Design and Analysis of Experiments
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Lecture-50
Fractional Factorial Design : Fold - Over – Design

Welcome to the lecture on fold over design.

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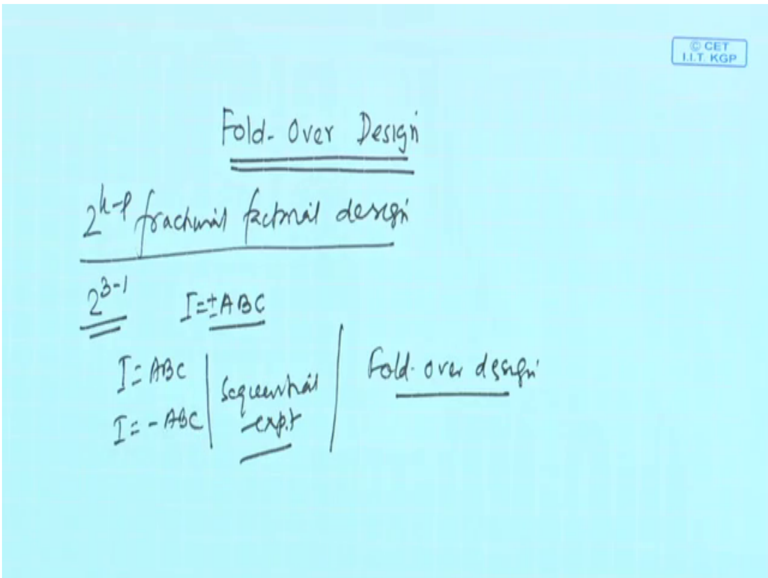

So, far in last few lectures; we have completed, rather we can say we have discussed 2 to the power k minus P fractional factorial design fractional factorial design ok. So, what you have seen that, in the very basic lecture; which we have given you 2 to the power 3 minus 1 fractional factorial design and where the defining relation was I equal to ABC, and we have that we have seen taken that I equal to in fact, plus minus ABC and the first fraction is I equal to ABC and second fraction is I equal to minus ABC. And we have discussed the concept called sequential experimentation sequential experimentation ok.

So, this sequential experimentation it can be done or is done depending on the purposes that you wants to fulfil. So, in more general way this can be expressed as or this can be achieved by fold over design fold over design.

So, what are the; consider important issues in fold over design.

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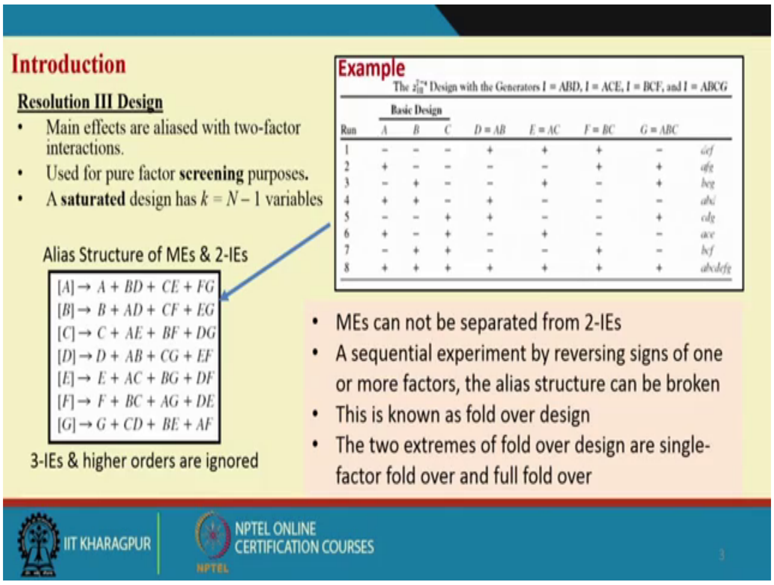

Let us see the slide first; and let us start with an example. Suppose you are interested in a resolution 3 design and the case for demonstration is 2 to the power 7 minus 4 resolution 3 design, if I say 2 to the power 7 minus 4 resolution 3 design, 2 to the power 7 minus 4 resolution 3 design, 2 to the power 7 minus 4 resolution 3 design and here you see that; what is the number of generators P?.

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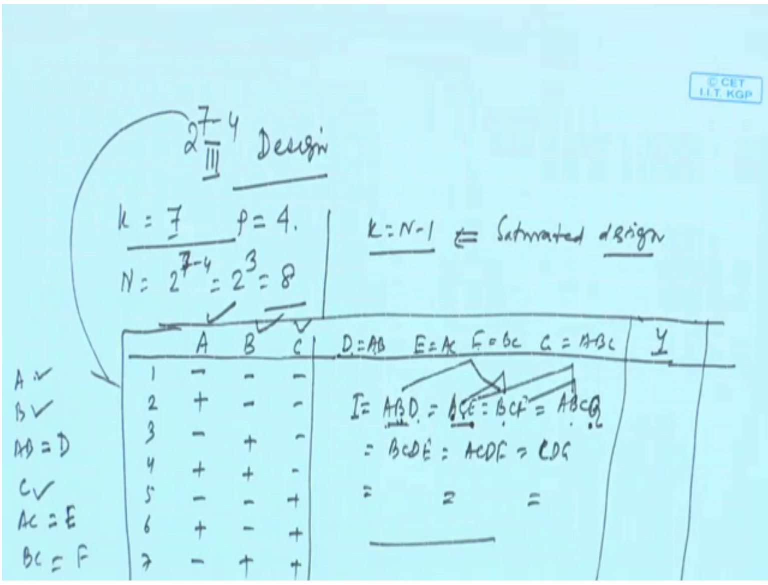

P equal to 4; what is the number of factors k; k equal to 7, what is the number of runs N; N equal to 2 to the power 3, 7 minus 4 equal to 2 to the power 3 equal to 8. This is a case, where k equal to N minus 1. So, this type of design is known as also called saturated design saturated design. So, how do you construct this design; first it should be a to basic design will be 2 to the power 3.

So, 1, 2, 3, 4, 5, 6, 7, 8 minus, plus, minus, plus, minus, plus, minus, plus, plus, plus, minus, minus, plus, plus, minus, minus, minus, plus, plus, plus, plus and then remaining how many factors are there. So, D will be (Refer Time: 04:01) factor. So, there will be 7 factors ABCDE; A, B, C, D, E, F, G; 1, 2, 3, 4, 5, 6, 7. So, you how many parameters you require to estimate here 7; excepting that one the mean value, because A parameters; usually in this case you have found out in if it is a 2 the power of 3 design; we estimate AB, AB, C, AC, BC, ABC.

So, A fine, B fine, C fine, now this can be AB can be I can say that AB can be confounded with D, AC can be confounded with E, BC can be confounded with F, ABC can be confounded with G, so; that means, this equal to AB this equal to E equal to AC, F equal to BC and G equal to ABC. So, this is your design. So, 2 to the power 4 minus design and now you know how to get the signs for D, B, C all those things are multiplying AB all in this is what is what is the construction. hm.

Then, if you do the experimentation what will happen you will get some y value y total value. So, now what will be this; what will be this all those things. Suppose, if you if you fill up all the plus minus here, you will get the structure you see AB plu, minus, minus, plus, plus like this. So, there are first run where D E F positive. So, you are writing that is DEF; second run A F G so; that means, these are the these are the treatment combinations ok.

And on other words also this will denote the total of responses; that is what we have seen in the first lecture on this issue ok. So, it is it is; obviously, a resolution 3 design and, because if you see find out the defining relation D equal to ABD; AB mean ABD is one, from E you are getting ACE, from F you are getting BCF, from G you are getting ABC G plus; if you multiplied this ABD and ACE, if you multiplied A will cancel out BCDE; BCDE. So, you multiplied these two. So, B will be cancelled out then AC DF ACDF multiplied this ABD and ABAB will cancel out CDG.

So, similarly multiplied these two multiplied these two mult0iply these two will be getting few more ok; so fine so you will be having a different generators. Now we all know that P plus 2 to the power P minus P minus 1; number of defining relationship you will be getting ok. Now, using this design; if you run the experiment and what results you get from estimation of main effects point of view, considering the interaction effects are higher order interaction means third and higher order interactions are negligible.

So, what will be the alias structure? So, your alias structure will be; suppose if we if we neglect the third and higher order interactions, then you will you will see that your alias structure will be A will be aliased with BDCE and FG, B will be aliased with B AD CFEG C like this. So, this is your alias structure and how you get the alias structure all of you know you just multiply the defining relation lesson by the particular factor. If you multiplied A with ABD, then you will be getting BD, A with ACE you will be getting CE, A with BCF you will be getting CF, A equal ABCG you will be getting ABCG means you will be getting I equal to a the BCG that is not the third order you do not require.

So, this is; what is the alias structure you are getting? So, all the main effects are aliased with second order interaction effects cannot estimate uniquely. So, under such situation what you require; you require sequential experimentation, if you want to uniquely estimate the main effects or some of the interaction effects. So, these sequential experimentation is possible by switching over the sign of the basic designs. For example, if here A is minus plus this is the case.

So, if you if you just switch over the sign by reverse the sign either may be there one effect one factor or maybe all the factors or maybe combination of factors; once you reverse the sign from the first fraction then what will happen; you will have a you have you will get another set of experimental data and the combining the two you will be able to de alias some of the effects.

Primarily we are interested to de alias the main effects and second order interaction effects, because third and higher order effects we usually consider they may be negligible considering the sparsity of effort principle, but many a times third order higher order may be important and in that case the switching of the sign will be accordingly none ok.

So, these kind of design when you do one fraction of experiment, then when you go for the next fraction which is required. So, depending on the purpose you switch over the sign of one or more factors and then this kind of design is known as fold over design. So, basically you are folding over to another side. So, there are two extreme of fold over design is single factor fold over and full fold over ok.

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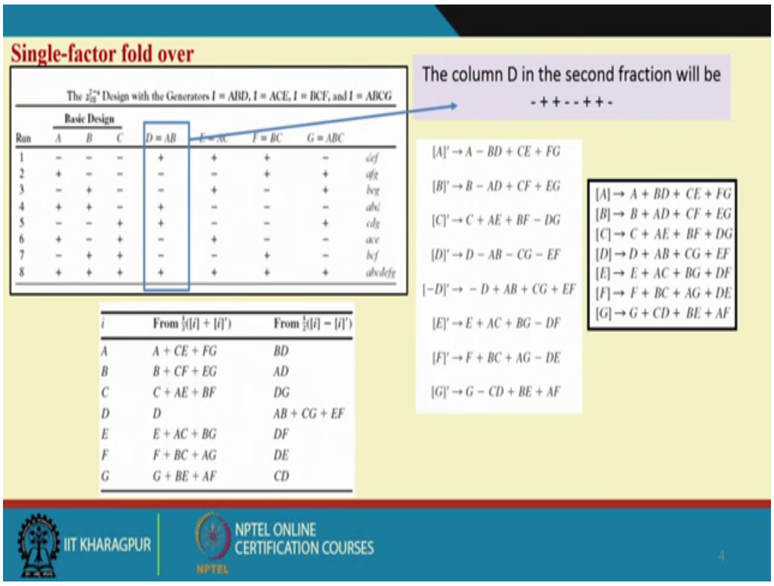

So, let us see one interesting example here. This example we have discussed earlier also 2 to the power 7 minus 3 we have discussed ok; this we have not discussed; we have discussed resolution 4 1 here it is that is the this; design is 2 to the power 7 minus 4 resolution 3 D I. This is what I think the to do a similar same thing same design.

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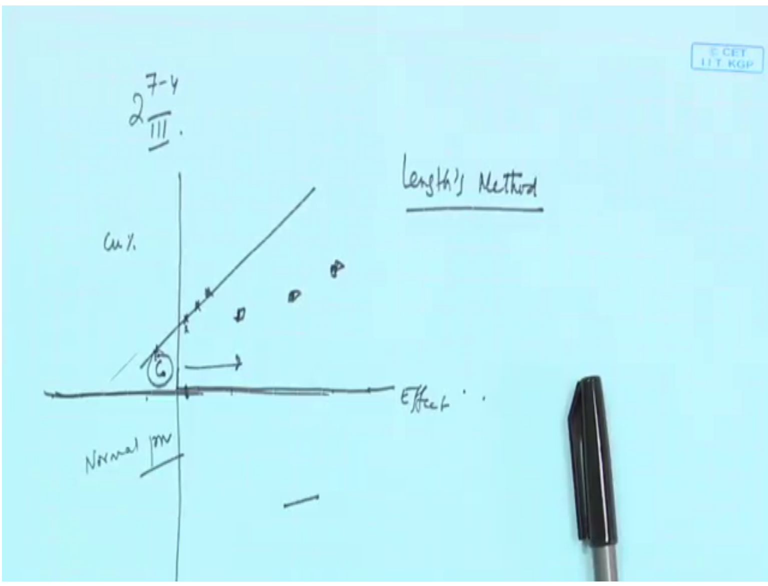

So, here what happened when you use the first fraction, where ABCD like this and correspondingly confounding other main effects with some interaction effects and you are generating the plus sign algebraic that sign conversion table or I can say design matrix ok.

Now, here what happened; suppose you do one thing you do the sign reversal for D; sign reversal for D, if you do the sign reversal for D. So, plus, minus minus plus plus minus minus like this will be reverse to minus plus plus minus minus like this, then what happened your earlier you see what was there here you have taken D equal to AB, then what you are doing here D equal to minus A B ok.

So, instead of; so D equal to AB here it will be minus AB and all other things remain same. Now, if you use this and if you do the experimentation. So, here DEF that you in the first experimentation you will get this alias structure, when you go for the second experimentation you will be getting this alias structure. You see what is happening here; wherever D is there that minus N is coming. The first fraction is giving A plus B D plus C plus F G; now B D here it is minus.

Similarly here AD is minus and here no DG is minus and here D is plus all other minus or if you are using minus D, then D minus D and all that plus. So, if you do minus D; if you make it minus minus D ad plus then this is the case so; that means, we are basically talking about a single factor sign reversal case here. So, that is why related to D the plus is becoming minus and accordingly this is the alias structure.

So, this is basically single fold over, because you have done the sign reversal of one factor; that is D. Now from the first fraction the alias structure is this right hand side, second fraction when you will go for sign reversal of column D, this is the alias structure; if you if you use these 2 and do little as a arithmetic operations, what you are getting? You are getting that for you see that D is dealiased.

The main effect D is dealiased and ABC other things are aliased with a is aliased with some other second order interactions and other main effects are also aliased with some of the second order interactions, but some of the secondary interactions are not aliased with other second order interactions may be aliased with higher order, but they are negligible.

So, this gives you a better picture; intentionally you have basically done the sign reversal for only one factor; factor D and that is why; the uniquely you are able to effect estimate of effect of D. Now, if you do it for the all the factors what will happen given the two data set you will be able to dealias all the main effects so; that means, when you do the sign reversal for one factor and then do the experiment again you have two sets of experimental data using these you are in a position to find out find out the effects; effects in the sense the effect of interest uniquely can be estimated ok. So, for then other one is the full fold over design where you do all sign reversal.

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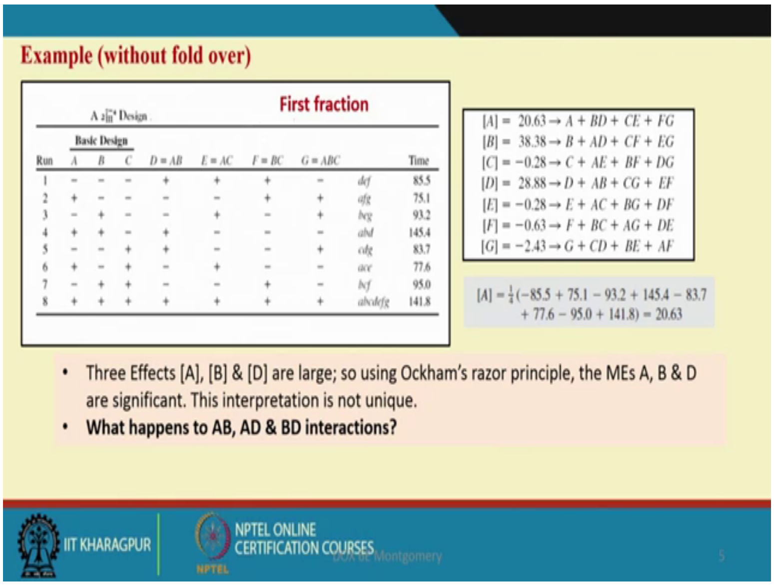

So, le let us see that the same thing here 2 to the power 7 minus 4 3 design and here suppose you have done the experiment; using the first fraction first fraction then you have done the experiment and you got these are the experimental results. So, if without going for the sequential experimentation, if you use this one your estimate is like this. So, if you see the alias structure and the estimated value that AB and D which are basically high value. So, we can say AB and D are large.

So, now, using the Ockham's razor principle, we can say AB and D the effects are significant, but this is not the unique interpretation, because there may be the second order interactions also, because A and B significant mean A B interaction may be significant; similarly B and D significant mean BD interaction may be significant and AD interaction may be significant. So, then if A, but as A is aliased with BD so, but just by seeing this value you can say using a Ockham's razor principle, that is significant obviously, but what will happen to BD you cannot uniquely say that what happened to BD.

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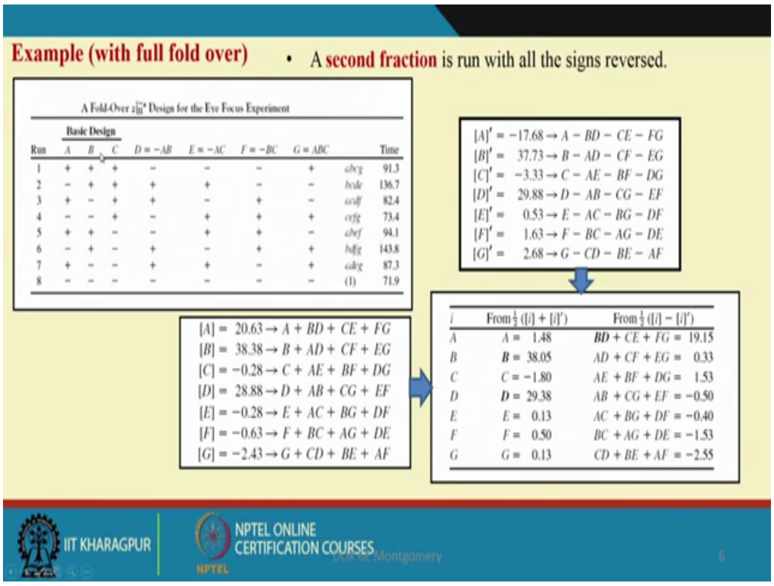

So, as a result suppose you go for a full fold over design means; here you see that ABC sign conversion from here minus, plus, minus, plus, minus, plus like this minus, minus, minus, minus, all those this ABC the basic design the sign conversion taken place plus, minus, plus, minus, plus, minus like this. And as a result you see that D also all cases minus, minus A; that means, here AB AC and BC they are basically sign reversal is taking place and G, because of ABC this sign reversal is; now if I go to G it is minus, plus, minus, plus here what happened plus, minus, plus, minus is it is also sign reversal taken place.

So, you have done the using first fraction your experiment and this is the result. Now you have gone for full fold over and then your this is your results. So, from the first fraction, what happened; from the first fraction you got this, from the second fraction you got this. Now if you if you use this arithmetic that half within bracket I and plus I dash that is mean first fraction and fold over fraction second fraction then first fraction minus second fraction, what are you getting? You are in a position to you are in a position to find out all the main effects parameter uniquely estimated.

In addition what you are finding out that the second order interactions are; they are basically grouped in a three aliased in a group of three and you see that BD CE FG 19, but other values are very very less so we can say this is, what is the significant one this BD CE and FG, but if you see the first order interactions; first main effects not first order main effects only, if the B is significant D is significant, but A, C, E, F, G they are not significant ok.

Now, here if you see that the only the first fraction results A is 20 and these; so they are using these you have said or I have said that a is significant, because seeing this value high A, B and D this they are significantly high. So, I say A is significant, but after doing fold over, when we got the second fraction and dealias the effect of A, but the effect of A is only 1.48 so; that means, then where from this 20.63 is coming; see there is an one interaction called BD, and if you see the total data after fold over design, so you see that B effect and D effect are significant so; that means, it interaction effect will become significant, because here A plus and other effect like C, E and all other main effects are not significantly large.

So, if B and D effect is significant, then BD interaction is significant and here although A is 1.48. So, from your 20.63; so BD it BD plus CE plus FG is 19.15. Now, what we are trying to say as CEFG all other effects main effects are negligible, it is quite likely that C E and F G effects also will be less, but as B and D effects are significantly high it is quite possible that BDF significant BD interaction effect is significant and as a result here the BD is bold means we are saying their second orientation BD is significantly large.

So, [vocalised noise] suppose this is just by seeing this seeing this values A, B, C, D, E, F, G and by seeing that; what is the value? What I can I say that you can go for normal probability plot also all those things, if you do normal probability plot how many estimates are there, suppose from there here you can say that A, B, C, D, E, F, G and BD also from here you can take, and then if you do normal probability plot it is quite likely that A, C, E, F, G.

And this will basically form a straight line and this B effect D effect and BD effect this will be falling apart means what I mean to say even if you do the hub normal also, and then if you go for cumulative percent probability and here effect it is quite likely that that ac it is quite likely that ac hmm there will be may be something like; this which is basically C is minus then A is 1 point E is 0.13 somewhere here hmm.

So, minus 1.8 here, 0.83 here 0.50 and another one that G, then 0.50 somewhere here and then your 0.1, 0.13, 0.50, then 1.48 maybe somewhere here. And in this manner, if you go you plot this line and AB is 38 point i think somewhere here B is 29, 29 may be somewhere here and BD will be 19 may be somewhere here 19 ok. So, it will be 19 cumulative probability will be this somewhere here.

So, what I mean to say if you go for go for normal probability plot you will find out that that these effects are these are significantly higher energy result they will not follow that hmm your plot this is normal probability plot normal, but if you go for half normal then the C and negative value will become positive and accordingly you do ok. So, otherwise you use lengths method lengths method.

So, lengths method we are not discussing, but in the in the length method is a two stage method and you Google it and actually you do some some kind of study with yours; so this is what is basically fold over design.

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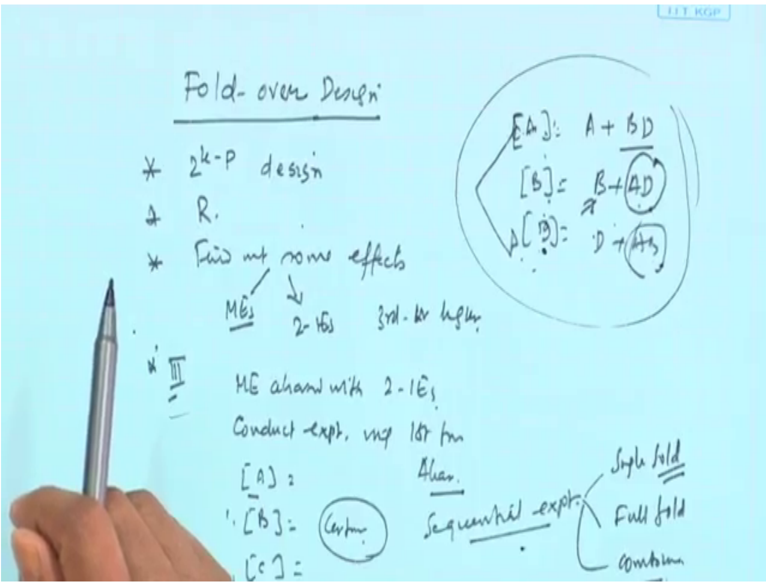

Now let me let me do this one that what is; I am just giving you the gist or summary fold over design, first is you have fractional factorial design. You have chosen a particular resolution and your interest is find out some effects may be primarily primary interest will be all the mEs; mEs is main effects then your second order interaction effects say early may be third and higher order interaction; third or higher order interaction depending on the resolution, what will happen; ultimately suppose if you choose resolution 3 design, then your main effects will be aliased with second order interaction effects after the first fraction. So, you conduct experiment conduct experiment using first fraction.

So, you get some useful structure. So, like this and there are some useful values and some alias other alias structure also you will get. So, by is by observing these you will have certain idea; some preliminary idea will be there, they will like here we said that A B and D are significant, but what happened as A is interacted with BD also and B and D is significant. So, it is difficult for a for example, like here you see a let me go back again here A B and D their values are high.

Now, as and if I see the alias structure A is interacted with BD aliased with BD CE and FG CE and FG fine, because CE CE FG their main effects are not a large, but BD both main effects are large. So, their interaction effect may be large. So, that is the interesting structure. So, A if I say; that means, primarily a plus BD that is the issue and again as B this one is large and D is also large. So, as if we assume that B and D are significant and then again B case you see that B plus A D and A and B they sell this one with alias is large this means this is also large this may be large.

So, then the question is whether B is large or AD is large hmm. And third one is also if you go for D; in the D case that D plus AB is there. So, as A and B this with alias is large we do not know that A large, B large or AB interaction is also large. So, that is what is the interesting structure you are getting; if you get this kind of structure then what you require you require to go for sequential experimentation; and in sequential experimentation you may you can go for single fold over or full fold over or combination means single means you will reverse only one factor sign reversal full means all and combination means some will be sign reversed not one not full in between ok.

So, the then you will get another faction and when you have another faction you see the results, suppose here you see the other faction A A value minus B positive and D positive.

So, here also see a positive B positive D positive, but their large value. The large value part is remain same here, but B and D they are positive, but A sign reversal has taken place here. So, this indicates that the interesting, where these two indicates that there is a problem with A; and when you use these two results combine that to data set and then finally, finally, dealias the factors, then what you are getting you see you are getting A effect is negligible, B and the effect is significantly large and so then from here you are getting BD effect with some alias in 19.15 so, BD effect is also large that is what is the function?

So, that is that is what why this fold over design is important. And I hope that you are in a position to understand; that what is fold over design and why fold over design is used and how fold over design will help you in dealiasing the some of the main and second-order interaction effects and also how to interpret the interpret the effects whether they are significant or not.

And; obviously, you will be heavily using and that probability plotting for effect hmm for effects and to get the value whether significant or insignificant or not ok. I think this is enough for fold over design and please remember I am following the book of Montgomery and you all must purchase the book and follow read the chapters corresponding chapters and you will definitely understand this and I find this a wonderful book.

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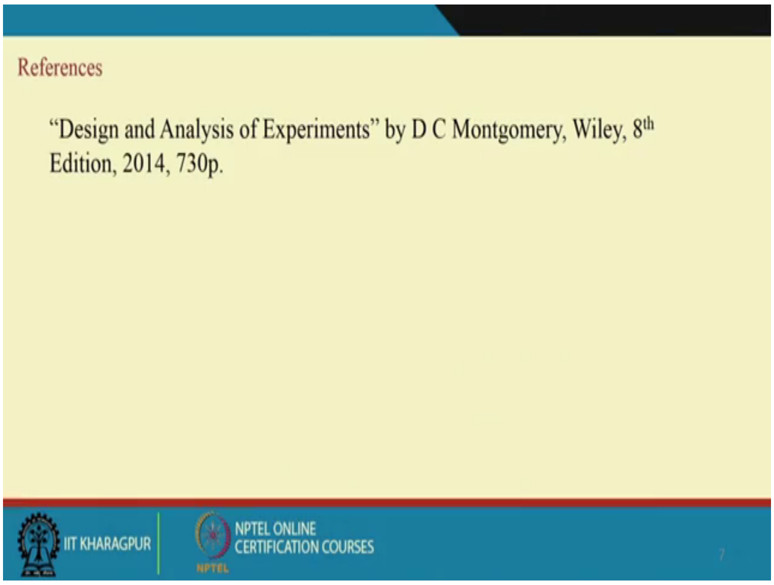

Thank you very much.