Design and Analysis of Experiments
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Lecture - 48
Alias Structure in Fractional Factorial Design: Regression Approach

Hello, welcome to the lecture on Alias Structure in Fractional Factorial Design, using Regression.

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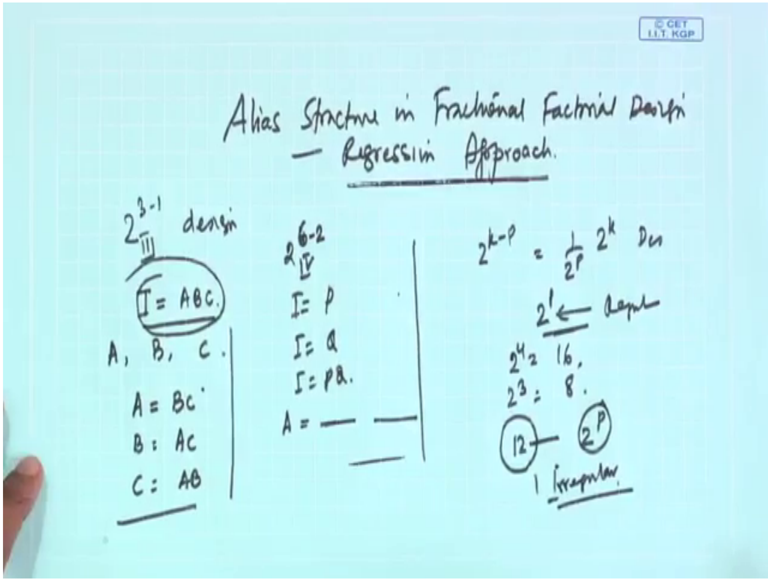

So, that mean you require a defining relation, either one defining relation or two or more defining relations and, and then those defining relations will help you to identify the Alias structure; that means, what are the, effects that are confounded that is known. So, it may. So, happen that there will be certain situations, when you may not get the defining relations and another one is that, that under certain, simple condition, this defining relations work well.

There may be situation, complex situation, when the defining relations will become complicated and you may not be able a, get a that, that desired one. So, alternatively you can use regression approach to find out the Alias structure. In this half an hour lecture, I will show you, how Regression approach will be used to find out the Alias structure and with, with, with two small examples.

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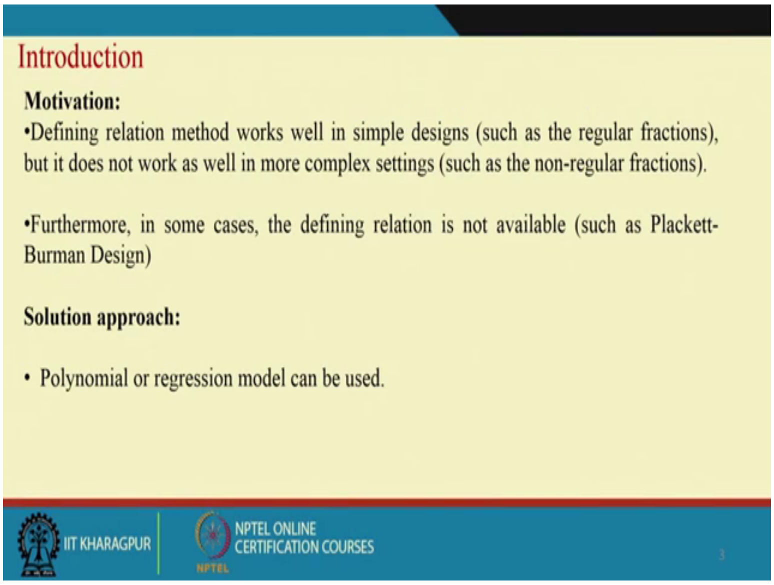

I told you the motivation, motivation is that there may be situation when, which will be complex design will be complex for example, non regular fractions and there are situations, where design is simple, which is regular fraction. For example, if I consider 2 to the power K minus P and then which is basically you, 2 to the 1 by 2 to the power P 2 to the power K design.

So, that mean 2, 2 to the power, power something, this kind of design is the regular design, if P equal to P equal to 4, then it is basically 16 runs, if P equal to 3, it is basically 8, but suppose, a design, where we are interested having the, 12 experimental runs, it can be represented in terms of 2 to the power P. This is irregular design, irregular. So, under irregular design case. So, ah, defining relationship will not work and again in some cases the defining relations is not available like that is Plackett Burman design. So, under such situation, how do find out the Alias structure or how do we estimate the parameters, when you go for Fractional Factorial design that is, is the discussion point.

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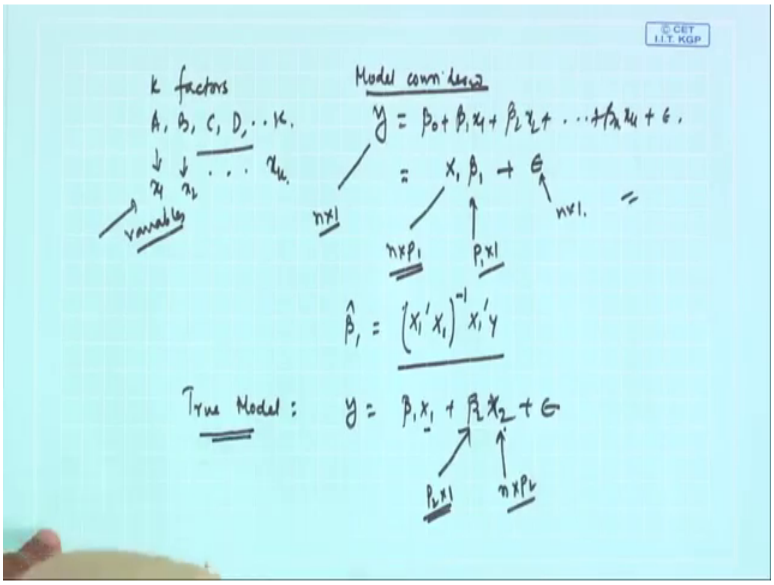

Now, suppose, you have K factors, A B C D, like up to K factors you have so, correspondingly, we can say, we have X 1 X 2. So, like this X K variables, X K variables. So, you can, you can feat a equation main effects model. Let it be the only, the main effects Y is basically, let it be beta 0 plus beta 1 X 1 plus beta 2 X 2 dot beta K XK plus Epsilon.

Let, we will define it like this, this is nothing, but X 1 beta 1 plus Epsilon, where Y is n cross 1 n number of observations X 1 is basically, n cross P 1. Suppose, P number of effects, when we use the only, the main effects, interpretation are not considered. So, that mean P number of parameters considered here beta is P 1 cross 1 number of effects to be estimated and this is n cross 1.

So, you know that in this if I write, then under this situation beta 1 cap is equal to X 1 transpose, X 1 inverse, X 1 transpose, Y that is the, that is a formula, you have used earlier. So, this is, we are saying, this is the model, you have considered. Now, if I go for fractional factorial in design here, what happened, you have considered only the main effects all interaction effects, you have been removed ok. So, that mean they are in, in this design, in fractional effects are not significant, but maybe the full model is a different one, the full model or the true model, probably true model, probably Y equal to beta 1, X 1 plus beta 2, X 2 plus epsilon.

Then what is your beta 2, beta 2, this maybe P 2 cross 1, the remaining, remaining factors, which are not included in the consider model, considered which is true for the true model, which is as considered and then, this may be n cross P 2, the additional design matrix. So, X 1 and X 2. This combine the full design matrix, what is the additional design matrix? So, if this is true, what will happen then, what will happen to the expected value of beta 1.

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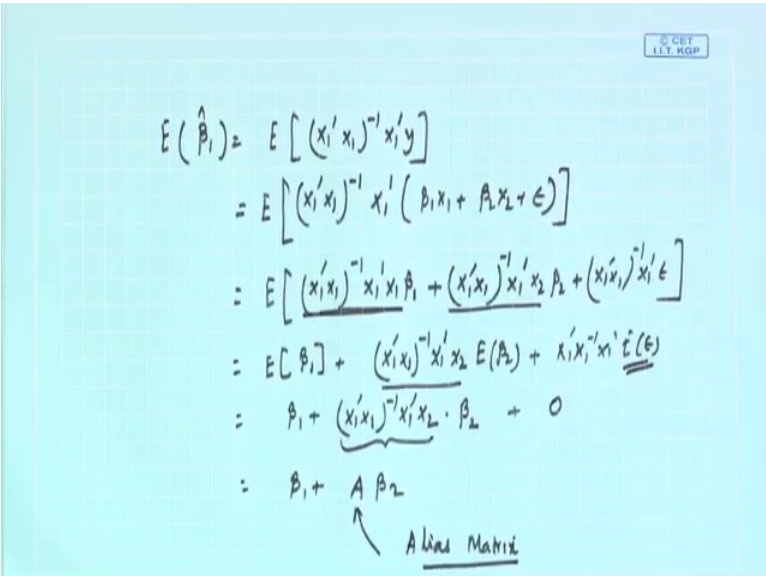

What is expected value of beta 1 cap, you all know, it will be expected value of X 1 transpose, X 1 inverse X 1 transpose Y, because beta cap is this. So, then, then if you write down, this further then this will be X 1 transpose X 1 inverse X 1 transpose, what is the Y true value, Y true model is this. So, you write down it, this is as beta 1 X 1 beta 2 X 2 plus Epsilon. So, then this will go to as X 1 transpose X 1 inverse X 1 transpose X 1 beta 1 plus X 1 transpose X 1 inverse X 1 transpose X 2 beta 2 plus X 1 transpose X 1 inverse X 1 transpose Epsilon.

So, X 1 transpose, X 1 into X 1, this is I. So, you can write this as expected value of beta 1 plus this is the fixed quantity, will come out X 1 transpose X 1 inverse X 1 transpose X 2, expected value of beta 2 plus X 1 transpose X 1 inverse X 1 transpose, the expected value of Epsilon. So, what will happen? This will be beta 1 and this will be X 1 transpose X 1 inverse X 1 X 1 transpose X 2 into beta 2 plus 0, because expected value will be 0. So, these quantity, if we put at A then this is beta 1 plus A beta 2 this, a matrix is known as Alias Matrix. A matrix is known as Alias Matrix. So, using Regression approach, you are able to find out the Alias matrix.

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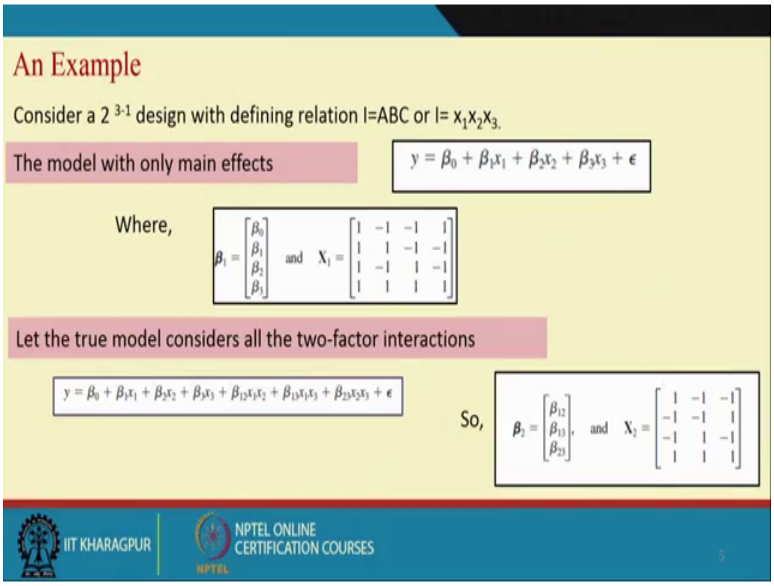

Now, let us see with an example, the example is 2 to the power 3 minus 1 design, 2 to the power.

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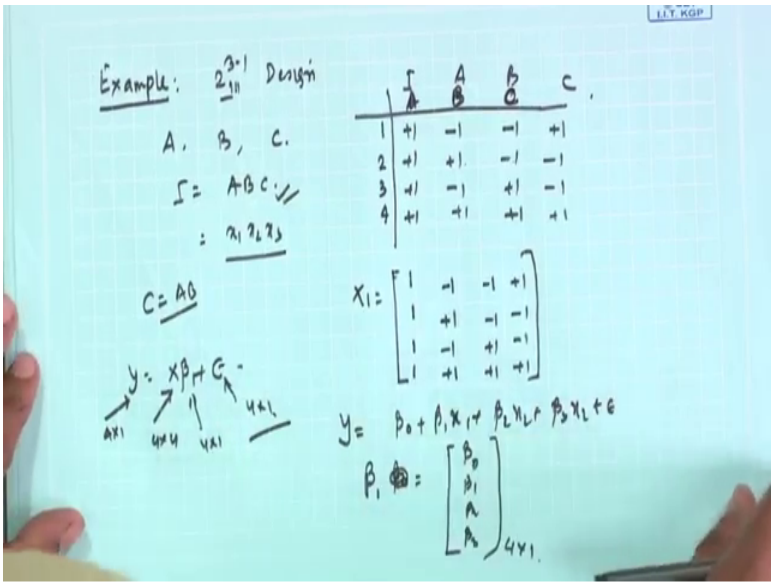

So, example, example is 2 to the power 3 minus 1 Fractional Factorial design, design, number of factors A B and C defining relation I equal to A B C or we can write like X 1

X 2 X 3 let. So, under this situation, what is the design matrix? So, it is 2 to the power 2 full A B and C ok, A B and C and I will also be there.

So, I will write here, this I will write, I, this I will write A, this I will write B, and this I will write C, but you have how many observation 1 2 3 4. So, I will be all plus 1 then this is minus 1 plus 1 minus 1 plus 1, this is minus 1 minus 1 plus 1 plus 1. Let for the time being suppose now, you have to choose the, where you will keep that C, which 1 is, where it is plus minus this thing. So, let us assume that using this concept C equal to if it is A B; we will assume the same thing to show that it is coming from Regression approach, also to prove it.

So, there has been C is plus 1 minus 1, this into this minus 1, this into this minus 1 then plus 1. So, what is my design matrix, then X 1, X 1 is 1 1 1 1, then minus 1 plus 1 minus 1 plus 1, then minus 1 minus 1 plus 1 plus 1, then your plus 1 minus 1 minus 1 plus 1, this is my X 1 and let the corresponding Y Regression model is Y equal to beta 0 plus beta 1 X 1 plus beta 2 X 2 plus beta 3 X 3 plus Epsilon.

Then beta 0 is our B A sorry, beta 1 is beta 0 beta 1 beta 2 and beta 3. So, 4 cross 1. So, that mean we have considered a model Y equal to X beta 1 plus Epsilon, where Y here is 4 cross 1, this is 4 cross 4, this 1 is 4 cross 1, this is 4 cross 1, then what you have not considered, you have not considered the second order interaction, third order interaction, one third order interaction is already ah, interacted with the confounded, with the average. So, remaining part is the 1, the second order interaction part is not considered, I have.

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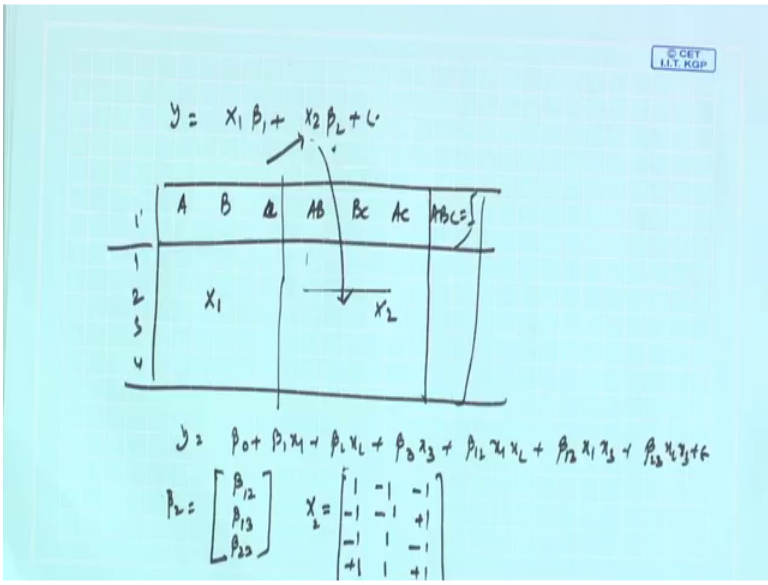

So; that means, if I considered that my true model is Y is X 1. So, this is X 1, this 1 is X 1. So, now, if we consider that X 1 beta 1 plus X 2 beta 2 plus Epsilon, where X 2 talks about the other, other terminology. So, what is the full design AB, suppose, C AB BC AC ABC. So, what we have considered, we have considered ABC only. So, that mean, this portion, we have not considered. We have 5 1 2 3 4. This is what X 1 is considered, but this is X 2. Now, this is X 2 ABC is equal to I with the beta 0.

So, now if the true model, where basically X 2 is this design matrix and beta 2 is basically the, all the interaction parameters. So, then our Y beta 0 plus beta 1 X 1, beta 2 X 2, beta 3 X 3, beta 1 2 X 1 X 2, beta 1 3 X 1 X 3 plus beta 2 3 X 2 X 3 plus Epsilon. So, the suppose, this is our full model. So, under such situation what will happen to beta 2? Beta 2 is your beta 1 2, beta 1 3 and beta 2 3

What about X 2, X 2 is this, this 3, 3 by 4 that. So, you know ABC is known. So, AB BC all those things, you can, you can write down and ABC is known AB BC all those things can be found out, accordingly and that value is basically, 1 minus 1 minus 1 plus 1 then minus 1 minus 1 plus 1 plus 1 then minus 1 plus 1 minus 1 plus 1. So, this is our. So, that mean beta 1 2 to the AB, then it is AC and it is BC. So, if you use the first model, if you use this model, where the true model is this, then definitely, definitely the Alias structure is there, what is this Alias structure ?

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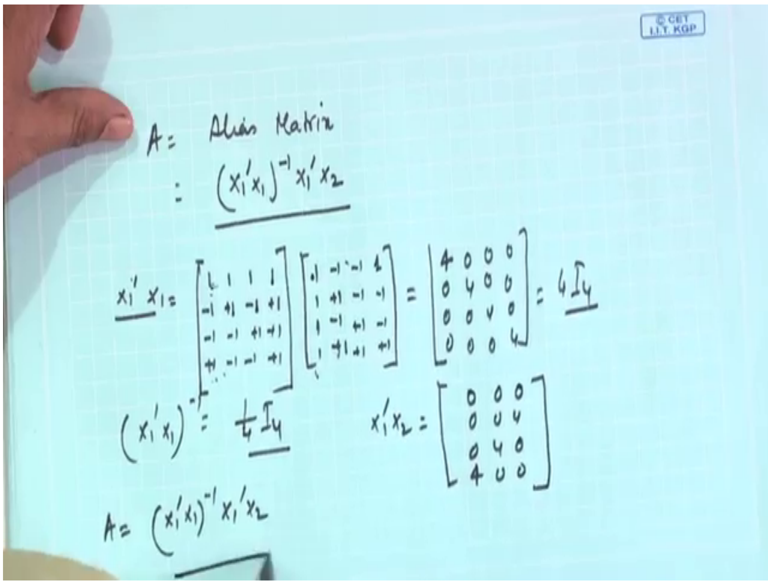

So, you all know A is the Alias structure, A is the Alias matrix, A is the Alias matrix. So, this 1 is X 1 transpose X 1 inverse X 1 transpose X 2. So, what is your X 1, X 1 is this, what is X 2? X 2 is this. So, X 2 is 4 cross 3 X 1 is your 4 cross 4. So, now, you find out X 1 transpose X 1 inverse and find out X 1 transpose X 2 multiplied, multiply the 2 ok. So, now, what is X 1 transpose X 1 let us see.

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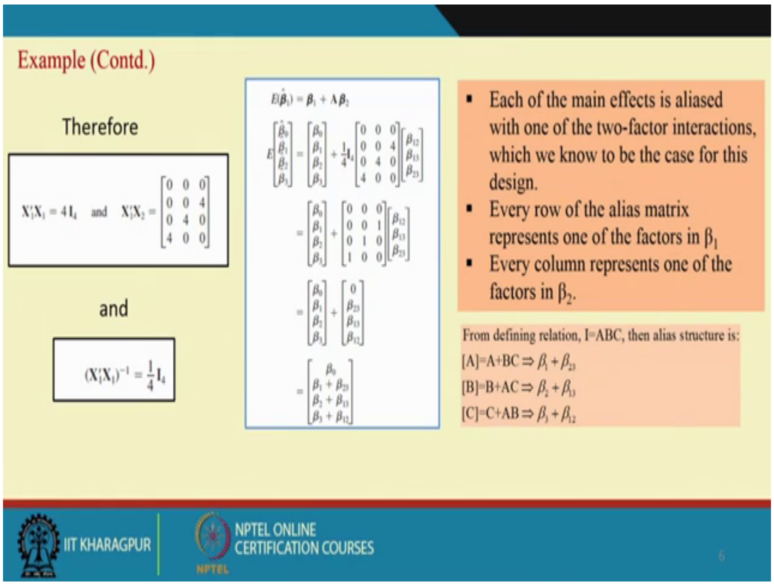

X 1 is or I can write X 1 transpose, X 1. So, we can write 1 1 1 1 minus 1 plus 1 minus 1 plus 1 1 1 1 1 minus 1 plus 1 minus 1 plus 1 minus 1 minus 1 plus 1 plus 1 then plus 1 minus 1 minus 1 plus 1, this is X 1 transpose, then what is X 1 1 1 1 1 minus 1 plus 1 minus 1 plus 1 minus 1 minus 1 plus 1 plus 1 1, which 1? This 1 plus 1 minus 1 minus 1 plus 1.

So, if you multiply this, what you are getting? Now, all 1 into 4. So, you will be getting 4, this into this, then this into this plus and this into this plus minus will cancel out. So, ultimately you will get, you will get 0 0 0 0 4 0 0 0 0 4 0 0 ze 0 0 0 4, which is 4 into I 4 ok. So, what will be then X 1 transpose X 1 inverse, it will be 1 by 4, I 4 then the same manner you will find, you X 1 transpose X 2, value will be your like this, will be 0 0 0 0 0 4 0 0 4 0 4 0 4 0 0 you will get. Now, if you multiplied, now X 1 transpose X 1 inverse X 1 transpose X 2, which is a matrix, what you will be getting, that is of importance. Let us see this slide that is better, you see that a is X 1 transpose X 1 inverse, which is 1 by 4 I 4 into X 1 transpose X 2, which is the this, if you multiply this 2 you are getting, you are getting this 1 0 0 0 0 1 0 0 1 0 1 0 and this thing and when you multiplied with this beta 2, then you are getting this and allowing this, you are getting this.

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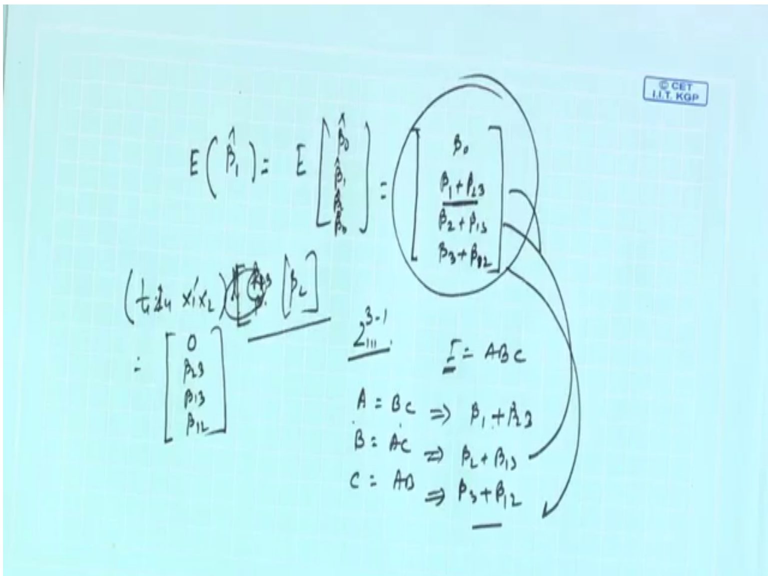

So, then what is the final one? Final one is your expected value of beta 1, which is your expected value of beta 0 cap, beta 1 cap, beta 2 cap, beta 3 cap, this you finally, got beta 0 then beta 1 plus beta 2 3, beta 2 plus beta 1 3, and beta 3 plus beta 1 2 that is what you are getting ok. So, it is a, just matrix multiplication at the end. So, I am expecting that you will be able to find out 1 by 4 I 4 into then X 1 transpose X 2 that value and that value is nothing, but nothing, but 0 beta 2 3. So, that value means what I mean to say 1 by 4 I 4 into X 1 transpose X 2. This value is nothing, but 0 beta 2 3, beta 1 3 and beta 1 2.

So, 0 beta 1 3 beta 1 3 sorry, with reference to your this multiplied by; obviously, the beta two part 2 3 beta, a beta we write this beta value only beta 2 value. What is the beta 2 value that you have already seen? Beta 2 values are your beta 2, first equation, this is what is beta 2. So, I am sure that you are, you will be in a position to compute it. So, this is what is our now, Alias structure sure.

So, now, if I go back to 2 to the power 3 minus 1 design, where I equal to ABC so, what I got multiplied with A BC equal to BC B equal to AC C equal to AB. Now, I want to relate the this, with this. You see A equal to BC is not this, this is not beta 1 plus beta 2 3 is not. The second one is this beta 2 plus beta 1 3 is not the third one is this beta 3 plus beta 1 2. So, that is what is the; Regression approach for Alias structure determination. So, let me read out each of the main effect is aliased with one of the two factor interaction, which we knew, which we know to be the case in this design from using defining relations that is what I have shown in here every row of the Alias matrix represent one of the factors in beta 1 Alias matrix, if you see what is this Alias, this matrix and then every column represent one of the factors in beta 2. So, there are 4 every row for beta 1 that is a four factors, a four parameters and beta 2 case, there are three interaction parameters. So, three.

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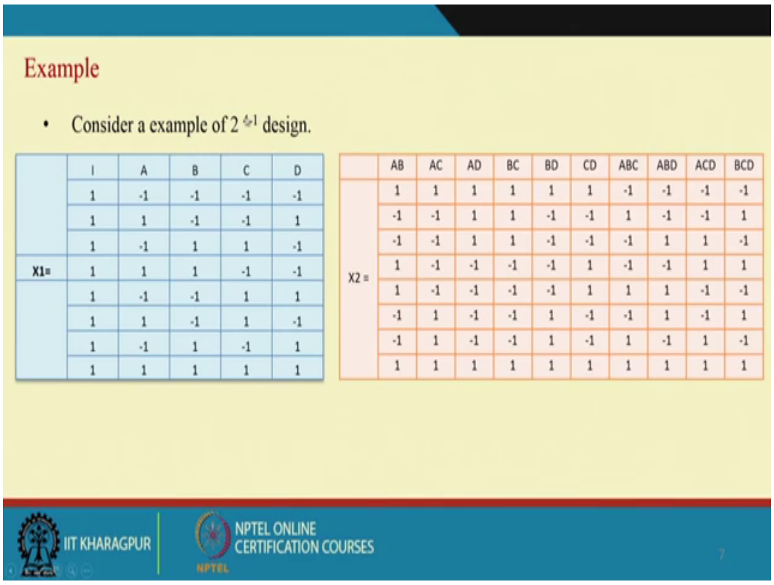

	I	A	B	C	D
	1	-1	-1	-1	-1
	1	1	-1	-1	1
	1	-1	1	1	-1
X1=	1	1	1	-1	-1
	1	-1	-1	1	1
	1	1	-1	1	-1
	1	-1	1	-1	1
	1	1	1	1	1

	AB	AC	AD	BC	BD	CD	ABC	ABD	ACD	BCD
	1	1	1	1	1	1	-1	-1	-1	-1
	-1	-1	1	1	-1	-1	1	-1	-1	1
	-1	-1	1	1	-1	-1	-1	1	1	-1
X2 =	1	-1	-1	-1	-1	1	-1	-1	1	1
	1	-1	-1	-1	-1	1	1	1	-1	-1
	-1	1	-1	-1	1	-1	-1	1	-1	1
	-1	1	-1	-1	1	-1	1	-1	1	-1
	1	1	1	1	1	1	1	1	1	1

Now, let us see another example suppose, you are considering 2 to the power 4 minus 1 design here, there are four factors ABCE and you also know that here, what will be the I, I ABCD will be I, if you consider that D equal to ABC that ABCD will be I in the same manner ah, if you multiply this, this, you will get this design, if you do not know, but you can use that minus plus some way and ultimately you will get the Alias structure, using Regression equation that may not be the good one, but it; obviously, if you choose D effectively, the best one you will be getting. Now, suppose, the we are basically considering that only the main effects are they interested and the other things. We are keeping under X 2 ok. So, suppose that the first model like this, reduced model is not good one, actually it is the total impact is there.

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So, then what you will do? You will find out the Alias structure by using X 1 transpose, X 1 is the and X 1s transpose X 2 is this and then X 1s transpose X 1 n bar is 1 by 8 into I 5 her it is I 5, because you see A B C D and I so; that means, that I cross 5. So, that is why 8 8 8 8, these are coming, because it is 2 to the power 3 full, full 2 to the power K factor here,. So, then X 1, this matrix if you see that, you will see that the first 1 2 3 4 5 6, this column, there everything is 0 and later on we will see that these are related to the 2 A interaction parts that AB AC AD BC BD CD like this.

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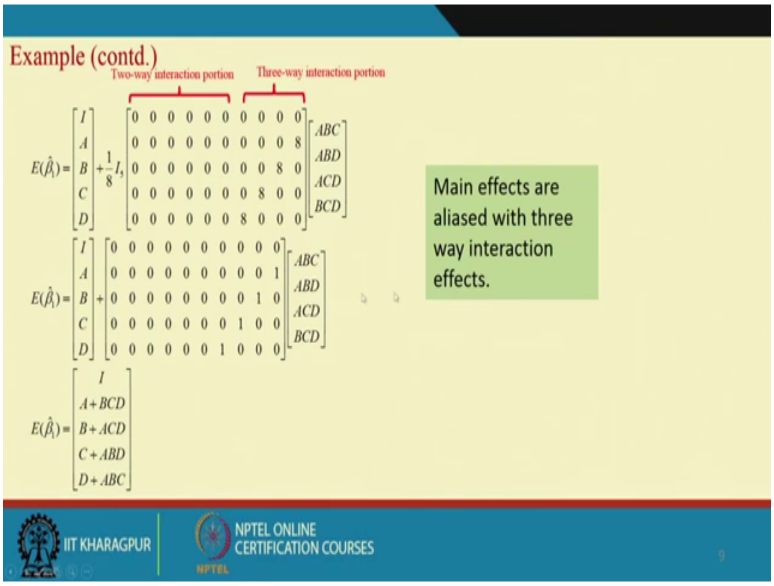

So, if you find out beta 1, here basically, please remember this beta 1 is is the main effects path. So, then if you, if you find out the expected value then using the Alias structure 1 by 8, I 5 X 2 X 1 transpose X 2. So, this one, if this is that the Alias structure, you see what is happening here in the two way interaction parts, there is nothing only 0 and 3 way interaction part, there are some values and once you complete these matrix multiplication and you are getting that beta 1 is this is not the same thing, what we have got in 2 to the power 4 minus 1 with I equal to ABCD.

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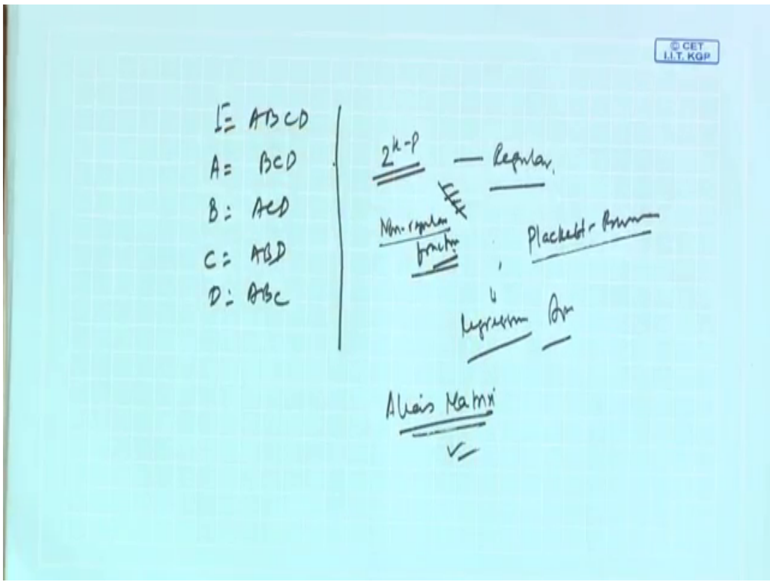

If I equal to A B C D multiplied by A, these will be BCD multiplied by B. It will be ACD multiplied by C, it will be ABD and multiplied by D, it will be ABC; it not need be same, yes as see the same thing, what you are getting here.

Here also we have used ABCD like this just to show the show the, 1 to 1 relation, but otherwise you can use beta 0 beta 1 2 3. So, this is beta, beta 0 beta 1 plus beta 1 2 3 beta 2 beta 1 3 4 like this. So, main effect are aliased with three way interaction and here you see no to a 2 way interactions are aliased with the main effects and that is what we have seen earlier also.

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Suppose, we, we, we do in a different way. Suppose, we are interested only in the interaction effects, in the first model and maybe the remaining part is coming under the and another part and, and in, in the same way, if you find out the alias structure.

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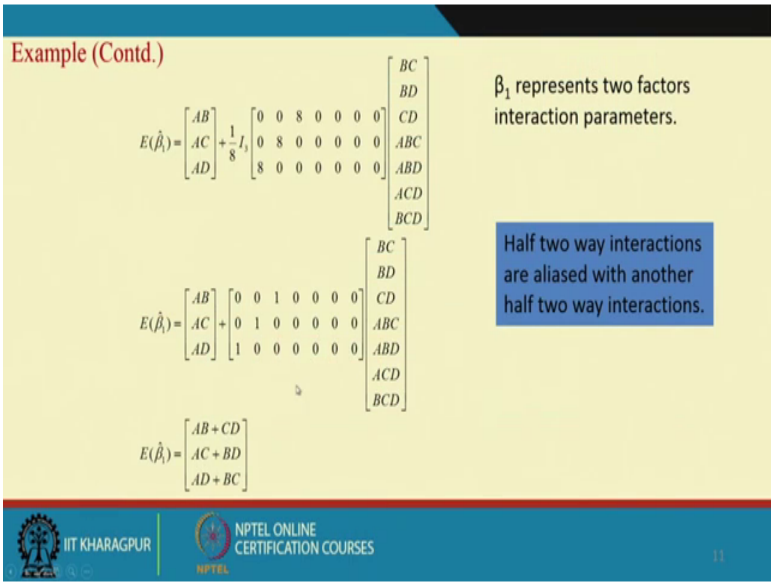

And then what you will get, you will get the Alias structure of these and when you find out the expected value of the interaction parameters and beta 1 represents 2 factor interaction parameters only AB AC and AD, here AB AC AD. We have not considered the ah, BC BD and other things let it be, we are considering only these three.

So, then what is, you are getting AB expected value of beta 1, just let me go back. Yes, we have considered only three AB AC and AD. So, then you see that this are interacted or aliased with other, second order interactions.

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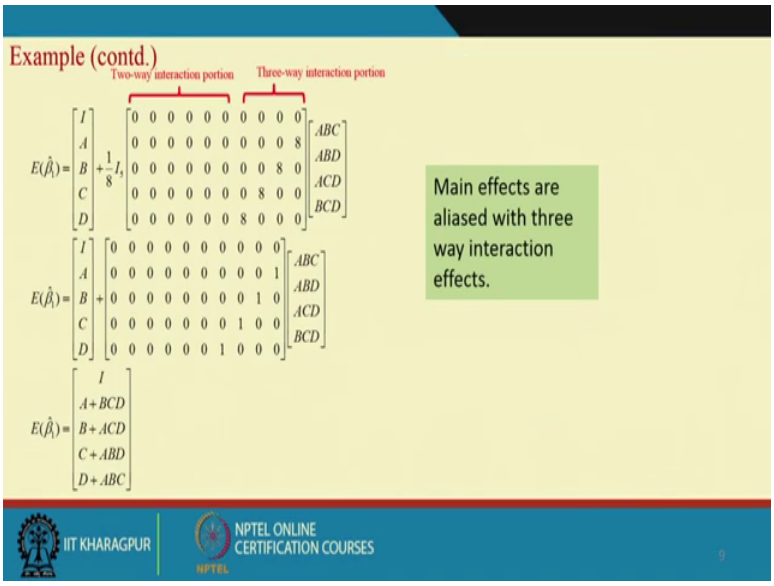

So, half two way interactions are aliased with another half two way interactions, that is what you can see using defining relations ok.

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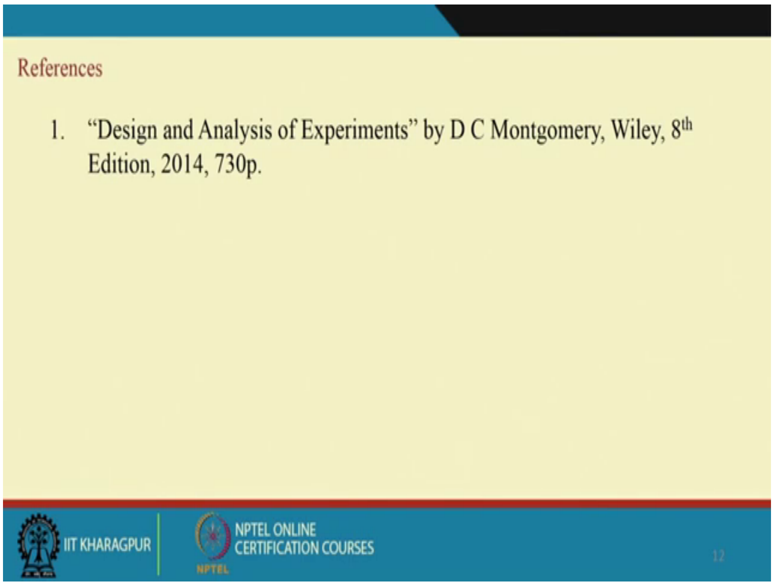

So, this is the way you have to understand that under a situation, when you do not have the defining relation or it is basically, a irregular one then the design is irregular, then what will happen ? You name this problem in finding out the Alias structure and the Regression approach is useful one and you please follow it.

So, to conclude I can tell you that fractional factorial design is very-very important. So, 2 to the power minus B, fractional design is the general 1 and you will find out ah, that when we write 2 to the power K minus P, it is a regular one, but there is no guarantee that you will be going for always design, which is 1, 1 by, 1 by 2 to the power P fraction, it maybe situation, where you will, you want a suppose, 1 12 fraction. So, in that case what will happen? It will not be a regular one.

So, there will be regular and there will be you know fraction, non, non-regular fraction ok. So, regular and non-regular fraction. So, non-regular fraction case defining relationship will be complex, there are certain other design like Plackett Burman design, Plackett Burman design ok. So, in this design what we will find out that the defining relation is distinguished. So, under such situation suppose, if we, if you do fractional factorial design, how do you get the Alias structure that is what the Regression approach is explained here, Regression approach and how you will do Regression approach here?

It is basically you think of a reduced model and in a full model or a better model, ideal model, true model, and then under true model find out the, the expected value of the parameters, considered in the ah, model, actual model what you have used, then you will find out that the expected values are aliased accordingly, as per the theory. It will be made and that, in that way you will get a Alias matrix, Alias matrix and using Alias matrix, you will be getting the Alias structure. So, that is what is for this lecture.

So, be very careful read chapter Montgomery very- very seriously and. In fact, all the chapters of Montgomery design analysis of experiments, which are under this course must be taught by you I mean you must read, because what I am giving you in this lecture, they are from that book and you it may. So, happen that some of the things you will not understand, you will just by going through the first ah, by listening first for every lecture, you require to listen several times, this is one you require to read the text given in that book. Second third one is you will require to solve the exercises given in the book and forth even if that will, you do not understand some other things, better you Google it, you will get some information, but be careful that the Google information, there will be a lot of junk information. So, you use effectively the forum post, your questions, we, we will answer it.

Thank you very much.