Design and Analysis of Experiments
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Lecture – 44
Blocking and Confounding in 2 k Factorial Design (Contd.)

Welcome. We will continue with blocking and confounding in 2 to the power k design; 2 to the power k factorial design.

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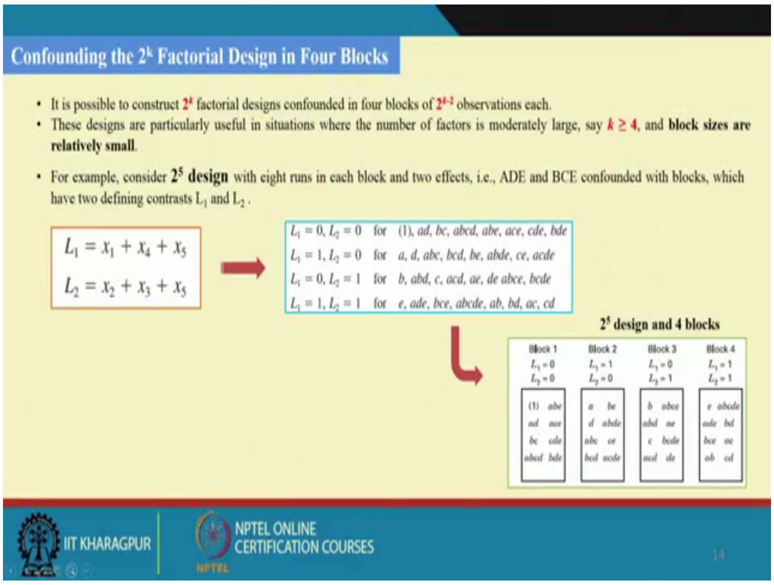

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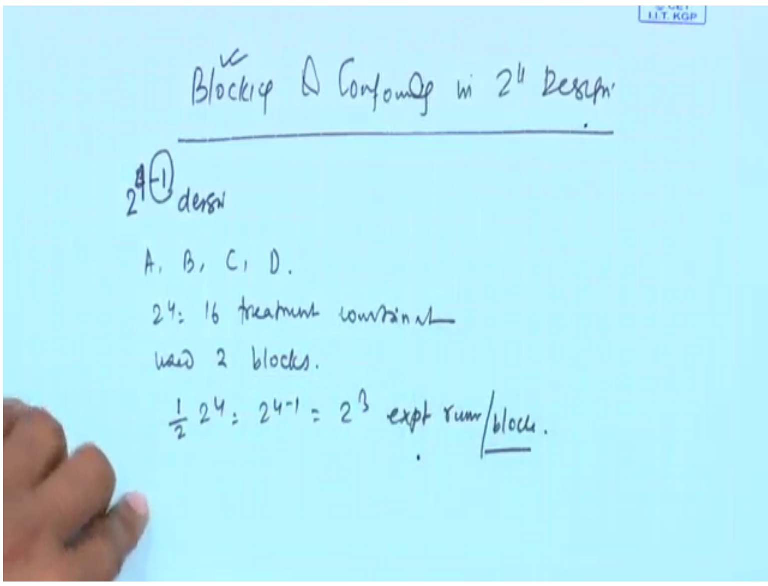

So, if you recall my last two lectures, where we have discussed the concept of blocking and confounding in 2 to the power k design with an example. So, finally, I have shown you that 2 to the power 3, I think 4 minus 1 that design. So, this minus 1 for blocks. So, I have shown you a case where 4 factors, 2 to the power 4, 16 treatment combinations; and we have shown you that we used 2 blocks. So, that is why that design was basically per block half 2 to the power 4 equal to 2 to the power 4 minus 1, 2 to the per 3 experimental run per block.

And, and with single replication case with 1 block and with 2 blocks, the results were shown. And, with probability plot, it was demonstrated that what are the factors that are significant, and how you will compute the errors and all those things. Experiment, the example was given in such a manner that you know previously, what is the block effect. And then the how the block is confounded with higher order interactions. And from there we have shown with block and without block the effect of estimate of higher order effect like in this case A B C D. And it is shown that actually in such design situation, they are confound it; and that confounding effect is clearly visible, that is what we have shown.

Now, we will continue with this, but instead of 2 blocks, if we go for 2 to the power P blocks, what will happen?

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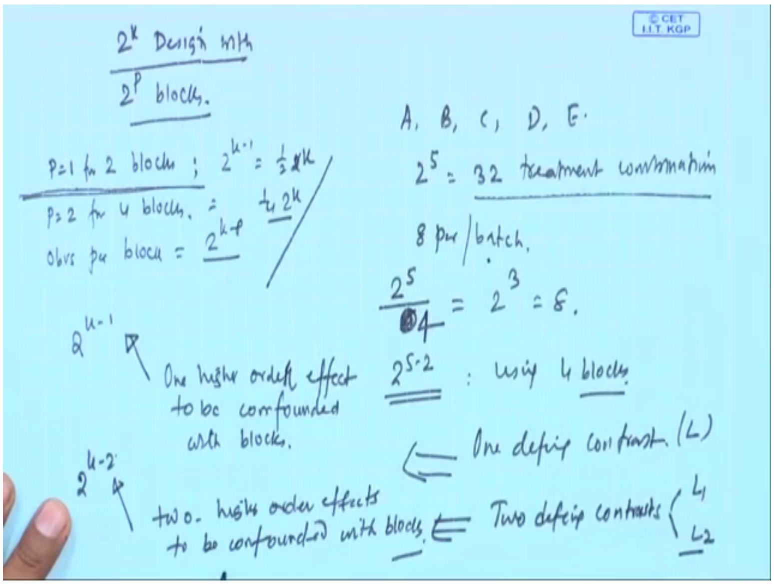

So, 2 to the power k design 2 to the power with 2 to the power P blocks. So, here if you use 2 blocks then your P equal to 1. If you use 4 blocks, your P equal to 2. So, that means, the observation per block is 2 to the power k minus P. So, that mean observation power block here in p equal to 1, it is 2 to the power k minus 1 or 1 half of 2 to the power k. Here 1 fourth of 2 to power k. So, this is what is the things we have discussed so far particularly I have talked to you about this much, P equal to 2.

Now, when you have more number of factors, suppose let there are 5 factors A B C D and E; even each with at 2 levels what you require you have 2 to the power 5 equal to 32 treatment combinations. So, it may so happen that, only now if it is the case of raw material let it be. The 2 batches of raw material different batch will not be sufficient, because they are not able to accommodate all those runs 16 per batch. So, you may require 4 batches. So, 8 per batch that means what you are doing? You are basically making 2 to the power 5 by 8. I think it is 2 to the power 5, 8 per batch. So, that mean you are dividing this by 4 so, that you will get 2 to the power 3, which is 8.

What I mean, you are making it 2 to the power 5 minus 2; kind of design with blocking and we are using 4, this is possible using 4 blocks. So, that mean what is the, in in case of 2 to the power k minus 1 where we are using 2 blocks, you require 1 higher order effect to be confounded with blocks. When going for 2 to the power k minus 2; this 2, you require two higher order effects to be confounded.

So, that means, in order to assign, who is of the treatment combination to be assigned to which of the blocks. As you have seen earlier that, you require to know who is effects to be confounded. In 1 in 2 blocks case, we say the highest order interaction that to be confounded. Then what will happen; 2 to the in this case? When 2 higher order effect, they may be highest and the next lowest one; that there will be several combinations. So, you can choose any one of them. Or depending on your, that domain knowledge, you may find out that some of the higher order interactions may not be highest order, but they have least significance. So, in that case you choose any two of them, but you have to choose two.

When one higher order effect is confounded, you have one defining raise contrast L. When you have two higher order effects to be confounded; you will have two defining contrasts, may be L 1 and L 2. So, in this lecture first I will show you how the 4 blocks case will be constructed. So, let me read out. It is possible to construct 2 to the power k factorial designs confounded in 4 blocks of 2 to the power k minus 2 observations each.

These designs are particularly useful in situations where the number of factors is moderately large, and block size is relatively small.

Situation is, you have more number of factors, but your resource is less in amount. Like raw materials per batch can accommodate less number of runs; or less number of experimental runs can be completed. For example, consider 2 to the power 5 design with 8 runs in each block and 2 effects, A D E and B C E confounded with blocks, which have two defining contrasts L 1 and L 2. So, let me explain.

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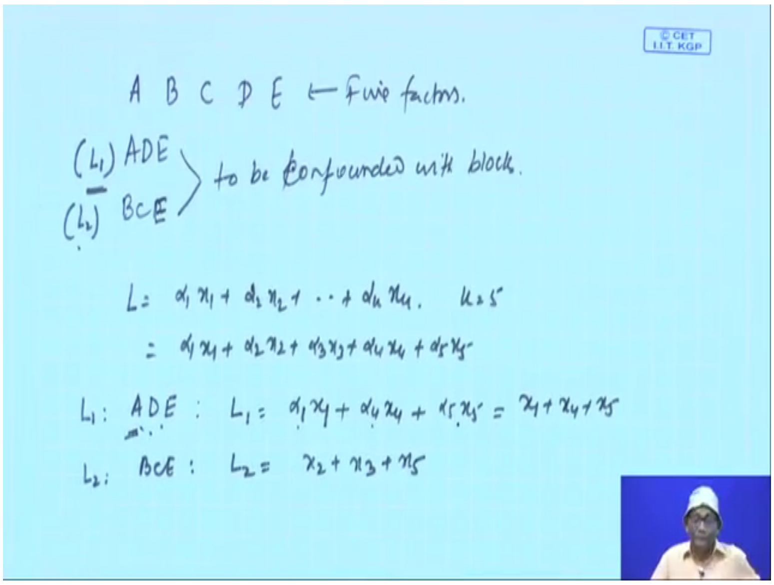

Now, so, we have A B C D and E that is 5 factors. Suppose you think that that A D E, and your B C E. These are the two higher order interactions to be confounded with block. So, how many defining relation; suppose for A D E it is L 1, and for B C E it is L 2, two defining relations. So, we know that L equal to alpha 1 x 1 alpha 2 x 2. So, like this alpha k x k in this case k equal to , this will be alpha 1 x 1 alpha 2 x 2 alpha 3 x 3 alpha 4 x 4 plus alpha 5 x 5.

Now, then what is L 1? In L 1, our defining that factors to be confounded is this. So then, our L 1 equation will be alpha 1. A is there, so x 1, plus B C is not there, only D is there. So, alpha 4 x 4, plus E is there, fifth one alpha 5 x 5. And in this case this both alpha 1 alpha 4 alpha 5, they are basically assuming one. So, we can write this, x 1 plus x 4 plus x 5. Similarly for L 2 which is B C E, we can write L 2 equal to x 2 plus x 3 plus x 5. So, now, what you require? You require to assign blocks. What way the blocks will be assigned. So, now, for L 1 also, suppose, how many effects are there here?

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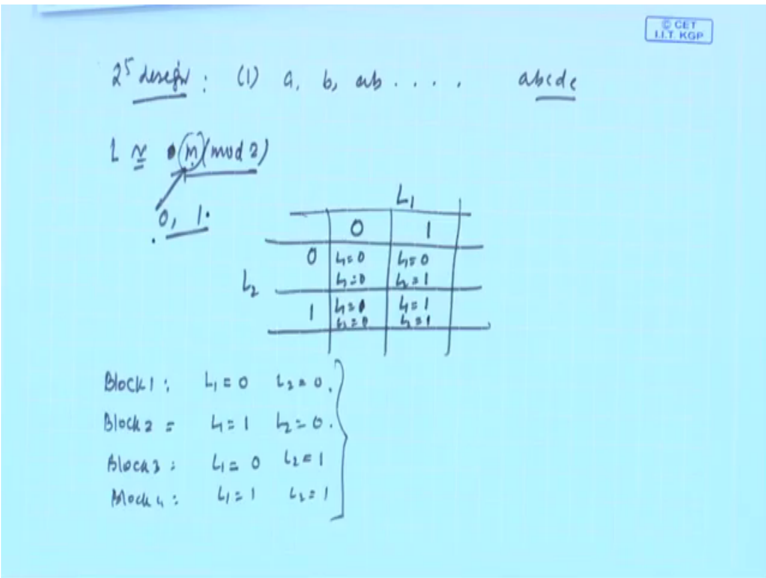

2 to the power 5 design. So, that mean 32 effects. Starting from 1, a, b, a b, that sends a b c d e so, many treatment combinations, and so many effects.

So, how to assign these many treatment combination to the 4 blocks? We have seen earlier that, whatever L value you got, we basically converted into; suppose let it be m mod 2 kind of things. And then we found out that this one will take 0 or 1 value. And, accordingly, 1 block will be assigned to all treatment combination where, this value is 0, and another treatment comb all other will be to this.

So, that mean for the two defining contrasts will give us 2 cross 2, 4 different combinations for the treatments to be assigned to blocks; means if I consider L 1, this 0 and 1, and L 2 also 0 and 1. So, then what is happening here? You see you are getting a situation like this where, this is L 2. So, L 1 0, L 2 0 L 1 equal to 0 L 2 equal to 1, L 1 equal to 1 and L 2 equal to 0 , L 1 equal to 1, L 2 equal to 1. So, that means, our assignment 4 blocks.

So, block 1, it will be L 1 0 and L 2 0 those many things will go to block 1. Now block 2 will be, L 1 equal to 1, and L 2 equal to 0. Block 3 L 1 equal to 0, L 2 equal to 1. And block 4 let it be 1, 1 and 1. So, this is the way you can assign that 4 blocks to different treatment combinations, and accordingly you do conduct the experiments. So, using this concept, what you are getting, you see that block 1 is assigned to 1 2 3 4 plus 4 8 treatment combination. They are different than block 2, different than block 3, different than block 4.

So, as I told you that there will be 32 treatment combination, and all those treatment combinations are assigned in this manner. So, this is this is what is the generalization of the case. So, you can go for 2 to the power 6 minus 3 also. So, in that case, 2 to the power 3, 8 different blocks will be used. And you require basically, in that case basically, 3 different defining contrasts. And accordingly, you proceed and first you assign the treatment combinations for blocks, and conduct experiment, get the data, and use the analysis the way we have explained earlier.

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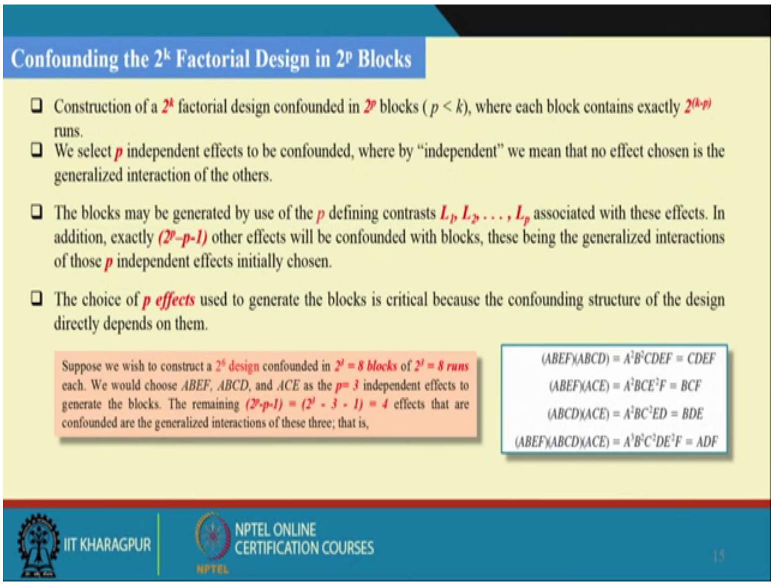

So, the generalization is now confounding the 2 to the power k factorial design in 2 to the power p blocks. So, construction of a 2 to the power k factorial design, confounded in 2 to the power p blocks; where each block contains exactly 2 to the k minus p runs.

We select p independent effects to be confounded, where by independent; we mean that no effect chosen is the generalized interaction of the others. The block may be generated by use of p defining contrasts; because it is a 2 to the power in 2 to the power p blocks power. So, L one to L p associated with these effects. In addition, exactly 2 to the power p minus p minus 1, other effects will be confounded with blocks these being the generalized interactions of these of these p independent effects initially chosen. So, now, see that, what we are what we mean by 2 to the power p minus p minus 1, other effects which are actually confounded, these are the case.

Suppose we wish to construct this 2 to the power 6 design, confounded in 2 to the power 3, 8 blocks 2 to the power 3, 8 runs each.

In that case suppose A B E F, A B C D and A C E; three independent effects to generate the blocks. The remaining 2 to the power p minus 4 effects that are confounded with these are given here. So, A B C, A B E F, and A B C D, and A C E these are the three interactions which are basically confounded with blocks. Now that will basically create another 4 effects to be confounded.

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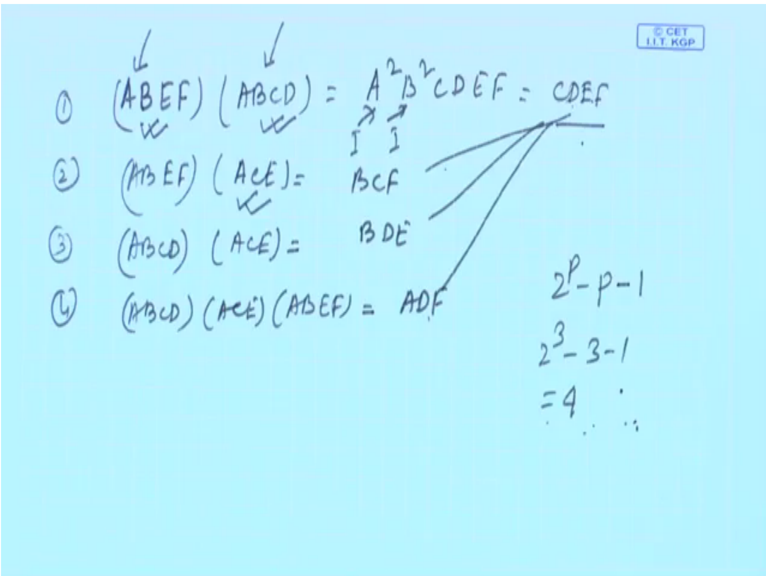

For example, if you multiplied A B E F into A B C D. So, this is confounded with block, this is also confounded with block. The multiplication will give you A square B square C D E and F. And these will become 1 I. So, that means, this is giving you C D E F. This is the other one which is also confounded. Second one; suppose here you multiplied A B E F into A C E. Third one; A B C D into A C E and Fourth one; A B C D, A C E and A B E F. So, all those things, suppose this will give you B C F, this will give you B D E, and this will give you A D F.

So, these are the other 4 factors; effects not factors. These are other 4 effects which are confounded with blocks. Not only A B E F, A B C D or A C E, this general structure will give you another 2 to the power p minus p minus 1..

So, here p is 3. So, 2 to the power 3 minus 3 minus 1 so, that mean 4. So, 1 2 3 4 other effects. Not only the 3 p equal to 3. So, 3 plus 4, 7 different higher order effects interaction effects, they are confounded with blocks. What does it mean? Those effects cannot be estimated uniquely. So, this is the general things, please keep in mind.

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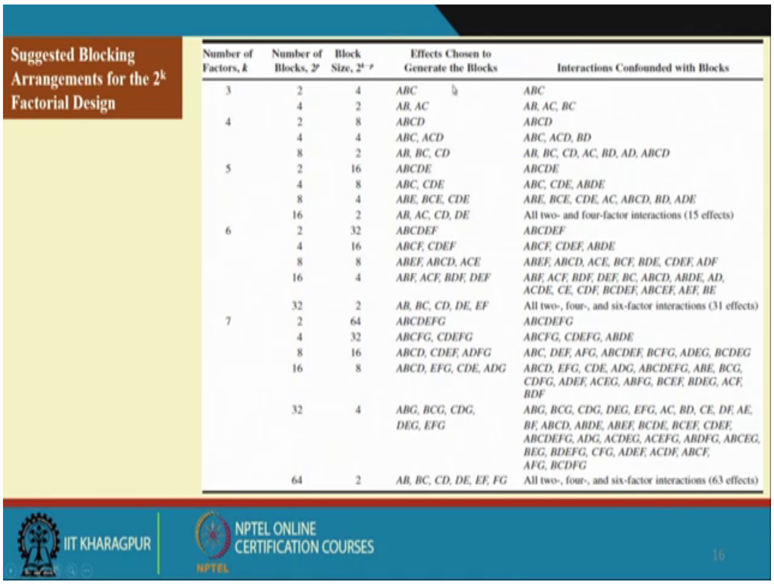

Number of Factors, k	Number of Blocks, 2^p	Block Size, 2^{k-p}	Effects Chosen to Generate the Blocks	Interactions Confounded with Blocks
3	2	4	ABC	ABC
	4	2	AB, AC	AB, AC, BC
4	2	8	ABCD	ABCD
	4	4	ABC, ACD	ABC, ACD, BD
	8	2	AB, BC, CD	AB, BC, CD, AC, BD, AD, ABCD
5	2	16	ABCDE	ABCDE
	4	8	ABC, CDE	ABC, CDE, ABDE
	8	4	ABE, BCE, CDE	ABE, BCE, CDE, AC, ABCD, BD, ADE
	16	2	AB, AC, CD, DE	All two- and four-factor interactions (15 effects)
6	2	32	ABCDEF	ABCDEF
	4	16	ABCF, CDEF	ABCF, CDEF, ABDE
	8	8	ABEF, ABCD, ACE	ABEF, ABCD, ACE, BCF, BDE, CDEF, ADF
	16	4	ABF, ACF, BDF, DEF	ABF, ACF, BDF, DEF, BC, ABCD, ABDE, AD, ACDE, CE, CDF, BCDEF, ABCEF, AEF, BE
	32	2	AB, BC, CD, DE, EF	All two-, four-, and six-factor interactions (31 effects)
7	2	64	ABCDEFG	ABCDEFG
	4	32	ABCFG, CDEFG	ABCFG, CDEFG, ABDE
	8	16	ABCD, CDEF, ADFG	ABC, DEF, AFG, ABCDEF, BCFG, ADEG, BCDEG
	16	8	ABCD, EFG, CDE, ADG	ABCD, EFG, CDE, ADG, ABCDEFG, ABE, BCG, CDFG, ADEF, ACEG, ABFG, BCEF, BDEG, ACF, BDF
	32	4	ABG, BCG, CDG, DEG, EFG	ABG, BCG, CDG, DEG, EFG, AC, BD, CE, DF, AE, BF, ABCD, ABDE, ABEF, BCDE, BCEF, CDEF, ABCDEFG, ADG, ACDEG, ACEFG, ABDFG, ABCEG, BEG, BDEFG, CFG, ADEF, ACDF, ABCF, AFG, BCDFG
	64	2	AB, BC, CD, DE, EF, FG	All two-, four-, and six-factor interactions (63 effects)

Now, a table that suggested blocking arrangement for 2 to the power k factorial design. So, you if your number of factors 3 to more, and then how many blocks you want to use, and then what will be the block size. So, if there are 3 factors, number of block; if is 2, then block size will be 4, means 4 observations per block.

If you want to use 4 blocks, then it will be block size will be 2. And then ultimately, you in order to create those block; in order to assign the treatment combination to these different blocks, you must have the generating principle; that means, the effect to be confounded. So, in case of 3 factors, 2 blocks; A B C is usually the interaction, which is confounded with blocks. If it is 4 blocks then, A B and A C can be confounded. And 4 blocks, you can choose A B C D. 4 factors, 2 blocks you can choose A B C D too, generate blocks in the same way, because these are the some examples given to you that, what way you can do.

So, depending on the number of factors depending on the number of blocks to be chosen, the block size will ultimately be different, or the block size will mention will dictate that what will be the number of blocks to be chosen.

But you quickly please keep in mind, from analysis restriction point of view, you have to choose 2 to the power p blocks; means a multiple of 2 (Refer Time: 22:29) not that 2 to the power 1 equal to 2 blocks, 2 to the power 2 equal to 4 blocks; like this. Not that, you will use 3 blocks, 5 blocks, it should be a 2 to the power p kind of things. So, this is the guideline. So, and when you conduct experiment, please follow this guideline with blocking.

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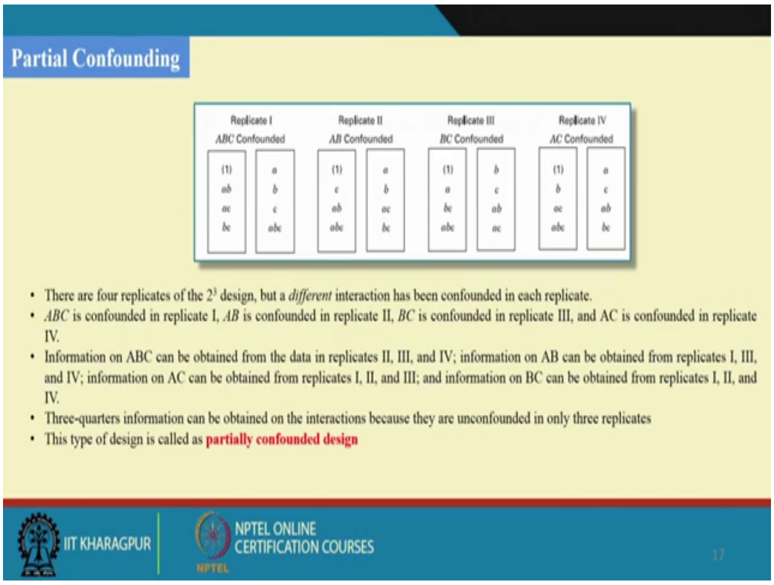

Now we will discuss something in the blocking, actually which is basically known as partial confounding. So, let us see this slide. Just you see that, what are the things written on the slide. So, there are four replications, replicate I to IV. And you see in the first replicate, A B C is confounded. In the second replicate, A B is confounded. In the third replicate, B C is confounded. In the fourth replicate, A C is confounded.

So, what are the advantage of such confounding? So, information on A B C, now can be obtained from replicates II, III, and IV. Replicates II, III, IV are not confounded with A B C. Similarly information on A B can be found out replication using replicate I, III, and IV. And B C can be found out with replicates I, II, and IV. And A C can be found out with replicate I, II, and III. So, that mean this is separate kind of arrangement. You require, suppose that four different replicates, may be 4 blocks; and you can confound you with different effects. So, as a result you have partial information, for though the effects that are already confounded with some of the replicates.

So, what happened in this case, you see that 3 quarter information can be obtained on the interaction, because they are unconfounded in only three replicates. This type of design is called partially confounded design.

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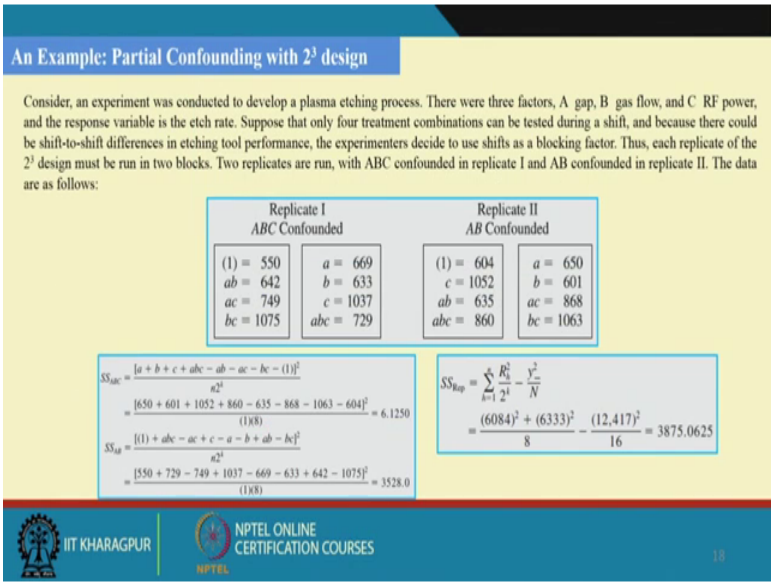

Now, let us see one example here. So, it is basically the plasma etching process. There are three factors, A, B and C. This also we have seen in the book of Montgomery. We have taken material from his book only. And the response variable is the etch rate. Suppose that only four treatment combination can be tested during a shift, because there could be shift to shift differences in choosing in the etch rate. I think the etching tool performance, the experimenters decide to use shifts as a blocking factors. How many shifts? Three shifts. So, three blocks. Thus each replicate of the 2 to the power 3 design must be run in 2 blocks. Two replicates are run with A B C confounded in I, and A B confounded in II, the data shown below.

You, the case is that, you have three factors. You have three shifts; and in each shifts suppose four treatment possible. So, in that case you use shift one as a replicate I, shift two is replicate II, and again you can use shift three. So, no problem, but here what happened that, two replicates; one A B C confounded, and another A B confounded, are basically used here. Now when A B C confounded, these are the responses under different treatment combination.

When A B confounded, these are the responses under different treatment combinations. And you know, if A B C confounded, then how to find out the replicate, and A B confounded also, how to find out the replicates; that means, here what happened you see that from here, you can find out A B, and from here, you can find out the A B C estimates.

So, suppose S S A B C, if you want to find out, this is what is basically the formula for find out the sum square A B C, and you see what you are using, from where you are using the data, a is 650. You see here, a is 650; 6 0 1 6 0 1 then 1 0 5 2, 1 0 5 2. So, that mean you are using information from replicate II, to find out S S A B C. You are using replication using information from replicate I to find out S S A B.

So that mean, it is helping you to find out the information, the statistics needed for those effects which are already confounded with some of the replicates. And, you also find out that, S S replications, because replication wise; now the confounding is different, and replication wise also there different. So, replication is also contributing towards the sum squares.

So this is what the way, the partial confounding is done; and the way the calculation is made.

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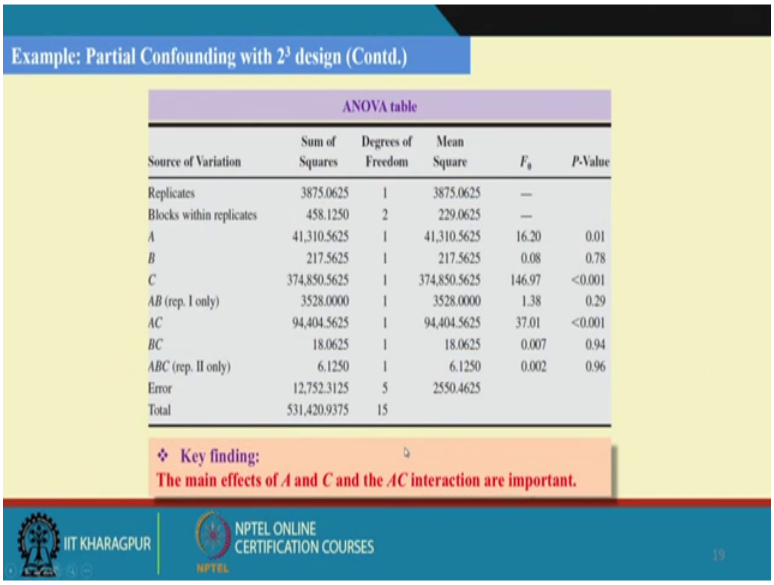

So, now the sources of variation is changed. See in the starting point when we start with 2 to the power k factorial design, we say that sources of variations are the main and interaction effects of the factors and error. When you introduced blocks, we said that these plus blocks. So, now, what happened when we introduce this kind of partial confounding case that replicates are confounded with different effects? So, then replicates is another source of variation.

So, you are in (Refer Time: 29:32) all those source of variations to be included, their sum square and degrees of freedom to be computed, then you do the F test, for what? For the effect parameters we are least interested with the replicates and blocks. We are interested in the effect of parameters, because controlling the blocks and the replicates effect; you are estimating the effect parameters. And more precisely actually importantly, here what is happening? The error value computation is changing, error S S.

And everything is changing; the degree of freedom is changing. So, eventually the F 0 values for all the factors, main effects and interaction effects; they are also changing. Their P value is changing, and their whether significance levels is also, accordingly changing. And you have to find out which one is significant, and which one is not significant. From this, we found that the main effects of A C and A C interactions are important using this P value.

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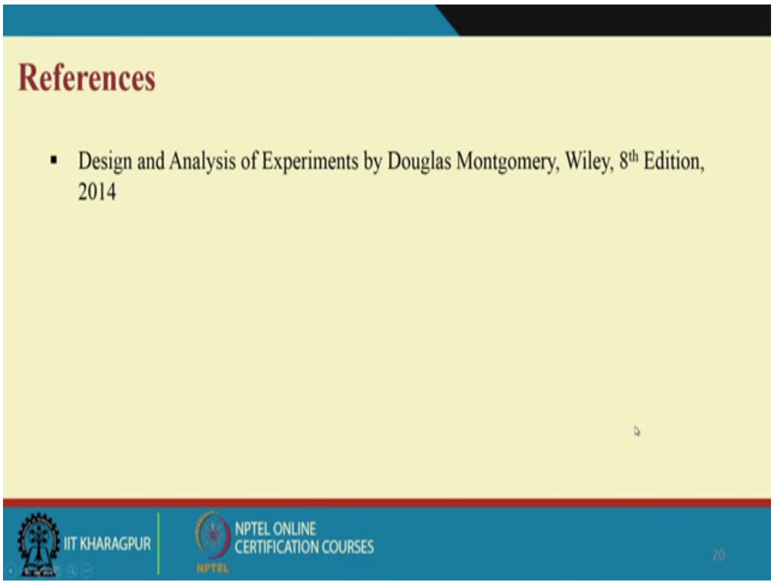

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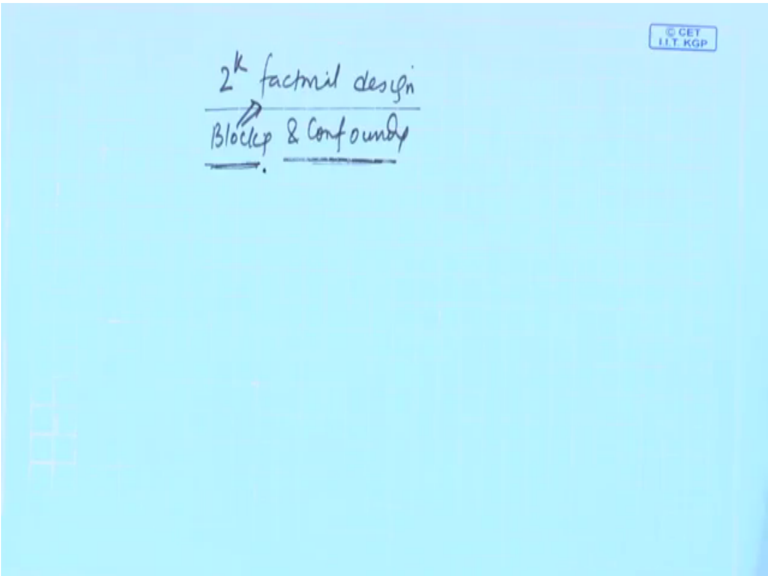

So, thank you very much. I told you that, this is chapter seven from Montgomery. And it is basically to just summarize that, we say that, we were talking about 2 to the power factorial design; from last few lectures. And in the last three lectures, we introduced and the blocking and confounding concept confounding concept in 2 to the power k factorial design, and we have explained that; when blocking is important and why blocking is necessary. And also we have seen that, where due to blocking under different situation, how confounding takes place; and under the blocking and confounding situation, how do we you define different treatment combination to different blocks.

And then, how do you estimate the treatment effects, main and interaction effects; and under this situation, how the things are basically changing with reference to the error estimates, and as a result the F value also will change. So, all those things are very very important for experimental design; and I am sure that, because of these three lectures, you will be in a position to do proper justification in you, actually perform such design in the laboratory level, or at the field level may be. So, if you have any questions enquiries, please use the forum for which we are there to help you, and we got some of the questions, and we are replying to all those questions.

Thank you very much. Have a nice day.