Design and Analysis of Experiments
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Lecture – 43
Blocking and Confounding in 2 k Factorial Design (Contd.)

Welcome, we will continue blocking and confounding in 2 to the power k factorial design.

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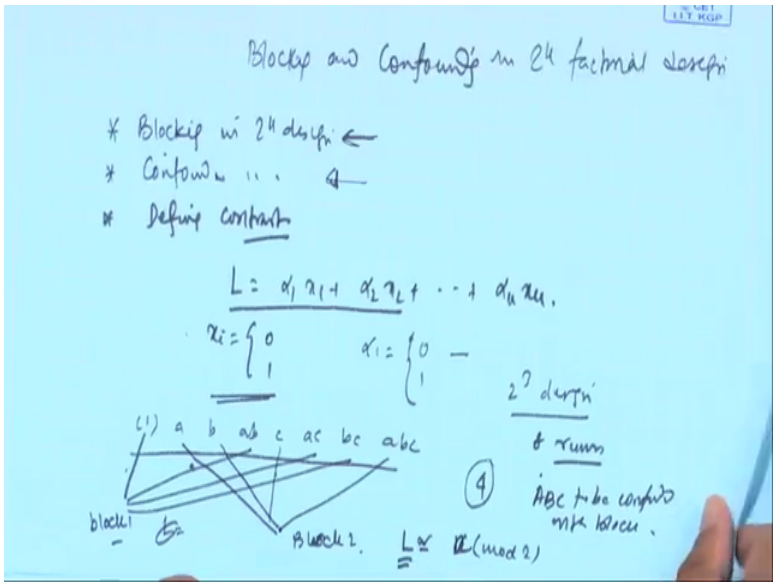

And in last class what we have seen that I have discussed what is blocking in 2 to the power k design. And then we also discussed what is confounding in 2 to the power k design. And we also discussed defining contrast defining contrast which will help in assigning different experimental combinations or treatment combinations to different blocks ok.

So, what we have seen basically in the first case you have seen that how the blocks are used as replicates. And when every block is able to accommodate all the treatment combination; in case of confounding you have seen you have seen that even every block is not able to accommodate all the treatment combinations. So, you found that there are some of the effects parameters which are confounded with the blocks and usual guideline is that it should be the higher order interaction that should be confounded.

And then we have we have used defining contrast where i said the different contrast is alpha 1 x 1 plus alpha 2 x 2. So, like this your alpha k x k if there are k number of factors and x i basically can take values 0 or one and depending on the from the contrast the n matrix you will find whether this effect is at low level or high level and alpha i will become 0 or 1 again if it is if it is present if that particular i th factor is present in the defining in the effect to be confounded and are absent in the effect to be confounded depending on that 1 or 0 value will be assigned to them.

So, and then with a 2 to the power 3 design case 2 to the power 3 design case I have shown you that how you we require 8 runs; 8 runs suppose you have insufficient materials you can go for two to the only 2 to the power 2 runs per block so; that means, 4 runs per block.

Then what are the treatment combination starting from 1 then a, then b, then a b, then c, then a c, then b c, then a b c. So, these are the 8 treatment combinations which are the treatment combination we will go to block 1; and which are the treatment combination we will go to block 2. So, using this concept like L equal to if effect to be confounded is a b; a b c to be confounded with block confounded with block with block then we have seen that 1 a b, a c b c will go to block 1 and a, b, c and a b c will go to block 2. How? You have used this your written first you find out the value of L and there that you write in terms of L mod 2 or I can say a mod two no problem if L is just to avoid this.

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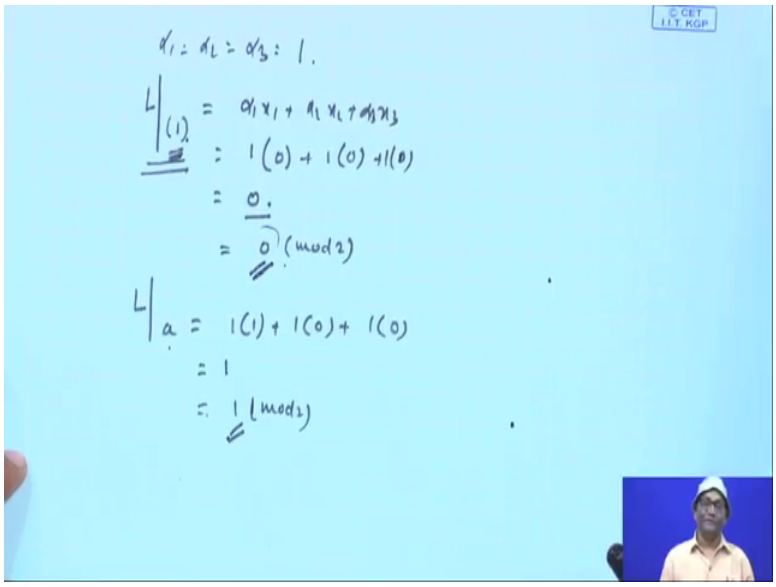

For example, we have seen in order to if a b c is confounded alpha 1 equal to alpha 2 equal to alpha 3 equal to 1 and in that case.

And if I am interested to find out the what will be which block it should treatment 1 combination 1 will be run that will be written like this. So, alpha 1 x 1 plus alpha 2 x 2 plus alpha 3 x 3 were alpha 1 and in one treatment combination all will be at low level plus 1 0 plus 1 1 0; then this will give you 0 value. So, this value is 0 now its equivalent writable will be 0 mod 2.

So, suppose if we want that what will be the block for so; that means, what is the block for a then in that case it will be 1 into 1 plus 1 into 0 plus 1 into 0 which is 1 and this can be written that 1 mod 2. So, this two; that means, when you just see that way that is going to 0 or is going to 1. So, all those treatment combination where this value is 0 will be put with block 1 and combination with 1 will be put with block 2. And in this way you are basically defining the treatment combination for two different blocks; that is what we have discussed so, far.

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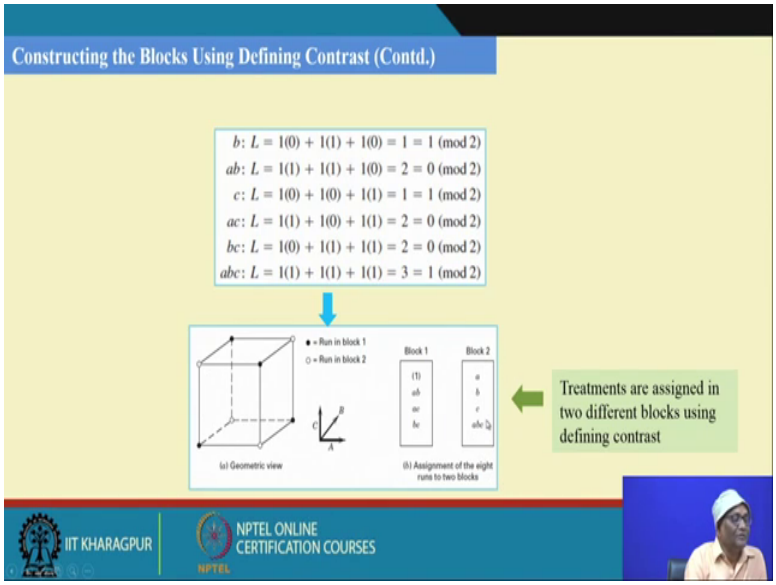

And geometrically you see that that one then your this one is a c and this one is a b and this one is b c; these are all with block 1. So, run in block 1 and others run in block 2 this is the first one.

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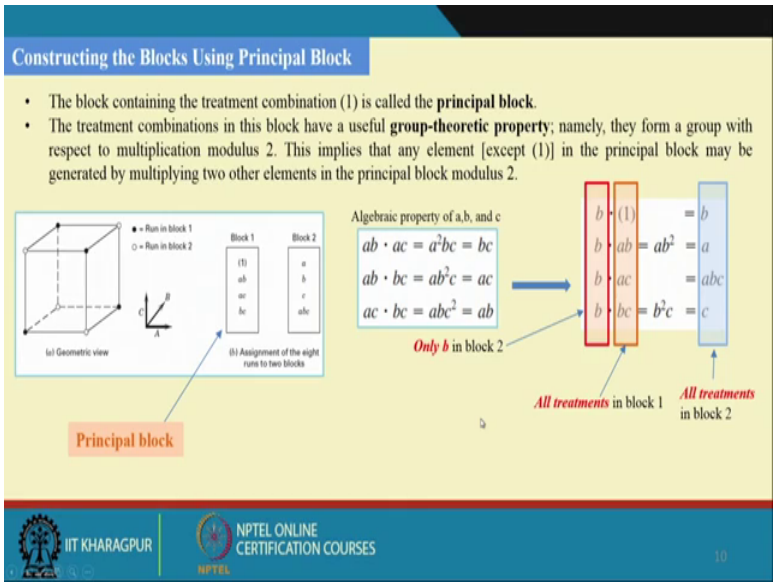

So, another concept is there the same thing that how which treatment combination will go to which blocks that can be done using the principal block concept. So, the block containing the treatment combination is 1 is called principal block. The treatment combination in this block have a useful group theoretic property namely they form a group with respect to multiplication modulus 2; this implies that any element except within bracket 1 in the principal block may be generated by multiplying two other elements in the principal block on modulus 2 ok.

So, what we mean to say this is our first block because this is the principal block it contains 1. Now if you multiply the other the treatment combinations here; then ultimately you will be getting another treatment combination in the same block like if you multiplied a b with a c you will be getting b c; if you multiply with a b with b c you will be getting a c.

If you multiplied a c with b c you will be getting a b and you will get the; you will get you will get you it what I mean to say that as you have one a b a c b c these are assigned to the first block. Now if you multiply it with any other treatment combinations which are not in this block. So, what will happen you will get the treatment combination in the other blocks; how it is.

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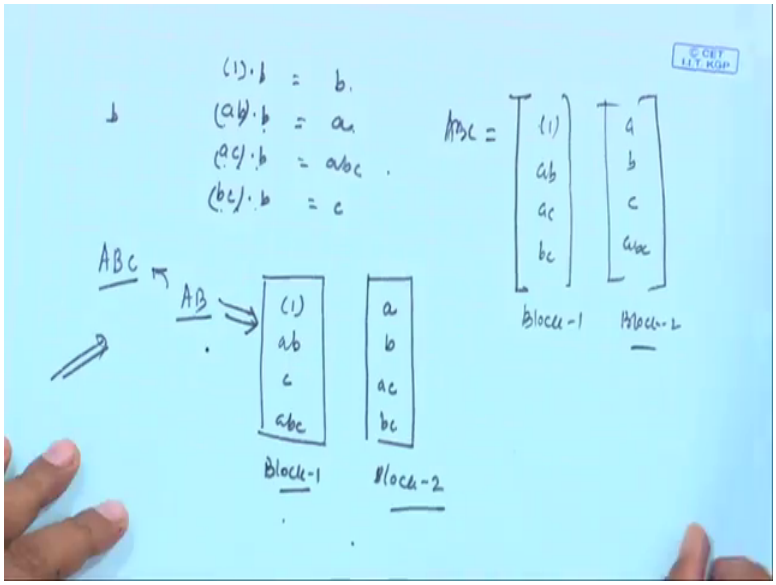

Suppose in the first principal block 1 then; we say a b then a c and b c; so, the other treatment combination.

For example b if you multiplied with b everywhere, what you are getting here? You will be getting here b; a b square it will give you a, a b c will give you a b c; b square c will give you c. So, c that a b c a b c which are the treatments that are not assigned to principal block. So, they will go to the another block; so, that sense even if you multiplied this with this with suppose a you will also get the same thing ok. So, this is the concept.

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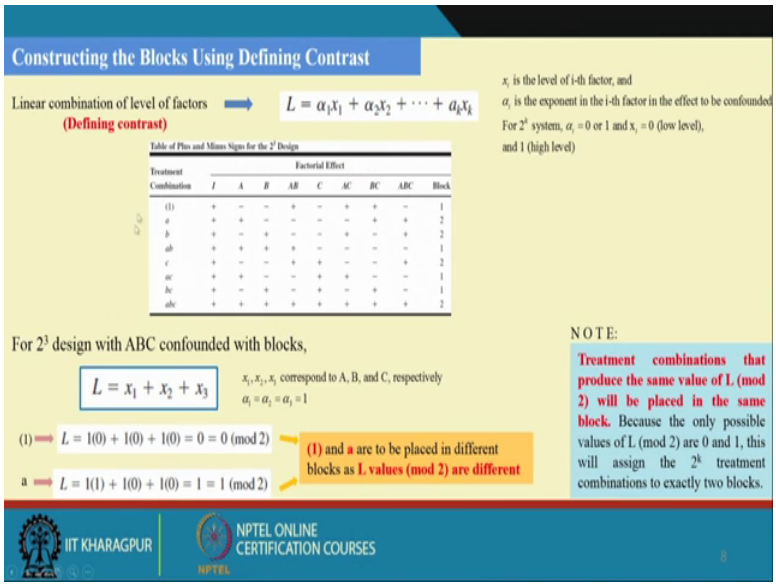

And another simple one the first one what I explained; but if you see here you see that we say that effect to be confounded with block. So, you when you create this algebraic matrix you will also get the contrast for ABC and here you; obviously, in all the columns you will say you will get plus or minus sign only.

So, now here the minus will be assigned to block 1 and plus will be assigned to block 2. So, all there will be only minus plus and equal number of minus and equal number of plus. So, minus tit treatment combinations will go to block 1 and plus treatment combination will go to block 2; obviously, with respect to the effect to be confounded. If you are effect to be confounded is AB, then the assignment of block will be different; so; that means, this plus 1 the first one will go to block 2, second third will go to block 1 then again block 2 then another to block 1 and block 2.

Now if you say that the block which contains the treatment combination 1 within bracket 1 is a principal block ah; then our principal block will be here if AB is confounded AB is confounded then this is minus one then ab then c and then abc ok.

So essentially what I mean to say here I mean to say here; instead of ABC if your effect to be confounded with block is AB and then the principal block will be will contain 1 then your ab principal block will contain 1 then ab then c and then abc. So, what will be the another block? This is your block 1 let it be block 1 goes to this, another block you

multiply with a then this will be a, this will be b this will be a c then this will be b c you see this will go to block 2.

So; that means, effect to be confounded is very very important and accordingly the assignment of the treatment combination to the block. So, if AB to be confounded this is the case if ABC to be confounded then you have already seen that AB; ABC to be confounded then you have seen that this is 1 a b a c and b c that is block 1 and another one will be others one a b c and a b c this is your block 2.

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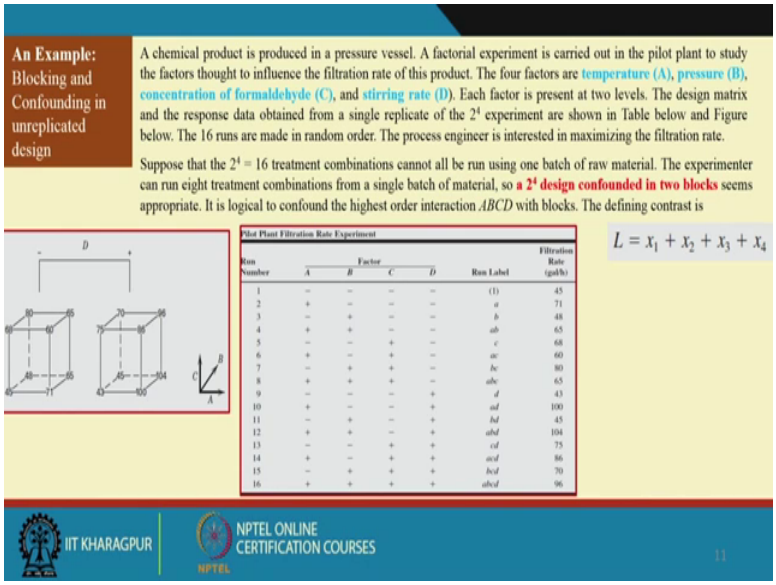

So, with this background let us see one example and let us estimate the effects as well as you see that the which of the effects are significant which of the effects are not significant develop an over table like this. So, we have 4 factors A B C D; they are temperature, pressure, concentration of formaldehyde and steering rate. And each with two labels; so, you know that there will be 2 to the power 4 means 16 treatment combinations.

And you can you can develop this algebraic matrix A B C D and that plus minus combinations all those things are known to you what happened you take the first column minus plus minus plus minus plus second column minus minus plus plus plus plus then third column that is 2 to the power j minus 1; 3 minus one two 4 minus 4 plus 4 minus 4 plus like this.

And you also know that how the wrong level will be defined with reference to that a b c this kind of this alphabets. And suppose and these are the these are the wrong level and suppose you have done once because you have the restriction with the (Refer Time: 13:42) restrictions and all those things. So, 1 1 runs other way I can want replications, but another restrictions is that you cannot do it with one block, you have you have to do it with two blocks the reason is you cannot go for 16 runs experimental runs in one block; so, here block is the resource.

So, you require let two blocks to be used so; that means, under each block there will be 8 differ 8 runs. So, 8 different treatment combinations will be allotted to block 1 another 8 treatment combinations will be allocated to block 2 ok. And this is what is this is this is the geometric representation of these 2 to the power 4 design and this is what is our this and suppose what will happen there will be two cases ah; one case will be one case will be when you have you are going for block only one block another case will be going for two blocks.

So, the although the this example is for two blocks, but we have seen this earlier with only in case of single replicate with one homogeneous set and that time; the filtration rate which is the response variable what are the data obtained with reference to different experimental or treatment combinations are like this; these are the values.

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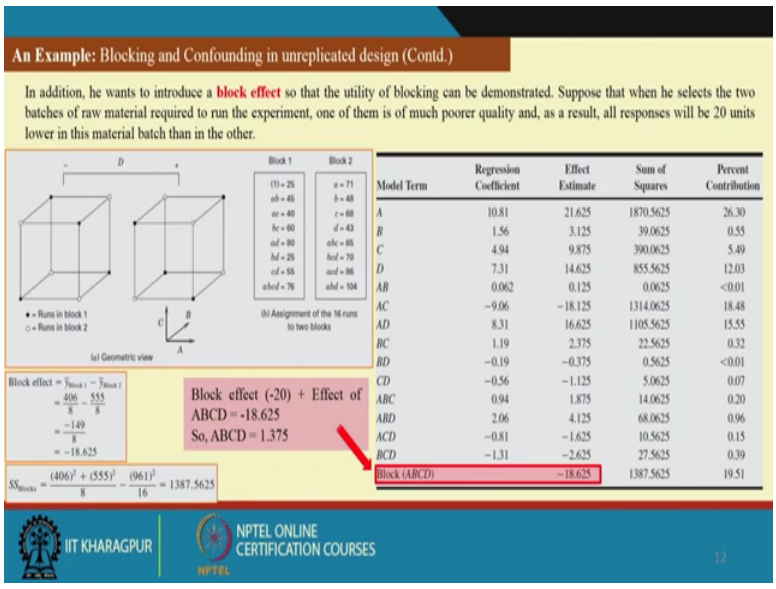

Now, as we have the restriction with blocks also; so, we are using two blocks and then in that case we are assuming one more important fun concept that we are saying that if you run in block 1 using block 1 or using that batch of raw material are run with block 2 there will be a difference.

What is the difference? Here we said that that in addition suppose you want to introduce block effect. So, the utility blocking can be demonstrated in that case when the two blocks of raw materials required to run the experiment; one of them is much poorer quality. And as a result all responses will be 20 units lower in this material batch than the other; so I am repeating this case.

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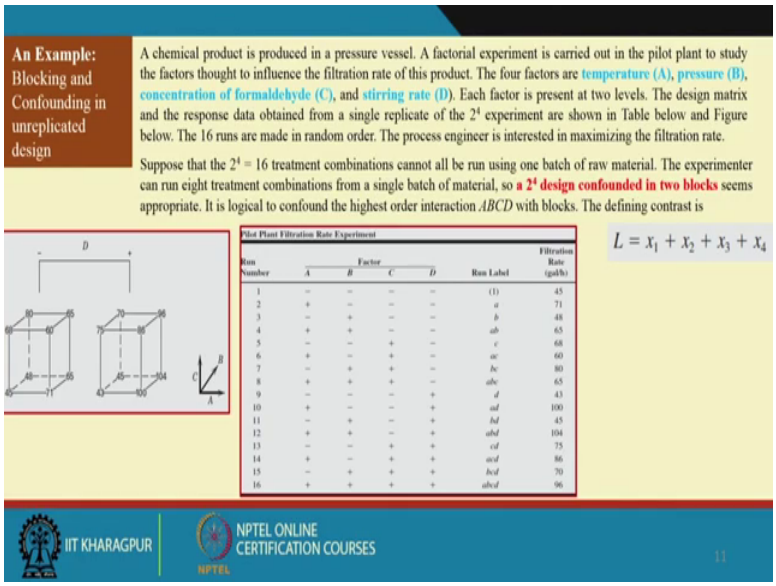

First one you see this data point. So, we have 2 to the power 4 design.

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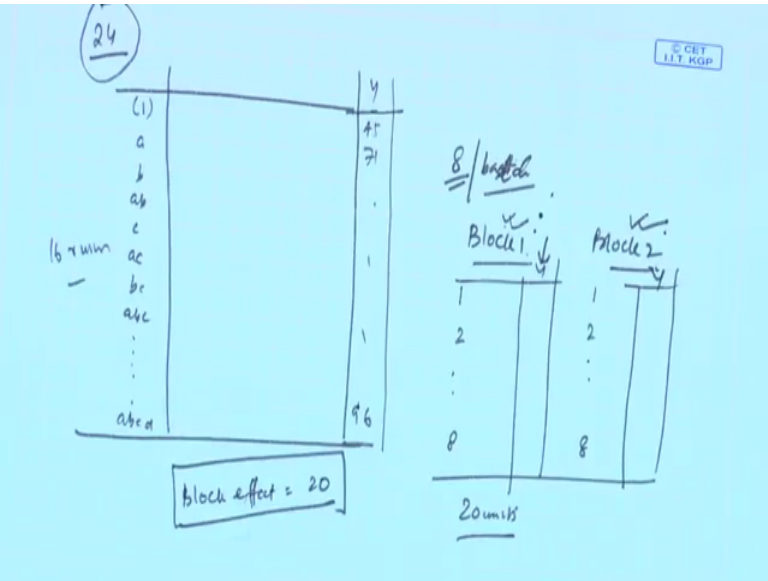

So, we have one a, a b, c, a c, b c then your a b c; so, like this ultimately you will go to a b c d. So, 16 runs suppose you are you have sufficient raw material in such a manner that every batch will be able to accommodate this 16 runs, but you do not you can you do not have more than that. So, one batch here one batch is used and your response variable is measured response variable is measured this measurement like this first one is 45 is the response second one is 71. So, like this 96 is the last one this is the response ok.

Suppose you want to conduct the same experiment another time let it be, but when your raw material is the batch is not sufficient enough to accommodate all the 16 runs; you require to do with two batches of raw materials. Because only 8 treatment combination can be accommodated per batch b a t c h; so, as a result the blocking concept is coming into picture.

So, that mean each block batch here represent one block; so, we use two block; block 1 and block 2. So, they mean two batches of raw material; so, here you will be having 1, 2 like 8 runs here also you will be having 1 to 8 runs then you will be having the response values y here also you will be having the response values y. Suppose you further assume that further assume that that the original batch is also available and it is assigned to block 2; it is called block 2. And block 1 is a much poorer one which is basically taken from may be some other supplier and then what happened it has effect because it will reduce the response values.

So, what is the reduction? Reduction let it be there 20 units reduction because of poor quality. So, as a result what happened? So, what happened the response will change, you see here in the block 1 treatment combination 1 it is 25, but earlier example it was 45. So, 45 minus 25; 20 that is 25. So, this block is poorer than block 2 and that block effect is 20 units that is that; that means, if you use block 1; that means, batches of raw materials representing block 1 your response will be 20 units less and that is what is the block effect.

When a force purposefully we have said first the block effect is 20 and suppose these are the data; now your question is first question is that how do we assign the 16 rounds to block 1 and block 2; 8 per block. So, that concept we have already discussed we said that you go by defining contrast or you first find out which effect to be to be confounded and then that effect find out the contrast for that effect plus and minus sign and then what you do then all block 1 will go to plus minus sign and block 2 go to plus sign and that since or using defining contrast you find out.

So, you using that you found out that this one ab, ac, bc, ad, bd, cd, abcd all will go to block 1 and others will go to block 2. Block 2 means the original batch of raw material which is of superior quality than block 1 ok. So, what will happen to the estimates? You see the regression coefficient here. So, we have A, B, C, D all those A, B, C, D all everything that effects and a regression coefficient we have seen.

And you see that a earlier example that example we have I think we have actually explained in some previous lectures where these two blocks you have not used, but with single replicate we explained. And there all you see that all the effect values they are same and they are sum squares but the difference is that this block A B C D and block are compounded here please see this red arrow here this component.

Now, in previously you found out the A B C D effect is 1.375 here as A B C D and block both are compounded the effect estimate is basically what happened it is minus 18.625 this is the effect. So, what is this what is the why it is happening because block and A B C D that interaction they are confounded each other; they are inseparable, but in this example that design is made in such a manner that the block effect before hand we assume that it is 20. So, now, then what is the A B C D effect if the last row here it is

talking about both the both the effects together and as we already know the block effect is minus 20; then block effect paralyzed effect of A B C D is this.

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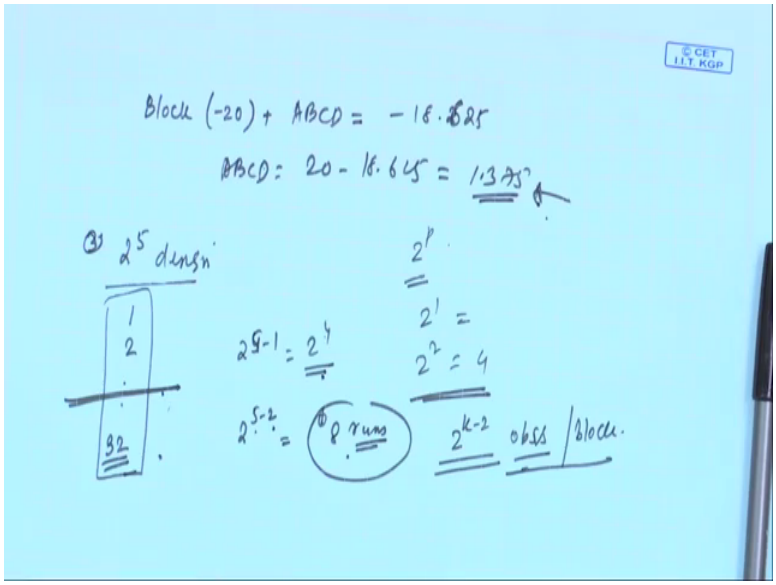

So, block effect that is minus 20 plus A B C D this confounded and this is 18.625. So, then what is A B C D effect is 20 minus 18.625 which is 1.375; this is what you will see we have obtained in earlier example using this data set single replicate case.

So, with single replication we have seen that the error cannot be estimated because we do not have any observation left no degree of freedom for error. Here what happened instead plus we have we have included one more thing that is what we are saying that the two blocks are used; so, one of the again one of the higher order interaction here the highest order interaction A B C D is confounded with the block here.

So, ultimately one moralistic some age impose and error is not estimated here; then what you will do? You will basically follow the same way we have done earlier that you find out the relative contribution in terms of person contribution. And then negligible contributors can be neglected then develop a probability plot or the effects. And find out the effect which are which are far away from the straight line and those effects will be considered as significant.

And then other effects which are lying on the straight line in the p p probability plot all those will be will be neglected and their effect will be considered a 0 effect as error measure.

And then the traditional ANOVA that f test all will be done ok. So, put that what you require to know also you require to know in this case what is the block effect. So, it is similar it is the same thing you just find out the block total. So, you have two blocks 1 and 2 find out the average for block 1 and average for block 2 and then the effect of the block will be the error the difference between the effect. So, y bar block 1 minus y bar block 2 which is minus 18.625 which you have seen here how we have computed the block effect.

Now again S S block you can find out that there are two totals for block 1 and block 2. So, those will be square and; obviously, that will be their average will be computed. And then finally, it will be the corrections will be corrected factors value will be subtracted and we will be getting this value and this is the this value.

So that mean for all A B C D everything you are using contrast you find out the effects per block use this find out the block effect block effect is confounded with A B C D effect also find out S S block using this formula is sum of b i by a i square by the some multiple of n minus y dot dot dot dot square divided by the sum of all divided all the observations these are the concepts already we have discussed. So, you see the relative contributions of A B C D everything the percentage contributions is written here and these are the effect values.

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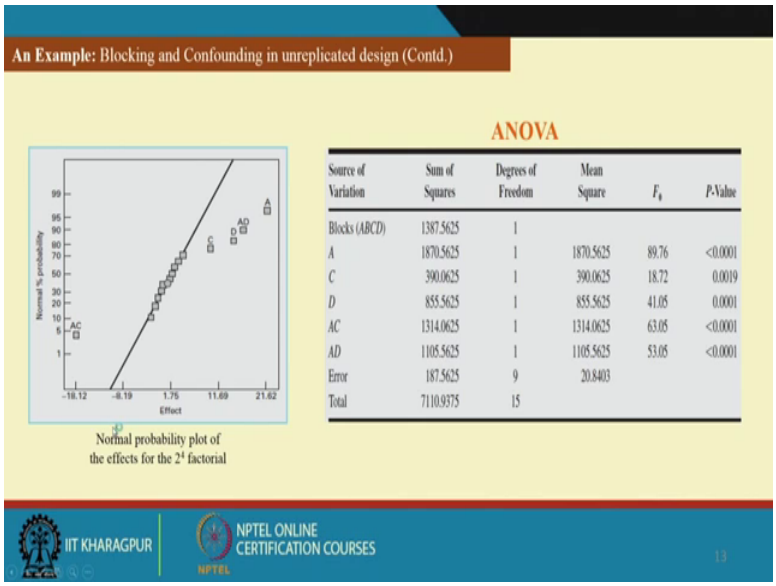

So, when you construct probability plot of the effects value; you will be getting something like this. And here you see that most of the effects values are lying on a straight line and there are few like A C D A D and A C you see they are away from the straight line so; that means, they are different they are not they are significantly contributing.

And as a result as a result see A, C, D, A C, A D plus block is already there and then error will be rest of the that effects which are not significant; they are total sum square sum square is taken and this is the error sum square. Now you are in a position not only you know the effect of block confounded with A B C D, but also you are in a position to know what is the error sum square and the resulting degrees of freedom for error.

And you will be you are in a position to do f test for to see that whether these effects are significant or not. And here you are doing this you see you are getting F value is very high F values and all these values are all these effects are significant. Now you can say that in this particular example case A C D A C and A D they are significant parameters; And if you want to do some subsequent analysis those things to be taken into consideration.

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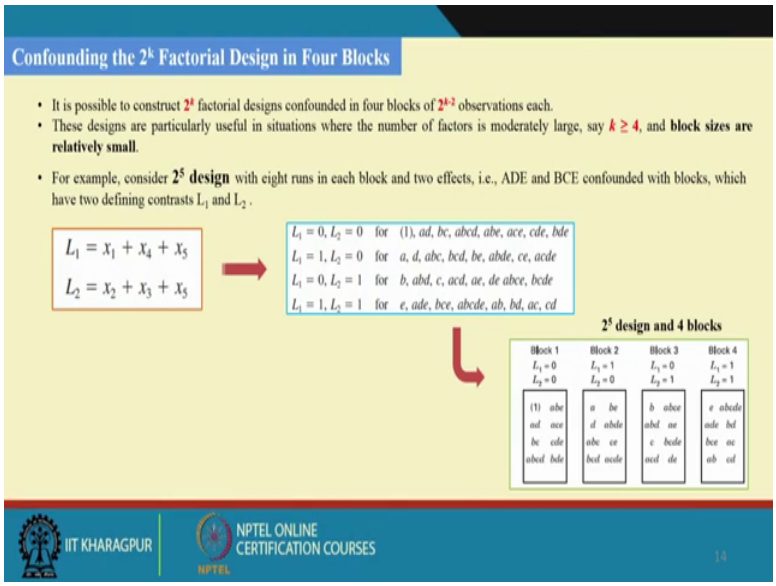

So, now, we want to see that suppose two blocks are not sufficient means what I mean to say you want to do suppose 32 suppose you have 2 to the power 5 design let it be like this. So, you require 1, 2 like these 32 experimental runs; if you want to use to 2 blocks case, then each block will contain 2 to the power 5 minus 1; that means, 2 to the power 4 runs; however, this is not possible you have to use something like this 2 to the power 5 minus 2 that mean what I mean to say I had mean one fourth of 32. So, 8 runs; so, to the power 5 minus 2; 8 runs means each block each block can accommodate only 8 runs not 16 runs.

So, if 1 batch of raw material can accommodate 32 runs; you do not require 2 blocks. If 1 batch of raw material in this case or one type of resource can accommodate here in with differences in the 16 runs; then you require 2 blocks, but if that will not happen then what happened the next is that why I say that 2 to the power p blocks k. So, you have to go 2 to the power 1; 2 blocks 2 to the power 2 then 4 blocks.

So, 4 blocks case you have to go; so, 4 blocks means each block will accommodate 8 runs. So, this is really that is why this is known as here we are saying that 2 to the power 2 mean 2 to the power k minus 2; this is what is the observations; observations per block; when there are k blocks and you want them in; sorry when there are k ferrets and you want to use 4 blocks because your situation is such that only 4 that 4 blocks can accommodate this at least 4 blocks can accommodate.

So, in that case each block will contain 2 to the power k minus 2 objects ok. So, see that in such cases what happened.

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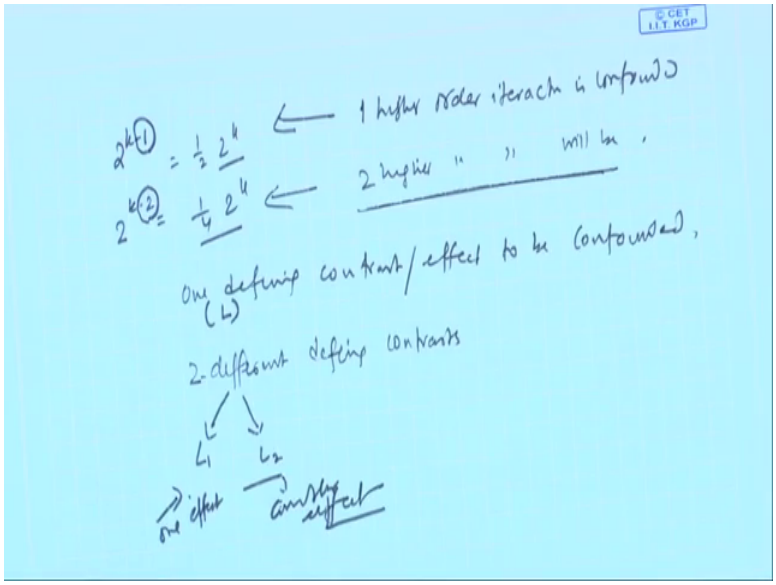

When we are when we are using two blocks that time we are saying 2 to the power k minus 1; that means, 1 by 2 into 2 to the power k. When we are using 2; 4 blocks we are saying 1 by 4 into 2 to the power k. So, like this; so, here 2 is very important here 1 is very important. In this case in this case what happened when you are using 2 blocks that 1 in higher order interaction is confounded 1 in higher order interaction is confounded.

But here what happens 2 higher order interaction will be confounded. So, every confounding case will give you one defining relations defining contrast per effect to be confounded. So, when there are 2 higher order interaction confounded. So, you will have 2 different 2 different different defining contrast ; so, here if we say it is L here it will be L 1 and L 2 ; so, L 1 with 1 1 fact if 1 effect; L 2 with another effect.

Now, see for example, consider 2 to the power 5 design with 8 runs in each block and 2 effects what are the two effects ADE and BCE are confounded with blocks which have two defining contrasts L 1 and L 2; obviously, a c d means suppose if I say a is x 1 d is x 4 e is x 5 then L 1 is x 1 plus x 4 plus x 5. So, in this lecture first I will you how the 4 blocks case will be constructed ok.

Let me read out it is possible to construct 2 to the power k factorial designs confounded in 4 blocks up to the power k minus two observations each; these designs are particularly useful in situations where the number of factors is moderately large and block sizes are relatively small; situation is you have more number of factors, but your resource is less in amount like raw materials, per batch can accommodate less number of runs or less number of experimental runs can be completed.

Thank you very much; have a nice day.