Design and Analysis of Experiments
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Lecture – 49
General 2 k-p Fractional Factorial Design

Hello, welcome.

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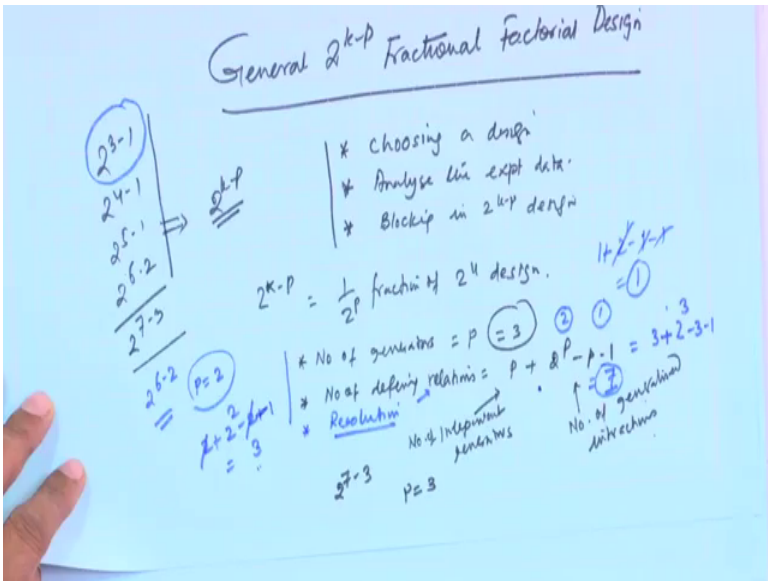

Today, we will discuss the general 2 to the power k minus p fractional factorial design. In fact, we have discussed on the fractional factorial design you know in first few lectures and being in very important one we are trying to give you as much as possible on this fractional factorial design. So, far what we have seen, you have seen 2 to the power 3 minus 1, 2 to the power 4 minus 1, 2 to the power 5 minus 1, like 2 to the power 6 minus 2. So, many such fractional factorial design; 2 to the power 7 minus 3, such fractional factorial design.

So, all those things in one word each to the power k minus p fractional factorial design. This lecture you can say the generalization of the previous lectures and we assume that after going to this lecture you will be able to relate to the previous fractional factorial design lectures as well as giving a fractional factorial design you will be able to a construct the design, you will be able to interpret the results of this design.

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What are the steps in this case? First step is choosing a design. So, what should be the correct or appropriate fractional factorial design choosing a design and second one is once you choose the design you will run the experiment and get the experimental data then you will analyze the design.

Analyze the experimental data, and you may find out there will be situations where even in fractional factorial case also you cannot complete all the 2 to the power k minus p runs in homogeneous condition single homogeneous condition and it is a such situation you require to employ blocking. So, we will give an idea of blocking in 2 to the power k minus p design.

So, these three things I expect to complete within thirty to forty minutes of time and as most of the basic things are already discussed. So, I will primarily concentrate on the generalized general issues so and giving you a generalized kind of things formula and explanation so that, so that it will justify the topic of today's lecture.

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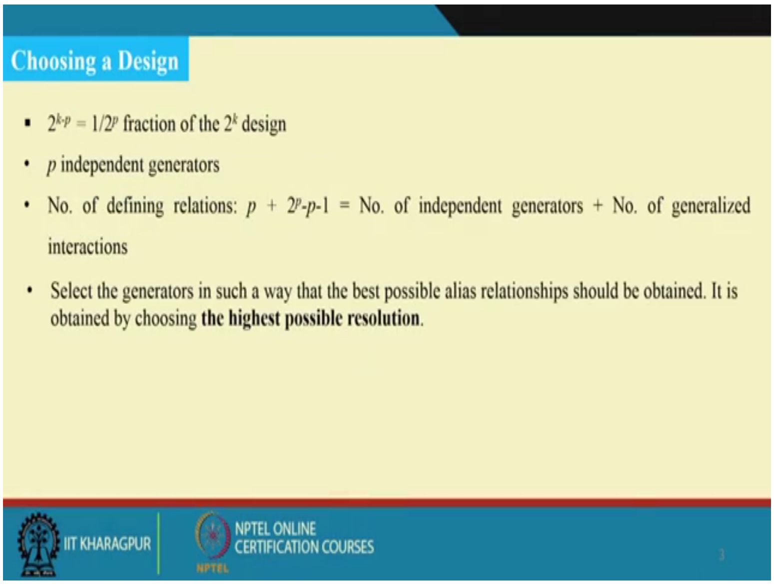

So, by 2 to the power k minus p fractional we say 1 by 2 to the power p fraction of 2 to the power k design. So, here few important things is that one is number of generators number of generators equal to p, second is number of defining relations, this will be p that is independent from independent general generators that is number of independent generators plus 2 to the power p minus p minus 1. These are basically the number of generalized interactions number of generalized interactions.

So, what does it mean? Suppose, for example, my design is 2 to the power 7 minus 3, then what is p? p equal to 3, p equal to 3. So, number of generators will be 3, and what is the number of defining relation that will be 3 plus 2 to the power 3 minus 3 minus 1. How much it is? 2 to the power 3; that means, 7 ok. Suppose, your case is 2 to the power 6 minus 2 then p equal to 2. So, number of generator will be here 2 and number of defining relations will be here 2 plus 2 to the power 2 minus 2 minus 1. So, 2, 2 cancel that mean 3, ok.

So, you have seen earlier. Suppose, if you consider this design, p equal to 1 defining relations will be p means 1 plus 2 to the power 1 minus, p equal to 1 minus 1. So, this will cancel out. So, it is 1. So, in this manner you go and understand. So, what is the number of generators and number of defining relations ok.

Now, find that which general that there can be depending on the situations depending on the p value fraction there will be different generators, ok. So, you have to choose the generators in such a manner that you will get the best earlier structure, that means, the minimum earlier structure I can tell you for the factors of interest and this is possible when you go for highest possible resolutions.

So, that means another one is important one is the resolution very important one is resolution and all of you know that resolution will be understood from the defining relations and if you are suppose you have 7 defining relations so, what is the least number of words what in which relation the number of word is the minimum that will be your resolution.

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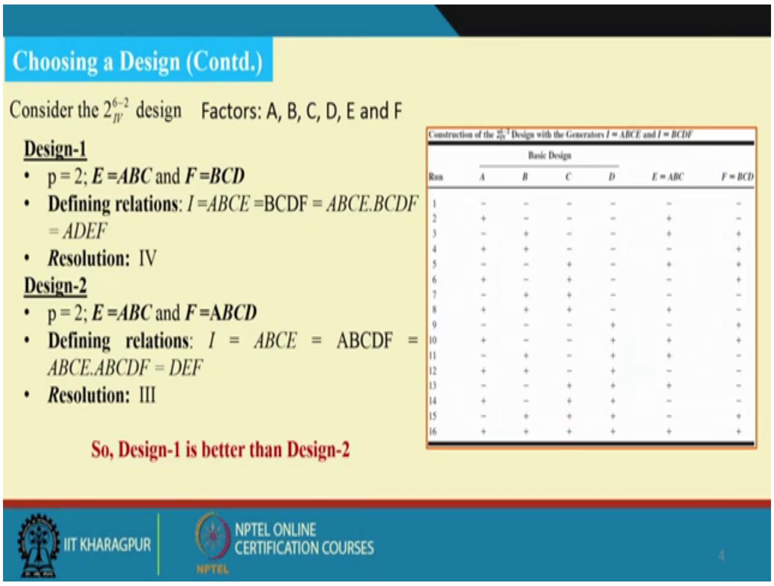

So, now let us see a case, let your you are interested to construct 2 to the power 6 minus 2, 4 resolution for design, ok.

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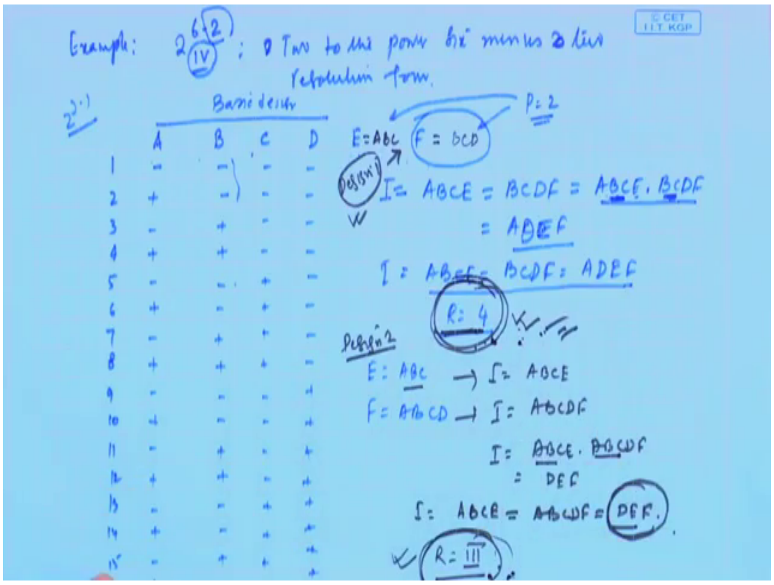

So, your example is 2 to the power 6 minus 2, 4 design. How do you read it this is 2 to the power to the power 6 minus 2 resolution 4. So, how do you construct this design this design you construct you first start with there are 6 factors A B C D E and F. So, it should be full factorial in 2 to the power 4 to the power 4 means 16. So, 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16.

So, first you consider 4 factors and develop the basic design. Here it will be minus plus minus plus minus plus minus plus minus plus minus plus minus plus minus plus. Second one minus minus plus plus minus minus plus plus minus minus plus plus minus minus plus plus. Third one minus minus minus minus plus plus plus plus minus minus minus minus plus plus plus plus. Fourth one minus minus minus minus minus minus minus minus plus plus plus plus plus plus plus plus, ok.

So, you know that is 2 to the power j minus 1 minus the followed by plus and that will repeat. So, that is why it is minus plus 1 after another it is 2 minus 2 plus 4 minus 4 plus 8 minus 8 plus. That is the basic design.

Now, you will create generator. So, you can write down E equal ABC equal to ABC and F equal to let it be BCD, ok. So, ABC will be this three column sign will be multiplied and you give fill up this column and for F fill up this column. So, as P equal to 2 here P equal to 2. So, you have 2 generators this and generator this, ok.

So, what is the defining relation different relation you will get using I equal to if you multiplied this with E E that is ABCE if you multiplied this generator with if you will get BCDF and they are these are independent generator and they generalized one he will find out ABCE into BCDF, that means, what is this value A B and C will be cancelled. So, it will be AC ACEF EF. So, that is your defining relations. So, defining relation is then equal to I equal to ABCE BCDF A, sorry, it is AD AD, BC is cancelled out EF, ADEF. Now, what is the resolution here? We want resolution 4 and you see that in the defining relation that minimum number of words is 4. So, resolution equal to 4.

Now that means this is what is chosen here. Now, instead of this instead of suppose F equal to BCD if you choose generators like this E equal to ABC and F equal to ABCD. So, I can say that this is my design-1 this and here design-2 because you can choose the like this. So, it is possible that E will be compounded with ABC that interaction F will be compounded with ABCD that fourth order interactions, ok.

So, under such situation what will happen to I from here you will get ABCE from here you will get ABCDF and generalized one you will get ABCE into ABCDF. What is this ABC; ABC will be squared and it will be I. So, you will get DEF. So, that means, your defining relation is ABCE equal to ABCDF equal to DEF, ok. So, what is the resolution here the mean the in the defining relation them the least number of words is three here. So, your resolution is three here.

Now, given this situation, which one you will you will prefer? The first one because of resolution 4; what is the meaning of resolution 3? What is the meaning of resolution 4? Ok. So, here what happened main effects are not aliased with other main effects, but main effects are aliased to with two factor interaction effects and two factor interaction with two factor and higher. Here main effects are not interact aliased with main effects and two factor interaction effects it will be interlaced with three factors two factor interactions will be aliased with other two factor interactions here.

So, if you employ these you will not able to uniquely estimate the main effects and two factor interaction if it higher order interaction effects are other higher interaction effects are negligible, but if I consider third and higher order interaction negligible here then you are in a position to estimate the main effects because main effects are not aliased with other main effects and two factor interaction effects. So, as a result uniquely you will be able to estimate the main effects considering this. So, that is why this is a better design ok.

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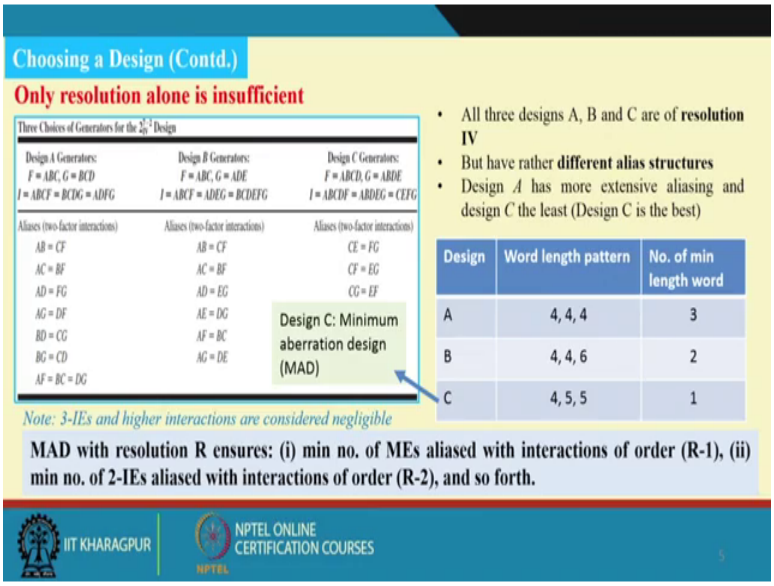

So, but many times what happen you will find out that only resolution is not sufficient enough. For example, you consider 2 to the power 7 minus 2 resolution four design. We are assuming that ok, resolution 4 will fix. Now, under this situation for resolution four case you see how can you generate the design?

So, here how many factors are there? 7 factors, 7 factors; that means, A B C D E F and G, 4 plus 3 7 factors. So, what is the design you want to choose 2 to the power 7 minus 2 four resolution design.

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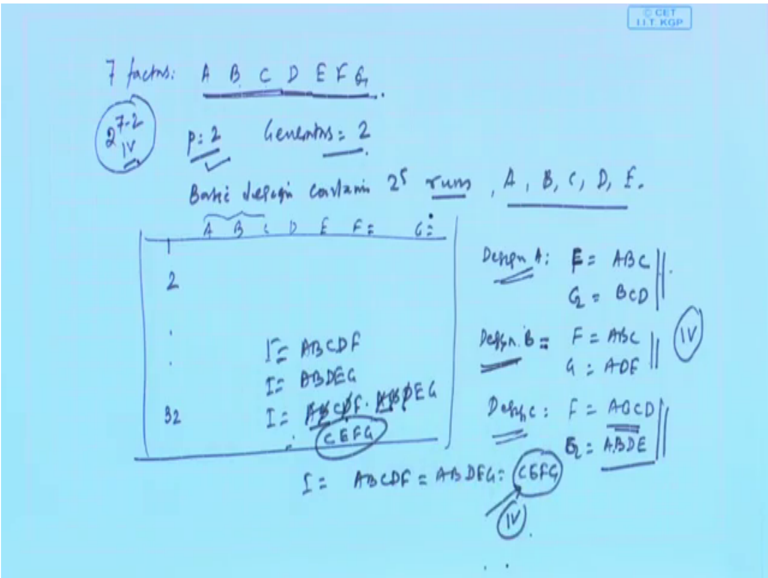

So, 7 minus 2 what is p? p equal to 2. So, your how many generators we needed generators needed is 2. So, as it is 2 the power 7 minus 2, what is happening? We have 7 factors A B C D E F and G our design is 2 to the power 7 minus 2, four resolution 2 generators.

So, it will be first basic design contains basic design contains 2 to the power 5 runs. So, A first 5 factors B, C, D and E. What will happen 1, 2 like this up to 32. Your basic design A B C D and E, now, you have to choose F equal to and G equal to. So, there are plenty of options you can choose if some you can it is it is it is desirable that the this will be at higher order interactions confounded with higher order interactions. So, 3 and more are considered to be higher order interactions.

So, you can you can generate if by interacting with this ABC or confounding with ABC, G also confounding with BCD so, that means, let us create three design. Suppose, design A where E equal to is confounded with ABC not E F and G with BCD. Second one let design B design B you can do it is ABC and ADE F with ABC, G with ADE another design C is basically you can go for 4 way interaction that confounded with 4 way interactions ABCD and ABDE.

So, if you choose this design and you find out what is the resolution choose this design find out the resolution choose this design find out the resolution. All those cases you will be you resolution will be four. For example, this you have seen for example, this 1 you consider here I will be ABCDF, I will be ABDEG and I will be ABCDF into ABDEG. So, AB AB will go, D will go so, that means, C will be there this and this so, C E FG. So, defining relation is ABCDF equal to ABDEG equal to CEFG. So, this one is in minimum word defining relation which is four. So, resolution is four. So, now, here also you similarly you can prove this is four, this is four.

Now, question is that will all the design gives you the similar results or you in some cases it is better? So, that is what is to be discussed. Now, go back to the slides again you see first design, second design and third design here you see that all the defining relations only 4 words here 2 4 words 1 5 words here 2 5 words and 1 4 words. I think here one is 6 words in the second one.

Now, once you know defining relations you are in a position to find out the aliases. So, using design A you are going to get these many aliases for two factor interaction assuming that the three, three and higher order interactions are zero.

And, if you go for design B you are getting little better. Because here number of aliases for two factor interactions are little less if you go for design C you see that you are getting much less aliased because CFG, CFEG and CGEF. Now, what happened if we just compare these three design with reference to word length that is the defining relations as well as with reference to minimum word length word you see that for a it is 4, 4, 4 length, minimum length word is 3 for B it is minimum length word is 2 and 4, C minimum length word is 1. So, 4 is the minimum length word.

Now, the design which is having the minimum length word least minimum length word this is known as minimum aberration design. So, this minimum aberration design is the best one. Here, A is having the most extensive aliasing and C is have a having the least aliasing relationships and C is having the minimum number of length words least number of minimum length words and C is the minimum aberration design.

Now, the advantage of minimum aberration design is minimum aberration design with resolution R ensures minimum number of main effects are aliased with interaction of order R minus 1 and minimum number of two-way interaction effects are aliased with interactions of the order R minus 2 and so forth. What does it mean?

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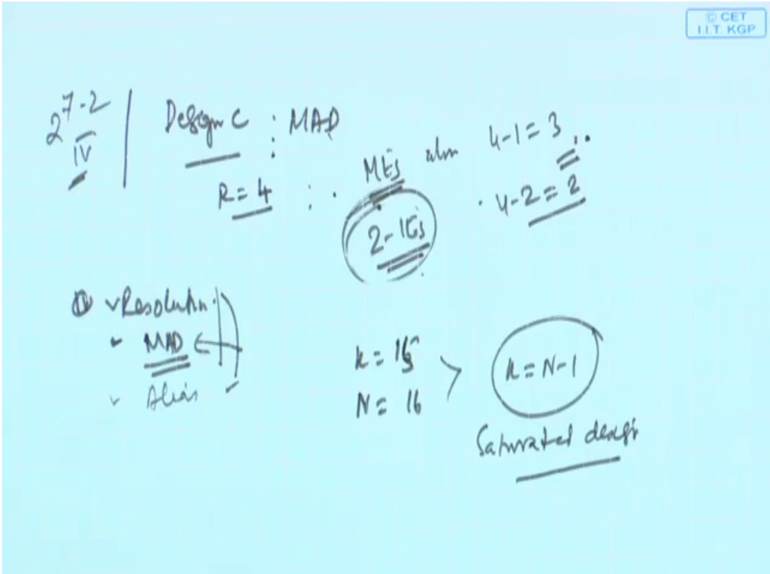

Our example is 2 to the power 7 minus 2, ok. So, if you see it if you take design C this is our minimum aberration design. So, what is saying that. So, what is the resolution R equal to 4 here. So, this says that that the ME's that main effects aliased with R minus 1 means 4 minus 1, 3 in this case the main the minimum number of main effects will be aliased to with R minus 1 button 3 third order interactions.

Similarly, if you go for the second order interactions this also R minus 2, that means, second order interaction. So, minimum number of second order alias interaction alias to with other second interaction. So, that is what is you will be getting. So, that is known as minimum aberration design and very very important.

Sincerely what happened you will see the resolution for choosing a design and another one is that what is basically the minimum aberration design MAD. Alias structure plays a significant role alias structure same resolution with different alias structure it is possible. So, resolution if it is same then see alias structure which is giving the least alias structure and also finally, choose the and this case finally, choose the minimum aberration design.

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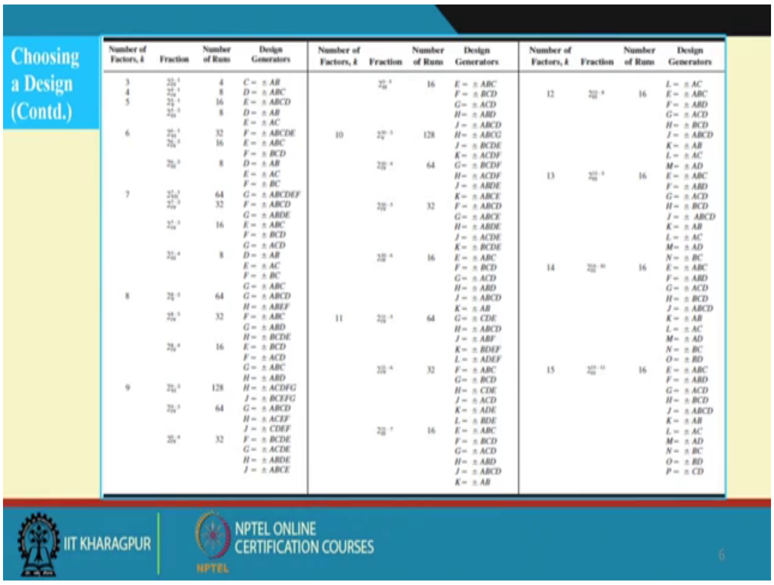

So, here a table is shown where different kinds of may 2 to the power k minus p designs are given number of factors k. So, it start from 3 to I think 15 this much it is given and then what are the possible fractions or fractional factorial that is possibilities given. For example, if I consider number of factors 7 then you see that the possible fractional factorial is 2 to the power 7 minus 1 that is 7 resolution 2 the power 7 minus 2.

It is 4 resolution to the 7 minus 3 resolution 4 and 2 to the power 7 minus 4 resolution 3. So, resolution 3, 4, 5, 4 and higher resolution is possible. So, if you consider here 2 to the power 15, so, these are to the 15 minus 11 resolution 3 with 16 number of runs. So, 15 number of factors with 16 number of runs, it is possible. So, we have 15 factors k equal to 16 15 and number of runs N equal to 16. So, that mean k equal to N minus 1 this type of design is known as saturated design and very popular in screening experiments.

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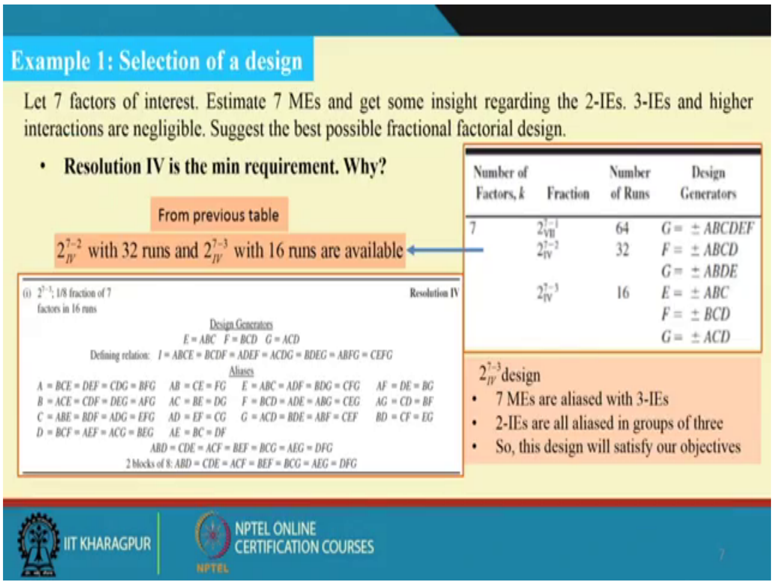

Now, how do use this table? I will give you one example. Suppose, you are you have 7 factors of interest and you are interested to estimate the 7 main effects and to get some ideas about the second interaction order interaction effects, third order and higher order interaction effects are negligible, lead you who have process knowledge you know that these process the third and higher order interactions further for the control factors or the factors of interest are negligible. So, then what will be the best fractional factorial design.

So, we are saying the resolution four will be minimum requirement the reason is a resolution four main effects are not aliased with second order and third interaction effects it is aliased with third order and interaction effects and second order interaction effects are aliased with other second order interaction effects also, but if you go for resolution three what will happen that minimum main effects are also earliest with second order interaction effects.

So, in that case you cannot estimate the 7 main effects uniquely, but if you choose resolution 4 design you will be in a position to uniquely estimate the 7 main effects and you have some idea about the second order interaction effects as third and higher order interactions are negligible here.

So, then what do you will do the previous table what I have shown a portion of this table is shown here. So, will go for number of factors 7 and there are possible that what are the resolutions from four resolution four point of view two options are available resolution four 7 minus 2 and 7 minus 3. So, then which 1 you will choose if you choose resolution 2 to the power 7 minus 2 4 design then you require 16 32 runs if you choose 7 minus 3 for resolution four design you require 16 runs. Is will 16 run sufficient other way round or 16 run if the less number of runs is always better from experiment point of view. We will 7 minus 3, 4 will give us the minimum requirement. Let us see these if you go by 7 minus 3 design then these are the E, F, G these are the generators and defining relations there are one two three four seven defining relations.

Now, if you using this defining relations you will be able to find out the alias structures you see that 7 main effects are aliased with the third three third order interactions and the second order interactions are aliased in group three, ok. So, you can esteem as third order and higher order interactions are negligible.

So, you can estimate A B C D E F G uniquely considering that the third order interaction effects are 0 and you will see that the alias the interactions alias the second order interaction alias in three, that means, AB AC AD AE and their aliases some estimates you are getting. So, we can say that this design is such satisfactory because we want to know some amount of information about the second order interactions and full information about the main effect. So, this design is this design is satisfactory.

Now, what will happen if you go for 2 to the power 7 minus 2 resolution four design you require 32 runs and they are what happen once you will find out the this structure alias structure you will find out what that many more second order interactions some better information you will get, but as per as the requirement is concerned here it is sufficient. Which one is sufficient? 2 to the power 7 minus 3 resolution for design sufficient to satisfy our condition, ok.

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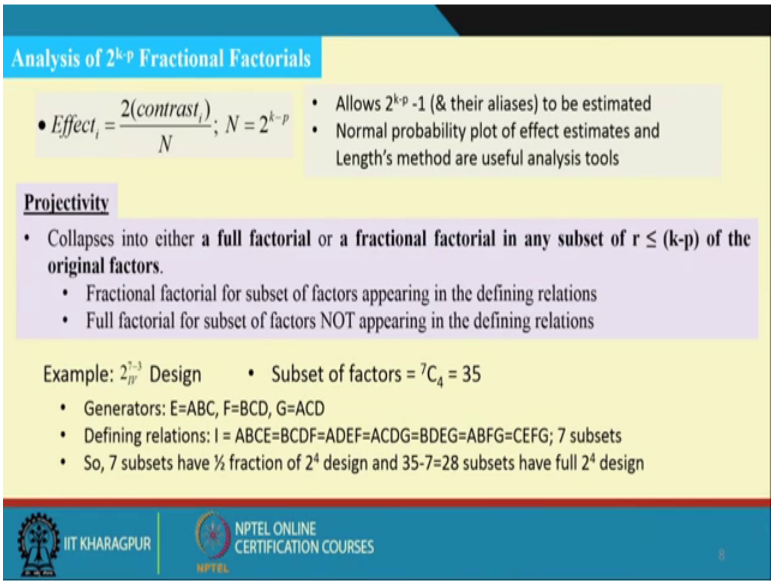

Now, I will explain that the second part is analysis. Under analysis you first find out the estimate of effect.

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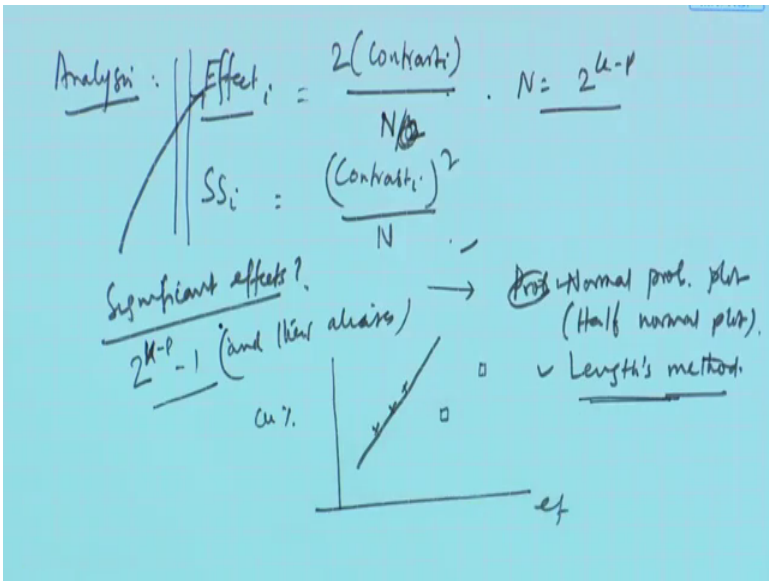

So, effect i. So, all of you know this is the contrast i divided by to any let it be N by 2 or other way you can write 2 into this by N, where N in this case is 2 to the power k minus p. So, another one may be you will be interested to sum square. So, this will be contrast square contrast i square divided by N. So, these are the things you require to compute, ok. So, once you know the effect and the sum square what will happen you require some kind of another analysis which is known as which are the effects that are significant effects.

So, what will happen here you will see that so, you will estimate the effects 2 to the power k minus p minus 1 and their aliases. So, you will estimate 2 to the power k minus p minus 1 and their aliases. This number of effects you will estimate and then you want to know which are the effect significant. The concept is you go for probability plot normal probability plot normal probability plot, which is very known half normal sometime half normal plot. Half normal means what that you take the mod value of the effects, ok.

Another method is known as lengths method and we have discussed normal probability plot and we have shown that this is cumulative percentage and this side effect then the effects which are basically along these, straight line they are in significant there will be some effects here and there which may be significant.

So, that is what is our normal probability plot. In the length method they will give you some this is basically subjective because depending on the graph, so, how what is the distance that is not, but only thing we are saying that the effects which are away from the straight line they are significant, but lengths may thought what it will do it will give you it will give you a sub objective assessment, ok. So, we are not discussing length method because of scarcity of time, but you may go for lengths method, ok.

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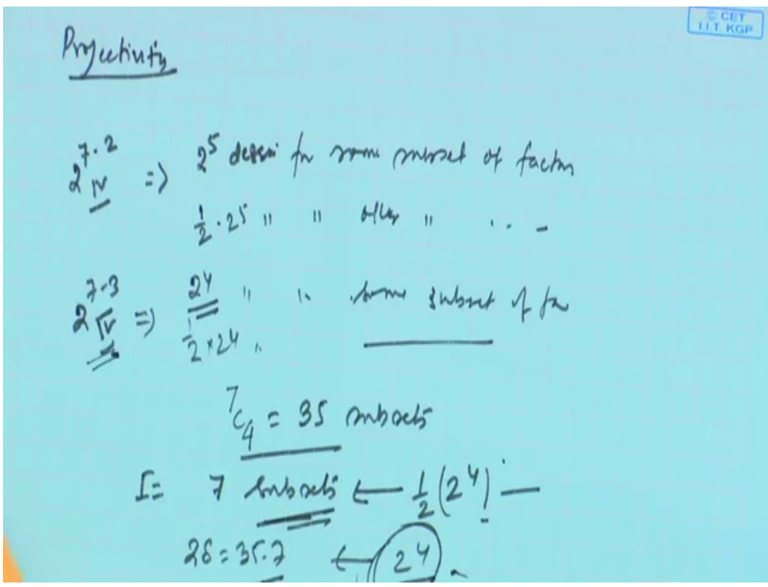

Under analysis another important thing is projectivity ok. So, we have discussed about projectivity also. Let us see the slide projectivity. If you go for it 2 to the power k minus p fractional factorial design then these collapses into either a full factorial or a fractional factorial in any subset of r, where r is less than equal to k minus p of the original factors. What does it mean? It mean the oppose you are basically talking about 2 to the power 7 minus 2, four design let it be.

So, then what happened you will you will find out from here 2 to the power 5 design for some subset of factors and fraction maybe one half or 2 to the power 5 design for other subset of factors. If you consider a 2 to the power 7 minus 3 suppose 4 design. So, here what happened you will get 2 to the power 4 design for some subset of factors and maybe 1 half into 2 to the power 4 design for some other subset of factors.

So, there are two subsets which will get that 2 to the power k minus p in this is full factorial and some will be will be having the fractional of that factorial design. The factors which comprise the defining relation subset of factor comprising the depending relations those subsets will not be will not get the full factorial they will get the fractional one and the other subsets which is not in the defining relations they will go for they will get they will have this full factorial kind of things. So, collapses into either a full fraction factorial or fractional factorial.

So, fractional factorial for subset of factor appearing in the defining relations and full factorial for subset of factors not appearing in the defining relations. So, as we start with these 2 to the power 7 minus 3 four design, ok. So, what is the subset of factors total subset we require 4 at a time. So, this will give you 35 subsets. If you see the defining relations you will get that 7 subsets will appear in the defining relations.

So, that means, 7 subset those factors those subsets will have 1 by 2 into 2 to the power 4 4 fractional design and other 28, that means, the equal to 35 minus 7 these have 2 to the power 4 full factorial design, means this will collapse. If you take these mini subset to the 4 to the power 4 that subset 7 the 25 8 sub sets subsets of four so, they will have full 2 to the power 4 factorial design, ok.

So, this is what is given in the slide you see, if we consider example 4 to the power 7 minus 3 resolution four design number subset of factors is 35, generators we have seen earlier that we have used ABC, BCD and ACD defining relation you see from these three generators you can you can find out 7 subset of defining the relations. So, ABC, BCDF like this give a 7 subsets. So, there are there are another 30, 28 subsets so, these 7 subsets have half fraction of 2 to the power four design and another 28 subsets have full 2 to the power 4 design, ok.

So, the projectivity part is important one. So, that mean you are now in a position to know which are the factors having in a lower dimension in that four dimensions I have full factorial and which are the factors in a four dimension have half fraction of 2 to the power 4.

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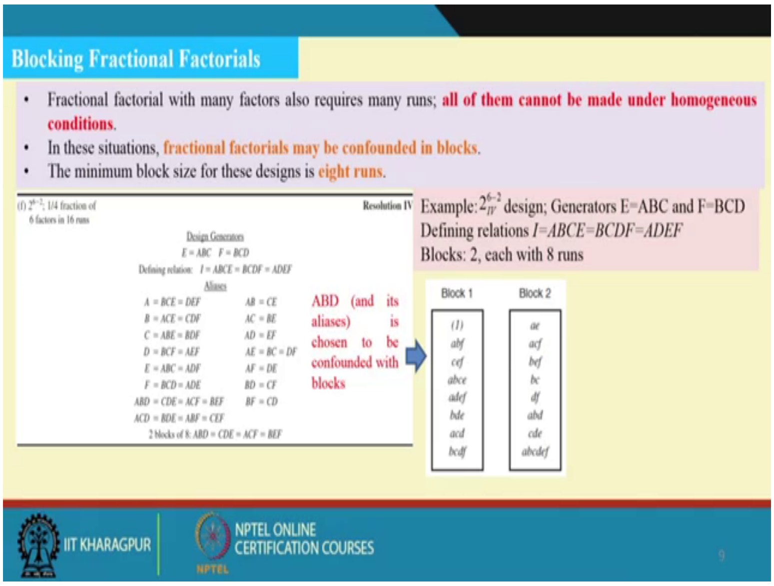

Then third one which is also very very important is blocking ok, blocking.

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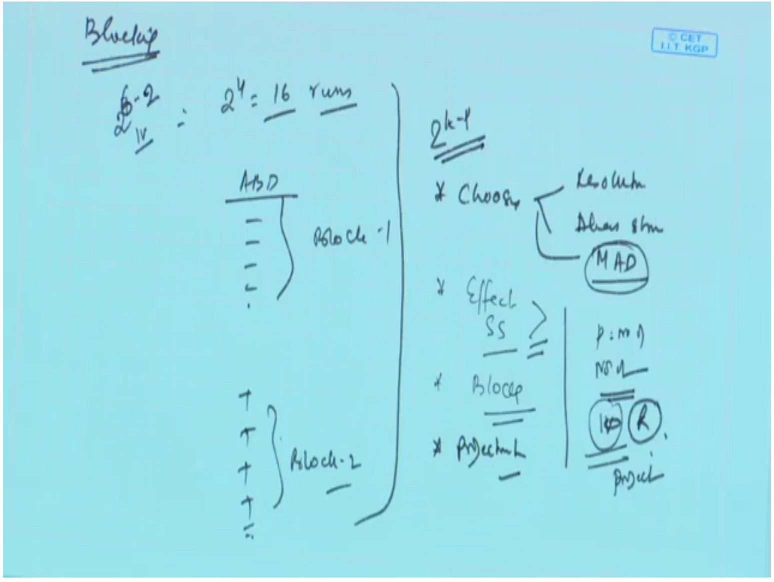

So, all of you know blocking. As I told you that if you go for 2 to the power even 7 minus suppose we will take 7 minus or 6 minus 2 let it be 6 minus 2 maybe resolution four design. So, that mean you require 2 to the power means 16 runs. So, there may be if each run takes a lot of time. So, you may not get that everything the 16 run can be completed in one shift you may require two shifts or the mid quantity required if it is large.

So, maybe one batch of raw material will not be able to make it possible. So, you require two batches of raw materials in. So, that mean even in fractional factorial design you adopt you require you may not be able to create the set of homogeneous condition so that all the experimental runs are possible. So, there what you have to do you require to go for blocking, ok. So, fractional factorials may be some of the effects will be confounded with blocks, ok.

So, under 2 into 2 to the power 6 minus 16 in case suppose, we require 8 runs per blocks suppose 16 runs we require here and we require 2 blocks because our capabilities 8 per shift or 8 per batches of raw material or dependent available operator it is such. So, then you see if we say this one then this 2 to the power 6 minus 2 resolution four design and you know the design there will be two design generators, there will be three defining relations and you are getting these alias structure ok. So, in the defining relation ABCE, BCDF and ADEF, these three subsets which is having which will be having at the four fact dimension case four factor case fractional factorial design, but rest of the rest of the your that four subsets they will be being your full factorial design that mean full factorial 2 to the power 4 design. So, that is what is the concept.

So, now here what happened, so, you have to choose the block. So, if you see the third order interactions suppose, ABC and ACD suppose if you choose A BD or any of it aliases to be blocked with the blocking variable then what will happen, you can you can create two blocks suppose if you if you if you block ABD ABD or confound ABD with blocks then you will be getting these structure. So, block 1 will go for this because you will take ABD plus ABD minus.

So, from the design you have to find out the ABD column and then you will be having all minus then there will be all 8 plus. So, this will go to immediately be block 1 and this will go to let it be block 2 ok. So, that is the way you have done blocking in earlier cases the same thing you are just you have to find out the appropriate interactions to be confounded here in this case ABD or any of the aliases can be blocked. So, absolutely no problem here so, this is what is your concept of blocking here.

So, in general, the concept of blocking is same that when there are you have paucity of material to say experiment in homogeneous condition paucity operator, paucity of time, you require to block what you require to have is smaller wave fraction of experiment will be conducted under a certain homogeneous condition and other fraction in the another homogeneous condition and in order to get the meaningful results so, you require to confound certain higher order interactions with the blocks and then choose that which are the experiment to be run in first block which are experiment in the second block or so forth, ok. So, this is what is our blocking.

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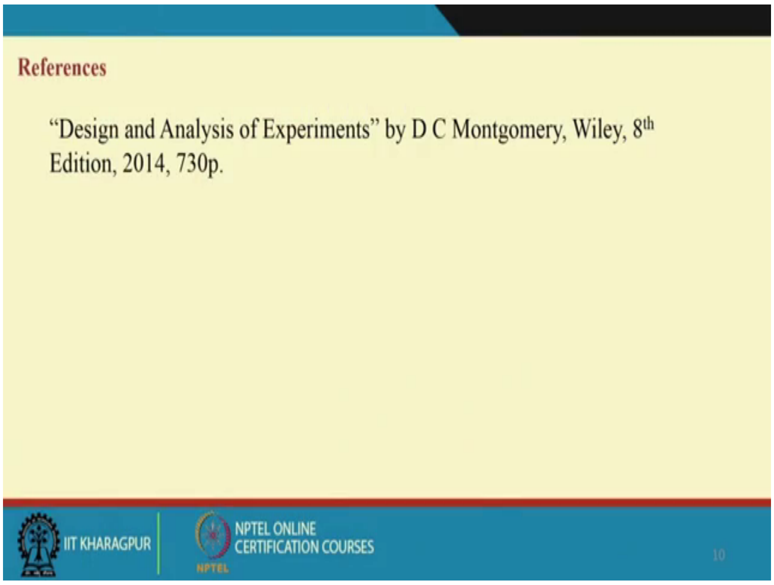

So, we have discussed very precisely the 2 to the power k minus p fractional factorial design. I said that choosing a design is very important. Here one of the issue is resolution, that is a very important one, that alias structure alias structure and this will ultimately lead to minimum aberration said aberrated design.

So, these this is the best one; the reason is, it will give you better alias structure and then you have effect estimate, you have some square that using contrast you do it, and then do not forget the concept of blocking also this is also very very important. In addition you must know p number of generators, that number of defining relations and how to compute resolution.

So, these are very very important resolution and projectivity is another concept which we have discussed here. Projectivity it is another concept which we have discussed; this also a very very valuable information for experimental design.

Thank you very much.