Design and Analysis of Experiments

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Lecture – 51
Plackett – Burman Designs

Hello, welcome. We will discuss today special kind of design called Plackett-Burman designs. Let us see the topics.

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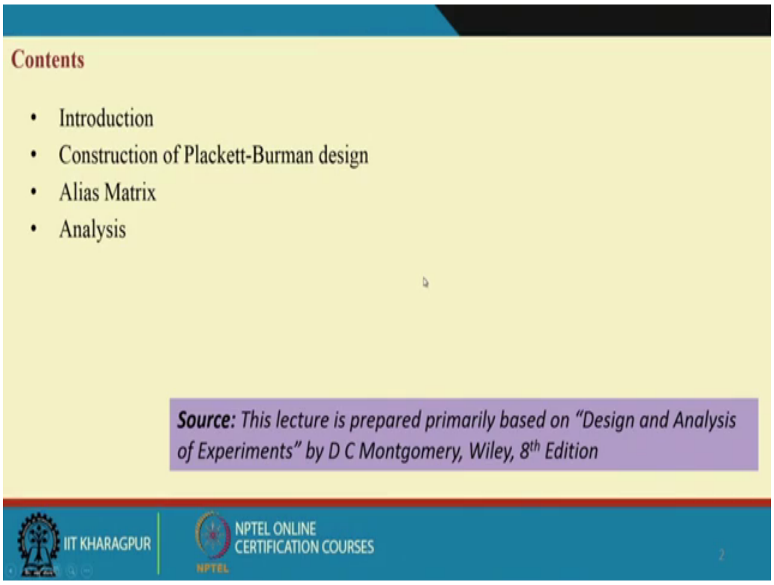

The contents of today's presentation introduction, then construction of Plackett-Burman designs how we will find out alias matrix in Plackett Plackett-Burman design and then how do you analyze the data that are resulted out of the experiments using Plackett-Burman design.

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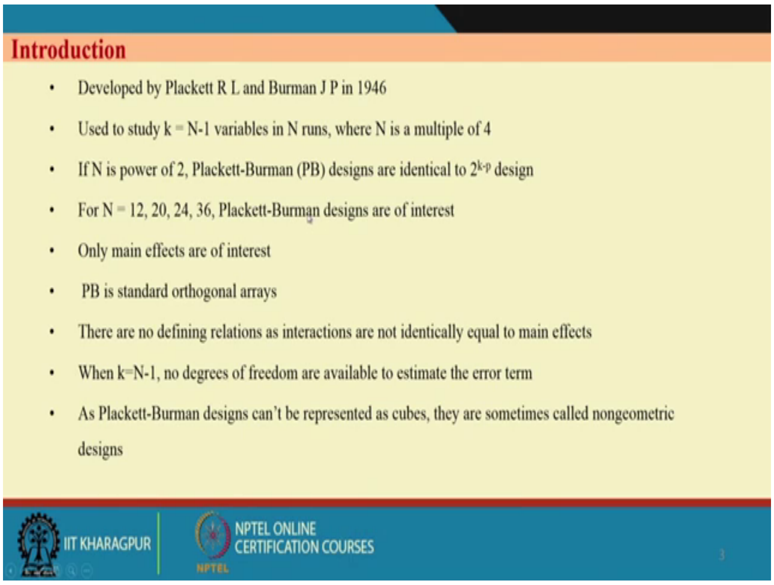

So, Plackett-Burman design was developed by Plackett R L and Burman J P in 1940 and 46, this kind of this kind of design is used to study k equal to N minus 1 variables, where N is multiple of 4.

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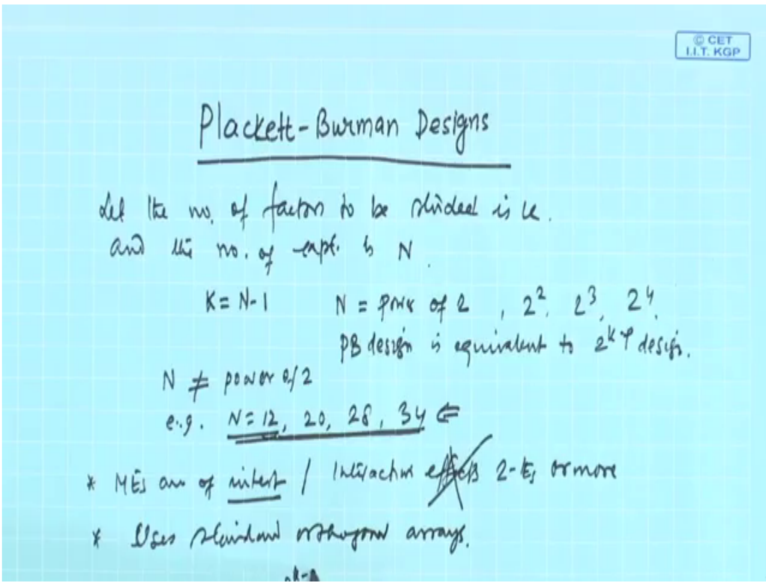

So, let the number of factors be factors to be studied each k and the number of experiments is N. So, Plackett-Burman design is used when k equal to N minus 1, when N is power of 2 like 2 to the power 2, 2 to the power 3, 2 to the power 4, then this Plackett-Burman design or PB design is equivalent to fractional factorial design, ok, but there are some cases when that N will not be a power of 2 not be power of 2. For example, if N equal to 12 this is not power of 2, ok. So, when N is not power of 2, so, in that case this is this is a special design special design and Plackett-Burman design is applied for such situations when N equal 2, 20, 28, 34. So, this kind of design these are the most popular one for Plackett-Burman design.

So, in Plackett Burman design often the main effects are of interest are of interest main effects. So, we are not interested in interaction effects interaction effects whether it is two way interactions or more this is not of interest, ok. So, it is it uses standard orthogonal arrays, uses standard orthogonal arrays. Another interesting issue here in Plackett-Burman design is that when we use fractional factorial design like 2 to the k minus p design.

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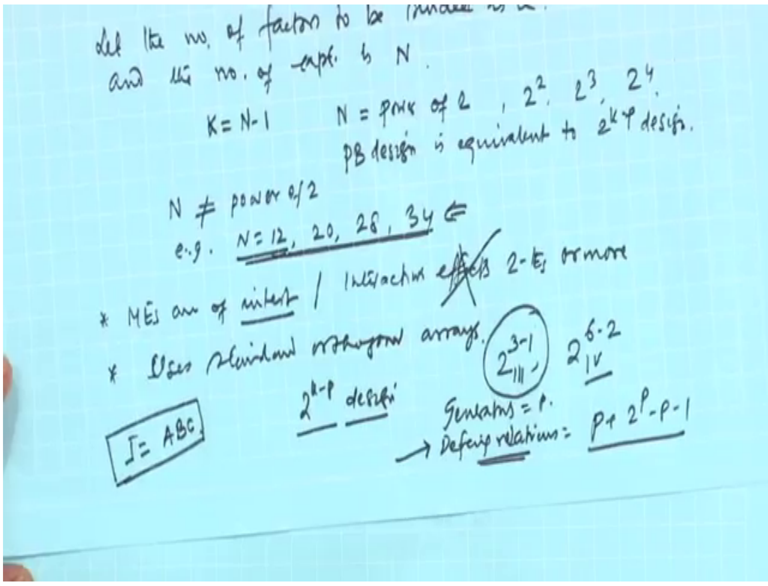

Suppose, you are using 2 to the power k minus p design for example, 2 to the power 3 minus one or 2 to the power 6 minus 2 whatever may be the resolution III or resolution IV, so, what happened you have generators. So, equal to p and defining relations defining relations equal to p plus 2 to the power p minus p minus 1, but in Plackett-Burman design or the design where Plackett-Burman design is what I can say valid.

So, they or use mostly used I cannot say valid mostly used so, they are the defining relations are not available; that means, if I use 2 to the power 3 minus 1, three design then you have seen earlier the defining relation I equal to ABC and you know that how to now obtain the 2 to the power 3 minus 1 design and conduct the experiments, but in this case in plack case PB case Plackett-Burman case this defining relation are not known. So, what happened the alias structure, cannot be found out by knowing the defining relation and then multiplying every factors. So, you require regression approach to find out the alias structure, ok.

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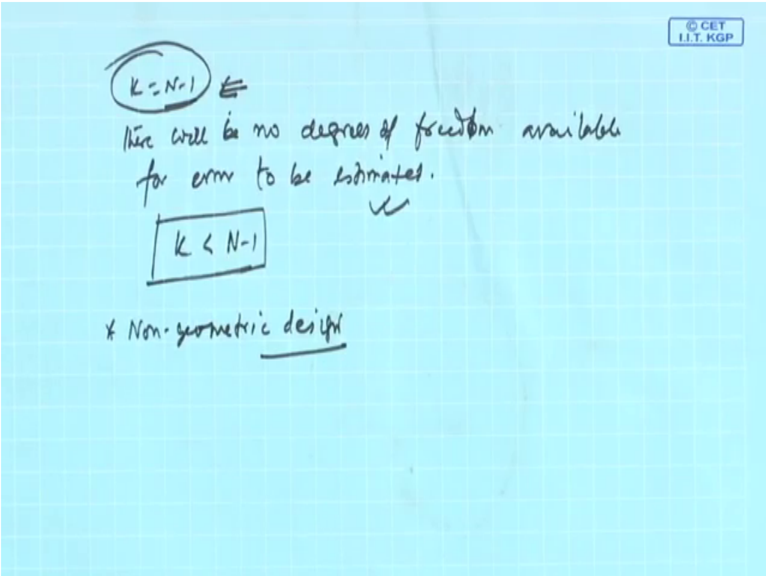

As k equal to N minus 1, and this is basically the case where PB is used so, you see there will be no degree of freedom available, there will be no degree of freedom degrees of freedom available for error to be estimated if situation is like this error cannot be estimated.

So, what you require then if you use k less than N minus 1 then depending on the type of effects is of interest. So, there can be some source of error estimation, ok. So, we will discuss those things and another important one is that if you go for 2 to the power k or 2 to k minus p design you will find out that this design can be represented in terms of cubes, but this Plackett-Burman design where N equal to 12, 20, 28, 34 you cannot represent that in terms of cubes. So, as a result this type of designs are known as non-geometric design, non-geometric design.

So, with this let me read out from the slide. So, it is developed by Plackett and Burman in 1946. Used to study k equal to N minus 1 variables in N runs, where N is multiple of 4. If N is power of 2 Plackett-Burman design are identical 2 to the power k minus design for N equal to 12, 20, 24 36 PB designs are of interest only main effects are of interest PB standard orthogonal arrays there is no defining relations as interactions are not identically equal to main effects when k equal to N minus 1, no degrees of freedom are available to estimate error terms as Plackett-Burman designs cannot be represented as cubes they are sometimes called non-geometric designs, ok.

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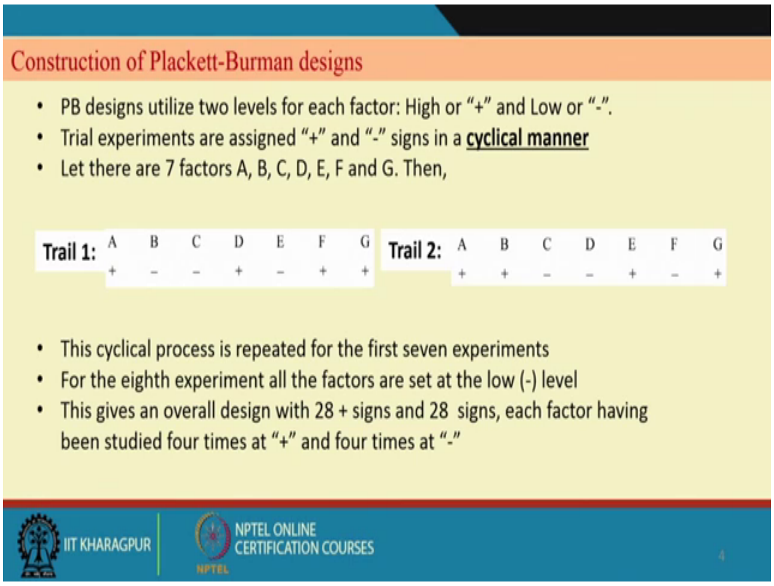

Now, second is that how do you construct Plackett Plackett-Burman design. Plackett-Burman design or PB designs also utilizes two levels level high or plus low or minus trial experiments are assigned plus or minus signs in a cyclical manner, ok. So, what is this cyclical manner?

So, just you assume that there are 7 factors A, B, C, D, E, F, G also you say that the trial 1 can be conducted keeping A at high B at low C at low D at high E at low F at high and G at high. Then to maintain the cyclicity your next trial will be started with plus then my plus then minus. Suppose, what you are doing then where you are ending in the first trial you end with G plus.

So, this will base this plus sign will start in the next trial first followed by the again the from the remaining signs from the left, ok. So, what do you mean you have started with first a plus then minus minus then plus then minus then plus and plus.

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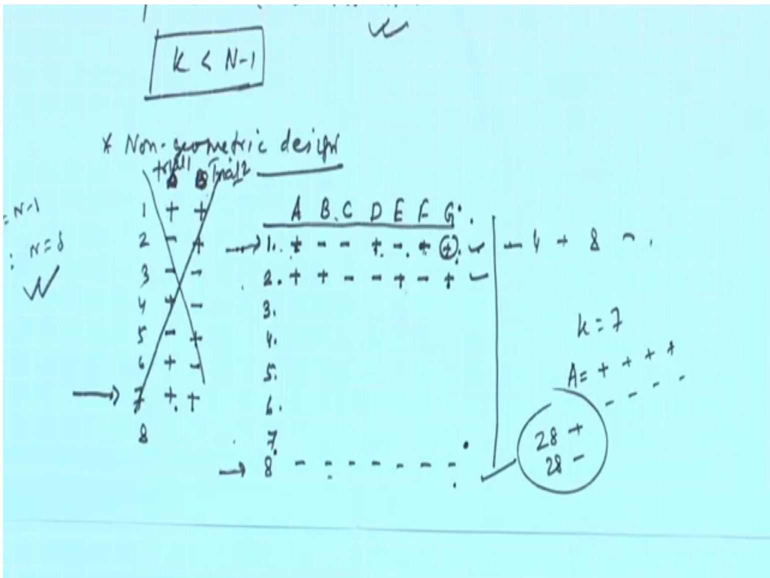

So, 1 2 3 4 5 6 7, so, it is k equal to N minus 1, where k equal to 7. So, obviously, N will be 8. So, there will be one there will be one more one more row fine. So, first a you start like this what will be B? B will where you just see the seventh case run. So, save a 7 it ended with plus you start with plus here then you repeat from here. So, then plus plus, then two minus minus minus, then again plus, then minus, then plus, ok; So, what I mean to say here I think I think I have given it a little differently you just write down in this manner suppose I have A, B, C, D, E, F, G, 7 factors.

So, my trial 1, so, actually this is my trial 2 and this one is trial 2 that is what I mean to say. So, in trial one you see ultimately A is plus minus minus plus plus minus plus minus minus plus minus plus plus and the second will start from here. So, plus then you start from here plus minus minus plus minus plus, ok. So, in this manner this what I am saying that when the first one and second one start and you repeat this process ok.

So, as a result what happened what happened I say this is my trial 1, this is my trial 2, then trial 3, then trial 4, trial 5, 6, 7 ok. So, you see that the trial one started like this A, B, C, D, E, F, G and trial 2 like this, then you do this cycle order you complete and ultimately you complete up to 7, ok.

So, then what will happen ultimately you see in the first trial how many plus 1 2 3 4 and how many minus there are 3 4 plus 3 minus ok. So, you repeat this process and then for the up to seventh and for the eighth run put all minus put all minus. Let me repeat forget about the there is some notation difficulties.

So, you have seven factors you are you started for the trial one, that means, the first experiment and you are keeping the factors either positive or negative, suppose this is this is a start, then for the second one what you will do you start where you end you start from there and continue from the previous in sequence, third one in the same manner start where you end and repeat this process. In that process what you do you finish the first seven first seven experiments and the last one will be all negative all at low levels.

So, what happened this gives you 28 plus signs and 28 minus signs here we have k equal to 7, there will be for each factor there will be four number of plus and four number of minus if you if you do this. So, four number of plus and so, all together there will be 28 plus and 28 minus with reference to this example, k equal to 7. Now, question is that how do you start with this when a why this one plus minus all those things.

So, that orthogonal array and concept and all these things have used and, but ultimately Plackett and Burman maybe they have given some starting some starting trial designs and the procedure is this, once you then there may be different combination, but please making that it will be orthogonal arrays and then once you start with the first trial, first experiment you do the second experiment will be done in maintaining the cyclicity and following and third experiment fourth experiment like this k equal to k number of that experiment first and then the last one what will happen the last one will be will be all the factors at low level, and this ensure orthogonality this ensure that there will be equal number of high and low value per factors and it is basically least number of experiment is required.

So, all those things makes this PB design very popular another one is that your in this case your it is not a 2 to the power type of experimental runs it is other than that 2 to the power case ok.

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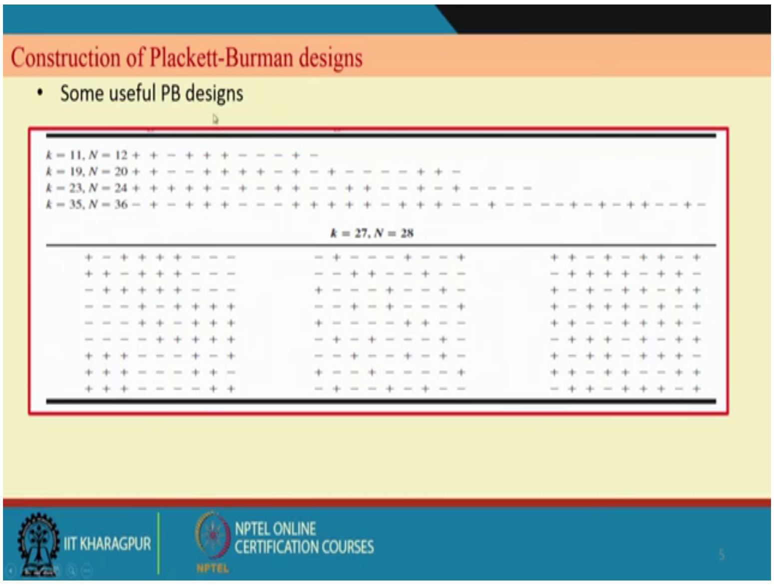

So, let us see some useful designs given by PB that for k equal to 11 urich an here you require 12 experiment to be run and the first trial is given like this. So, you use it to find out the second third fourth like this there will be 12 times and then k equal to 19 k equal to 23 k equal to 35 and k equal to 27.

So, these are these are the starting points given. So, when you start using Plackett-Burman designs it is we must know that what is the number of experiment number of factors and what is N value you want to choose and accordingly you select one of the initial trial a first trial from this the given plus minus signs and you follow the cyclic process and get the complete design, ok. This is what is presented here.

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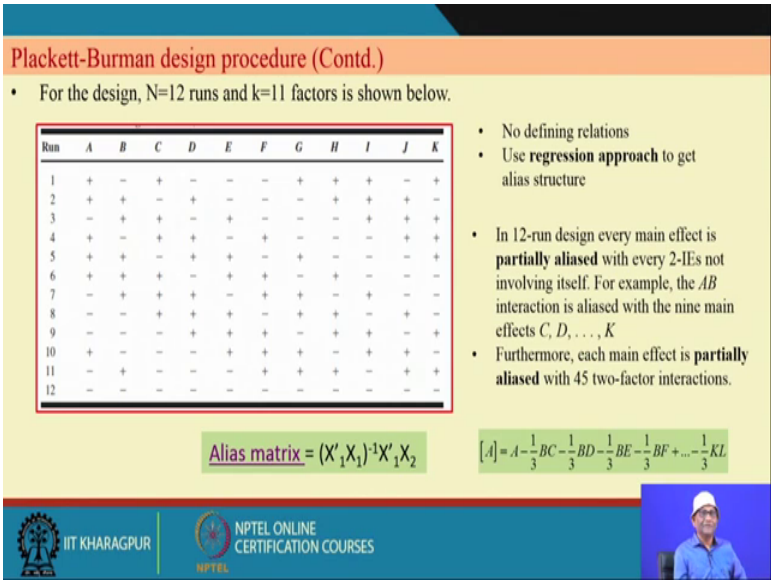

You see here we have 11 factors and that means, we require minimum 12 runs. So, what you have done you have run number 1 to 12 and this side 11 factors. Now, the first trial plus minus plus minus minus minus plus plus plus minus plus, ok; Suppose, this is what is the things you have started with, then the second one will be where you have ended you have ended with plus. So, plus then you start from this then plus plus then minus then again plus then 1 2 3 minus 1 2 3 minus then 1 2 3 plus 1 2 3 plus minus. So, here you ended with minus you start with minus again, ok.

So, minus then again start from here plus plus plus plus one minus then one plus then three minus then three plus. So, like this. So, in this manner you go like this like this and up to how much k equal to 11 up to 11 factors here you continue this, ok. Then the last one twelfth one you see all are negative; that means, all at low level.

So, now, if you see every column of the factor factors A to K you see there are equal number of plus and equal number of minus here plus; that means, 6 plus 6 minus every column like this now if you take the dot product of the every column what will happen dot product of two any two columns you will get 0. So, plus means plus 1 minus 1 minus 1 that is the orthogonality, ok; So, in this case few more issues to be discussed we have how many factors? 11 factors.

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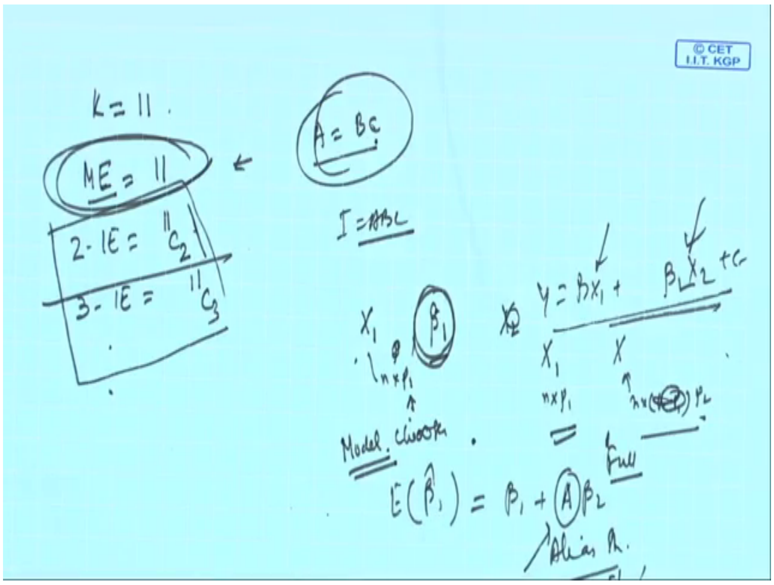

So, how many main effects? Main effects will be 11, how many two way interaction effects two way interaction it will be 11 C 2, how many three way interaction effects three way interaction effect will be 11 C 3 and so on, but please keep in mind that Plackett-Burman design is used mainly to get the estimate of the main effects so, that means, the second week 3 and higher or additional they are also negligible and we are a two and higher order are negligible or of not interest ok, but from our previous knowledge of fractional factorial design that that we know that there that must be defining relation to get the alias structure.

So, in this case there is no that equal sum with because of the main interacts main effects are aliased with second order interaction effects and in earlier case we have written that main effect A is aliased with second order suppose b c in that case will write like this, but in this design that kind of equal that alias structure is not there and or may not be there and. So, as a result what happened you will not get defining relations. So, I equal to ABC kind of things you will not get, ok. So, we use regression approach to get the alias structure.

Now, see here there are 12 runs and every main effect is partially el aliased with every two interaction effects not involving itself one of these in this design the thing is that you should find out the alias structure of A then A will be aliased partially aliased in the sense that with other second order interaction effects which the which do not involve A or where A is not there is not into consideration. So, that means, the AB interactions for example, the AB interaction is aliased with the 9 main effects involving CDE and k, ok. So, furthermore each main effect is partially aliased with 45 two factor interactions. Now, let us come to let us see that how this kind of alias structure is generated I told you in one of the previous lectures that regression approach to identify the alias structure we and there we have given you the alias matrix.

What we have done? we have run a partial model involving X 1 factors and another may be in X 1 is basically N cross N cross p 1 p 1 number of factors and another one X 2 another one full one involving X 1 and X 2 where this equal to N plus p 1 and this equal to N cross may be k minus p 1 that if there are total number of factors other rewrite we to the other effects.

So, p effects and p 2 effects, so that means, there is a there is a model which is basically run and there will be that is basically chosen and there will be another model which may be a full model or the correct model and then we found out here what happened the estimate is beta 1 for all this and let here it is beta 2 then the for the full model this will be with reference to beta 1 X 1 plus beta 2 X 2 y equal to plus epsilon.

Now, we have computed expected value of beta one cap and then what we have found out we found out this is beta 1 plus A into beta 2, where A is the alias structure. So, this is nothing, but X 1 transpose X 1 inverse X 1 transpose X 2, where X 2 coming from the design matrix of here and X 1 is the region matrix is this that we have discussed earlier and using this we have found out the alias matrix here.

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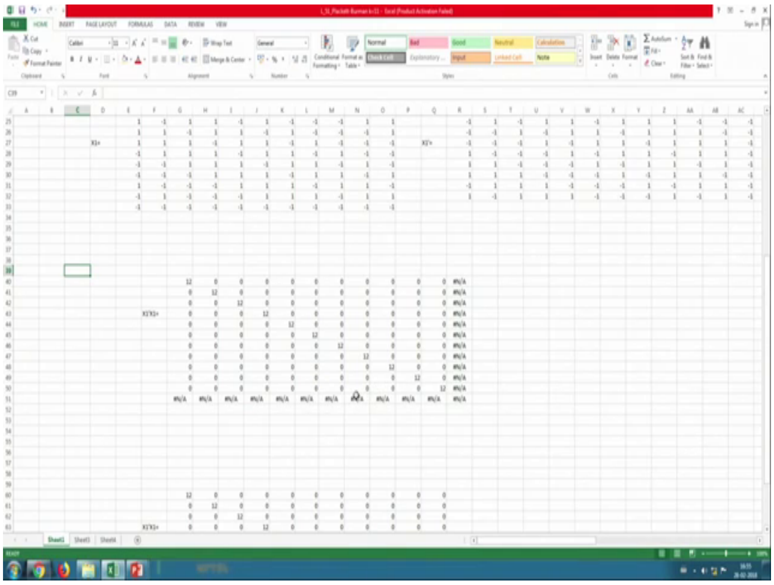

Now, because there are 7 11 factors it is a huge one. So, we have used excel sheet to find out this one you see A to K that what is this is what is basically and again 1 to 12 this is plus 1 minus 1 this is what the design matrix and from here you can compute also A, B, C all those things. So, there will be so many interactions second order interaction.

So, we are not considering the third or higher order interaction because PB design is not for that purpose now and what happened this is X 1 then we computed X one transpose and then finally, X 1 transpose X 1 when you do you are basically getting this matrix diagonal elements are 12 of diagonal elements at 0, ok.

So, if you develop a regression model using this X 1, so, this is the model you have chosen this is the model where the only the main effects are involved. Now, if you if you go for a full model involving all the interaction parameter then X 2 part will be this that only the second order interactions now when you multiply these things wait let me go back X 1 and X 2, X 1 transpose X 2. So, ultimately you will be getting this kind of this kind of letter see 0 0 4 4 all those things and finally, I will show you that that when you find out the alias structure you will be getting this kind of things.

Here you see that minus 0.34. 0.34 means it is basically 1 by 3. So, this is what is the alias structure computed using regression and if I if you see the if you go back to the presentation now you see that now a minus 1 by 3 BC, 1 by 3 we are saying 0.34 minus 1 by 3 BD minus 1 by 3 BE and like this. So, this alias structure is obtained using a regression approach. So, you just see that using excel and under the situation you will be able to find out the alias structure, ok.

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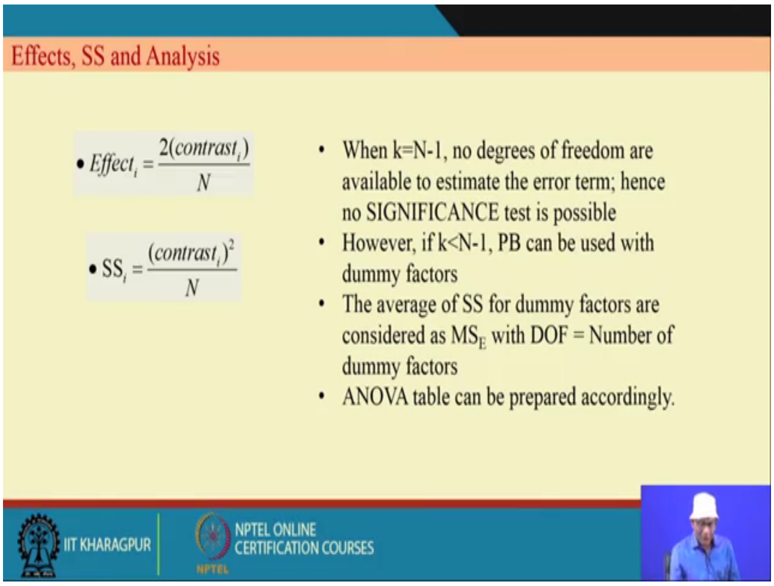

So, only alias structure is not sufficient you require to find out the effect as well as the sum square and as well as you want to analyze the data you want to find out whether the effects are significant or not and which of the effects are significant that also you want to find out. So, now, effect is the same formula and S S is the same formula what we have identified or what we have shown to you earlier now when there are when k equal to N minus 1, no degrees of freedom are available to estimate the error term hence no significant test is possible if you have k equal to N minus 1 that is the situation, but if you have k is less than N minus 1.

Suppose, actually you have seven factors and you are going for eight design then you have no degree available for apart from the main effects estimating other effects including the error terms, but if you use suppose k equal to 7, but you are using N equal to 12 and you are interested only in main effects then you have 5 different runs available and that will help you to identify the identify the errors provided the second and higher order interactions are negligible.

So, under such situation how the Plackett-Burman design is used Plackett-Burman design is used using dummy factors, ok. Then what do you do suppose, you have I will give you one example later on then once the dummy factors are known and then find out the sum square for the dummy factors and the sum of some square or average of some square of the dummy factors will be will be the mean square errors and degree of freedom will be the number of dummy factors then what happened you have the main effect estimates plus you have the error estimates. So, you are in a position to develop ANOVA table and then find out the significance.

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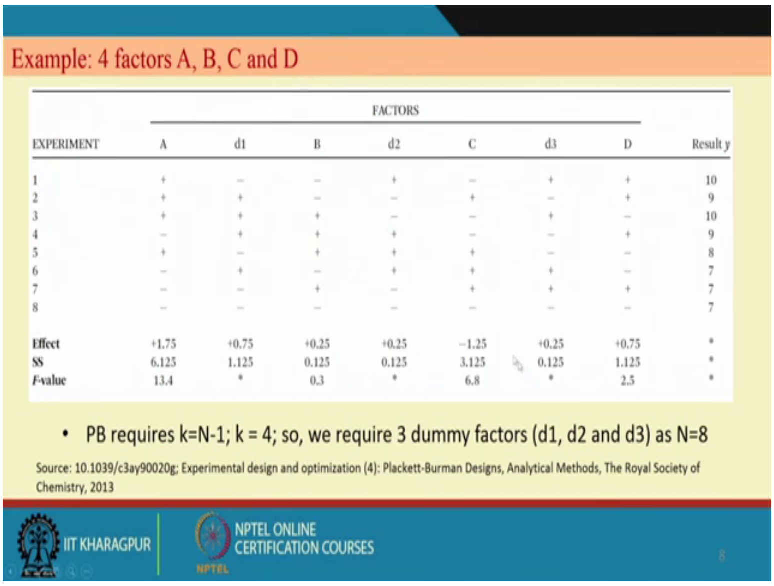

	FACTORS							
EXPERIMENT	A	d1	B	d2	C	d3	D	Result y
1	+	–	–	+	–	+	+	10
2	+	+	–	–	+	–	+	9
3	+	+	+	–	–	+	–	10
4	–	+	+	+	–	–	+	9
5	+	–	+	+	+	–	–	8
6	–	+	–	+	+	+	–	7
7	–	–	+	–	+	+	+	7
8	–	–	–	–	–	–	–	7
Effect	+1.75	+0.75	+0.25	+0.25	–1.25	+0.25	+0.75	*
SS	6.125	1.125	0.125	0.125	3.125	0.125	1.125	*
F-value	13.4	*	0.3	*	6.8	*	2.5	*

So, here is one example suppose you have 4 factors. So, if you go by k equal to N minus 1, then k equal to 4 means you require 5 runs, but here what happened who you are using suppose 8 runs. So, you have 3 excess runs. So, that means, you can use 3 dummy factors. So, this is one such design where A d1 B d2 C d3 D and this kind of structure is or layout is used and you have used Plackett-Burman that the design that cyclic one and once you have done experiment, let the result is like this for the first treatment 10, second treatment 9, 10, 9 like this, ok.

So, what do you do you find out the effect using the formula 2 into contrast by N and sum square is contrast square by N and then using this data you will be will be getting effect equal to this for a it is (Refer time: 30:53) 1.75 and like this and SS also like this and then what happen you can find you can use this information to find out the error values. So, in this case let me repeat PB requires k minus equal to N minus one and k equal to four. So, we require three dummy factors d 1, d 2, d 3 because we have used 8 number of this one I have taken from internet material source is this and you can download and read it.

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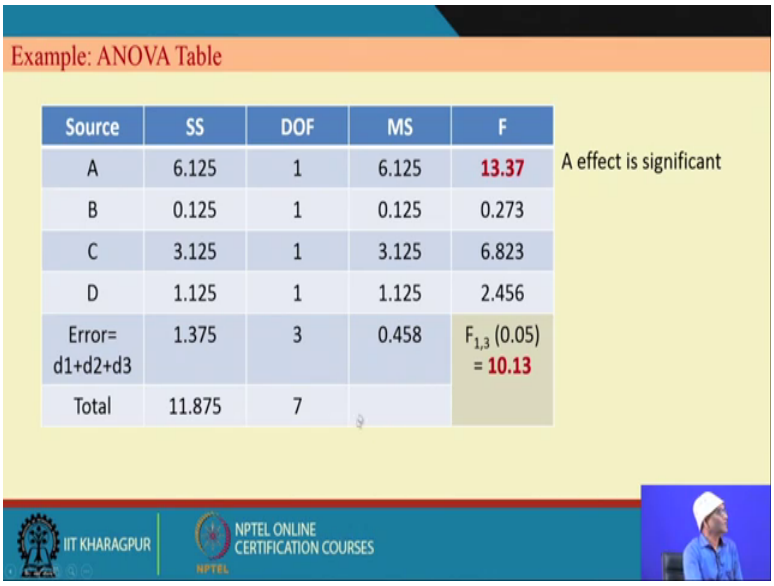

Source	SS	DOF	MS	F
A	6.125	1	6.125	13.37
B	0.125	1	0.125	0.273
C	3.125	1	3.125	6.823
D	1.125	1	1.125	2.456
Error= d1+d2+d3	1.375	3	0.458	$F_{1,3}$ (0.05) = 10.13
Total	11.875	7		

So, now, using this information SS and the degree of freedom. So, we have computed the ANOVA table. You see the ANOVA we are interested in the main effect we are saying the sources are the main effects only and then A B C D the SS what we have computed that is. So, written here and for error the SS of all the dummy factors. So, dummy factor d 1 SS is 1.125, d 2 0.1125 and d 3 0.125. So, if you add them 1.125 plus 0.125 plus 0.125, so, ultimately you will be getting that 1.375.

So, as every factor is having two levels the degree of a freedom available per factor is one, but here error is equated to three dummy factors having one degree of freedom each. So, total a degrees of freedom available for error is 3.

So, now, then using the traditional ANOVA procedure find out the means sums mean square which is sum square by degree of freedom and then your MSE is 0.458 then find out F value F value is image that effect by m is error; that means, for a it is 6.125 divided by 0.456, 13.37 in this manner the F values are computed.

Now, from the theoretical value using alpha equal to 0.05. F 1 3 is 10.13. So, you see if you see the F value for all the 4 main effects you find out that only A, F value for main effect A is more than the theoretical value. So, effect A is significant and other effects are not significant, ok.

Now, this is what is Plackett-Burman design. I hope that you understood it and you will be able to reproduce the same thing or recall this procedure whenever it is requested for.

Thank you very much.