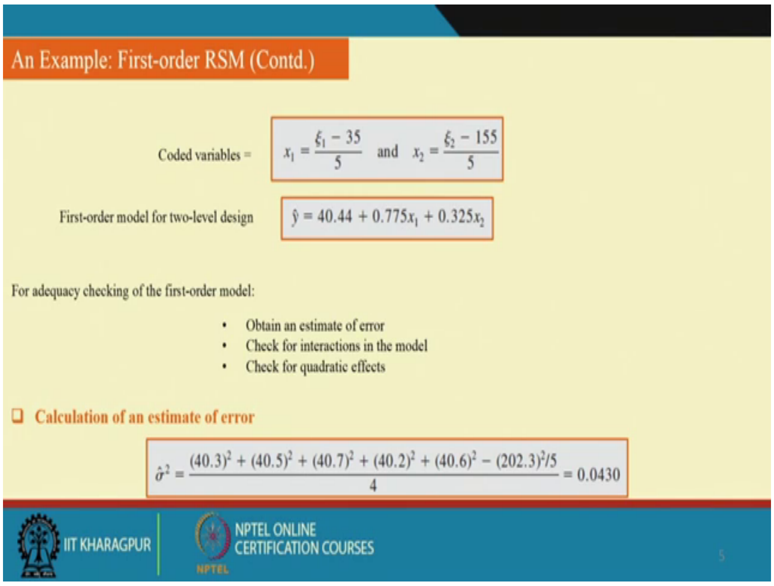

So, you see the coded variables from the original variables and first order model is fitted here what is first order model y equal to 40.4; 40.44 y equal to 40.44 plus 0.775 x 1 plus 0.325 x 2 that is y cap.

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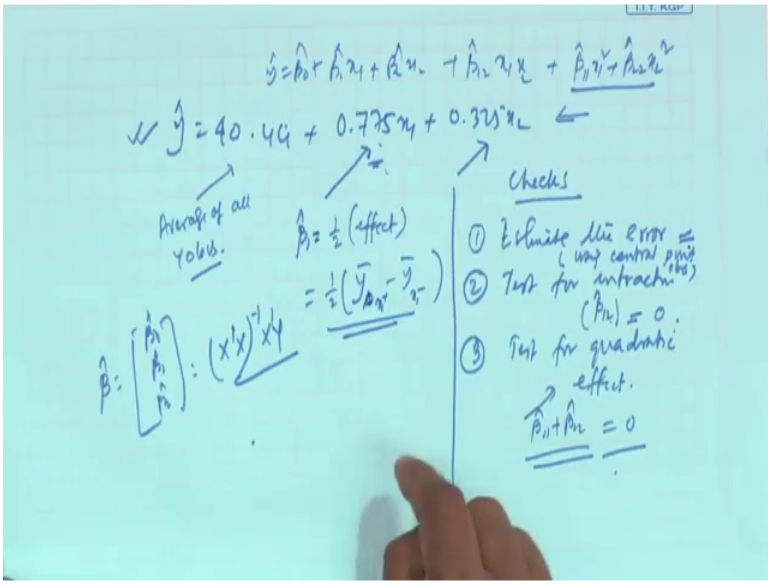

So, then what is this one? This is basically average of all y all y observations; how do you get the x 1 x 1 you get; that means, the sorry beta this is beta 1 this value how do you get this is beta 1 cap how do you get this is basically 1 by 2 into effect. So, how do you get the effect here?

Effect you get average the 1 by 2 y about if I say this is x 1. So, x 1 plus bar minus y x 1 minus bar in this way and you know how to compute the effect; so, similarly this one. So, when the regression coefficients will be half of the effect value that is what you have you know. So, accordingly this is done other way you can create the beta cap equal to where beta 0, cap beta 1 cap, beta 2 cap straightway going for regression X transpose X inverse X transpose Y and where the data are given in the previous slide.

Now, this is the first order model what you have fit. So, y cap is the expected value of y for different combination of x 1 and x 2. So, what you have to do now the you have to check whether the first order model is really fitted one at this current operating condition or not. So, in order to check the whether it is fitted or not what are the things you have to do? First you have to find out the error estimate, second you check the interactions, third you check the quadratic effects.

So, what are the things you have to check point for first order model checks; one you estimate the error terms errors, second one you basically test for interactions and third one is test for quadratic effect. So, in order to test all those things estimate of error is very important; estimate of error will be done using the central point data one central point or central point observations.

Test for interaction means in this particular model first order model that you have considered like this that y cap equal to beta 0 cap plus beta 1 cap x 1 plus beta 2 cap x 2, but there can be beta 1 2 cap x 1 and x 2. So, that is means beta 1 2 cap whether it is significant or not. And when you are talking about quadratic effect you may add something like this beta 1 1 x 1 square plus beta 2 2 x 2 square this cap.

So, I mean you want to test here that beta 1 1 cap plus beta 2 2 cap whether they are significant or not. So, if you find that this is equal to 0 or this equal to 0; the text is accepted then you consider the first order model. If you find that this and this two are not accepted that this hypothesis beta 1 2 0 and this 0 not. So, this model is not valid what you require to do you have to add this 1 is there ok.

This effects to be consider and in it may show happen that that is a local minima local optimum situation or then you have to go you have to do separately further experiments in other zone and then read ok. So, one after another we will now go for all the first one is estimation of error.

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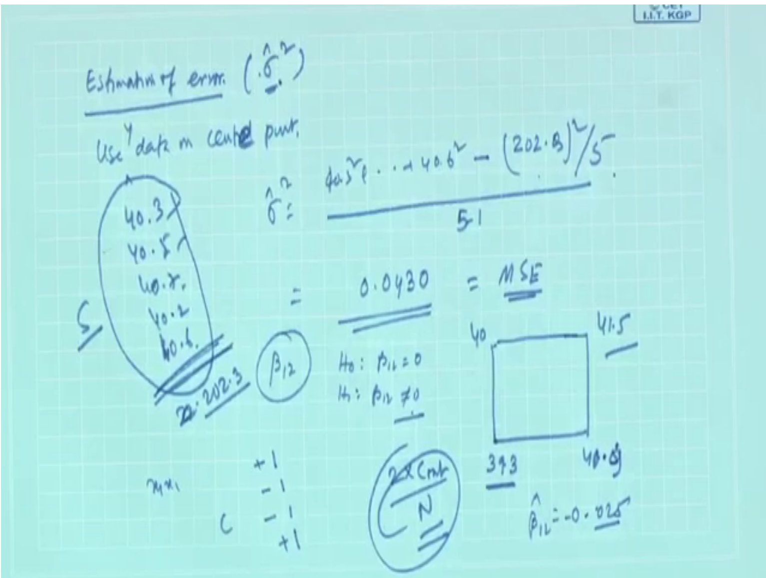

So, estimation of error estimation of error we will say sigma this is the mean; what you do? You will use the central point same data use data on y data y data on central point centre point you write. So, what is the y data are centre point where x 1 x 2 coded values are 0 how many data points are there? 40.3 40.5, 40.7 40.2 40.6 ok.

So, using this data point data can you not find out the sigma square yes you find out the mean of this data, deviation from the mean and then square divided by how many data points 5 data points of there you will find out. Other way that sigma square in generally this will be squares all square all the data points minus the average square by 5 divided by 4 using this formula.

So, that mean 40.3 square plus this square plus this square plus like this plus 40.6 square minus 202.3 square by 5 if you add them up it is basically 202.3 this square by 5 divided by how many observations? 4 5 minus 1; 4 this will give you 0.0430. So, this is the estimation of error this is basically mean square error ok. So, when you have discuss central point that one you have already seen this one.

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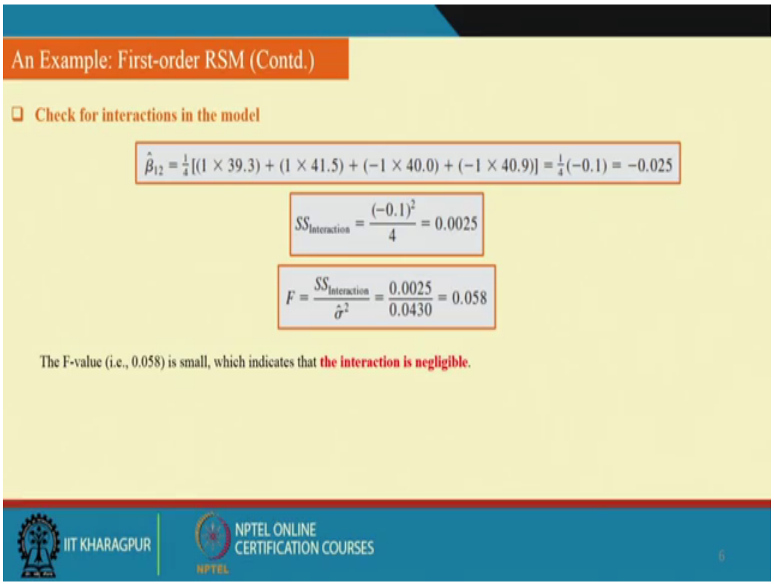

Now, what you want to do now? You want to find out the second one second one is your beta 1 2 that effect is there or not. So, hypothesis is h 0 beta 1 2 equal to 0 null hypothesis beta 1 2 not equal to 0. So, beta 1 2; how do you find out? You will find out from the factorial points. So, what are the factorial points values at low and high? So, when both at low both at low mean this point it is 39.3 when your first one is high value. So, x 1 at high there is 41.5 no 40.9 when x 2 high this is 40 and x 1 this and both at high 41.5.

So, the interaction effects will be both similar and both the average of this plus this by 2 minus average that in this plus this by 2 will give you the interaction effects. So, interaction effects will be accordingly you see that what happened or other way you know the you know the contrast you know the contrast. So, you please see this factorial points. So, minus 1 minus 1 that will be plus 1 contrast for x; x 1 x 2 contrast will be that mean plus 1 then minus 1 then minus 1 then plus 1 once you know this contrast. So, effect will be contrast divided by 1 2 into contrast by N; N is the total number of observations ok.

So,; so, using this contrast or the other way these two. So, for see what is happening 1 by 4 of this. So, 30 9, 41, 40 and this will give you that 1 by 4 into this because you are basically what is your contrast? Contrast is this one in between and you are estimating the regression coefficient means effect by 2 that is why initiate of 1 by 2 it is 1 by 4. So, beta 1 2 is your 0 point beta 1 2 cap is 0 point minus 0.025. So, as I told you whatever what is happening first you find out the effect 41.5 plus 39.5 divided by 2 minus 40.9 plus 40 divided by 2 that will give you the interaction effects AB.

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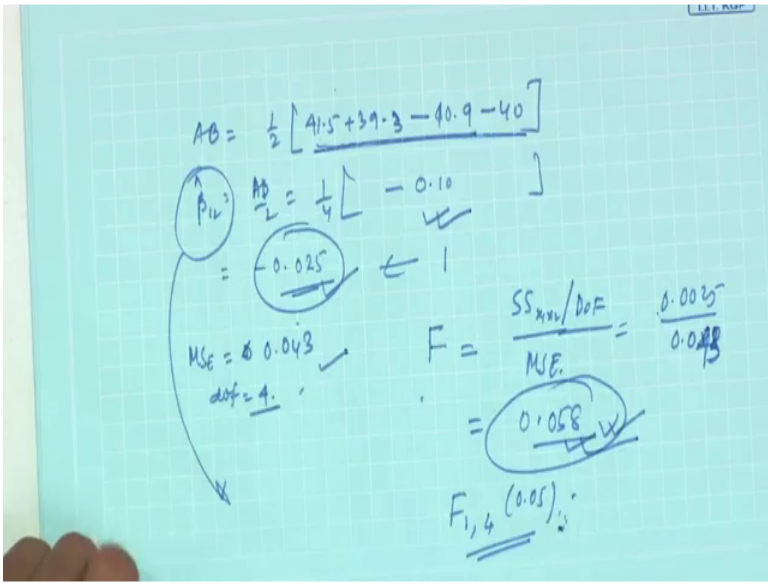

So, using this data you can find out AB equal to half of forty one point 5 plus 39.3 minus 40.9 minus 40 then beta 1 2 will be cap will be half of AB that mean 1 by 4 of this. Now in between this quantity this quantity is minus 0.10. So, beta 1 2 is 0 minus 0.025. So, this quantity is what is the degree of freedom for this degree of freedom is 1 and what is the MSE error degrees of freedom you got MSE value is 0.043 and degree of freedom is for this equal to 4 ok.

So,; so, now you know that whether this value is significant or not just compare this with your MSE ok. So, you find out structure now what happened in order to a get F you required to find out SS that AB or x 1 x 2 divided by divided by degree of freedom by MSE. So, SS how do you find out you know the contrast; so, contrast square divided by divided by 4 that will give you SS interactions is you just see here.

So, contrast square by 4 is the interaction SS interaction a for SS interaction by 1 and sigma square that you are getting 0025 by 0430 this is 058. So, this value is your 0.0025 divided by 0.058 now how much it is sorry 0.043. So, this is 0.058; 0.058 ok.

Now, this value if you compare what is there with F 1 I think 4 0.05 then you will find out this value is much higher than this. So, ultimately; that means, this effect is insignificant fine. So, interaction effect is not there interaction effect is negligible that is what we have done here. Now go to quadratic effect.

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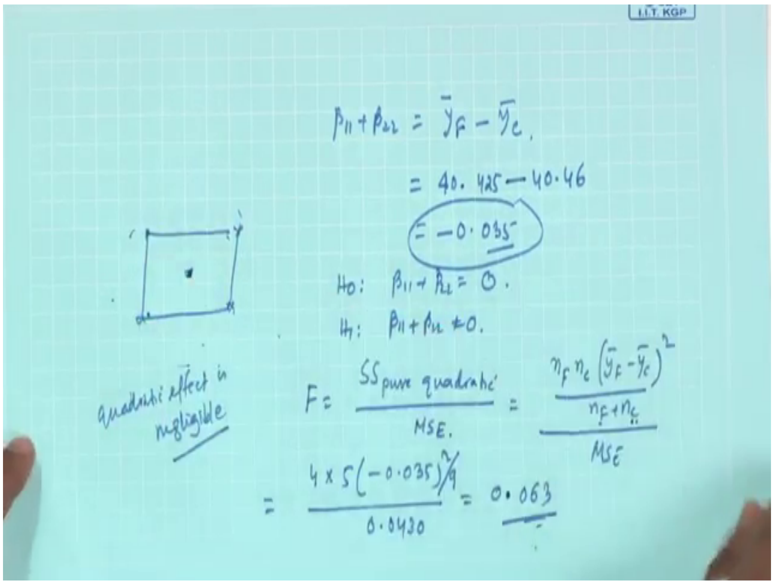

So, if you recall that we have discuss that if there is if there is center point and if there is no quadratic effect, then the average when you develop the response surface the average on the central points and average at the corner factorial point they will be same ideal case the difference will be 0. So, that is the case that mean if there is quadratic effect like beta

1 1 plus beta 2 2 in this case. So, that will be represented by your y factorial point minus y at centre point now y bar at factorial point is 40.425 and y bar a centre point is 40.46 this value is minus 0.035 ok.

So, what do you are doing here? You are creating a hypothesis null hypothesis that beta 1 1 plus beta 2 2 equal to 0; alternate hypothesis is beta 1 1 plus beta 2 2 not equal to 0. So, what do you required to do now you have to in order to conduct F test you required to know SS pure quadratic divided by obviously your, the sigma square that is MSE.

So, this is the computed now how do get SS pure quadratic? If you recall that SS pure quadratic we said that number of points at the factorial points into number of points at the center into y; then y F bar minus y c bar its square divided by n F plus n c and it has 1 degree of freedom divided by your MSE. Now how many points are there at the fact factorable is 4, center point is 5 now y y F bar minus y c bar is this. So, minus 0.035 square divided by MSE sigma square is 0.0430. So, this value is coming around 0 point this divided by n F plus n c is 9.

So, this value is coming around 0.063 which is again much less than that one 4 F 140.04. So, quadratic effect is negligible; so, you got that you factorial point design with center point has given you to estimate the error; even you data to estimate the error you have estimate the error you are in a position to find out whether the beta 1 that interaction effect is significant or not.

You are also in a position to find out that whether that quadratic effect is present or not fine. So, quadratic effect is not there interaction is not there that mean the first order model is a probably a fit one ok.

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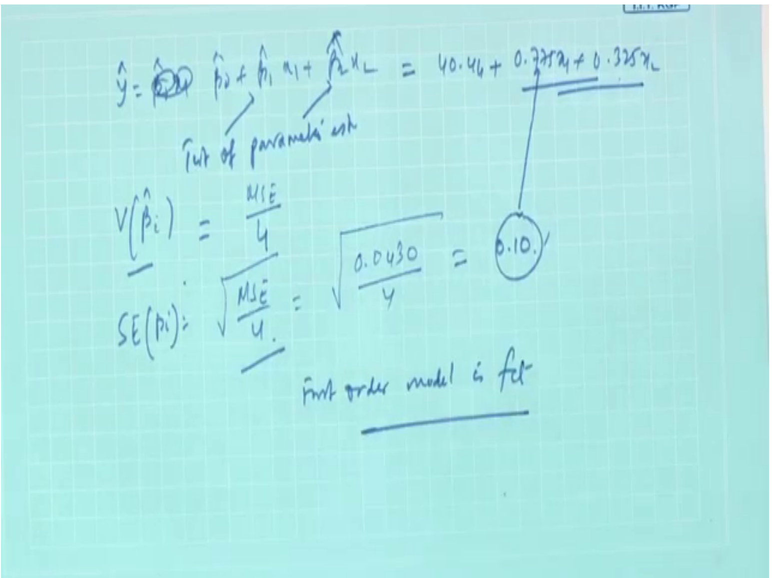

But what you require further that.that mean we are saying y cap equal to beta 1 cap x 1 let it be beta 0 cap plus beta 1 cap x 1 plus beta 2 cap x 2 this is a fit one. But whether this values the beta cap they are significant of not parameter estimates test of parameter estimate you have to do test of parameter estimates. So, what you require to do you require to find out the variance of beta i ok.

So, if you do not do this then you will not be able you have the estimated value, but variance you have to find out. So, if you can recall that when we talk about optimality issues in factorial design; there we have said that under this factorial design case the variability part is basically MSE by 4; here basically two to the power two design.

So, MSE by 4; so, then standard a error standard error of beta i you have to find out this is MSE square root of MSE by 4 because of this coding. So, you got that for all the variables are minus 1 to plus 1. So, the standard error of beta you will be equal for all the fact all the coded variable and this is nothing, but 0.0430 by 4 square which is 0.10.

Now if you see what is the estimated estimate value the regression coefficient value is 40.44 plus 0.775 x 1 plus 0.325 x 2. So, with reference to the estimated value than standard a good is very low and using t test you will find out that their significant. So, first order model is a is fit so first order model is fit.

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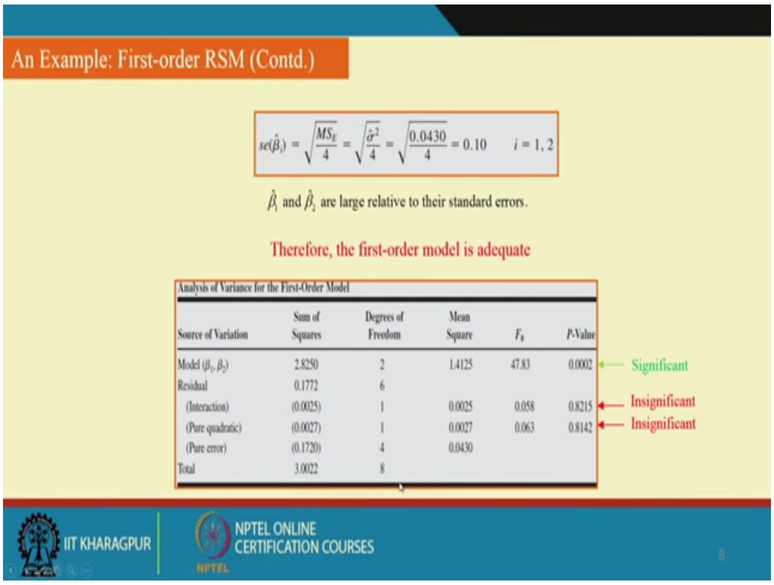

Now this is; what is your ANOVA table I will not discuss further ANOVA because you all know. ANOVA model is significant and a residual is this much and there are interaction insignificant pure quadratic insignificant and pure error is 1720 that you have you have seen also earlier. So, total is total is this and these ANOVA table also giving the support to the first order model is fit.

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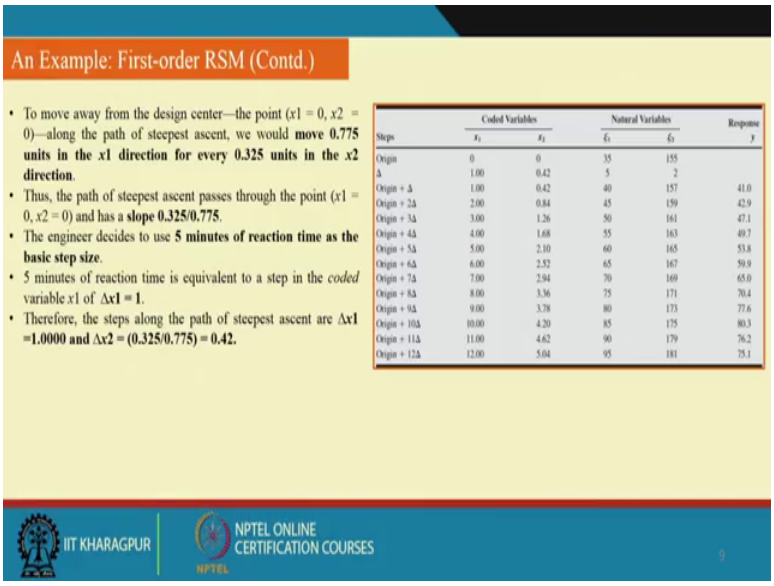

Once first order model is fit you got the direction; now how do you set the direction? The direction you see you see this side start with coded variable. In coded variable origin is 0 and you have done you have in natural variable is 35 and 155. So, you choose there size delta for which we one for the first x 1 or x 2? We are choosing x 1 here because x 1 is having the maximum contribution compare to minimum better contributor than x 2.

So, now what will be the step size; step size if you want to find out for x 1 then you have to go to the natural values that 35 with a starting point now the from process knowledge you have to find out whether the next experiment you will conduct at 35 plus 5 or 35 plus delta x 1 basically. So, through process knowledge laid it is 5; so, I mean delta x 1 is 5 if delta x 1 is 5 then what will be the delta x 2 value that will be guided by the regression coefficient what is this?

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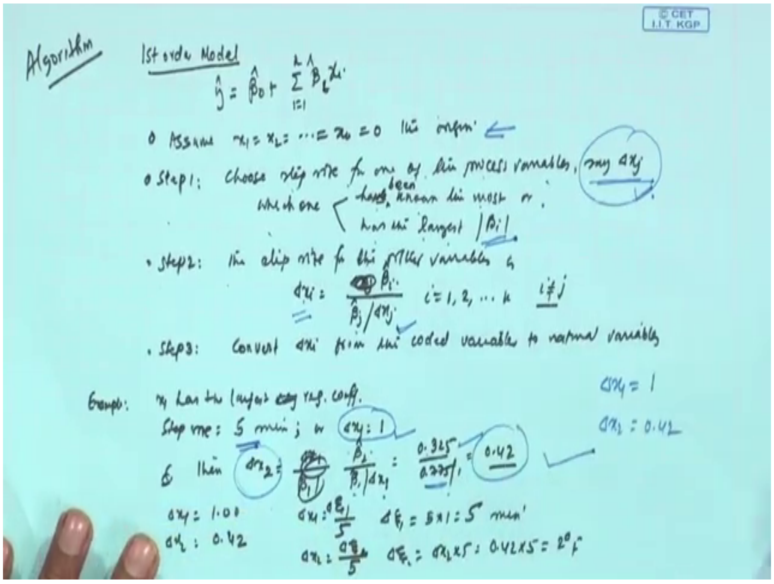

Let us go for this; so, this is this is the basically the steps. So, you start with origin then step one choose step size for one of the process variable say x j; which one you will choose has been known the most or has the largest contribution in terms of f x; then step size will be delta x i then you will you will choose this one.

Once you choose this one delta x j the next variable step size will be beta is the estimate of that value divided by that estimate of the chosen one and by its step size ok. For example, in this case what we are chosen? We are chosen that x 1 is 1 delta x 1 is 1 and step size for that variable 5 units, then for the x 2 it is basically beta 2 by beta 1 by delta x 1 point beta 2 is 0.325 and beta 1 is 0.77 by 1; so, this is 0.42.

So, what is your step size for x 1? Is 1 and step size for x 2 is 0.42. So, you see that in the in the in the in the in the slide what happen we are changing delta we have fixed in coded units and in natural units now origin plus delta origin is 0 0. So, this is coded unit and this is in the natural units and you conduct the experiment and your response value is this followed by followed by the second step will be 2; so, that mean 1; coded variable 1, 2, 3 this way your changing; x 2 your changing accordingly 0.42, 0.484 like this and the result in natural value is required.

Because actually when you do the experiment you have to said the para said the process parameters according to this values. And then after experiment you will get the y will and this is the case. So, this is the way the in sequential experiment is conducted and the algorithm is also what I given you that is the algorithm for conducting the steepest ascent from the first order model ok.

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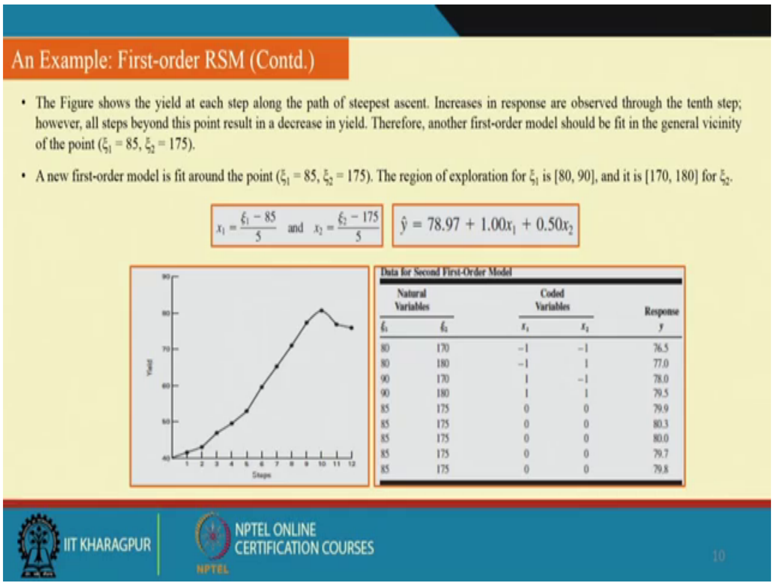

Natural Variables		Coded Variables		Response
ξ_1	ξ_2	x_1	x_2	y
80	170	−1	−1	76.5
80	180	−1	1	77.0
90	170	1	−1	78.0
90	180	1	1	79.5
85	175	0	0	79.9
85	175	0	0	80.3
85	175	0	0	80.0
85	175	0	0	79.7
85	175	0	0	79.8

Then if you plot this what you are finding out at the tenth step? At the tenth step this is this is the maximum value. So, that mean you got you gone to the new position what is this new position? New position is you started with 35, 155, 35 then 155 you have gone to this place 85 and your 175 the center point center values.

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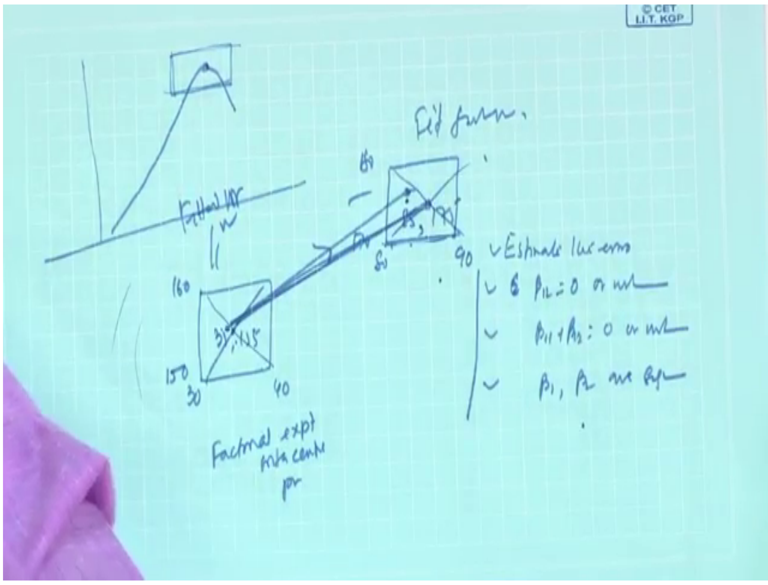

You started experimenting here your first order regression will laid you to there. Now why here is the here is the curvature along this line point you know experiment. So, along 175 and 85; 175 you do the other another experiment.

So, what you have done now then you are here now you are doing another experiment. Again the central point is 85, 175 and the factorial points are 80 to 90 and 170 to 180 here it was 30 to 40, then this is 150 to 160 your movement you have that improvement path list here 80 to 90 and then 170 to 180; you here you have you used factorial experiment with central point; that means, this is the case. So, factorial experiment with central point here; here also you do factorial experiment with central point. So, that mean you are going from here to here factorial experiment with central point and then. So, 80 170 and this is what is in a in a coded design is this ok.

So, one you do once you do experiment; you are getting the response values like this. So, what you to do here know at this new location you just see where the first order model is fit or not.

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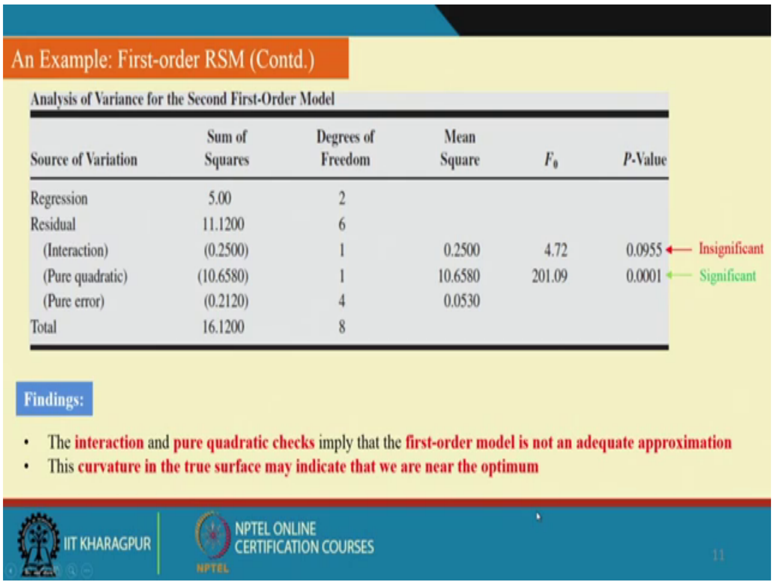

So; how you are doing this? You do exactly the same thing what we have done here. First here find out the estimate the error ok, then second you find out that test beta 1 2 equal to 0 or not; then test beta 1 1 plus beta 2 2 equal to 0 or not. And then test whether beta 1 beta 2 are significant or not exactly same manner the way you have fitted first order model.

So, here you fitted first order model, here also you fit first order model ok. And then what you will find out that from the diagram; we are seeing that the things are going like this then here and it is. So, there is in this zone there is quadratic effect; so, it is expected that first order model will not fit, but you have to test there is no check. Because if the zone is very large one you have taken a large one then what will happen may be first order model you actually fit there. So, whatever may be the case; so, you have to fit you have to again fit the first order model and the result is given here ok.

So, you see that although interaction is insignificant, but pure quadratic is significant. So, that mean first order model is not fit in the new operating zone. The interact finding say the interaction and pure quadratic checks imply that now first order model is not an adequate approximation, the curvature and in the true surface may indicate that that we are near optimum, but you do not know whether you are near first order model helps you go to the new operating zone and there when you after experiment you found out the first order model is not fit because quadratic effect is there. So, what you have to do now at this new zone you have to fit a second order model and then see that whether second order model is fit or not ok. So, whether if it is the quadratic effect is there if it is second order polynomial then you will get the optimum value there ok.

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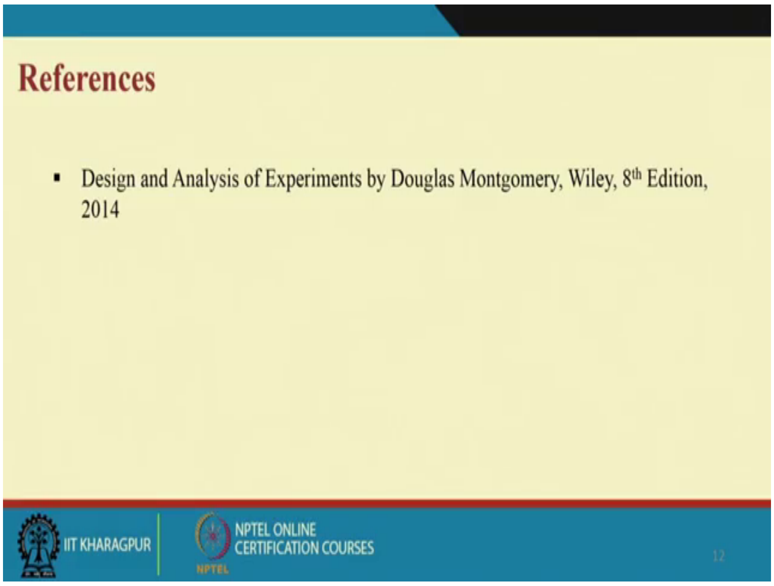

So, let me conclude. So, ultimately you know that we have you we have basically presented here the some portion some portion of the chapter 11 by design of analysis design and analysis of experiment of montgomery it nut shell what I we have discuss. So, for we say the response surface methodology is very very useful one and it is heavily used to find out the optimum zone of operation. Here response surface is methodology actually lay on regression there are will be first order regression, there are will be second order regression.

We have discussed so, far the first order model and there what we are shown that with reference to one example with two factors; we have shown that how you will fit the first order model, what are the conditions and then how do using the first here regression data first order model data; how you go for sequential experimentation and then how do find out the step size, conduct experiment then change this one after another. And finally, and then finally, you will get some plot like this suppose this is the point of optimum, now you have to again do another experiment from here to here and there is experiment you fit the first order model again.

If first order model is likely to be un fit here, but if it is first order model is fit again gives that first order model go for another domain of operation operating zone or another range for operating zones. So this is; what is our first order model for response surface methodology.

Next class we will discuss second order model for RSM.

Thank you very much.