Design and Analysis of Experiments
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Lecture - 45
Fractional Factorial Design: Introduction

Welcome in today's lecture, we will discuss fractional factorial design. I will introduce this concept introduction.

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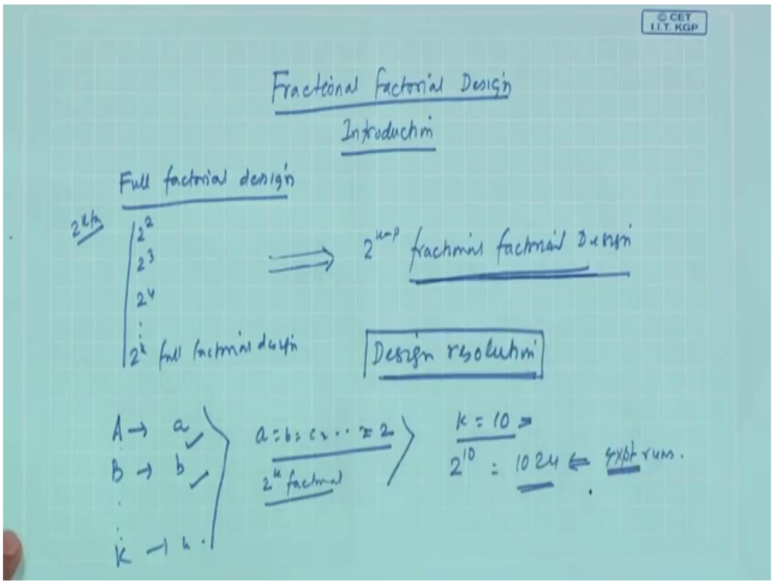

So, you we have seen earlier, factorial that is full factorial design full factorial design. So, it is 2 to the power 2 design, 2 to the power 3 design, 2 to the power 4 design, as such 2 to the power k full factorial design. So, we our today is from full to fractional it will be something like 2 to the power k minus p fractional factorial design when we go for fractional factorial design from full factorial design.

So, there will be lot of changes and there are conceptual issues there are estimation issues and there are design issues. So, all those things will be discussed in this introduction lecture on fractional factorial design. In subsequent lectures, we will show you the examples and calculations.

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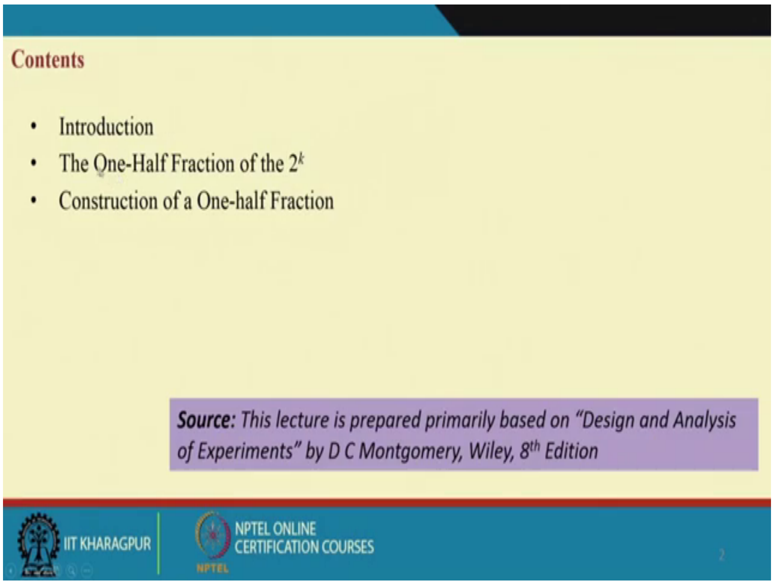

Today, we introduce this subject this concept and then I will go for one half fractional factorial design. And then how the one half fractional factorial design will be constructed one example will be shown to you. And then there is another important concept called design resolution.

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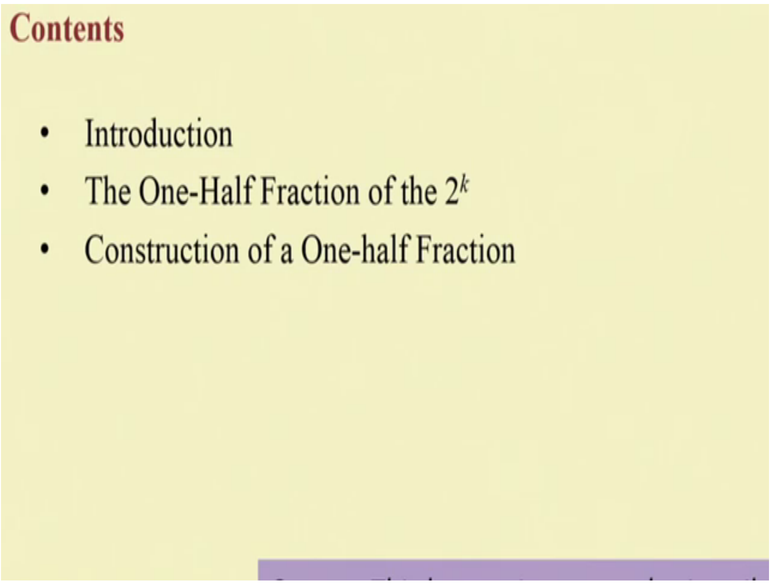

Design resolution will be discussed. It is it may take time, but we will try if not otherwise in the next lecture, we will start with design resolution.

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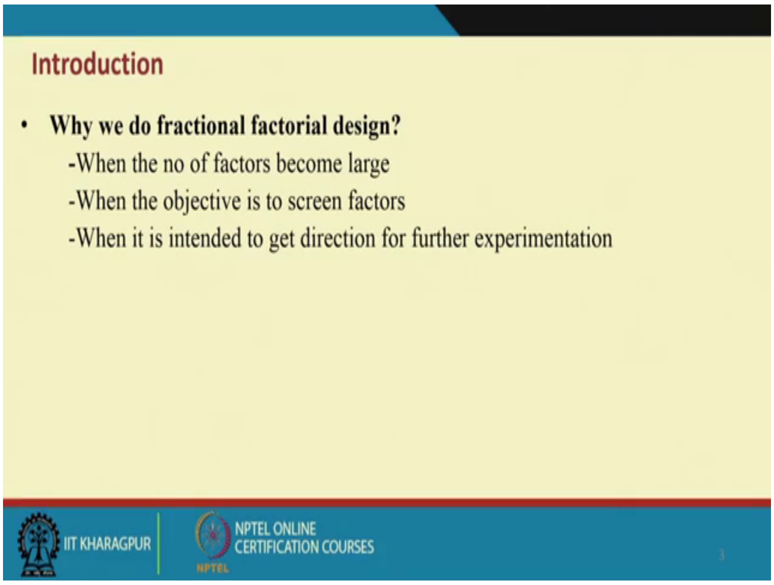

So; obviously, you may be thinking that, the lectures if you if you see from the beginning to today's lecture that initially we have started with several factors. And each factors with several levels and then from there like we have started with suppose factor A with a number of levels B with b number of levels. So, like this K factors with k number of levels then from there we have gone we have reduced this levels to 2 a equal to b equal to c or do those things will be equal to 2. So, level 2. So, then 2 to the power k factorial design, that is evolved 2 to the power k factorial design is evolved.

We said that why 2 to the power k is required because if we go for a level for factor for a b level for factor for b and as such there are many factors with many levels. Then the number of treatment combination will become very large and it is impossible for experimenter to conduct experiment from the cost point of view, time point of view, resource point of view. And it will be so; costly that maybe the it is say it will be a feasibility issue also for conducting such experiment. So, we can we say that 2 to the power k factorial design from full 2 to the power k factorial design. We have considered so; these are all 2 to the power k factorial design.

Now, even into 2 the power k factorial design suppose a if k equal to 10 then 2 the power 10 means it is 1 0 2 4. So, these many experimental condition these many experimental runs with single repeated it is required. So, if I have k number of factors each at 2 levels.

So, a require a large number of experiment to be conducted. So, that means, here also there is problem for experimentation physical experimentation.

So, what do you require to do that? See is there any other way to reduce the number of experimental runs without losing much the information that we want to achieve or the objectives that we want to accomplish.

So, this is possible, if you go if you reduce the number of runs further and that concept is fractional factorial design , fractional factorial design. So, when we go for fractional factorial design, these are the situations. When number of factors becomes large, you cannot conduct large number of experiments when the objective is 2 screen factors. So, it is it is basically to see that, which are the factors that are having effects, which are having not effect, to screen out, the trivial factors for further analysis or experimentation and when it is intended to get direction for further experiments.

So, that mean the fractional factorial design help you to find out find out that, what are the key factors that are contributing and at the same time it will tell you that in which direction you go for further experimentation. So, that you will get better results or when you go for fractional factorial design.

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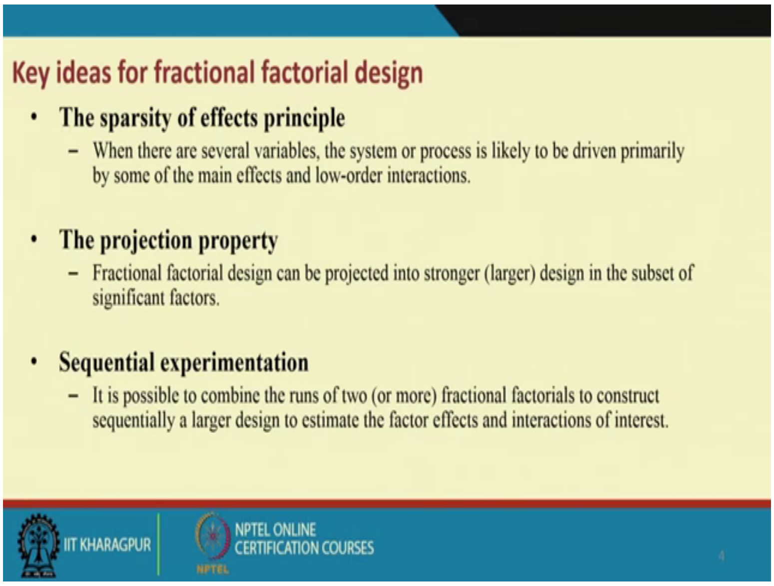

Now, the next question is fine. So, when I require I have large number of factors, I want to screen out the factors and I want the feature direction of the experimentation. I will go for fractional factorial design, but then you may be interested to know what do you mean by fractional factorial.

That is what I have not told you. So, I told you that if that 2 to the power k minus p fractional factorial design. That is what I told you. Just consider a case with 2 to the power 3 factorial design. So, how many treatment combination 2 to the power 3 equal to 8 treatment combinations ok. So, then if I write the factors are A B then AB I can write here then C AC BC ABC this kind of notation is seen earlier.

So, how many combination 1 2 3 4 5 6 7 8 if you recall we said this is minus plus minus plus minus plus minus plus this 1 minus minus, plus plus, minus minus, plus plus right then in this 2 when you multiply a and b you get plus minus minus plus plus minus minus plus. Then C will be minus minus minus minus plus plus plus plus ok.

If you recall that I said that 2 to the power j minus 1 minus plus minus plus alternatively when j equal to here 1 2 here 3 you see first factor second factor third factor. So, 2 to the power 3 minus 1 means 2 to the power 2 bit 4. So, 4 minus followed by 4 plus then what happened AC if you multiplied A and C this is plus this is minus this is plus this is minus minus then this one will be minus this will be plus this will be minus plus minus plus plus plus. So,1 2 3 4 4 plus 4 minus will be there. So, like this what will be the ABC if I multiply AB and C then this is minus then that mean AB into C.

So, AB plus minus minus minus minus plys AB into C minus minus, plus plus, minus minus, plus plus, plus minus, plus minus, minus plus, minus plus, plus plus, so minus minus 4 4 4 1 2 3 4 and bc minus minus, minus minus, plus minus, minus plus, plus minus, minus plus, minus minus, minus plus, plus minus, plus minus, minus plus, minus plus, minus minus, plus minus, minus this also minus then minus plus minus plus plus plus plus plus plus. This is what is the algebraic that sign matrix that what we have seen earlier.

Now, here you do some kind of readjustment. Suppose, you just keep ABC only plus one side top up and only minus 2 ABC plus plus plus plus then minus minus minus minus this differentiation this kind of difference you make and then you will get it a table like this.

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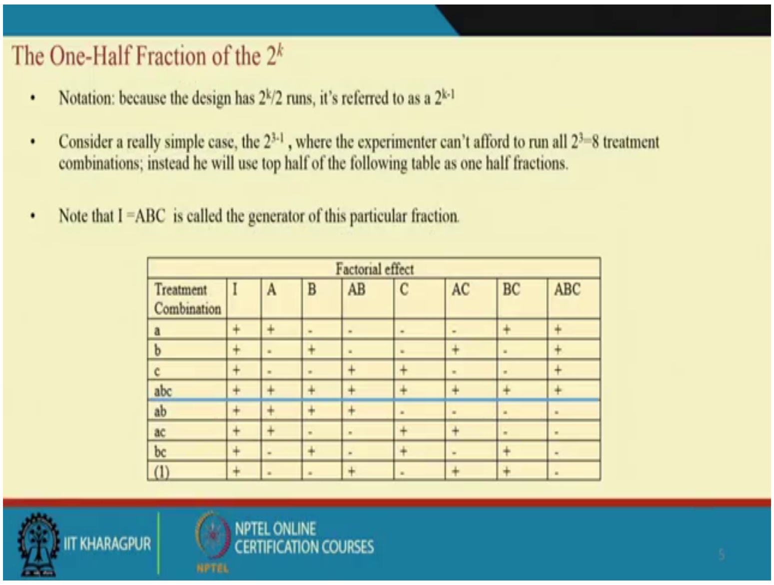

You see this is the table. In this table, we create 2 halves for ABC 4 plus with 4 minus. Now, let you will conduct experiments only with the combination ABC and ABC. Then what is the design is 2 to the power 3 minus 1, or 1 by 2 2 to the power 3 that is known as half fraction ok.

Suppose, you have you have 4 factors instead of 3. Now, you consider 4 factors ABCD this will increase. So, determent combination 2 to the power will be 16. Now, suppose that you can you can run only 4 experiments, only 4. And if you do this what are you doing then 2 to the power 4 minus 2 that mean 1 by 4 2 to the power 4 design you are bring that is one 4th one 4th fraction one 4th fraction. So, in general that is why.

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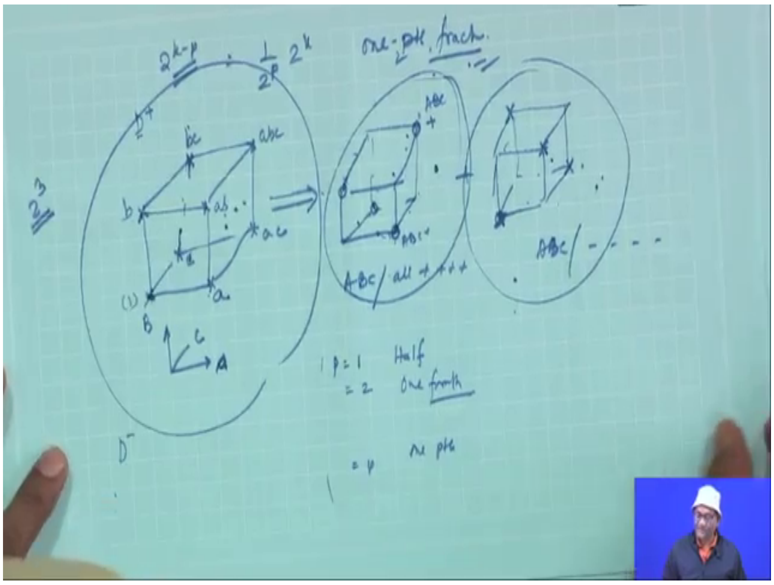

In general, if I we say that 2 to the power k minus b fraction means; that means, 1 by 2 to the power P 2 to the power k. So, one P th 4 that is P th fraction mean 2 to the power k fraction which is the (Refer Time: 14:10)

So, if I go by this way, you all know that the cube the geometric view also you know, geometric interpretation you know you ; that means, in case of in case of fractional factorial design , instead of taking all the factorial points for experiment. You split it into 2 halves, one half , where you maybe depending on this you may n plus another half. So, 2 halves here ABC all positives positive values 4 positive. Here, maybe ABC or 4 negatives. So, any one half you can consider and conduct the experiment. So, that is half fraction.

So now, but if it is 2 to the power 4 then if you want one 4th, you are basically finding out 4 different path segmentation, here if I consider if I consider in this case, if I consider this is my a factor this is our B vector and this side is C factor. So, then what will happen suppose, if I start with these this is one, this is your A. This is B and this one will be AB and this will be C this will be AC and now, with these one 8. So, AB BC then one more is there BC and ABC.

Suppose, if I go for you will see the slide now. So, if I go for if I go for ABC plus. Then you are experimenting ABC and ABC. So, where is a your A by BB where is CC and where is this then rest these are plus ABC plus like this. So, rest will be ABC minus where this then this one this one and this one this is called one half of 2 to the power 3.

Now, if you say one half of 2 to the power 4 then what will happen there will be one more factor D and d suppose this is for d positive similarly another one will be for D negative and D will be (Refer Time: 17:34). So, in general when you when you make it 2 to the power k minus b, that is known as 2 to the power k minus b fractional factorial design.

So, then what we have said? We have said so far that that, 2 to the power k full factorial design, may not be possible in many situations. Particularly when you have large number of factors under such situation, if the objective is to screen out the factors and to find out further direction of experimentation, then you can adopt fractional factorial design. Fractional factorial design, the concept is that there will be some ways to identify less number of experimental rounds. And one of the ways we have demonstrated here considering ABC all plus and all minus although I have given this arbitrarily that why all plus and all minus separately taken out. So, that mathematics I have not said or that concept I have not discussed so far.

I will discuss this, but please keep in mind that there is a scientific way to find out find out a fraction of the full 2 to the power k factorial design. That fraction can be half fraction where P equal to 1 P equal to 1 half fraction P equal to 2. Then it is one 4th fraction. So, like this 4th 1 P th can P equal to P 1 P th ok. So, this is what is known as fractional factorial design to repeat it again if you have 2 to the power 3 factorial design. It is the rule that you will conduct experiment all in all the factorial points.

If you cannot conduct experiment on all the factorial points due to resource (Refer Time: 19:50) you go for fractional factorial design. And there instead of all the corner points factorial points structural. You choose one half of these or one 4th of these one 4 theory is not and not good one, but one half of these and these then you will be doing experiment on selected point factorial point. So, that is known as fractional factorial design. So, how you come to the factorial points? Where experiment will be conducted? Is an issue and here using ABC all positive and all negative. We have shown the split of this 2 to the power k factorial to 2 to the power k minus 1 factorial ok.

Now, you will be interested to know that if I split this stuff the 2 to the power k factorial points into 2 to the power minus k factorial points. So, you are after doing experiment so; that means, you will not do experiment on both the both the case, but you will do experiment either here or here if you do both actually you are doing the same thing again. So, that is why you will take one of the 2 the question is; that means, whole different important these treatment combinations are not considered. So, then if I if I do not do all this whole in reference to this example, in the 4 different points like this and these, I will be losing information.

So, if I lose information then, how do I conclude from the results? Or how do I analyze the experimental data? And subsequently conclude from the analysis that what are the factors that are affecting? What are the factors that are not affecting? So, these there will be or there will be there will be concepts. So, then your question will be then how the fractional factorial, why fractional factorial design works that is an important issue. Fortunately, we have some important properties and that will help us conducting fractional factorial design. What are those properties?

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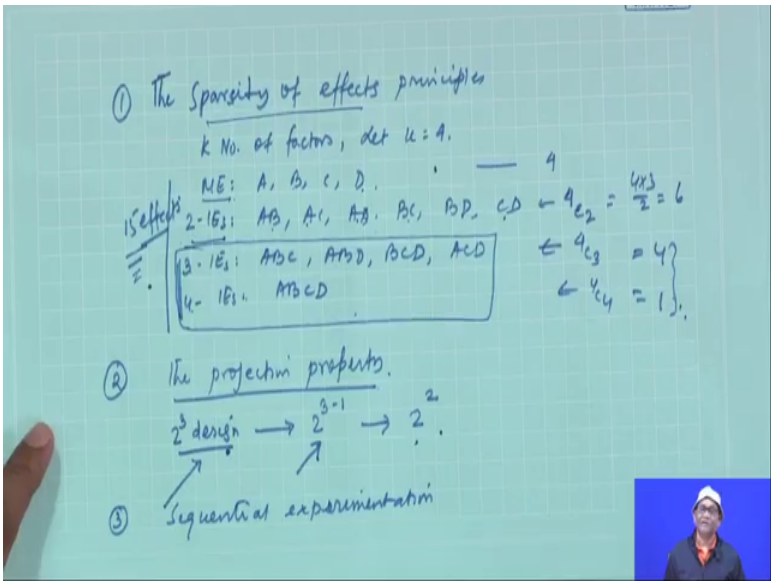

First property is the sparsity of the sparsity s of effects principle. This is the first, what is this the sparsity of effects principle? If you have large number of factors suppose you have k number of factors, let k equal to 4. Then how many main effects main effects will be ABC and D how many 2-way interaction effects 2-way interaction effects will be AB

AC AD BC BD CD. How many 3-way interaction effects ABC ABD BCD ABC ABD ACD ACD BCD how many 4-way interaction effects AB CD.

So, we know this will be k equal to 4 this will be 4 c 2 this is 4 c 3 this is 4 C 4. So, 4 C 2 means 4 into 3 by 2 it will be 6 1 2 3 4 5 6 this will be 4 and this will be 1. So, 6 plus these are 15 interactions will be there 15 effects the sparsity of effects. So, there are 15 effects some are lower order effects like main effects, lower order interaction effects and then higher order. Sparsity of effects principle is that in general the higher order interaction effects are negligible. What does it means? The contribution of higher order interaction effects on the response where you responses will become negligible. So, you may not a given a situation you may not estimate this third and 4th order interactions.

So now, you require 15 effects to be estimated if you do not do third and 4th order that mean 5 different effects you do not require to estimate. So, as a result even with the less number of experimental data you will be able to conduct some analysis and get some useful results. Second principle is the projection property. What is this projection property ? The projection property means, if suppose you consider 2 to the power 3 design. Now, you consider you are you are not able to do all done. So, you are going for 2 to the power 3 minus 1 design.

Although in 2 to the power 3 design case, this is this is fractional, but this is not this is not fractional in 2 dimensions. In 3 factor case it is fractional. Not giving you the full part if you project into the lower dimensions, then it will become full 2 to 3 power k factorial so; that means, what may what you; that means, you have full information all from the factorial points point of view in the lower dimension, even if you do fractional factorial at the higher dimensions. So, that is known that is a projection property means in lower dimension you have full information to get all effects at the lower dimensions. And the third property is sequential experimentation.

What does it mean by sequential experimentation? Sequential experimentation means here, this 2 to the power 3 design you split into 2 to the power 3 minus 1. Let you have you have done experiment with this the first stuff. And got some reasons you may not be happy in that case what happen you require conducting another experiment. So, you hope you consider the second part.

So, after conducting the second part experiments when you combine the 2 you have this full. So, this is what is that is full then. Suppose, when I conduct the first one and I you are happy with the results you do not require to go for the second one because your screening as well as for further experimentation point of view the information you got from this.

If you do not get from these, you go for the other half and then the combine these 2 combine will give you the full one your complete information as in terms of 2 to the power k factorial design. So, these 3 properties enables the experimenter conducting fractional factorial design. Because, this will give useful results it is not arbitrary it is a good one. Now, I come to that another one that one very important concept called that how defining relations or generator. So, you see the, this table 2 to the power 3 minus 1.

So, and this is this is basically 2 to the power 3 full design now we have we have written ABC in special manner. So, that you will get all 4 positives first followed by 4 negatives and if you see this table, you see this table there is one column called I in the, and if you see the symbol in I and ABC both are same they are plus plus plus plus they are plus plus plus plus in the first half.

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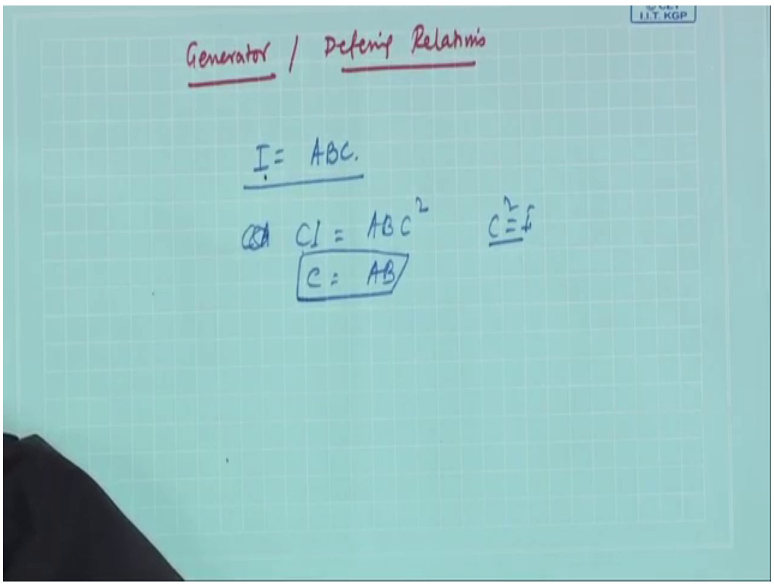

So, we can say I equal to ABC I equal to AB C ok. Now, before hand or other way I can say that earlier example I have shown suppose if I multiplied if I multiplied these I into C if I just CAC I then you will multiply these AB into C mean C square C I is C is equal to ab because c square equal to I C equal to I.

So; that means, if you see the plus and minus sign in the table. So, you see that C equal to AB, because a minus minus minus minus plus plus plus plus C equal to AB. So, if I do one half here. So, my design matrix or the factorial effect matrix will be supposed let us let us consider the top one let it be.

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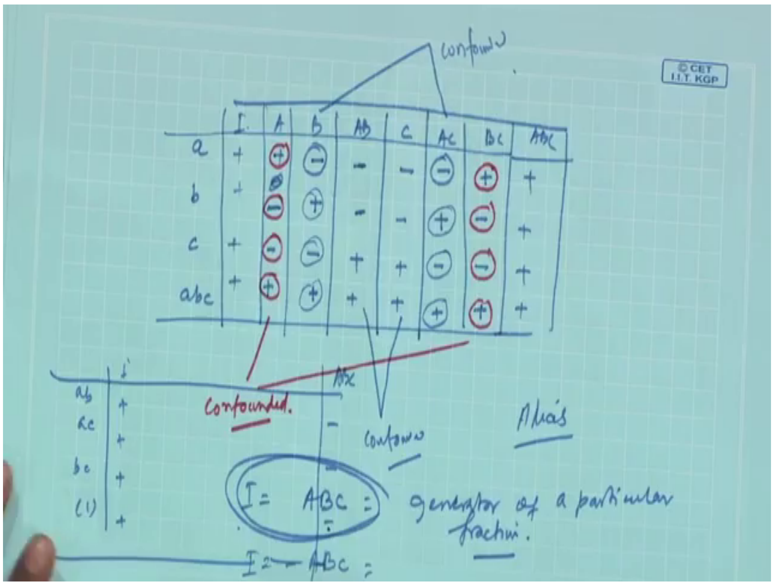

So, ABC ABC then I plus plus plus plus then your A it is plus minus minus sorry plus minus minus plus then B, B is your minus then followed by plus followed by minus followed by plus then you're A B is minus plus minus minus plus plus then c is your minus minus plus plus.

So, minus minus plus plus AC is minus plus minus plus BC is plus minus minus plus and ABC is plus plus plus plus ok. So, interesting pattern you see in the contrast and you all know that this multiplied by this will become give you the contrast suppose A here minus CA plus C the BC part plus minus minus minus minus plus plus so; that means, the contrast for these and this are same so; that means, what will happen here you cannot estimate A and B C separately they are confounded.

Second one if you see B and AC you see you see B and AC minus minus plus plus minus minus plus plus they are confounded. Similarly, this 2 are this 2 confounded ; that means, if you do one half you will not with this you will not be able to estimate all effects uniquely some of the effects will be confounded with others this is known as alias structures alias.

And here this concept I equal to ABC this is called the generator of a particular fraction. So, this half is created considering I equal to ABC if you consider I equal to minus ABC you will create the another half where things will be your another half will be AB AC BC and one.

This few we are not consider and all those things will be there what you will find out I all plus plus plus plus, but when you go for ABC. Here, they will be all minus minus minus minus so; that means, this is a very important concept called generator. This generator helps you in doing the fraction fractionalization and when you do such fractionalization such kind of fractional design, what will happen? You will find you will see that you are not in a position to uniquely estimate all the effects some of the effects are confounded with some other effects ok. So, with this I conclude the first lecture and. So, what we have discussed so far.

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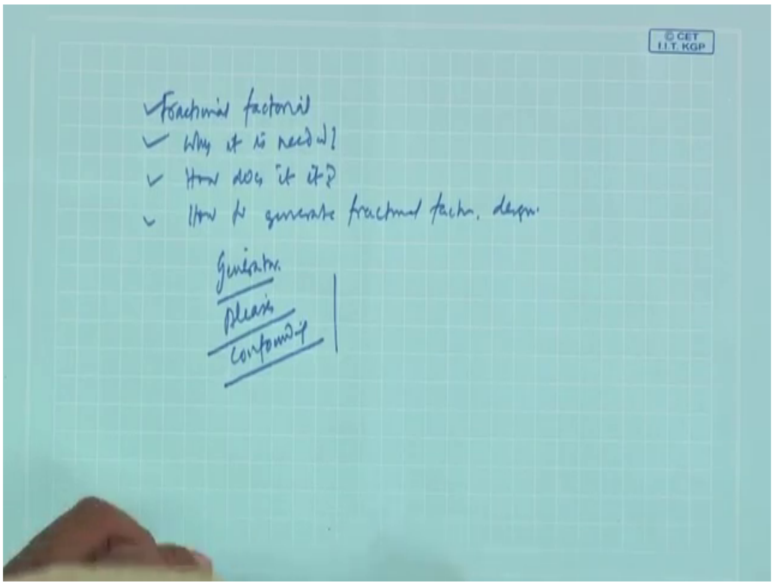

We have discussed that what is fractional factorial? Factorial and why it is needed? It is needed it is needed how does it work and how to generate? How to generate fractional factorial design? So, in this process also I talk about that generator talk about aliases, talk about confounding and all those things we have discussed. In the next lecture, I will show you that the estimation part, the effect estimation part. The with and an example and the design resolution with reference to fractional factorial design.

Thank you very much.