Design and Analysis of Experiments
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Lecture – 46
Fractional Factorial Design: Introduction (Contd.)

Welcome to this class. So, we continue fractional factorial design. In last lecture I have given you the concepts what is fractional factorial design and why it is needed, how does it work what is generator for fractional factorial design and in this process I told you that what is confounding and what is alias in this lecture we will continue with all those things.

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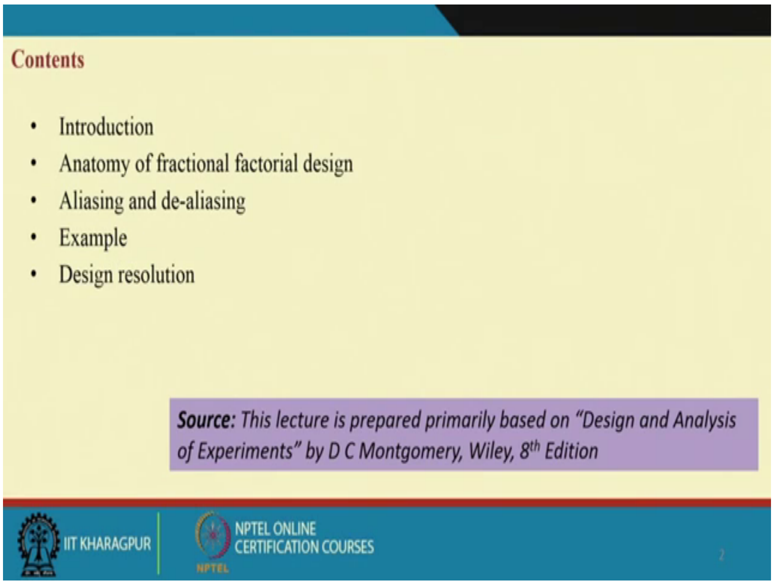

And, the content is we will start with the anatomy of fractional factorial design then aliasing and de aliasing with example and finally, design resolution.

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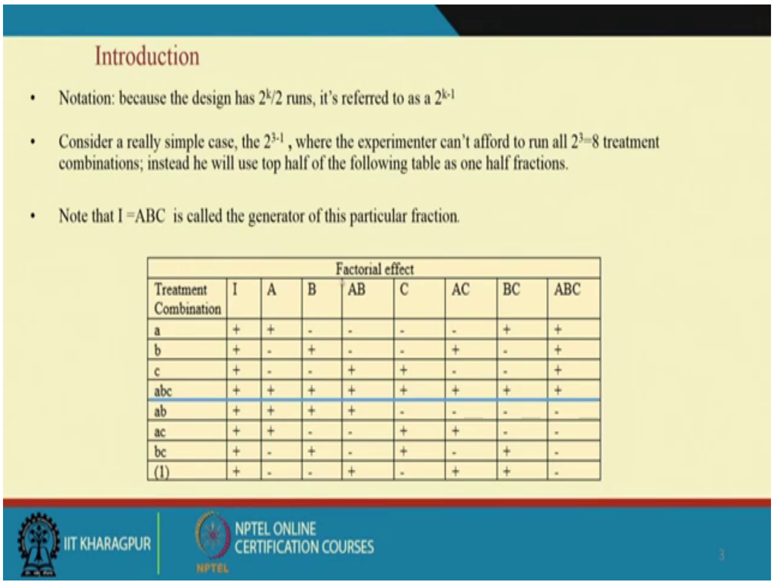

So, last lecture you have seen this slide what I have said that if you go for 2 to the power 3 factorial design there what happened you require 8, 8 runs; 2 to the power 3 design 8 runs.

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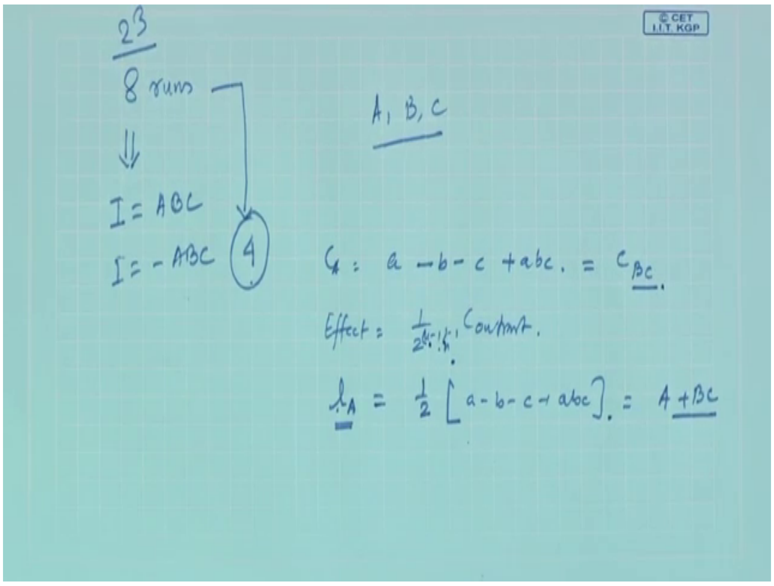

So, we have shown you that with generator I equal to ABC or I equal to minus ABC you can you can reduce the number of runs to 4 and in that process you definitely lose some information, but because of the properties like sparsity of fx principle projection property and sequential experimentation you are still in a position to analyze and get useful results.

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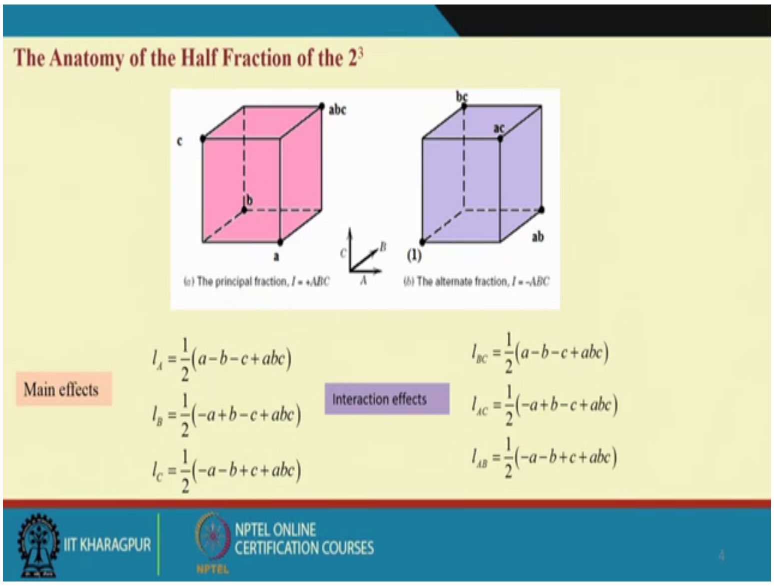

So, the 2 to the power 3 design with factors ABC let us see the geometric view you say the first half I equal to plus ABC you are conducting experiment at this factorial point this factorial point this factorial point and this factorial point and if you do this you will be doing you will be using this factorial this factorial this 4 factorial part.

Now, you have to find out the effect of a, effect of b and effect of c the main effects. Instead of writing that a equal to this here we are writing l A, l B and l C it has a specific purpose of writing l A and l B and l C which will be known to you little later.

Now, we all know that what is the effect is the contrast by 2 to the power k minus 1, here n is 1, so, 2 to the power k minus 1, ok. So, here what is happening you are actually doing because of fraction you are doing 2 the power 3 minus 1 means 2 to the power 2 design. So, in 2 to the power 2 minus 1 again; that means, 1 by 2. Now, a minus b minus c plus abc these are the this is basically the contrast for A as well as BC let me let me see, what will be the contrast for.

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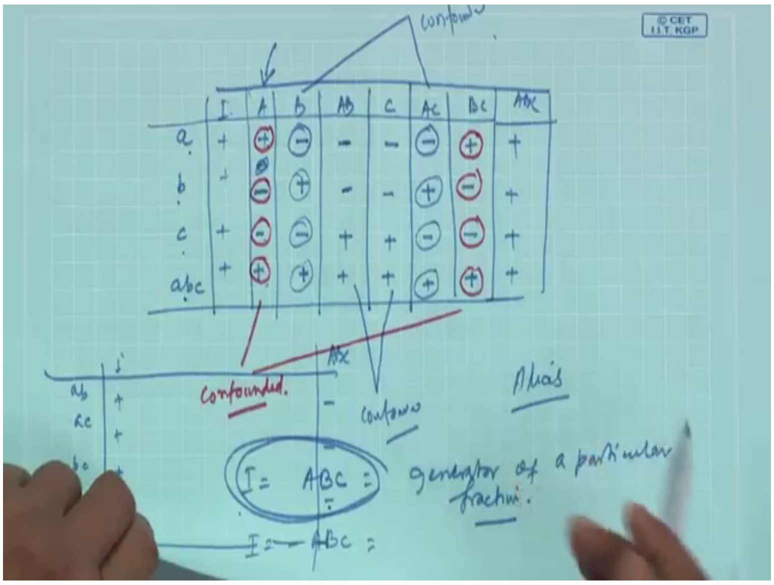

A this into this that is a this into this is minus a this into this is minus c this into this is abc. So, here your contrast for C A then a minus b minus c plus abc. I told you that a is confounded with bc if you see the contrast for bc you see that also A and this is basically equal to contrast for BC. So, we know the effect is 1 by 2 to the power k minus 1 into n contrast that you know earlier.

So, now, n equal to 1 k minus 1 here k equal minus 1 design; So, it will be this again minus 1. So, that mean 1 by 2 into here if I say this is the effect of a then a minus b minus c plus abc, but this also effect of BC include effect of BC. So, as a result we are writing only one intervene we are using the word l and writing l A because a's affected and BC is affected confounded. So, this is means this means we can say A plus BC there is a purpose.

So, once you have this kind of design you know the contrast you know the effect and in the slide itself slide you see l A equal to this, l B, l C. Now, from the table you can find out all the contrast and get the main effects and interaction effects ok; So, calculation is possible, but as I told you.

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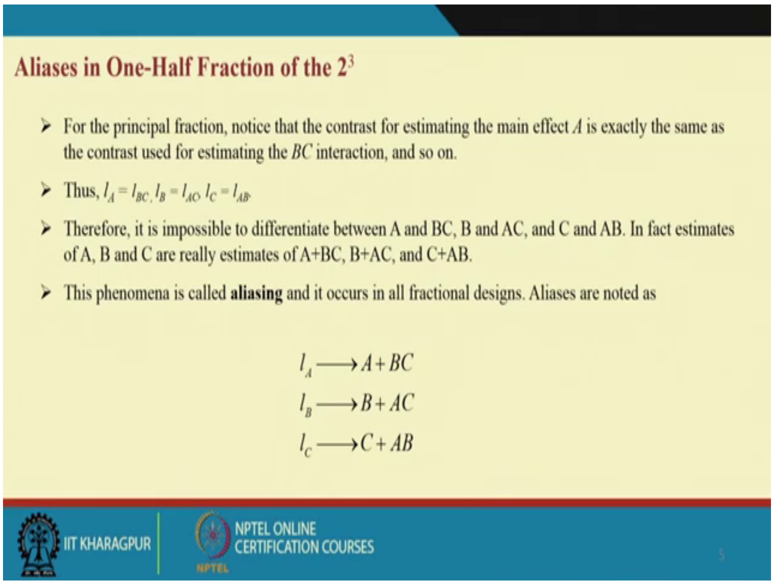

This calculation is alias, this is alias structure because l A is not effect of A only it will basically effect of A and BC that is why A plus BC, l B is B plus AC, l C is C plus AB. So, like this, ok.

So, if you if you see that for the that l A equal to again l A will be equal to l BC, l B equal to l AC and l C equal to l AB because of the contest structure you see. So, truly speaking there is confounding and then they are known as alias structure A with BC, B with AC and C with AB, ok.

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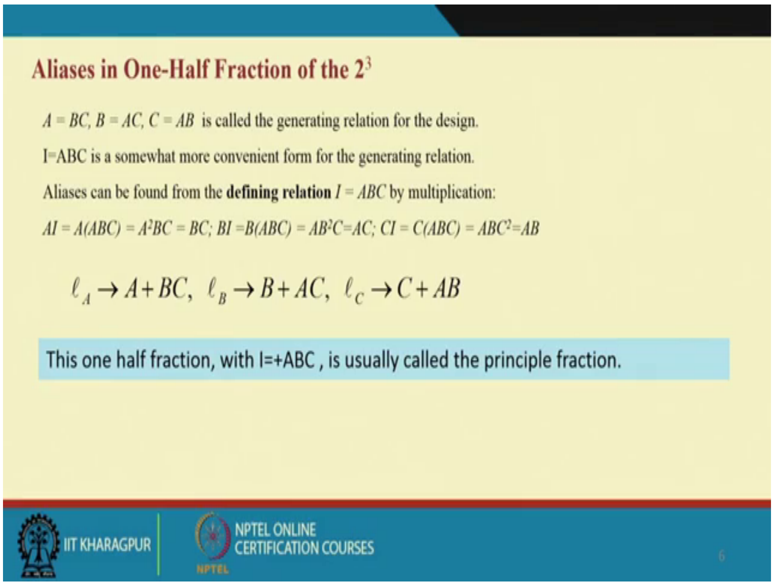

Now, how do get the alias structure, ok. How do get?

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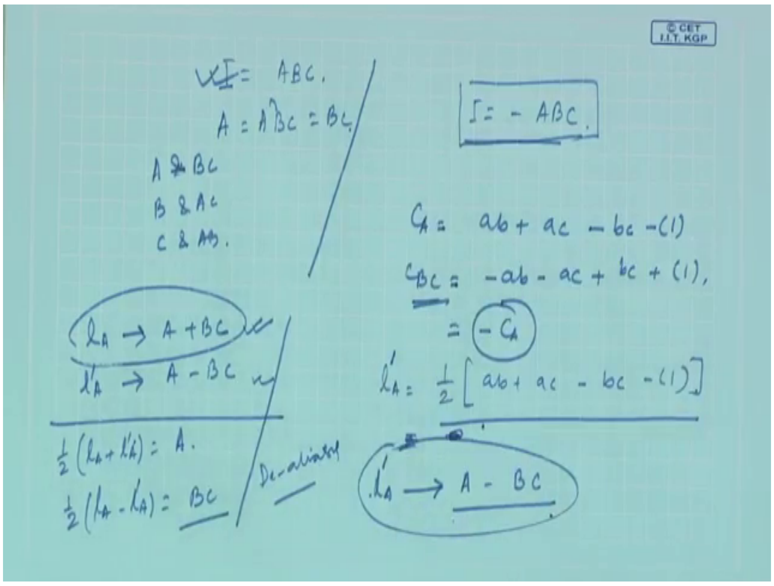

You see here your generator I equal to ABC. If you multiplied with a here you are getting BC because this is a square BC equal to BC. So, that is why we are saying a and BC are aliased A and BC similarly B and AC, C and AB, and if you will find out, ok. So, this is this is what is the alias structure in this design.

Now, what do you do instead of using this you use I equal to minus ABC if you use I equal to minus ABC then let me go back to this so; that means, this side the lower portion half you are using here what is the contrast for A, A contrast for A is your ab plus ac minus bc minus 1; then what is the contrast for BC; C BC if you see the contrast for bc is minus ab minus ac plus bc plus 1.

What is happening then; that means, this is minus C A. So, as a result if you use this half the second half then when you compute l A which is 1 by 2 ab plus ac minus bc minus 1 this is nothing, but also minus this is this is basically minus of this minus of C A is there. So, BC, C BC ok.

So, as a result what happened this l A when you compute like this ultimately relates to l A relates to A minus BC. So, using this half you are getting l A equal to A plus BC using this up you are getting l equal to BC. So, we will use a dash here that is all.

Now, if I use l dash A then it will be A minus BC. So, if you do two fractional design then in the first design using I equal to ABC and second is equal to minus ABC in the first design you get this estimate and the second is and you get this estimate then you are in a position to uniquely estimate A and B because half of l A plus l dash A will give you A here and half of l A minus l dash A will give BC here.

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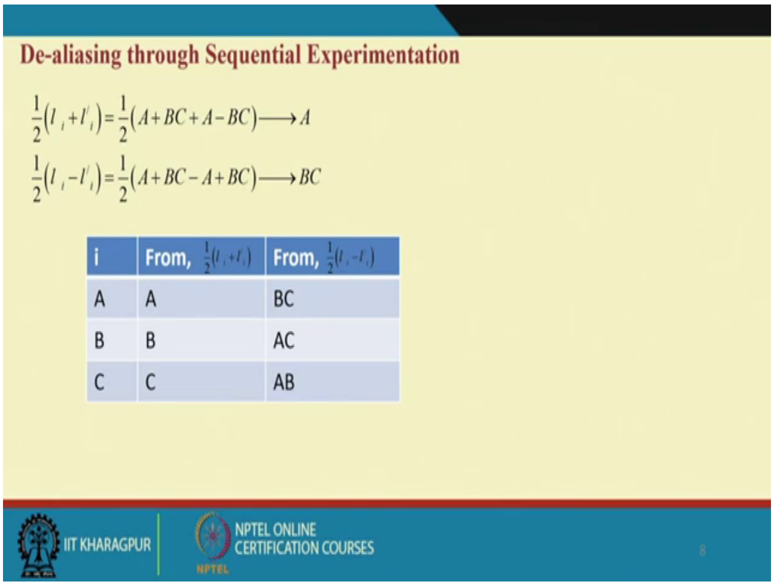

So, that is the advantage as I told you that if you go for sequential design first half maybe using these second main using this and then what will happen combining the two you will be getting the unique estimates ok. So, if you see this table using half l i plus l i dash i i from ABC and l i minus l i dash you will be able to find out the main effects and the second order interesting effects. So, this concept is known as de-aliasing this concept is known de-aliasing and either this one when you use one half and we get like this or like this is the aliasing one.

So, in order to de alias you require to have the other fraction also when it is half fraction when it is one fourth faction you require more, ok. Let us see one example.

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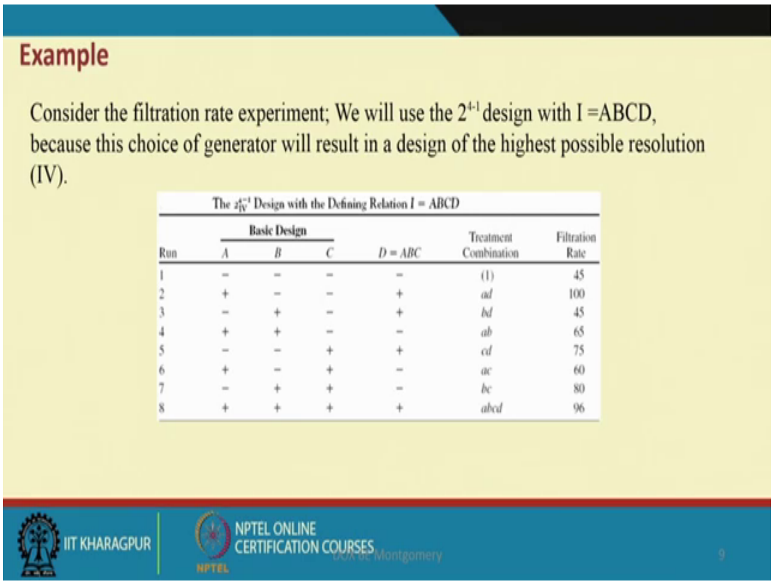

Here after doing so much of fractional things let us see one example. This is an example where there are 4 factors A, B, C and D and you require 2 to the power 4 means 16 experimental runs with single replicate, but you do not want to do this because you have problem resource problem.

So, what do you want you want to you want one flex one half fraction 2 to the power 4 minus one; that means, 2 to the 3 means 8 runs.

So, how do you do this it is given here.

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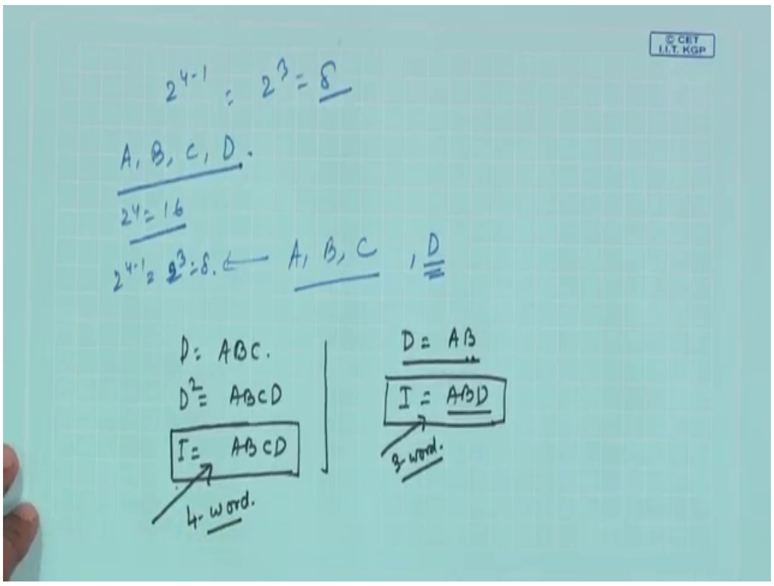

So, the proceeds procedure is if 2 to the power although I have A, B, C, D 4 factors and I require 2 to the power 16 runs for complete factorial d design here, but I you are going 4 to the power 4 minus 1 means 2 to the power three; that means, 8 runs. So, that means, at least in three factors A, B and C let this is full 2 to the power k factorial. So, then what will happen to D what will happen to D. So, it is said that D should be D D D should be related or D should be 8 it to the highest order interactions of A, B, C then it will be A, B, C.

It not be it should be in confounded with A, B, C third order interaction it maybe with ab also, but it is suggested that the inter the confounding should be with the highest order interaction for better result. So, if that means, your design will become like this you see here. So, for A, B, C you see that the full to the power k factorial written here minus plus minus plus up to 8 minus minus minus minus minus minus plus then you find out the highest order interaction A, B, C. So, you multiply although you are getting this.

You are saying that D will be set like this if you if you choose D like this then what is happening here your treatment combinations are this first one is the all low second one AD high and something like this and with this treatment combination you have done the experiment and you get the filtration rate that is the response values.

Now, you may say that what will be the value of AB AC BC you can multiply now you will be getting everything. So, if you use D equal to D equal to ABC here then if I multiplied this with D then it will be D square then it will be ABCD and D square means it is I equal to ABCD ok. So, I equal to ABCD this is what is the relation. So, then if you now, it is possible suppose you what do you do you multiplied all the columns here 1 minus you will find out that if you multiplied A, B, C and D you will get I mean all positives ok.

Ah let me tell you one more thing suppose if you do not if you say D equal to AB here what will happen then what will be I; I equal to ABD, please keep in mind the number of words in this relation this is very important. Here if I if I if D is confounded with ABC by design then I equal to ABC D this is 4 word you are getting 4 word relation if you instead of confounding with AB if you do a with ABC if you confound d the with AB then the defining relation here we have three word this word business is also equally important. So, I equal to this how many words here I three words and here four words, ok. So, now, let us do some calculation and then again we will come back to the concepts.

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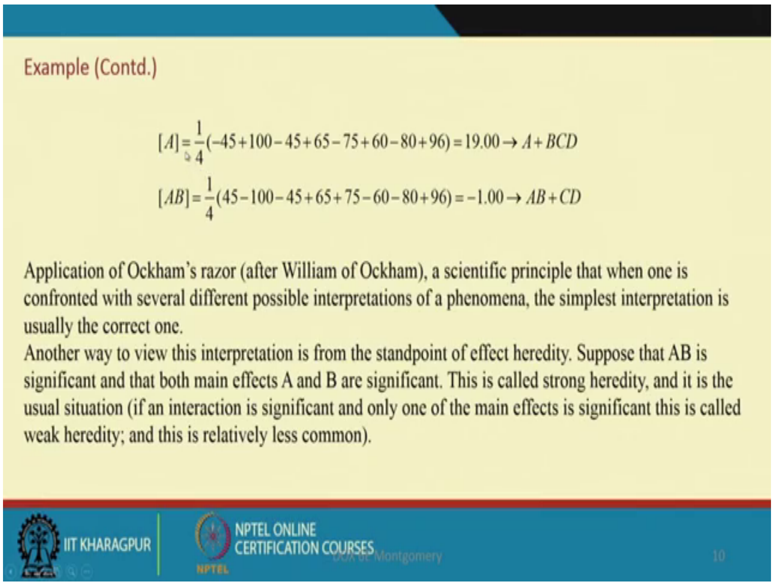

So, with this with these data; so, you want to estimate A.

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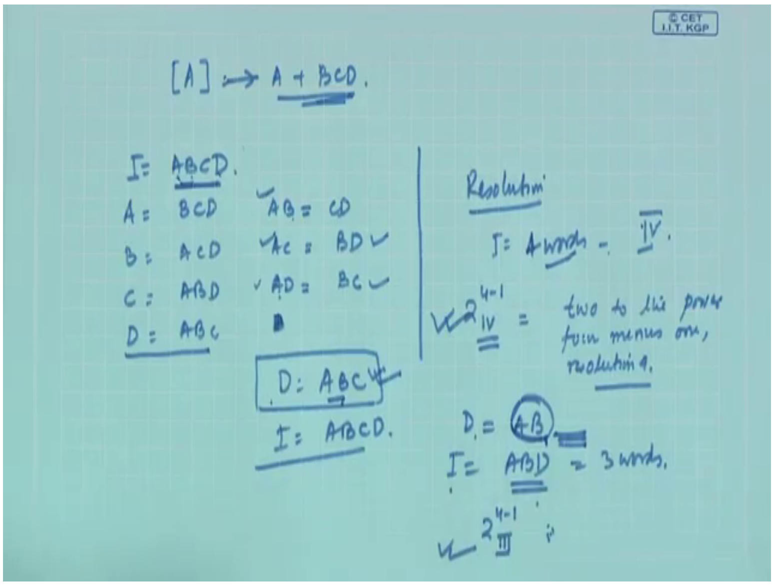

So, instead of A you will estimate A within bracket because this is nothing, but A plus BCD why because this earlier structure you have to find out you can find out this I equal to ABCD. If you multiplied this with A, you will be getting BCD; if you multiplied this with B you will be getting ACD. So, with C you will be getting ABD with D you will be getting ABC suppose you multiplied with AB what will happen AB, if you multiplied with AB you will be getting CD, if you multiply with AC you will be getting AC under BD if you multiplied this AB AC AD you will be getting BC, if you multiplied this with BC you will be getting AD, if you multiply this AB over AC over AD over BC already done BD done CD done.

Now, if you multiply this with ABC you will be getting d so; that means, multiplying the one or other factors you will be getting another factor in this kind of this is the aliasing structure, so, fine. So, suppose I want to from using this data using this data when you compute A you are basically computing A plus BCD. So, this is what a 19 AB minus 1 that is AB, AB also you can compute in this manner you can compute other effects also all the effects with the alias structure.

Here two important things to be noted here one is that the effect value whether it is large or small if effect value is large then the effect with this alias is significant. If the effect value is small then there the particular effect with alias that that may not that is not significant in the first case it is significant. Now, which one is significant are both of them significant or one of them significant and another one is not significant, how do you find out in the second case are both of them insignificant because the total is insignificant or individually they are significant and collectively they are insignificant because one going in opposite to the other direct direction.

So, it says it says little interpretation issue little complex from that point of view also, but we have some rescue one of them each Ockham's razor. Ockham's razor means that is basically by William Ockham who said that a scientific principle actually the that when one is confronted with several different possible interpretation of a phenomena, the simplest interpretation is usually the correct one, ok. What does it signify say if I suppose here we are saying A and B C D together the effect is 19. Now, again the perceptive principle says that the higher order interactions will be effect will be negligible compared to the lower order interaction.

So, it is quite obvious that A effect is significant as A plus BCD effect is significantly large nineteen. So, A is significant because from perspective of effort BCD effect will be much lower than A in some k in any case if we can found that BCD effect is negligible then we can take this 19 as effect value for k,. Another view point is the standpoint of effect heredity what is this suppose that AB is significant and that both main effects A and B are significant, in that case it is a strong heredity the reason is A and B are significant and AB also significant. So, another one is that suppose, AB significant, but one of the one of the A and B are not significant. So, then it will be weak heredity, ok.

So, this also this is basically less in less common that is usually this is less common. So, that that is why these two principle helps us to focus on the effect when it is estimated using this alias structure ok.

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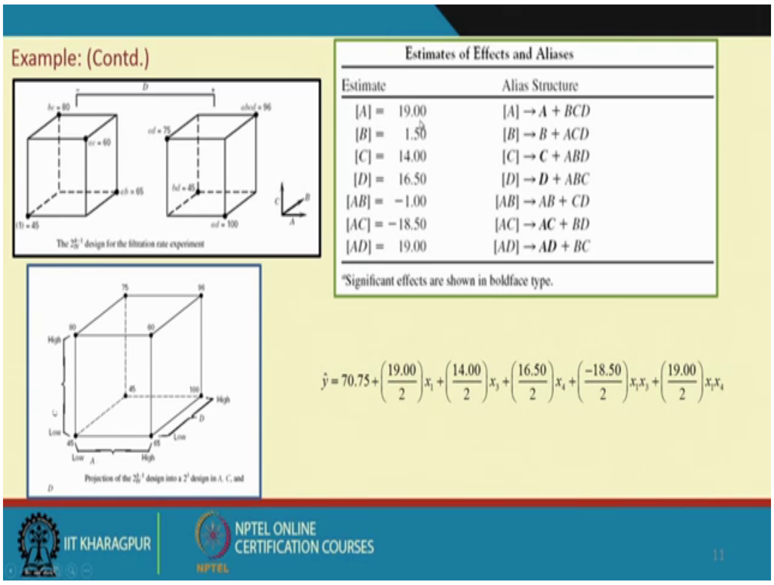

So, with this explanation now see the effects and their aliased structure A 19, B 1.5, C 14, D 16, AB minus 1, AC these like this. So, here A, C, D, AC and AD they are large. So, significant compared to B and AB it is really significantly large. So, as B value is low. So, we can we can assume that it is not significant and here AB is value is also very low, ok. So, this is also not significant.

Now, as AB is aliased with CD. So, AB plus CD you may say that this value is low maybe AB is in the positive side and a CD affecting negatively. So, that is why this is low, but this occurs hardly occurs this kind of things are you. So, so all practical purposes it can be assumed that A, C, D, AC and AD are significant and using this AC all those things this regression equation can be developed. Now, once you develop this regression equation now your another work you can do you can develop response surface, you can go for contour plot, you can set the parameters at different levels and to get that you can optimize this also. So, things are possible.

So this is what is in case of fractional factorial design how do you how do you design for fractional factorials then once you conduct the experiment the rate the experimental data how do you analyze using contrast only you analyze the effects and then from there as there are there are alias structures. So, you cannot separate out some of the effects from the other effects. So, using the Ockham's razor and heredity principles as well as the sparsity of effect principles, so, you are in a position to conceptually eliminate some of the some of the low value effects.

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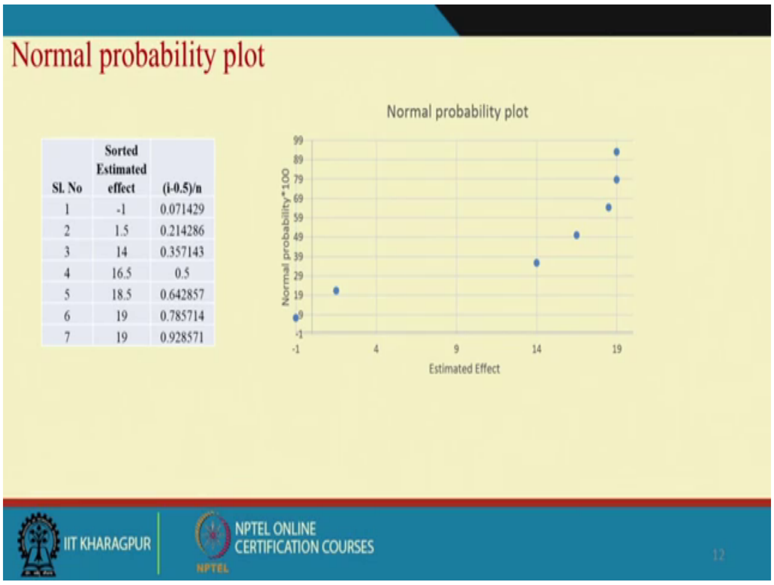

This you have seen earlier the probability plots also the normal probability plots and efforts to find out the which are the, they are significantly larger than others.

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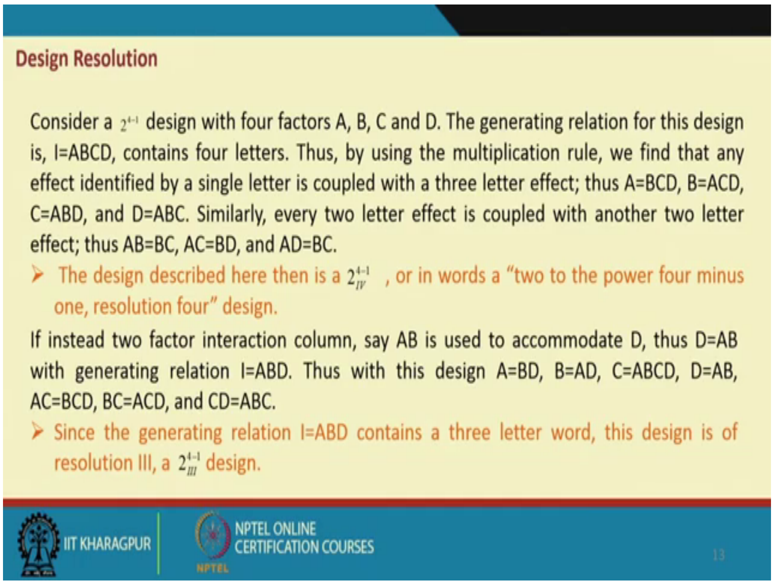

The last concept to into in this lecture is the design resolution.

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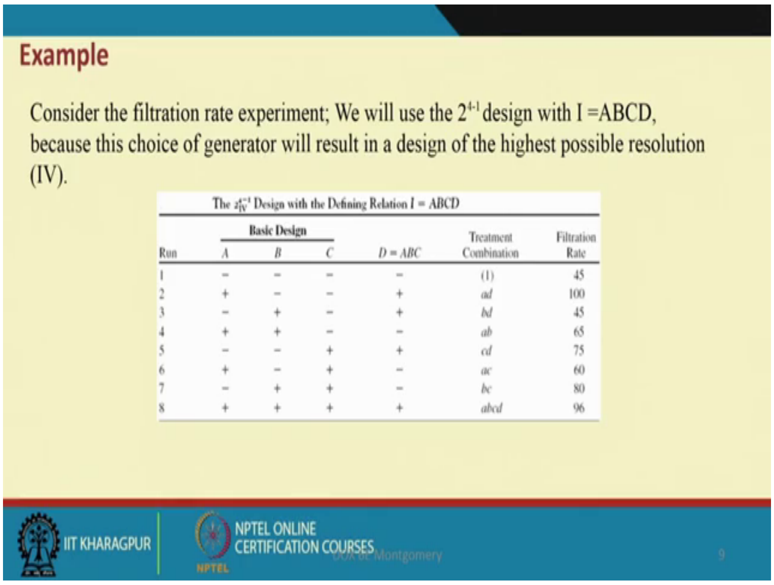

	Basic Design					
Run	A	B	C	D = ABC	Treatment Combination	Filtration Rate
1	−	−	−	−	(1)	45
2	+	−	−	+	ad	100
3	−	+	−	+	bd	45
4	+	+	−	−	ab	65
5	−	−	+	+	cd	75
6	+	−	+	−	ac	60
7	−	+	+	−	bc	80
8	+	+	+	+	abcd	96

So, you have seen example that four, four factor A, B, C, D in this particular case you see that the example where is the example now this example you have seen that we have considered four factors and I equal to ABCD either defining relation, ok. So, the generating relation or defining I equal to ABCD it contain four letters.

Usually these this talks about the resolution, the higher the resolution that is better the better the design. If I equal to content four words four words then it is it is region resolution is 4. So, that is why this in this example it will be said at 4 minus 1, 4; that means, what is then that 2 to the power to the 2 to the 2 to the power 4 minus 1 resolution 4. This will be written like this fractional factorial design 2 to the power 2 to the power 4 minus 1 comma resolution 4, ok. Resolution for this is the dn.

Now, as I told you that here why this the resolution each you got the high region that photo resolution because you have converted D with ABC when you started the design and accordingly you got I equal to ABCD. If by if instead of this if you say D equal to AB then I told you I will be ABD now, how many words? It has three words. So, here if you if you do this and accordingly the algebraic sign table will change, the contrast will change, alias structure will change everything will change then and then what will happen that this design will be known as 2 to the power 4 minus 1, 3.

So, this fractional factorial design 2 to the power 4 minus 1 resolution 3 ok. Obviously, this is better than this. The reason is here you see D is l D the confounded with higher order interaction in comparison to this here third order interaction here. Second order interaction partially of a principle says that if this is significant this will also be significant because AB will order. So, that this design is not good compared to this in general. So, it is recommended that you go for higher order designs, ok.

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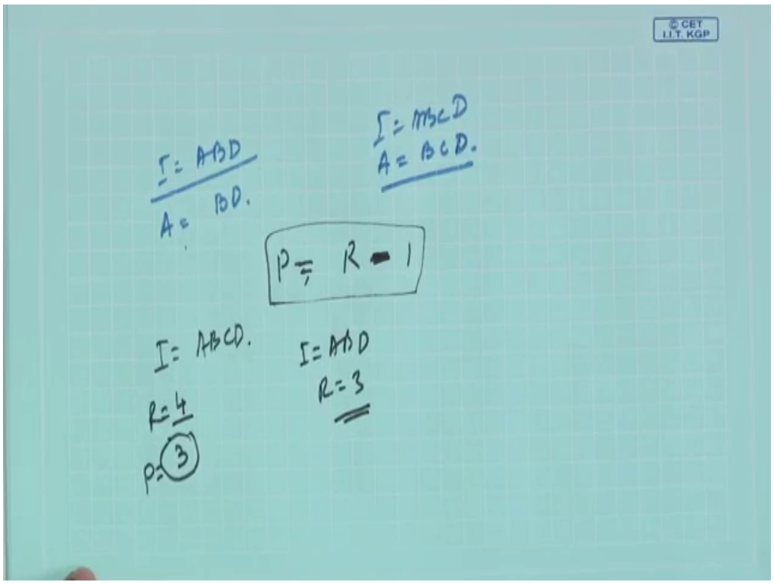

Now, suppose I will ask you question like this that given that I equal to ABD what is the alias structure if you multiplied this with a what do you get you will get B, but if I equal to ABC, if you multiplied with A what you are getting ABCD if you are getting BC DC the alias structures are also changing. So, depending on what kind of what kind of resolution you are considering that will ultimately change the idea structure and obviously, if you have enough expertise on the process the physics of the process and where you think that it is better to confound D with AB instead of ABC then, obviously, you go for this other design that is what is and what basically the common sense explanation.

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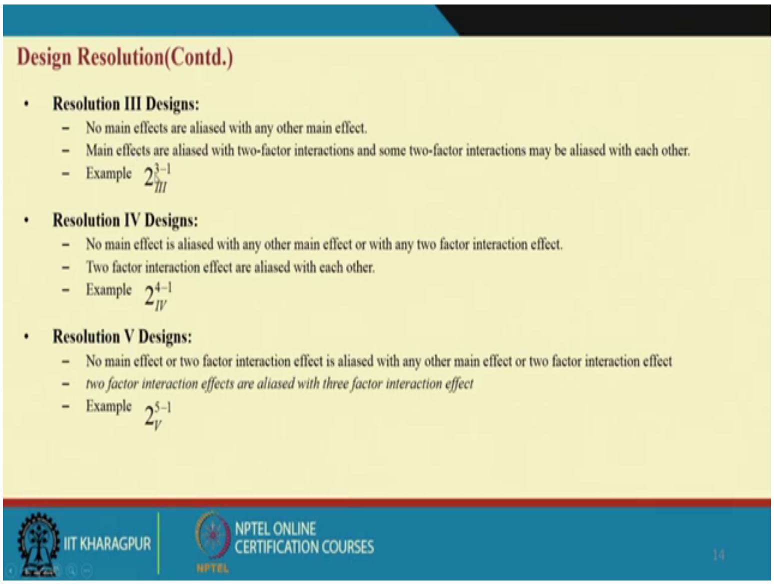

Some of the guidelines if you go for resolution III designs, no main effects are aliased with other main effect main effects are aliased with two-factor interaction and some two-factor interactions may be aliased with each other, for example is this. Integration 4 design no main effect is aliased with any other main or to order in two way interaction effects two-factor interactions effects are aliased with each other example is this, if you go for higher order, like resolution V design; no main effect or two-factor interaction aliased with any other effect main or two-factor interaction effects no two-factor interaction effects aliased with any three factor interaction effects example this.

So, from here you can understand the aliased how the aliased structures changing if you go for higher resolution and it is obvious that the more the resolution the better is the reason.

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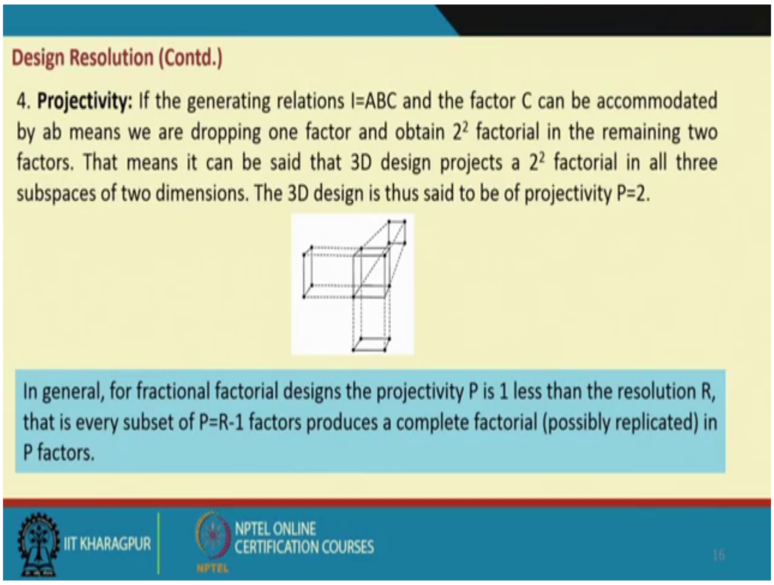

So, I already explained most of the thing like higher order interaction negligible.

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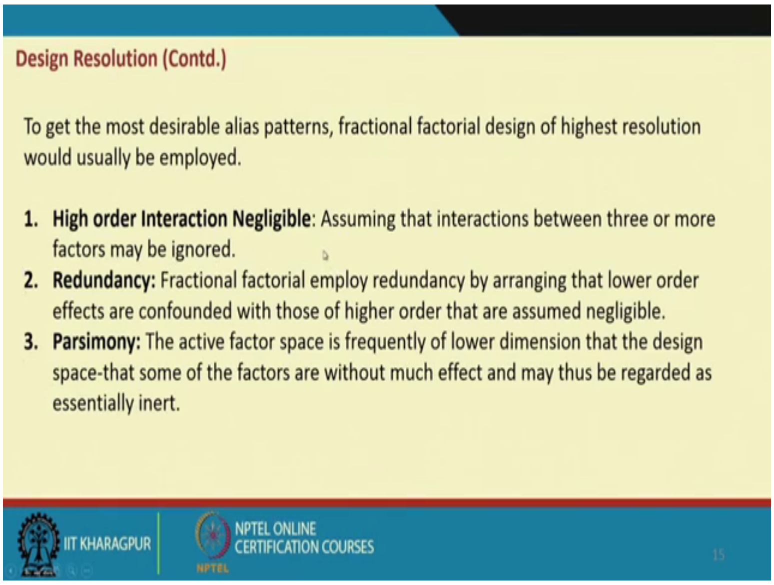

Redundancy that means, a fractional factorial design; Employee redundancy in arranging that lower order effects are confounded with those of the higher orders that are negligible; Parsimony the active factor space is frequently of lower dimension that the design space means actually if you have for many factors the design space is a bigger one, but actual actives that face that space basically that will be in that will be lower than the bigger one and then projectivity.

Projectivity is important one as I told you that any fractional factorial design of higher at higher order is basically full that to factorial again at the low order. So, that is known as the projectivity. So, if you project a higher dimensional issue in to do a dimensional issue you will have full information at the lower dimensional issue and there is there is interestingly suppose if it is 2 to the power 3 minus 1 design then projectivity is 2 because in 2 dimension it is it is full factorial all factorial points are captured.

And, now that is why whatever and it is there is a relation between the resolution and the projectivity that projectivity is resolution minus 1, keep in mind. If projectivity is P resolution is R then R equal to 4 P equal to R minus one P equal to R minus 1. So, so if you pure you are defining relation is ABCD then your resolution is 4, R is 4, if it is ABD, R is 3. See here in 3 projectivity will be 3 that mean in 3 dimension full information you are getting, but here you are getting full information in some of the cases only in 2 dimension. So, the solution has very good effect in case of lower order analysis also, ok.

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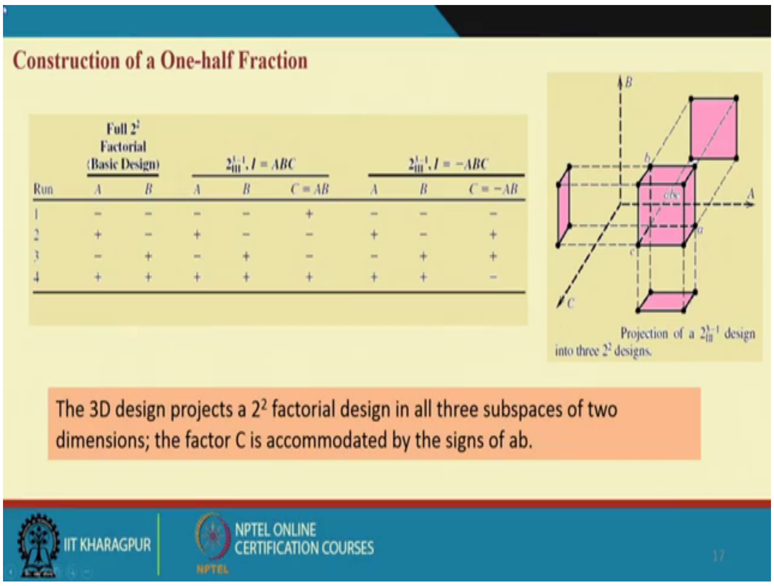

So, this is and this is the way; that means, the projectivity part we have explained this one. You see that 3 third 2 to the power 3 design and if you do even fractional factorial in one half of t you will in the at least in the AB level or here AC or BC level you will be having full information provided at each and that resolution is 3.

The lecture taken from Montgomery’s book, Design Analysis of Experiment; So, thank you very much and I equates all of you to carefully read the chapter of Montgomery for fractional factorial design, the concept another thing what we have discussed here. These are not simple, there looks to be simple, but they require a careful reading and I hope that you will be able to able to do the assignments and also will be able to score high, that is all.

Thank you very much.