Design and Analysis of Experiments
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Lecture – 42
Blocking and Confounding in 2 k Factorial Design

Welcome. Today, we will discuss blocking and confounding in 2 to the power k factorial design. You know the concept of blocking and we will introduce the concept of confounding and you also know what is 2 to the power k factorial design. So, within 2 to the power k factorial design, how blocking is done and when confounding arises and how to deal with such situations those things will be discussed in one hour lecture.

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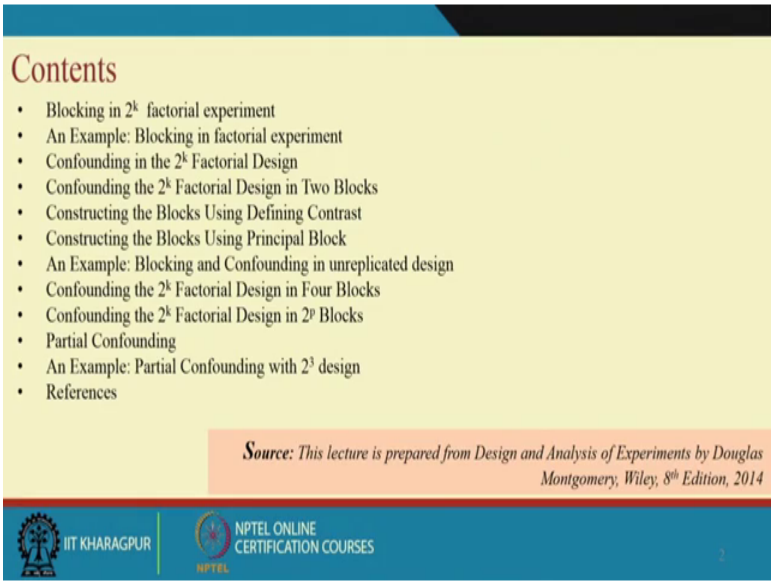

So, contents today's presentation; blocking in 2 to the power k factorial experiment we will discuss this with an example, then confounding in 2 to the per k factorial design then confounding a the 2 to the power k factorial design in 2 blocks, then how to construct blocks using defining contrast, then how to construct blocks using principle block concepts another example blocking and con[founding], confounding in unreplicated design, confounding the 2 to the power k factorial design in 4 blocks, then confounding the 2 to power k factorial design in 2 to the power p blocks, then partial confounding and example with partial con[founding] confounding using 2 to the power 3 design.

This entire all the topics to be covered by two lectures, half an hour to thirty minutes duration and the source of this lecture is the book written by Montgomery Design and Analysis of Experiments. So, if you go through the chapter in blocking in 2 to the power factorial k factorial design it is in chapter-7, in this edition.

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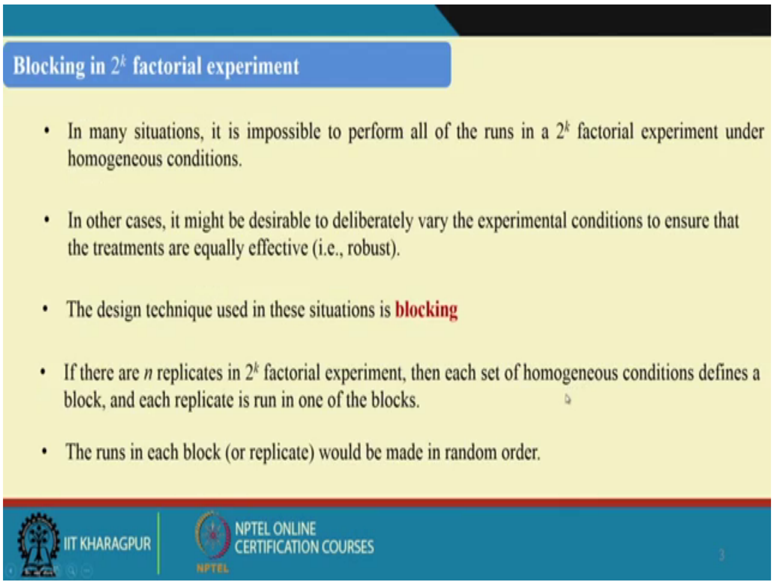

So, in many situations it is impossible to perform all the 2 to the power k factorial experiment under homogeneous condition. Other situations where the experimenter deliberately vary the experimental conditions so that they can ensure the use of this product or the item produced effectively under different situations this is what is in the concept of robust design. So, when you have insufficient resources such as raw materials or the operators or when you deliberately create different heterogeneous situations.

So, under such cases the design technique which is effectively used is called blocking to handle such situations. For example, if there are n replicates in 2 to the power k factorial design then each set of homogeneous conditions; define a block and each replicate is run in one of the blocks.

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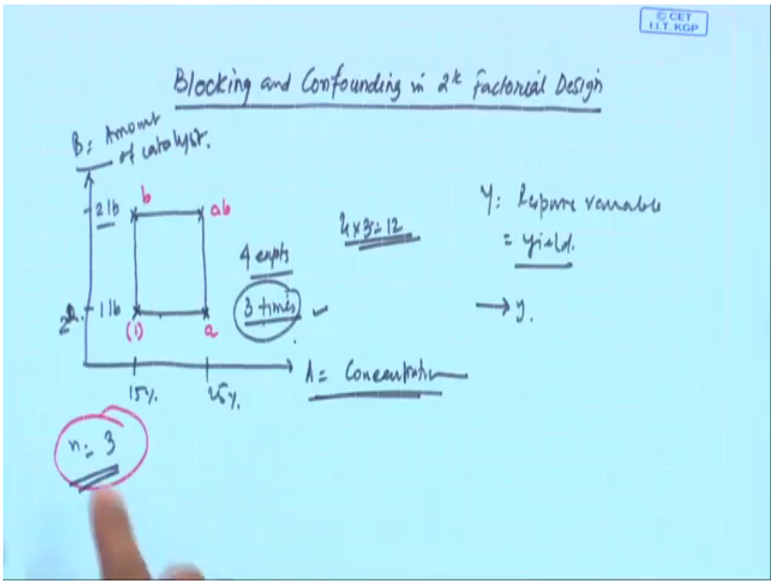

For example you consider a 2 to the power 2 design. So, then the all the settings complete settings all the factorial runs require 4 experiment to be conducted 4 experimental runs and suppose you want to replicate it 3 times maybe 3 times replications and suppose you do not have enough material to do 4 into 3 that is 12 experimental runs under this. In this case, maybe you take batches of raw material one batch with 4 experiment with the complete all the settings factorial settings will be covered, second batch second replication third batch third replication like this. So, that is what we had said that if there are n replicates in 2 to the power k factorial experiment then each set of homogeneous conditions defines a block and each replicate is run in one of the blocks.

The runs in each block would be made in random order, it is obvious. So, let us see one example.

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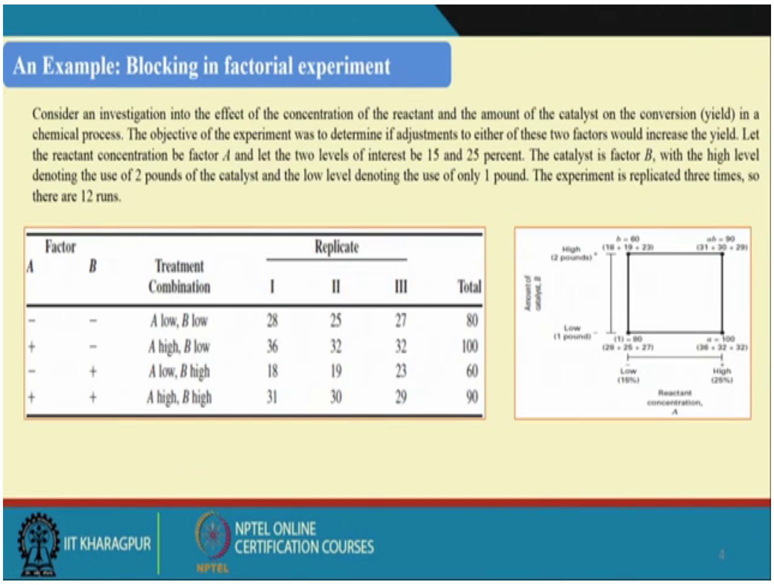

. So, let me read it. So, that you will get a feel of the experiment and then the subsequent slides will explain how the confounding blocking primarily and then confounding will be understood based with these examples. The first example is consider an investigation into the effect of the concentration of the reactant and amount of catalyst on the conversions in a chemical process. So, that means, you have two factors A and B; A is concentration and B is amount of catalyst amount of catalyst. So, you are interested to know the effect of these two on the conversion which is yield. So, Y the response variable is yield. The objective of the experiment was to determine if adjustment to either of these two factors would increase the yield.

So, you want to adjust either A or B or both in such a manner that that yield will increase to increase yield let the reactant concentration be factor A and let the two levels 15 and 25 percent. So, two levels are here 15 percent here 25 percent and for amount catalyst for catalyst amount with high level 1 pound and one level determining the use of 2 pounds of the catalyst and low level with one pound this we have discussed earlier also 2 pound. The experiment is replicated 3 times. So, n equal to 3.

So, you require 12 runs, this is a general case and even up 2 to the power 2 design and you have seen this example earlier we have explained and. In fact, we have done 2 to the power factorial design with this example we have explained everything. So, the this kind of result. This table also we have shown you earlier and the corresponding that A and B that in the graph geometric representation also shown to you earlier.

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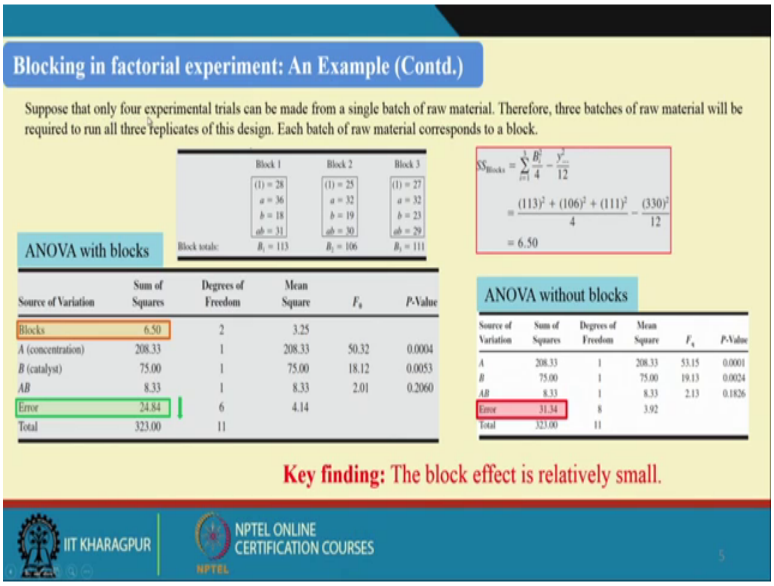

So, what is the difference here? Difference is suppose that only four experimental trial can be made from single batch of raw materials. So, the difference what we are finding out you have four settings 1, a, b, ab these are independent settings. So, you have the batch size raw material batch size in such a manner that you can conduct only four experimental runs. So, as a result what you require in order for full factorial design in one batch of raw materials you will get only that experimental experiment can be conducted. So, if as you require 3 replications at least you require 3 batches of raw material or it may so happen that you may not be or you will sacrifice some of the a factorial experiments per batch.

So, but here the procedure adequate is that let take complete experiments, that means, for all the settings per batch. So, as 4 experimental runs can be conducted every batches of raw material will be used to conduct this 4 experiments. That is what is given in this in this figure you see block 1 – 1, a, b, ab, block 2 – 1, a, b, ab and block 3 – 1, a, b, ab. So, overall 4 treatment combinations treatment combinations using 3 batches of raw materials where batches of raw material is representing blocks, ok. So, this is what is the what is blocking in experiment 2 to the power k factorial design.

So, you require you require suppose capital n number of experimental runs, but you do not have sufficient resources. So, what you are doing, you are using those you are taking separate resources like batches of raw materials one batch followed by after second batch followed by third batch like this and what you are doing here each batch is representing a block as well as a as well as replicates, ok.

Now, what will happen you require to find out the block effect is there or not. Second thing is that within under such situation what is the effect of a what is the effect of b and what is the effect of ab and how error is computed and then how you will test all those things. So, basically I want to tell you that you require to know develop the ANOVA table under this situation.

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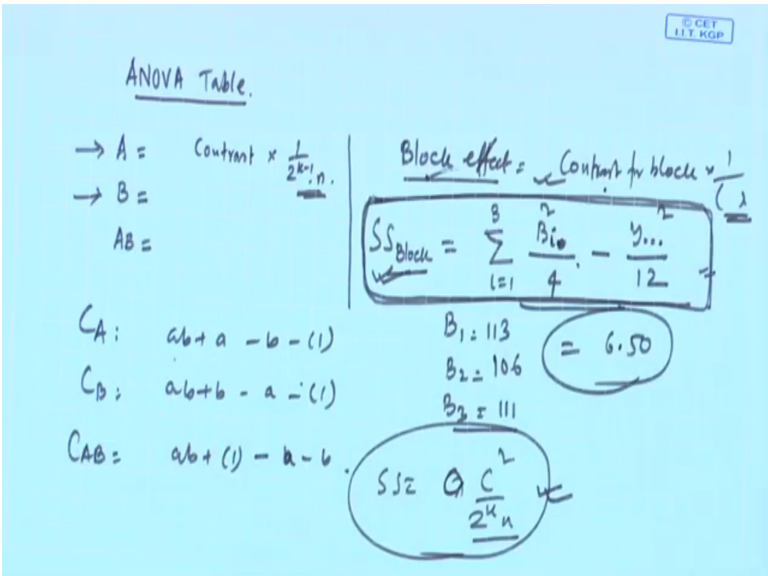

So, you know how to compute effect a, how to compute effect b how to compute effect ab. So, this is basically if you see that we say that contrast into 1 by 2 to the power k minus 1 into n. So, this is the general formula given to you. So, for every effect you have to find out the contrast and then you know that what here basically 2 to the power k minus 1, k is 2, 2 to the power 2 minus 1, 1 by 2n. So, you require to you require to you know that what is the value of n also n will be the replication, ok.

And, and also you have seen earlier that how to find out the contrast for a will be ab plus a minus b minus 1 that is contrast for a. Contrast for b, ab plus b minus a minus 1 and contrast for ab accordingly you have to find out means ab plus 1 minus a minus b and what are those ab, 1 and all those things you know that you if you from this from this table you can find out that what is a here you see 1 is 80, a is 100, b is 60, ab is 90 so, like this. This you have already done, ok. So, what you have not done here you have not done that what is the block effect? You see we have how many blocks 3 blocks. So, you create block total B1, B2 and B3. What is the block total for first block that is one 28 plus 36 plus 18 plus this 103, 13, for block 2 it is 106, for block 3 it is 111.

And, then you are finding out SS blocks SS blocks. So, i equal to 1 to 3 because what will be the SS block or block effect if I want to know block effect then definitely this is contrast for block into the that one that multiplier in terms of n ok, but here where other way we can use this formula that i equal to 1 to how many blocks, 3 blocks then in each block total you have found out B i let it be B i total B i square and then divide by 4 minus y triple dot square by 12, ok. So, B1 is what is B1 here? B1 is 113, B2 is 106 and B3 is 111. So, you will get block effect, but this is this is not this is basically block contribution block a bit, fine.

Now, I am talking about SS block talking about SS block. So, also you know that if you know the contrast you can find out the sum square, but here also you know that how to compute from the formula that is also known to you. So, here we are using the formula for SS block. So, let me repeat so that you will not get confused. So, the block effect and sum square block these are two different things sum square block is computed in this manner also using contrast we know that that if you want SS using contrast that is basically contrast square divided by 2 to the power k into n. So, this formula also can be used. Now, we are using the original formula what we have seen earlier. So, using this you will find out the SS block is 6.50, ok, 6.50.

So, now, let us come to the ANOVA table in the ANOVA table when you have different blocks. So, what are the sources of variation? Sources of variation will be, obviously, the factor and their interaction plus block here and error is always there. So, we have one two three four five sources of variation and for every source what is the sum squares? So, if for a it is 208, for b it is 75, for ab 8.33 we have seen this thing in earlier lectures that without block although these three we have seen and the result is given here. So, without block this is what we have discussed earlier without block. Then what is the error quantity? Error quantity is 31.34. You see the error here 31.34.

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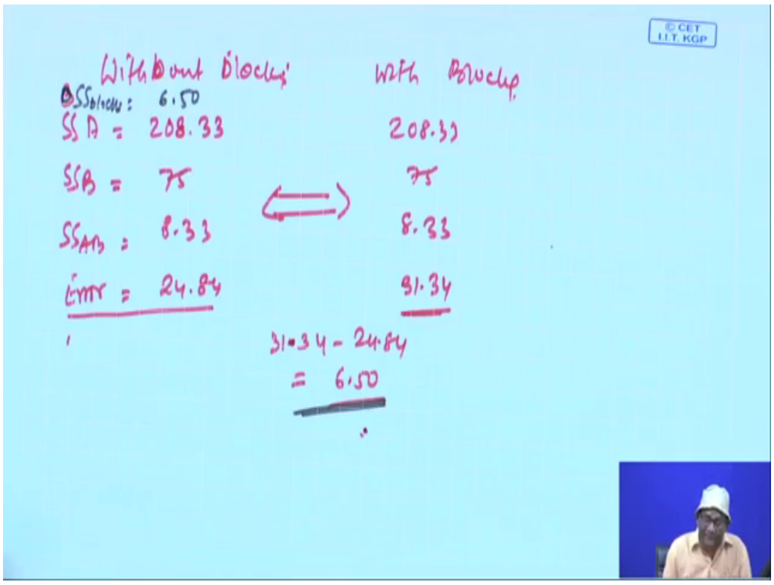

Now, without block error without block and with blocking; So, if you see a without SS A without block it is 208.33 it is without block and with block also 208.33 for SS B also it is 75 it is 75 for SS AB this is 8.33 this is 8.3 there is no difference. What about error here? Here error is 24.84, but here error is 31.34.

But, the difference lies if you compare differences in error. So, what is the difference in error between with blocking and without blocking that is 31.34 minus 24.84 which is basically 6.50, what is it? This is the effect of SS of block what we have seen earlier. So, as a result here block is there. So, the block SS block SS block which is 6.54.

Now, the value is significantly low I do not think this is very high value and it can be proven also through f test this is low so, that means, there is no block effect news batches of raw material are not significantly differing so that is the key finding here is that block effect is relatively small. But, the sole purpose here is you to say that with this example which we have tried to show you that how blocking is done in 2 to the power k factorial design.

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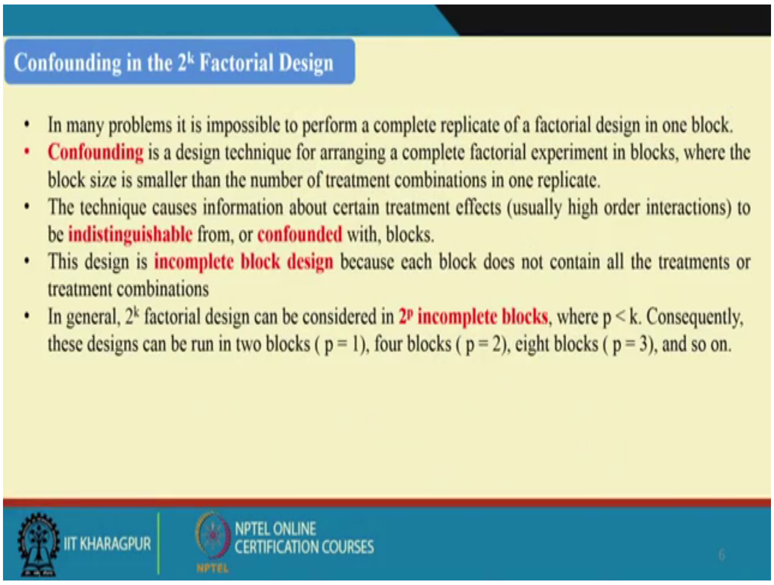

Now, come to the second concept called confounding. Let me read out, in many problems it is impossible to perform a complete replicate of a factorial design in one block, ok.

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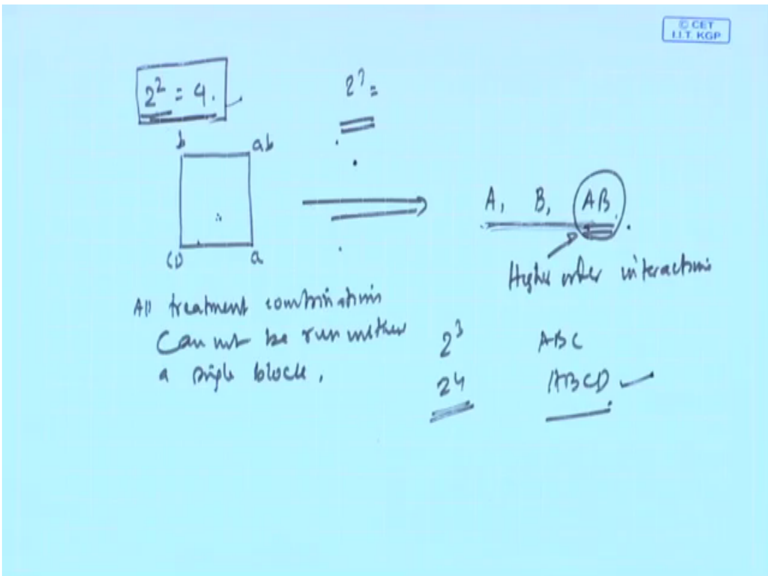

So, when I explained confounding with 2 to the power 2 factorial design. I said that you have batches of raw material and every batch can every batch is sufficient enough to conduct 4 experiment 2 to the power 2 equal to 4 experiment 4 runs possible here. Suppose, your batch size is not sufficient to a way to accommodate 4 experimental runs, but to be used for 4 experimental runs I mean what will happen? You cannot you cannot get a block which will be able to accommodate all the treatment combinations. So, all treatment combination cannot be run treatment combination cannot be run within a single block.

Please remember two things; first of all you have treatment combinations and you require to conduct experiment in all treatment combinations and also you want replications. So, precisely estimate the error. So, now, you have resource constant in the first case, last example what we have seen that you have batches of raw material which can accommodate all treatment combinations, then there what we have done is we consider blocks separate batch as separate batches of raw material every batch is a block.

Advantage there was that at least you can accommodate or you can run the experiment for all that possible treatment combinations. Here the situation is it is not only limited to block, but it also limited to the situation another situation that within a block all the treatment combination can be run because the size each the resource is not sufficient enough to conduct such so many runs. For example, within 2 to the power 2 design 4 runs 2 to the power 3 design 8 runs it is not possible.

So, then what happened confounding takes place and the technique what is used to overcome this situation is also known as confounding. So, confounding is a design technique for arranging a complete factorial experiment in block where the block size is smaller than the number of treatment combination in one replicate. So, even if you cannot go for all possible treatment combination per block, but because of confounding you can conduct this is a technique because you are confounding in the sense using confounding technique you can do it you can even do the experiments.

The technique causes information about certain treatment effects to be indistinguishable from or confounded with blocks. So, very interesting; You have in the in this example how many effects? Two main effects and one interaction effects; In order to estimate all the effects you require to conduct experiment all treatment combinations now what is happening here your raw material batch is not sufficient enough in this case you cannot conduct all 4 experiment per raw material per batch of raw materials.

So, what you are doing then? You are basically confounding your if you go for estimating the all the effects you will find out when you use several blocks use several blocks what will happen here the some of the effects cannot be estimated uniquely they will be confounded with the blocks, ok. So, some of the effects will be indistinguishable from the blocks. There they will be they are confounded and that is what is known as confounding and this design technique is incomplete block design because each block does not contain all the treatment or treatment combinations.

In general, 2 to the power k factorial design can be considered in 2 to the power p incomplete blocks, where p is obviously less than k. Consequently these designs can be run in 2 blocks when p equal to one 4 blocks b equal to 2 and 8 blocks p equal to 3 and so on. So now, key issue is that if in case of confounding you are not able to estimate uniquely some of the some of the effects. So, which effect you will be confounding with the blocks the redline is that higher order interactions. In this case obviously, AB if it is 2 to the power 3 design obviously ABC, if it is 2 to the power 4 design obviously, ABCD.

Sometimes it may so happen that only you require to sacrifice not only one parameter interaction parameter you may require to sacrifice more interaction parameters that means, more parameters will be confounded with the blocks in that case you will be you will be having a near to sacrifice more effects not to be precisely estimated or to be confounded with blocks.

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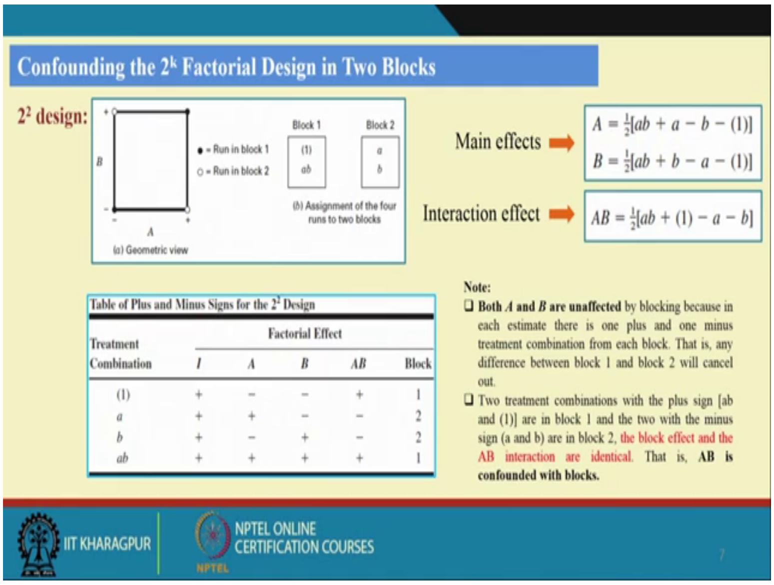

Here is the same example 2 to the power 2 design case. What we said that you cannot run 4 treatment combinations let you will run you have you can run two treatment combinations. So, you require two blocks; block 1 and block 2. So, in here in this example you see that we have we have dotted line the dots that is blacks and white dots these two. So, these two that is 1 and ab is run in block 1 and a and b is run in block 2. So, assignment of the 4 runs in 2 blocks this is the goal. What you require to do here? You you require to understand which are these treatment combination will go to block 1 and which are the other treatment combination that will go to block 2. If it is batch 2 two batches of raw material then batch 1 will be used for 1 and ab treatment combination and block 2 will be used for a and b treatment combination or other way, ok.

So, we have two things suppose I want to estimate the main effects you all know that that is the average response when the factor is at high level minus the average response factor is at low level if this is the case and this is the content and in this particular case A equal to 1 by 2 ab plus a minus b minus 1; B will be like this and C will be AB will be like this, this is known to you.

Now, come to this side you see that what is the block condition here. So, we have 4 treatment combination A, B and AB and if you see the contrast that is plus minus sign see that AB and block that AB plus 1 block 1 is assigned to AB and combination and block 1 to assign sorry block one assigned to ab as well as block assigned to 1 that is 1 and a.

So, now if I if you try to find out the average difference when AB at plus and AB at minus this is nothing, but basically that a the difference when the experiment is done using block 1 and block 2. So, that means, this difference and this difference are same and as a result what happened AB is confounded with block. So, you cannot separate them out and that is what is in here also.

So, as a result the few important notes one is both A and B are unaffected by blocking because in each estimate there is 1 plus and 1 minus treatment combination from each block. See, suppose you want to estimate effect of A, so, it is minus block 1 and plus block 1 means in each block minus plus is there low and high is there. So, as a as a result the effect is nullified.

Similarly, for B, but for AB it is not like this both high level or the both the factors are the same level that is with block 1 and both the say another le[vel]- and another one with block 2. So, the difference is not there that and two treatment combination with the plus sign and that is ab and plus 1 and 1 are in block 1 and two with the minus sign the block effect and the AB interactions are identical here that is AB is confounded with block. So, I hope that you understand the concept of confounding, you understand the concept of blocking, you have understood the concept of confounding.

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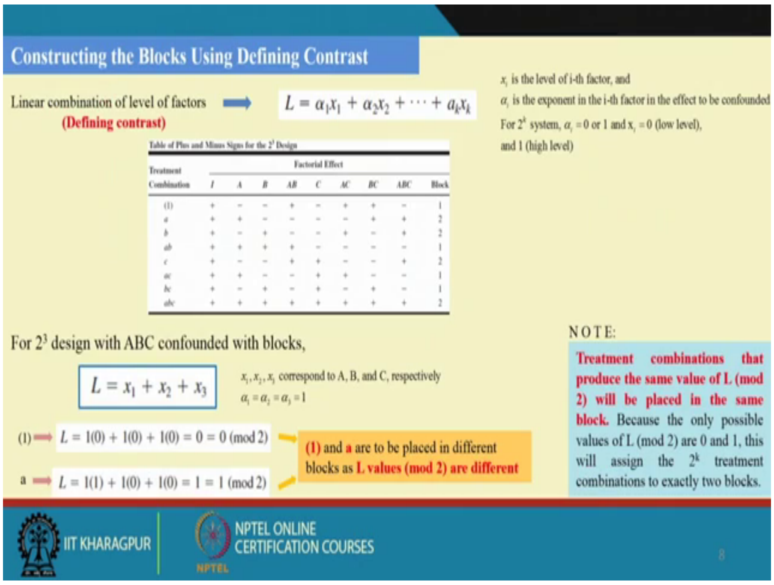

Now, finish this lecture just by discussing that using defining contrast how can the blocks will be constructed, ok. So, you all know that the contrast is the linear combination of the level of factors. So, this is what is written like this one equal to alpha 1 x 1 plus alpha 2 x 2 plus alpha k x k. So, what we are doing here?

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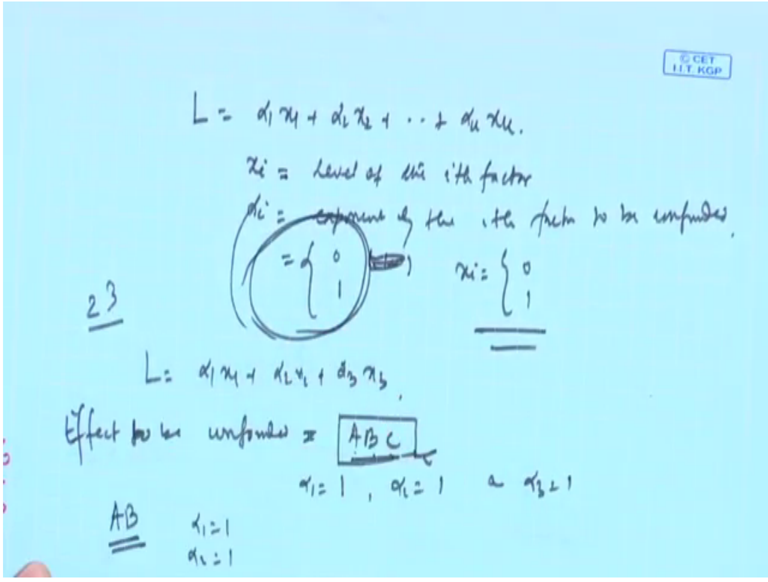

The contrast you were writing alpha 1 x 1 plus alpha 2 x 2 alpha k x k. So, if I say what is x i, this is the level of the i-th factor and alpha i is the exponent of the i-th factor to be exponent of the i-th factor to be confounded, suppose and it will take value 0 or 1 it will take value 0 or 1, ok. So, I will be 0 or 1 and x is 0, ok. So, 0 for low level when x i had low and x i and x i will be that we know either 0 or 1, ok.

So, now, come back to the slide again, in fact, we are in the seeing the slide only. So, if 2 to the power 3 design case these are the treatment combination and you know that what is this these are the these are the these are the basically plus minus sign and which will help you in defining the contrast, fine.

Now, how do you assign blocks suppose you require to assign two blocks. So, as I told you that using linear combination of factors it can be done, ok. So, first you determine that which effect to be confounded with the blocks. So, you your guideline is the highest order interaction if you ok. So, highest order interaction in this case is ABC, right.

So, then consider to the 2 to the power 3 design then l will be alpha 1 x 1 alpha 2 x 2 plus alpha 3 x 3. So, effect to be confounded the effect to be confounded equal to ABC. So, here all ABC in this case all the three treatment combinations are present other way other way I can say not all three complement combination. So, all the three factors are present in this interactions three factors into accessibility.

So, as a result alpha 1 equal to 1 alpha 2 equal to 1 and alpha 3 equal to 1. Suppose, if you think that no the effect to be confounded is AB in this case then alpha 1 will be 1 alpha 2 will be 1, but alpha 3 will be 0, that is what I said exponent of this will take 0 or 1 value.

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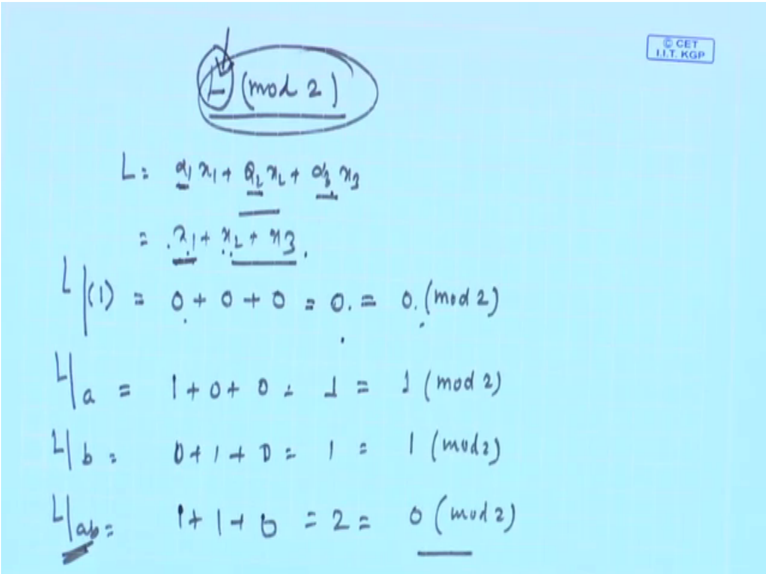

So, what you require to do now? You require to find out a interesting concept called suppose that L L mod mod 2 this one. L mod 2 this concept for example, or other way I can say that you just first find out L. L is alpha 1 x 1 plus alpha 2 x 2 plus alpha 3 x 3. In this case as to be as the effect to be confounded is ABC alpha 1 alpha 2 alpha 3 is 1. So, this is x 1 plus x 2 plus x 3 only, ok. Suppose, you now consider the treatment combination first treatment combination is 1 in this case, what is x x 1 value? x 1 value.

So, I want L for one then in this case x one value is 0 x 2 value is 0 x 3 value is 0 ok. So, then this will be 0, because alpha 1 1 alpha 2 1 and alpha 3 1. So, 1 into 0 plus 1 into 0 plus 1 into 0 this is 0. So, this can be written as 0 mod 2, ok. There is something like this minus this by 2, so, anything divided by 2 what will be the remainder will be either 0 or 1. So, now, similarly you find out that second one L at a. So, what will happen, this will become x 1 become 1 plus 0 plus 0 this is 1.

So, this can be written as 1 mod 2. Similarly, L at b this will be 0 plus 1 plus 0 that is 1. So, 1 mod 2 suppose L at ab we can write that L at a when a and both a and b are played what we write x 1, ok. So, there are many more treatment combination not only this. So, those so, that mean so, 1 plus 1 plus 0, so, this is 2. 2 will be 0 mod 2. So, in this manner you have to find out.

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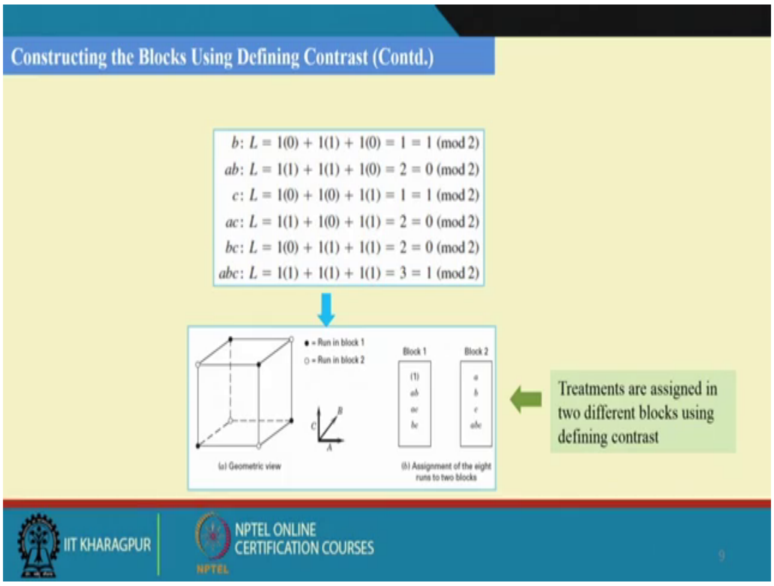

You see what we are given here I have written b; b is 1 mod 2; ab 1 plus 1 plus 0, 2, 2 0 mod 2; c will be 1 mod 2; ac 0 mod 2; bc 0 mod 2; abc 1 mod 2. So, ultimately you are clearly getting something like this L mod 2 and in this case obviously, this value this value will become either 0 or 1, because it will be divided by 2 only the remainder will be either 0 or 1. So, then you require to assign the two blocks. So, wherever you are getting 1 you assign to them in one block and 0 to another block, ok. Now, in this example you see that block one contains 1, ab, ac and bc. So, that means, block 1 all the 0 cases are assigned to block 1 and 1 cases are assigned to block 2.

So, this is what is you are assigning the blocks to the different treatment combination experiments. So, treatment are assigned in two different blocks using defining contrast, ok.

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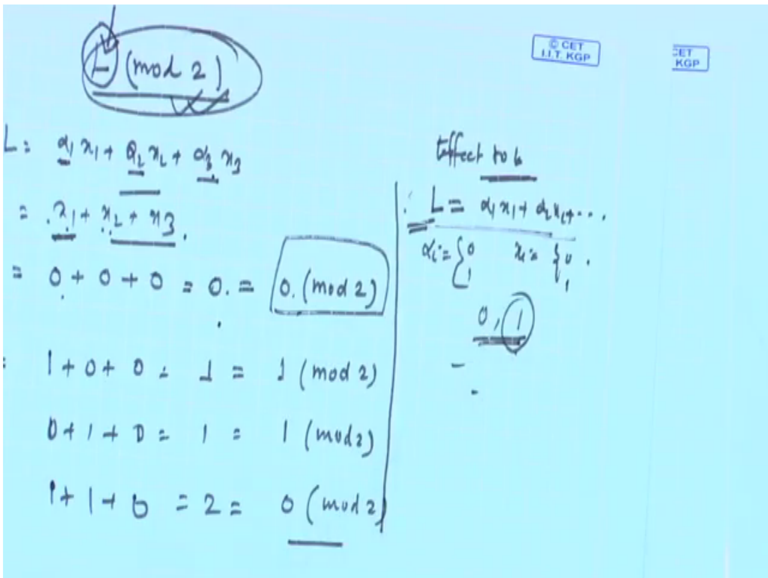

So, we will start again with contrast, but for the time being for the time being let me tell you that you have to create that which effect to be confounded and accordingly you find out L. L is alpha 1 x 1 alpha 2 x 2 like this and then you know there are two things; one is alpha i it will take 0 or 1 value as well as x i will take 0 or 1 value, depending on which treatment combination you are interested in and accordingly that and also the effect to be confounded you will get two things alpha values as well as x i values. Put here find out the value of L and that value you write in this terms that L mod or in this term L mod 2 in this term you write.

So, as the things will be divided the integers are divided by 2 the remainder will be always 0 or 1. So, you assign to two blocks. So, all the 0 treatment combinations will be assigned to one block and another one treatment combinations will be assigned to another blocks.

So, thank you very much. We will continue it in the next class.