Design and Analysis of Experiments
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Lecture – 52
Response Surface Methodology (RSM) – First Order Model

Hello. Welcome to the lecture on Response Surface Methodology under Design Analysis of Experiments.

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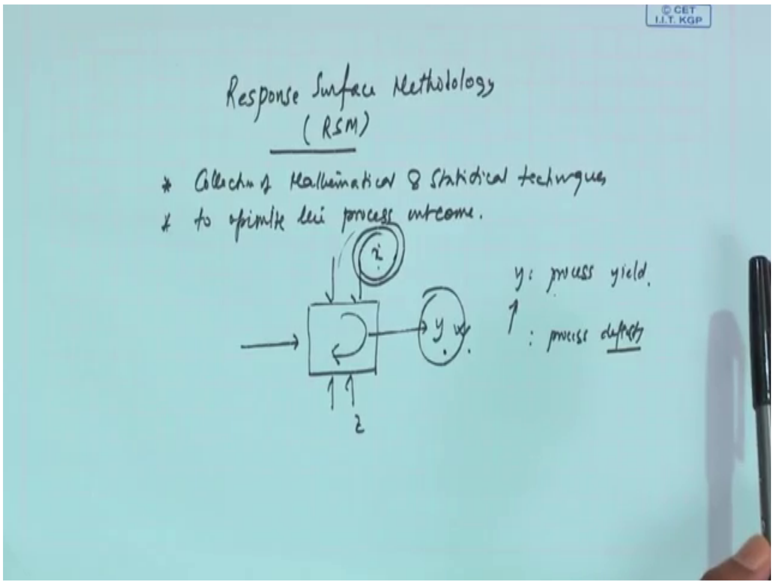

So, today I will introduce the concept called response surface methodology which is in abbreviated form we say RSM. So, RSM is a collection of mathematical and statistical techniques mathematical and statistical techniques this purpose is to optimize the process outcome means what I mean to say here that given a process with set of controllable factors you want to set the controllable factors in such a manner that the response of the process will be the maximum or minimum depending on the characteristics of the response in general we can say the optimum response value. So, you want to achieve the optimum response from a process this is my process inputs controllable add x uncontrollable z process steps you got output.

Now, question is that I want to optimize this output. So, where to set x, what is the zone of x, so that you will get the optimum y. So, what is my desired value desired value is the minimum or maximum depending on if y is higher the better or lower the better or there may be a situation where a target value is important. So, here we are basically talking about the optimum value; that means, is at the maximum or the minimum value if y is process yield then definitely you want to set your controllable parameters in such a manner that y will be maximum if y is that what I can say that defects process defects. So, then what you want you want to set x in such a manner that the amount of defect or the (Refer Time: 03:31) that will be at the minimum level.

Suppose, you are conducting an experiment and you want to see that the impurity level in a particular product will be the minimum. So, that time it is basically minimization case. So, response surface methodology will be will be useful to find out the optimum process output depending on depending on one on the nature of the controllable variables and the relationship between the variables will dictate will dictate or will lead to the situation to the level possible. What I mean to say, I mean to say that you want to set x in a particular zone or a particular range. So, that y will have been having a optimum value that is the purpose of response surface methodology. So, it is basically a optimization technique.

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So, what I will do today I will give you the general introduction to response surface methodology and then will show one method called method of steepest ascent when the purpose is maximization or method of steepest descent when the purpose is objective is minimization so, that will be discussed and with an example the first order response surface model will be discussed.

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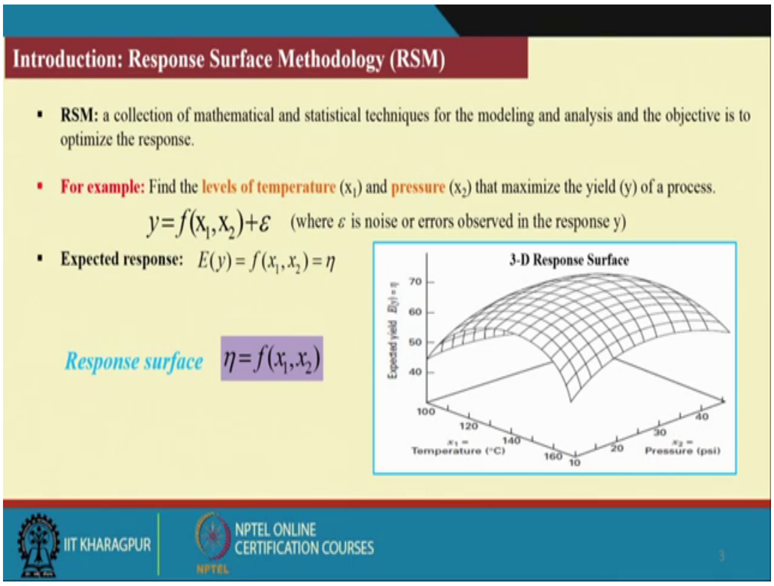

So, let us see what is RSM I said a collection of mathematical and statistical techniques this helps in modeling the relationship between y and x as well as it will help in analyzing the behaviour and finally, it will help us to optimize the response. So, that is what is response surface methodology. So, here I have given you a surface if you carefully look into this you see that there are two process variables temperature which is denoted by x 1 and pressure denoted by x 2. So, temperature and pressure that vary from certain range within certain range. For example, here in this graph 100 to 160 I can say if it is 100 maybe it is 80 to 180 and again that pressure 18 to that 50 in that range it is varying.

Now, and according if temperature and pressure is related to the yield. Then what happened you are getting the surface this surface, the curved surface here this one is known as response surface.

So if you take any point on the surface and project towards this temperature axis and pressure axis then you will get the value of temperature and pressure and the for this yield value is can be obtained if you if you keep the process temperature at that particular value and temperature pressure at that particular value. What do you mean we have we are giving we have given you here one is this was yield, I think this is yield.

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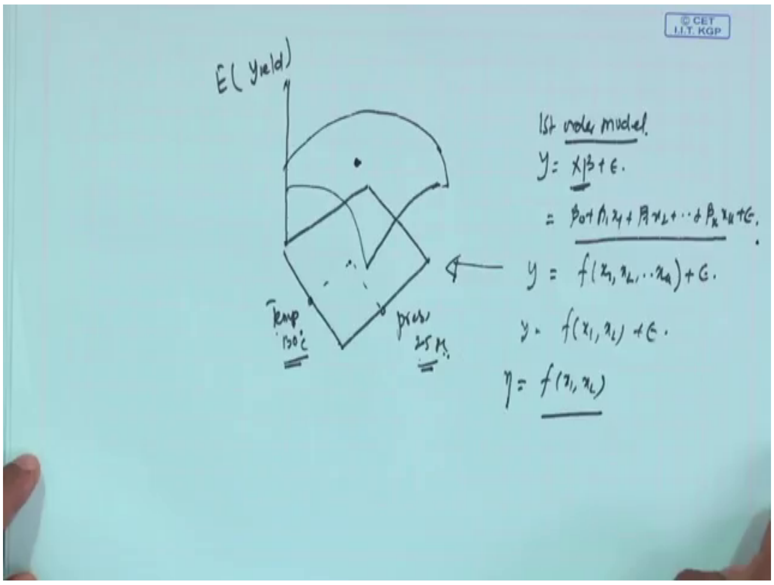

And, we are saying expected yield and then what happened then you see that this axis is your temperature this axis is your pressure and ultimately you are getting a diagram like this now if you take any point there. Here any point suppose if I say this is my pressure, this is my temperature any point here and from that point you when you project there suppose this projection is here and then you corresponding you get there. So, then this is these are the two values which for a suppose if this is I can say that 130 degree 130 degree centigrade and if it is this one is my 25 psi and corresponding this y value is this so; that means, if you keep the run the process at 130 degree centigrade temperature and pressure 25 degrees in 25 psi your yield expected yield value will be this.

So this is the way that response surface will be analyzed ok. So, this is the way response surface will be interpreted and that is what is in better manner that is given here this is my response surface. So, you want to find out that response surface this is the first thing and then second one is basically that depending on the nature of the nature of the output you want to maximize the output or minimize the output if it is a maximization of output you want to find out the point on this response surface, where the output is the maximum. And another I can say that you will you have to find out the global maximum a zone you want to create that if you operate your process within this that zone then you will have the have the near optimal near optimal that responses, ok.

So, few very few interesting things are that what is the response surface. So, all of you have gone through the regression. So, I have given you the regression like this y equal to x plus beta plus epsilon primarily in the regression chapter we have discussed the first order model that mean I can say that y equal to beta 0 plus beta 1 x 1 plus beta 2 x 2 plus beta k x k plus epsilon and this portion is coming under x beta and epsilon is there. So, in general in general this every; that means, this can be y can be written as f x 1, x 2 like x k and plus epsilon with reference to this example or y is f of x 1, x 2 plus epsilon.

So, then the response surface is this response surface eta is basically the expected yield for different values of x which is function of x 1 and x 2. So, so far whatever design we have employed experimental designs and the results what we got finally, everywhere I have shown you the regression equation that regression equation basically talks about the response surface. By first order regression equation or first order model and we say that that basically the interaction and the higher order effects are negligible only the main effects are of importance.

So, in regression the main effects we have considered, but if you if you see the nature of this nature of this response surface it may not be a main affect more model only. There may be there may be interactions or higher order affects some kind of maybe second order effects are there. So, by saying eta equal to f x 1 x 2 so, this may be a function of the main effects interaction effects and higher order quadratic maybe the quadratic effects. So, you want to find out first these whether the first order model or second order model or higher order polynomial are required that you will be finding out and once you have that response surface you also want to find out where the optimum response lies. So, that is the purpose of response surface methodology.

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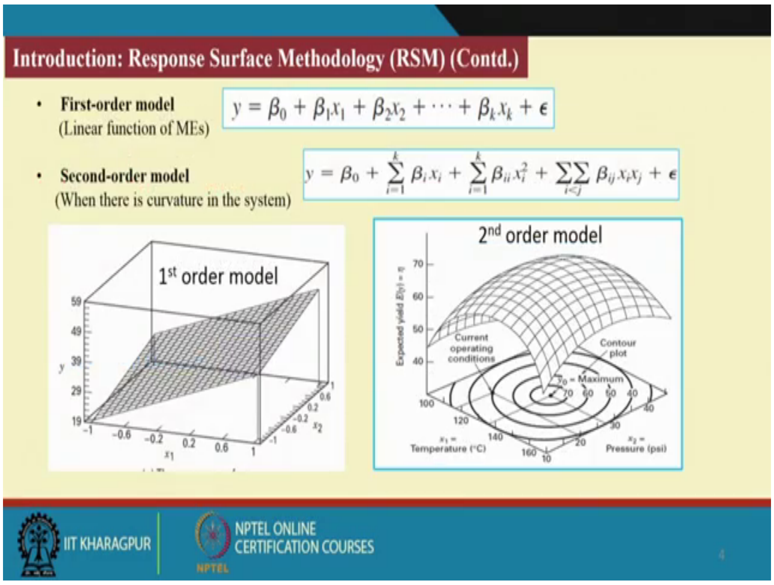

Now, what I will show you these response surfaces here one is your first order response and another one is second order response. So, first order response and second order response how it be looks like if there is no interactions and higher order effects. So, then the regress the response line will be this flat type this one ok. So, and if there is second order effect interactions then the regression line surface will be that kind of this curve will be there curvilinear surface.

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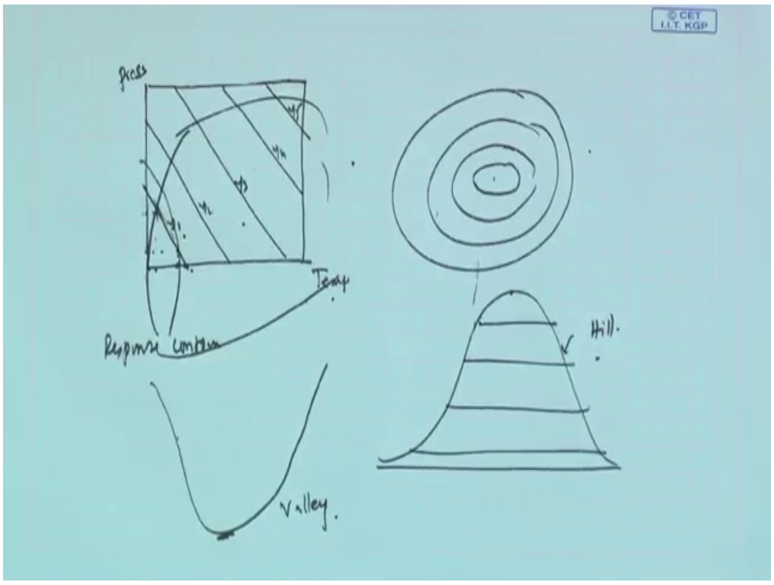

And, in the first case first case if you suppose you want to develop contours, I also explained contours earlier I said that suppose this is your temperature and this one is pressure. So, then y contour the response contours response contours that mean for certain values of temperature and pressures what happened there y response will be same for example, if it this that mean if temperature is this value pressure is at that value then response is y suppose this is y 1. So, any point on this line is having same value for response, but if I took take a point here the temperature and pressure that value will change like this is the process temperature this is the process pressure if I take this point the temperature is reduced pressure increased, but in both the operating values in terms of pressure and temperature give you the same value of y. So, this is the contour. So, maybe the second contour third contour like this. So this is y 2, y 3, y 4, y 5 something like this, ok.

So, if you do not have the interaction and the higher effects then your respond contours are contours will be like this the parallel lines means if the first order model is fit here, then if you if you develop the contour for the response you will be having parallel lines if your second order model is feed then you see the contour what is happening this is the first contour y, y is 70, this is the second contours y is 60, this is the third contour y is 50, fourth contour y is 40 like this the reason is the maximum value is lies somewhere here. So, so, the second case now as per then is the contour is like this maximum value here and your contour such contours are reducing like this.

So, it is some kind of some kind of hill some kind of hill here the maximum height then this height then this height then this height then finally, this height. So, you are getting this one with reference to this show this is nothing, but you just go periphery of the hill. It may be in edit basically your case may be just reverse, it can be it can be other way around also like this. So, in that case the response surface will be will not be will not be like this it will be like this what I mean to say it will be like this when the between point it will be down so, it is.

Now, if your maximization case suppose it is it is this is this is some kind of hill and it will minimization say this is some kind of valley. So, your response surface will give you depending on the number of variables obviously, that the surface will be different, but for the explanation point of view if we consider the two raise two controllable factors or variables and then if it is a maximization case and where the higher order interactions are they are including the higher order effects quadratic and higher order effects.

So, you will be having a hill kind of structure or valley kind of structure depending on whether it is a maximization case or a minimization issue. Directions are the rebels are not there then what happened it is a surface plane, it is a surface may be either surface will be plane surface some inclination will be there, ok. It is not that constant that is slope for example, here this surface, ok. So, in a small surface methodology it will help you in finding this surface and also helps you in identifying that optimum zone of operations.

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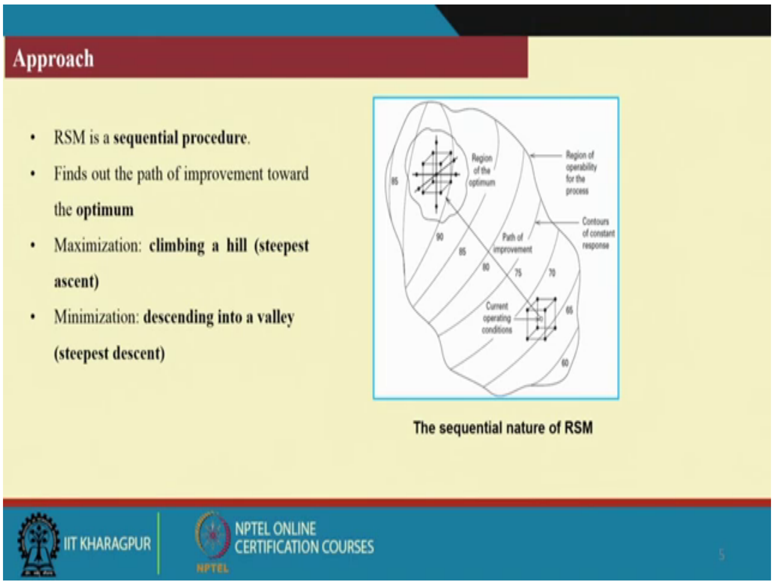

Ok. So, now, let us discuss that what is the approach adopted by in response surface. So, for this you require to have some more concepts what is the what are those concepts. So, so let us see let us write down change, ok.

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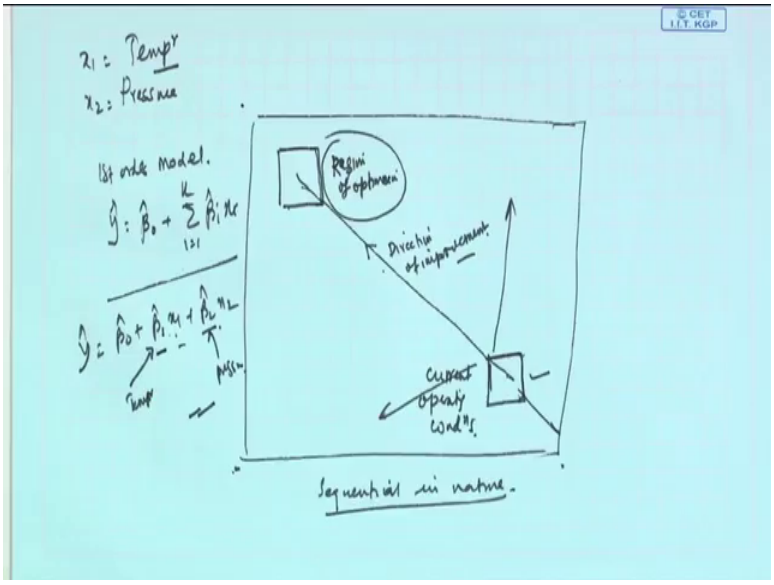

So, suppose you just think of for the time being you think of the same that pressure and temperature pressure may be x 2, temperature is x 1, ok. So, these are the basically process controllable factors this will give you the design space where you will open suppose you just think of. So, this is the world operating zone you can operate means temperature from here to here, pressure from here to here for that and you want to find out that should you work here operate the process here. So, or should operate the process here.

Suppose, you are operating somewhere, but without doing any kind this kind of experiment based on process knowledge and maybe previous experience you started with doing somewhere. So, there may be some technology changes and other things have happened or you may be thinking that no this is this zone this operating zone is inferior this operating zone is superior, but this will give you my the optimum response not this one.

So, how do you know? So, for that purpose that you must know; what is your current operating current operating conditions. For example, suppose you are doing here, then using response surface methodology what you want to find out first, you want to find out the direction of improvement. If I we assume that this is basically the this is basically the region of optimum then this response surface will give you these directions it will give you these directions.

So first you approach each that you know your current operating conditions then fit a do experiment here fit a model response surface model and then from there analyzing that model you find out the direction of improvement. So, this is our direction of improvement ok. So, that means, your one set of experiment is not sufficient one experiment is not sufficient one kind; you have to have several experiments. So, set of experimentation this is basically I can say you have to do sequential experiment experimentation. You do here find out the direction go to the second place again feed the response surface and see that whether optimum point is reached or not if not again you find out the direction from there and in that manner you finally, reach to the region of optimum. That is why response surface is a sequential in nature sequential in.

So, you see, that this is your current operating conditions. Now, what is there. So, you have and you know that in the operating that zone. So, you if you operate here somewhere here the process output value response value will be somewhere let it be maybe a 67 and now, if you go along this line suppose assume that you know then the 70, 75 these are all the contours.

So, that means, this is direction of improvement, but actually you do not know that and. So, what do you want you want this direction and response surface what it will do it will when you do conduct experiment here and then fit a regression model that regression model will help you to go either this direction or other this direction or this direction means maybe this direction maybe some this direction or this direction or this direction depending on the model will guide you this will go which direction you will go and the purpose is basically you want to reach the vicinity of the optimum.

So, that is why finds out the path of improvement towards the optimum now if it is a maximization case as he told you we are climbing a hill if it is a minimization case you are descending along the or into the valley that is the case. So, this is what I giving you.

So, now, there are a few more issues here that issue is that when I you are considering such a bigger zone and at some at some zone you are working and if it is not the point of optimum what will happen that you will find out that the lower order model will become sufficient to find out the direction of optimum. You do not require to go for higher order model, because this is this is the place where your optimal optimum value is not reached. This is the operating conditions where you are working sub optimally. So, under that is that condition because you want to go it is not a localized one even broader perspective. So, you will see that at the aggregate level what happened the lower order model will what.

So, and if this is not the point of optimum here you may fit a first order regression model and then from that from that model find out the point of optimum direction of optimum and then follow that direction, do the next experiment, again fit the model first order model. If first order model does not fit there, that means, you have to go for second order model, that means, there is a curvature or quadratic effect a point of optimum might have reached ok. So, if it is a maximization problem we will say that the method is the optimization method basically steepest ascent method, if it is a minimization one it is will be steepest descent method.

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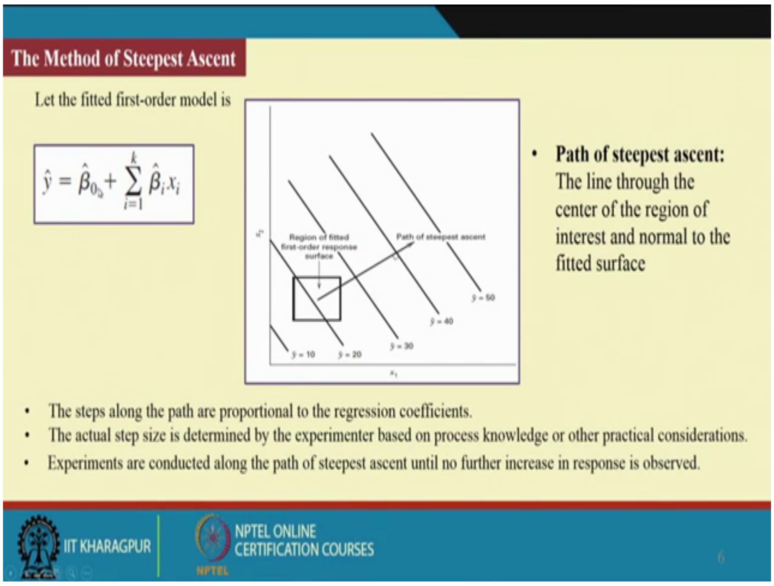

So whatever may be the case, but let us explain only the steepest ascent method because the distant will be there is the traverse, ok. So, let us see that suppose the first order regression model is 50 then your predicted value will be beta 0 cap plus beta i x i sum up this. Now, if you if you plot the contours. So, x 1 and x 2 as it is a first order model and that means, there is no interaction effect. So, you are getting a surface and where there will be there will be parallel lines which basically gives you the contour for response. So, here you see that y bar y capital to 10 y cap 20 y cap 30 like this. So, these are the contours. So, you are at present you are operating here.

Now, from the contour plot you know that where you should go you should go in this in this direction, but what is that direction. So, first where you are operating you have the origin. So, you take you take from origin a perpendicular distance towards this contour this these are all perpendicular to this you are getting is surface. So, perpendicular to that surface from the origin you move. So, normal to the fitted surface perpendicular is normal. So, that means, what is the path of steepest ascent the line through the centre of the region of interest and normal to the fitted surface. So, here you basically started working here is the centre and you have the surface. So, from here you take the normal path normal to this surface. So, you will be getting this is the direction.

Now, what happened the steps along the path are proportional to regression coefficient very important? So, you have beta 0, beta 1, beta 2, beta 3 like this so beta i and beta j. So, beta basically l you the how do you get this path using the regression coefficient the concept is that the paths are proportional to regression coefficients number one. The actual step size is determined by the experimenter based on process knowledge or other practical considerations. Experiments are conducted along the path of steepest until ascend until no further increase in response of some charge.

So, what I mean to say y cap is beta 0 cap plus sum of beta i cap and x i; i equal to 1 to k, this is our first order model. So, with reference to this temperature and pressure case we can write beta equal to beta 0 cap plus beta 1 cap x 1 beta 2 cap x 2. So, beta 1 is the regression coefficient for temperature let it be and beta 2 regression coefficient for pressure. Now, this beta 1, beta 2 will tell you which direction you will move, ok, because that that will be that the basically the steps what you take you should not take along x 1 or along x 2 only your beta 1 and beta 2 the coefficient will guide you whether you how much you will go along x 1 and how far you will go along x 2. So, that will be discussing definitely ok.

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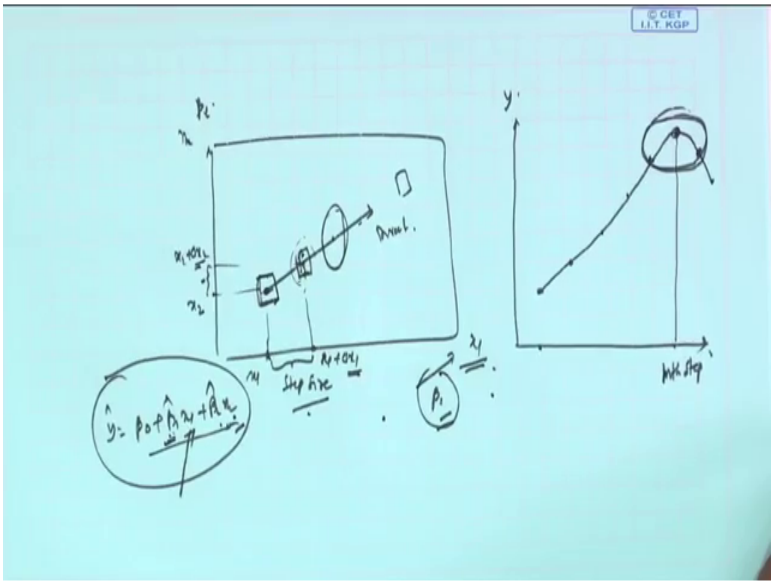

Now concept is that how far you will go if I say so, you this is the zone you have started with this and you are moving to this direction. So, so, that means, what happened you are first you are experimenting here then suppose you want to experiment here again. So, that mean how much you are moving from here you are you are here from x 1 point of view x 2 point of view we are here originally you want to go this.

So, this much movement and here it is this and you want to go this much movement. So, that means, these are known as step size. So, step size for x 1 you may choose or x 2 you may choose that ok, at this point suppose if it is x 2 then this one is x 2 plus delta x 2 and if this is x 1 may be x 1 plus delta x 1 if you choose delta x 1 then delta x 2 will be automatically determined because we have beta 1 coefficient here and beta 2 coefficient here with reference to the regression first order model.

Now, then which variable you will choose you will choose thus this is the step size for that variable where you have maximum knowledge or which is having the maximum contribution in terms of beta. So, that means, the you start with the variable to find out the go to the next step for experimentation you start with the variable which is having either the maximum beta coefficient or you have maximum amount of knowledge for that variable and then the other one will be determined accordingly because I you know y cap equal to beta 0 plus beta 1 x 1 plus beta 2 x 2 this is known.

Once you choose this delta x 1 and where the increases beta 1 per unit then what will be the change in x 2. So, that considering beta 1 cap and beta 2 cap you will be finding out the x 2 value. So, that is what we have a and I will tell you that the computation issue, but that issue is that the actual step size means in order to go along this line you require to you require to choose steps that mean how far you will go you may say I will I will do here it may not work. It may be the distance so high that in between there are points which are basically it is gives you maybe who knows that the maximum lies here not lies here.

So, this direction you got and you follow that direction and finally, conduct experiment everywhere; here, here, here, here, everywhere. And finally, you will find out the position where the maximum value will reach and when you go beyond that the optimum value will decrease, that is the concept I will used here in the method of steepest ascent. Experiments are conducted along the path of steepest ascent no further increase in response each object, what does it mean? Same thing suppose your present current operating zone is this and you got the regression line that what I can say that this one regression surface and you know the from the contour you know this is the direction is known. So, here you conducted experiment you got the y value response value suppose if I say this is response. So, then this is my first experiment and my response value is this.

So, these directions suppose you are doing here the second set of experiment your response value is this, third set of experiment here response value of this, fourth response value is this, fifth response value is this, the sixth response value is this, the seventh response value is this, the eight response value is this. So, what does it mean you are going along the direction you are going and finally, your curve is like this. So, this is the suppose if it is the k if that is m-th step this is this is the maximum one below this you have less value and above this also you have less and here you got a curvature we will then start with an example.

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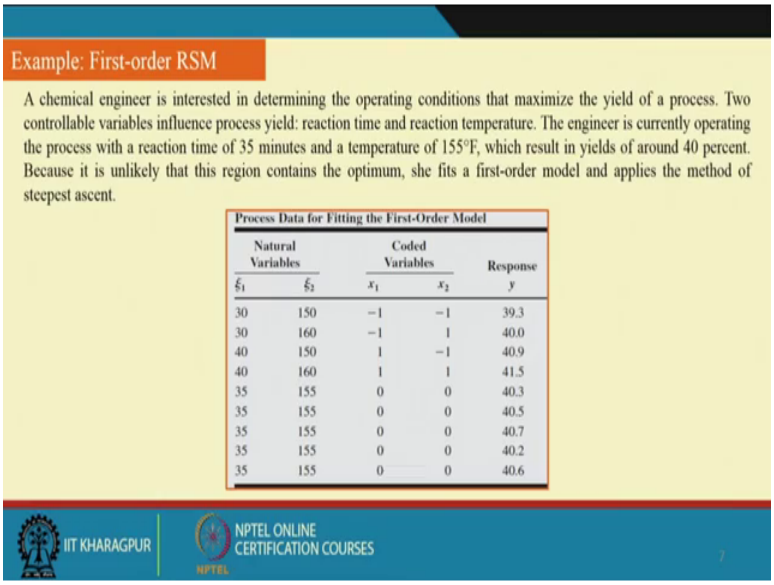

And, but I will complete the example in the next lecture. In this lecture I explain that what is the example and what the results we got, but the details of the results all those things will be explained later.

A chemical engineer is interested in determining the operate operating condition that maximizes the yield of the process to controllable variables including the process yield reaction time and temperature the engineer is currently operating the process with a reaction time of 35 minutes and temperature 155 degree Fahrenheit which result yield around 40 percent, because it is unlikely that this region contains the optimum she fits the first order model and applies the method of steepest ascent.

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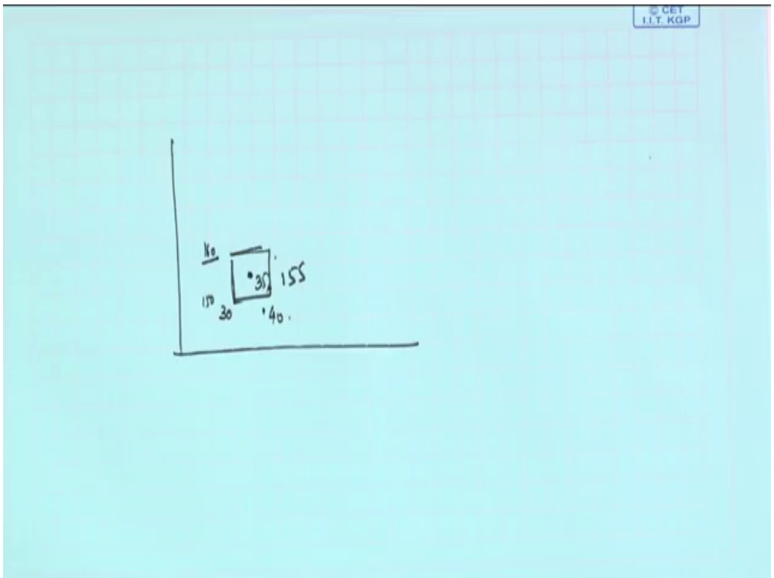

So, that means, suppose you are you are again reaction time and temperature reaction time and temperature and you are operating here operating in current operating in 35 minutes and 155 degree Fahrenheit; Fahrenheit this is the centre point. So, that means, I can say this 30 to 40 maybe this one is 150 to 160 something like this is the range you are working with. So, you conduct an experiment and get the values and that experimental result is given here. Here what happened? Here a factorial experiment with centre point is used and experiment is conducted, fine.

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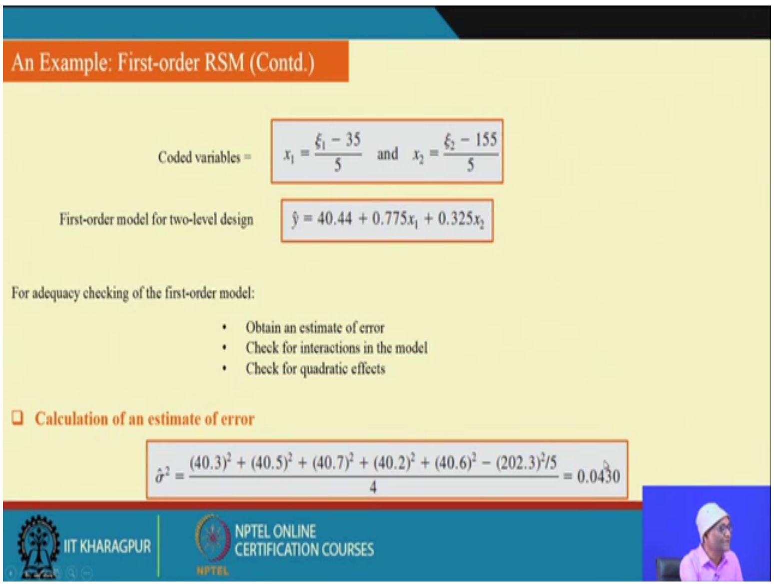

With this data what happened? The coding and that there are three kind of three kinds of things to be estimated one is the error estimate. So, error was estimated using the central data.

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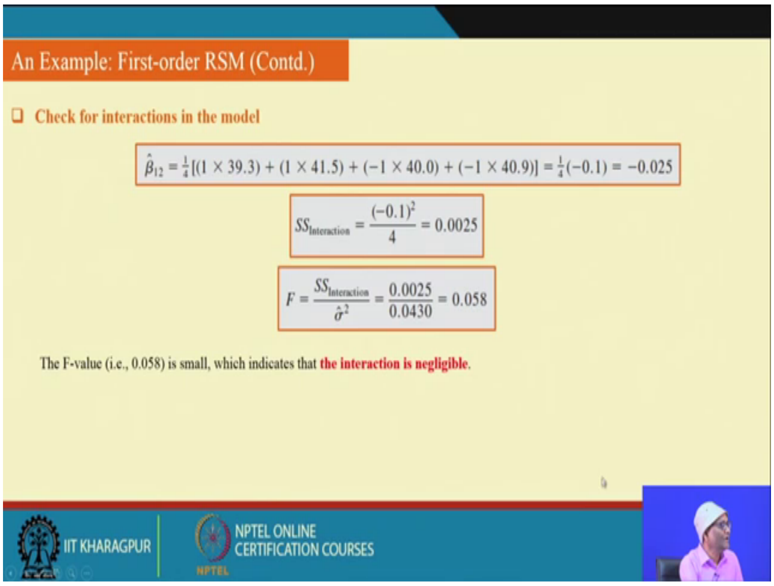

Then you interaction estimation; that is also estimated with the factorial data points.

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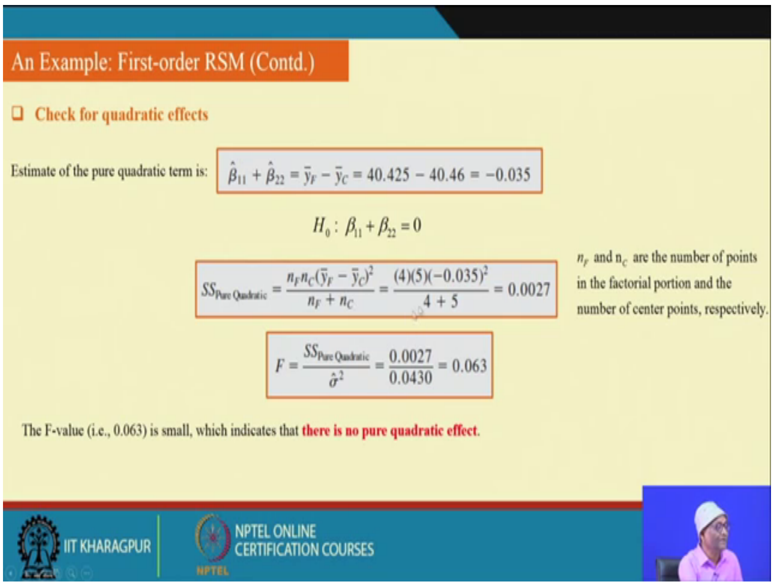

And, then through F-test, we say that interaction if it is negligible and then quadratic effect is estimated that how to estimate the quadratic effect one lecture we have already given you using the central point how quadratic effect will be estimated.

So, that was estimated and using F-test found out that quadratic effect is not there. So, interaction is not there quadratic effect is not there.

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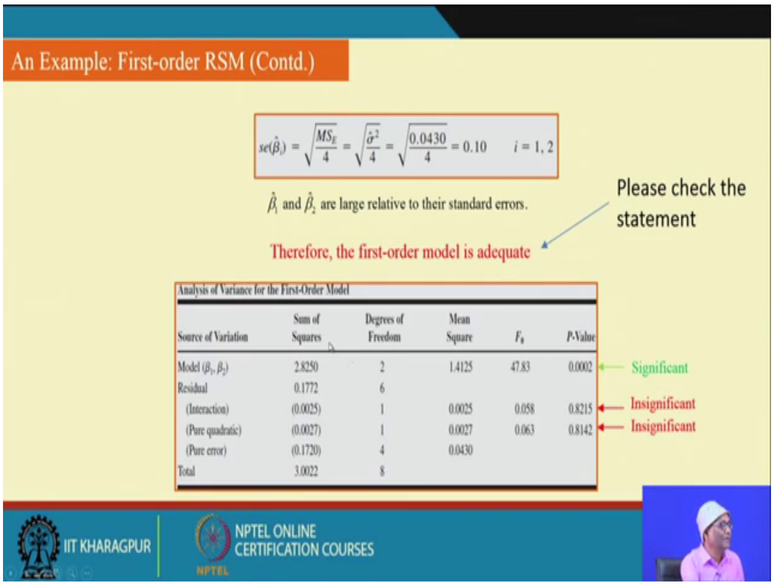

Then what happened the beta coefficients their significance was tested also, using the standard deviation of the beta coefficient and it was found that the standard deviation value is very low compared to the magnitude of the beta values like your our beta values are our beta values are 0.775 and 0.325 and standard error is 0.10. So, they are quite large with reference to the stand standard deviation. So, we can say that first order model is significant. That means, this model will help us in finding out the direction of direction of optimum.

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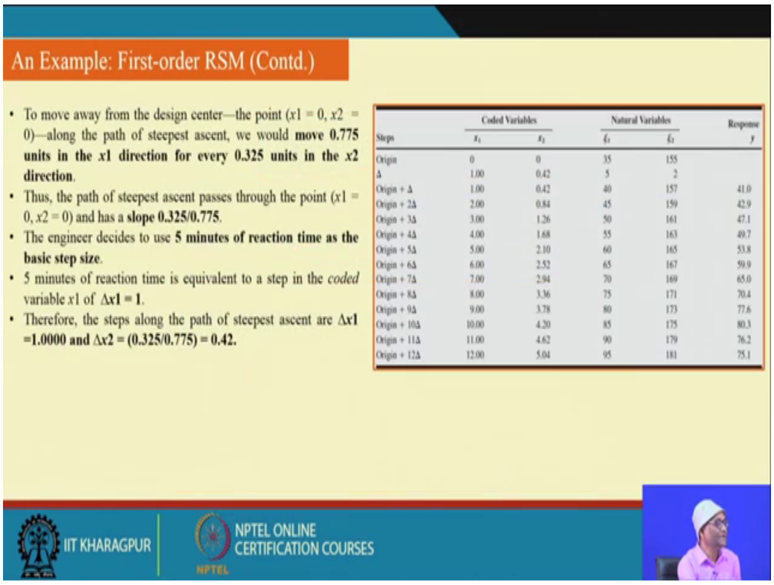

So, using these then as I told you that you first in the origin, set the origin and then you find out the step size and then use you go on changing the values of the two variables and conduct experiment one after another and get the response values. Once they all those response values you if you plot you will be getting something like this.

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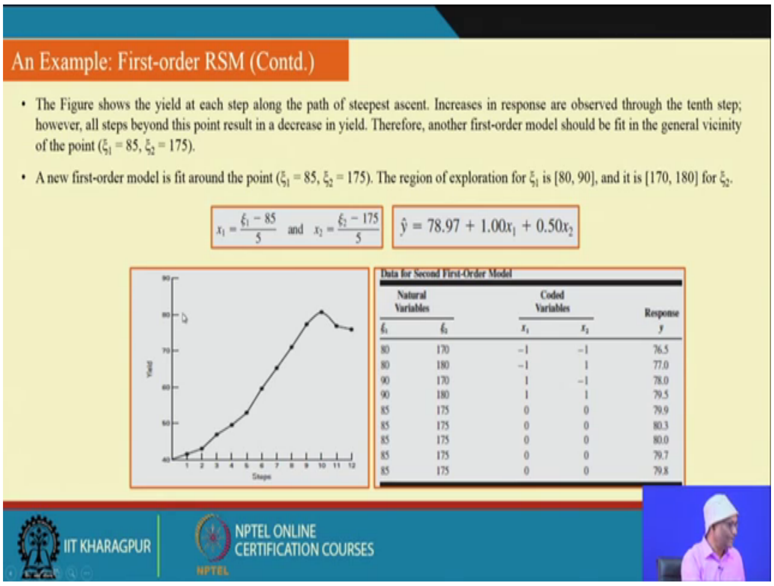

Obviously, big based on the first order model and the sub sequential experiment you have reached to a point where step size I think it is 10. This 10 step size here you are finding out that what is the x 1 and x 2 value and that is giving you that what is the value of y. It is y value is or more than 80 percent.

So, if you see if you see this suppose I say the tenth one; tenth one means your temp that natural variable the temperature and the reaction time and temperature there is an 85 minutes and 175 degree centigrade a degree Fahrenheit your that response values is 80.3. 80.3 This is the maximum value you reached. So, that mean earlier you are operating 30, but I think you are operating 35 and 155 so, but that is in failure zone you are better zone is definitely 85 and 175, because the yield is increased from 40 percent to 80 percent; so 80 percent.

So, now, what do you require to do, at this zone you conduct another experiment keeping the central centre at 85 and one 175 and again fit a first order response surface and you see that whether the first response surface is sufficient or not if sufficient that is not the optimum point. Then you have to go for again using that response surface regression values response of the surface find out the direction again and follow that direction, so long you are not getting if zone where a curvature effect is there.

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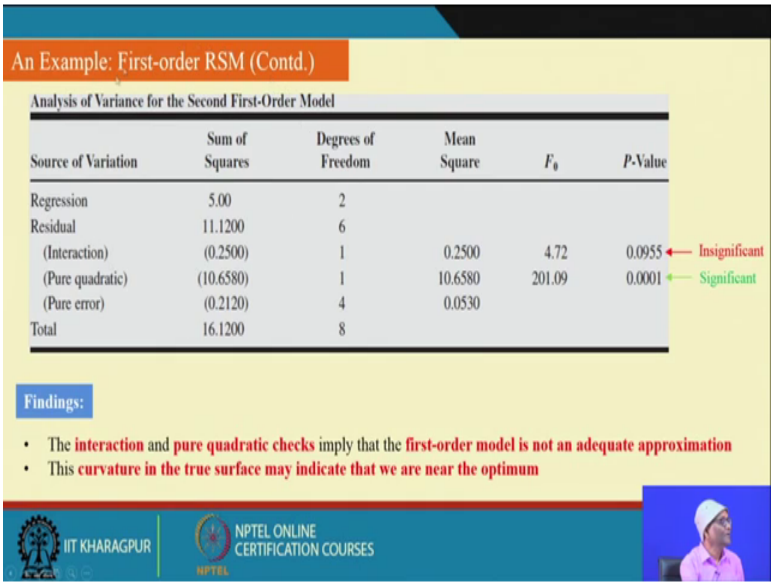

For example, the once we have fit the first order effect in that new zone what happened we found out that that pure quadratic effect is significant. So, it is in first order or not will not fit you have to go for higher order model, ok. So, this is what basically the way a response surface will work.

So, in the next half an hour what I will do I will elaborately explain the example the concept is given to you now. Now, next lecture we will explain those concepts with reference to the example and we will show you the all the calculations and one and results, corresponding results and their interpretation and then what you should do next ok.

So, to conclude I say the response surface methodology is a very powerful technique and heavily used in analyzing that experimental data, particularly to find out that zone optimum we want to go to the vicinity of the optimum and at the same time what happened once you know that you know zone of optimum you will say to your process parameters accordingly and you get the best results what is achievable from that from that process. You should not run a process at the inferior level it should, you should achieve the optimum on the best possible outcome or outputs from such a model, it is a wonderful thing.

Thank you very much.