Design and Analysis of Experiments

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Lecture - 04
Types of Experimental Design (Contd.)

Welcome to the 4th lecture of Design Analysis of Experiment. We will continue with Types of Experimental Design.

Why it is not going?

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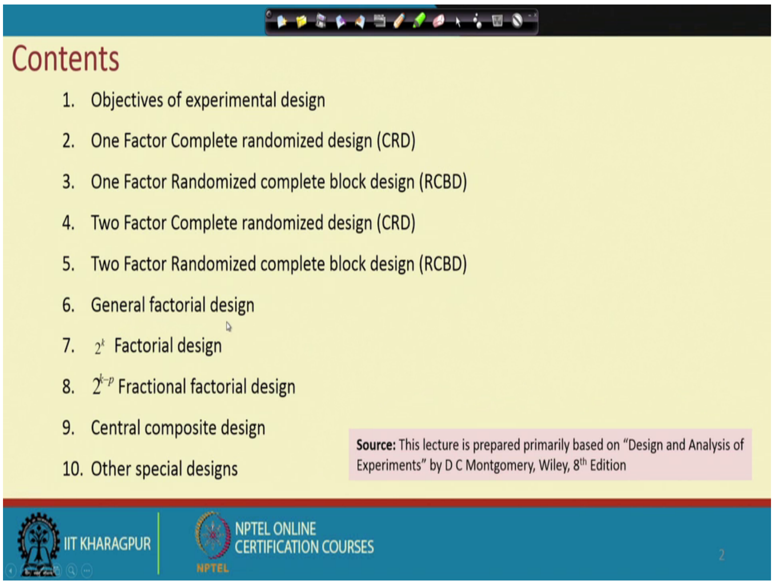

So, in last class we have completed up to general factorial design. So, by types of experimental design we mean that depending on the situation, different kinds of experiments to be conducted, and what are those experiment those types of experiment, which has scientific basis and designed scientifically we are discussed here.

There are many more designs which are available all will not be discussed most commonly used things are discussed here and accordingly last lecture in last lecture we have discussed up to general factorial design. Now, we will explain few more like 2 to the power k factorial design 2 to the power k minus p fractional factorial design, which is central composite design and we will name some of the other special designs which we will not be discussed here.

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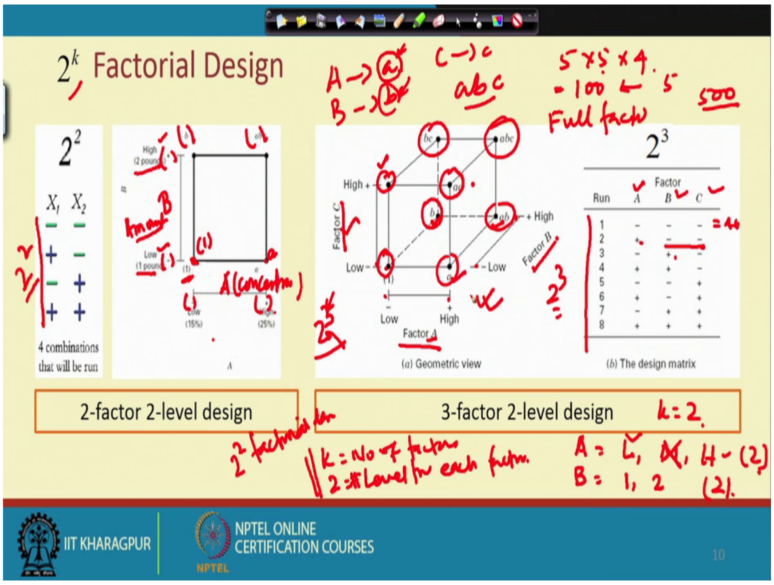

So, we have seen that in general factorial design. There are many factors each of the factors having different levels not necessary that the factors will be at 2 levels only, the factors can be have different levels like a factor can be factor a can have a level, factors b can have b level, factors c can have c levels, like this there are several levels for example, let.

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It is for example, let there are 3 factors A B and C, now if A has a levels, B has b levels, and C has c levels. Then how many treatment combinations will be there a b c, if we consider a is equal to 5, b equal to 5, and c equal to 4, then this is 5 5 25 plus 400 different settings.

Now, again is settings if we conduct 5 replications. So, you have to do both 500 experimental runs, only for 3 factors now if the vector is 5 or 10 or 15. So, what will happen? So, the number of experimental runs will be huge and as a result you might have not in a position to conduct the experiments, because cost is a issue, time is a issue, resource is a another issue. So, cost time resource other type of resource all those things putting together, we will make it impossible to conduct full factorial experiment full factorial experiment and if you think that there will be A levels of for a and B levels for b C level for c it is.

So, under such situation what you will do we will not conduct experiment yes, you can do experiment you can reduce the levels, you can make some systematic leveling also like, you can go for 2 to the power k factorial design where k is the number of vectors and 2 is level number of level for each factor, what does it mean in in our earlier example factor A 3 levels low medium and high and factor B 2 levels type 1 and type 2 filter. Now if we go by this principle or this method there in this k equal to 2 and what changes is required we may not considered middle medium clutter. We will consider low and high then this will also level 2 this is also of level 2. So, you have 2 factors each with level 2 we and the design is 2 to the power k means 2 to the power 2 factorial design.

For example, here you see that this is let this is factor A this side this one is factor B maybe this maybe some corner suppose in a chemical reaction this is the concentration and this is maybe the amount used pound. So, let if the concentration is 15 percent we denote by low if it is 25 percent we denote by high. So, either minus or plus so low minus high plus. So, this is basically 2 levels here also if the amount used amount material amount material is 1 pound it is low, if it is 2 pound it is high, this low is again minus high is plus this symbol it is also 2 levels. So, that effectively you have 2 to the power 2 means 4 treatment combinations.

First combination of these we are denoted by within bracket 1, this is your when factor A is at high, but factor B at low level this we are denoting by A, then this is a combination A treatment combination or the independent experimental setting where A is at low level B is at high level. So, this is denoted by B and this is one where both A and B are at high level this is denoted by A B. So, this kind of design is known as 2 to the power k factorial design in 2 the power k factorial design, please keep in time that we have k number of controllable factors and each of the factors are set to 2 levels, level low high or minus plus or minus 1 plus 1 that way sometime we will use coating variables minus 1 plus 1 that we will discuss when we go for detailed analysis of 2 to the power k factorial design.

Now, in this 3 factor case if A equal to B equal to C equal to 2 then this is the design. So, this is 2 to the power 3 factorial design what does it mean we have 3 factors factor A, factor B and factor C and factor B we have 3 factor A B and C and each at 2 level. So, this is k this is each has 2 level. So, level one this one low high, low high, low high. You have how many X A treatment combinations or independent or distinct experimental settings 8 2 to the power 3 here it is 4 2 to the power 2, because 2 factors. So, 1 to 8 now what is the first one when all are at low all low A at low level, B at low level, C at low level. When A at high level, other 2 at low level this is A, when B high other 2 low. So, this is your B.

When both A and B high C low this is the setting A B when C, at high other 2 low this is the setting this one these are we have already discussed. Now when both a and c are high b at low this is the setting similarly bc, similarly abc. So, now if you have 2 to the 4 factors each at 2 level either you have 2 different cube like this where we can what we will do.

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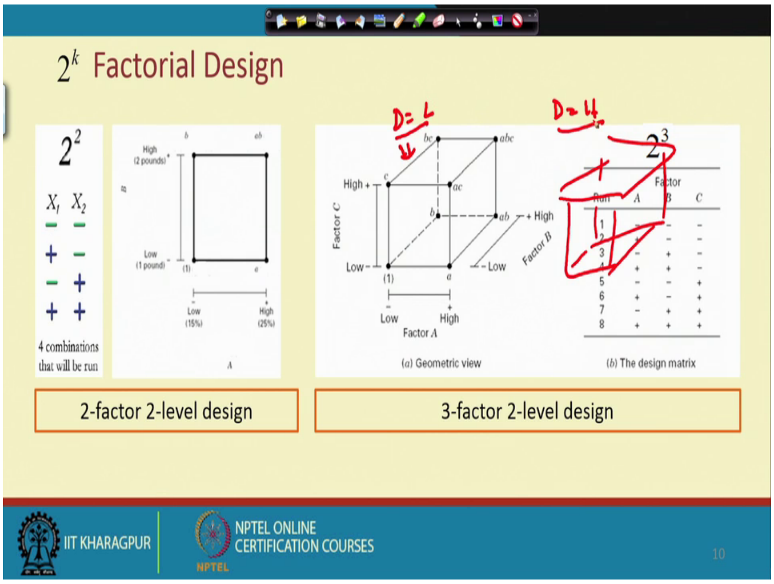

Show here, suppose another factor D is there D at low this kind of and then D at high another cube we will make another cube for D at high. So, if you have if you have further that k equal to 5. So, you cannot pictorially like this cubic representation not possible for how much higher level and not required also 3 level.

So, this is known as 2 to the power k factorial design. So, what is the difference between 2 to the power k factorial design and general factorial design 2 to the power k factorial design and general factorial design?

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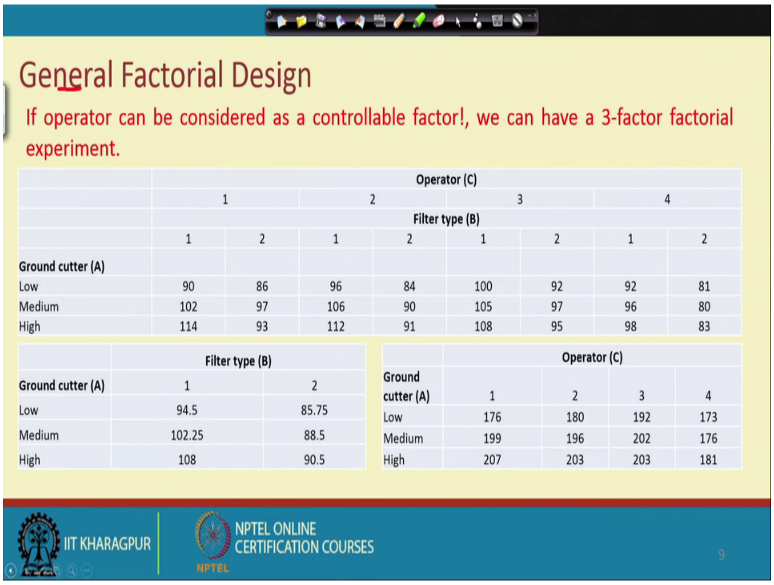

In general factorial design what you observe you observe that, the factors having different levels factors having different levels.

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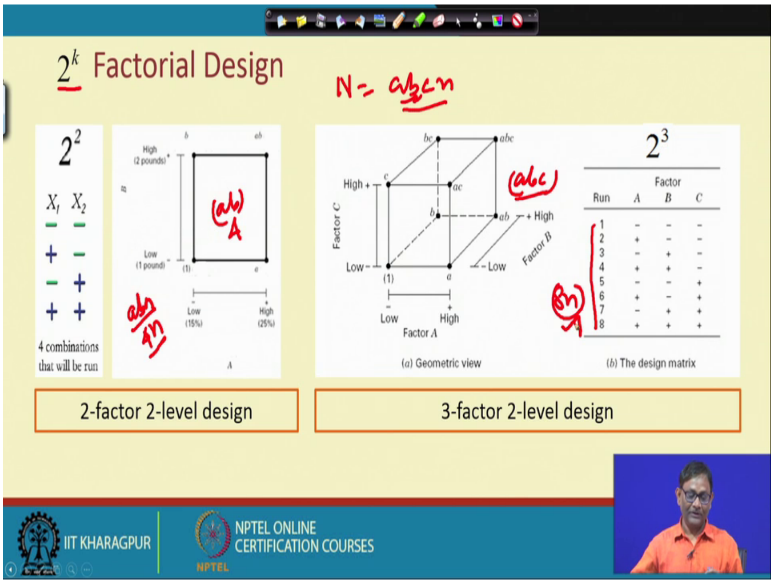

So, a with a level b with b level c with c level and with 3 factor case that the total number of experimental run will be a b c into N for 3 factors n is the replication, but here what will happen you require 8 into n abc equal to 8, here what happened if it is general factorial design it will be a b a n number of experiments and total treatment is combination ab here treatment combination abc. Now if you go for 2 to the power k factorial then here will be 4 treatment combination and you require 4 number of experiments to be conducted with n replications.

Here it is abc treatment combination 8 n 8 n number of total number of branch. So, now think of a situation I do know it (Refer Time: 13:04) disturbing can we.

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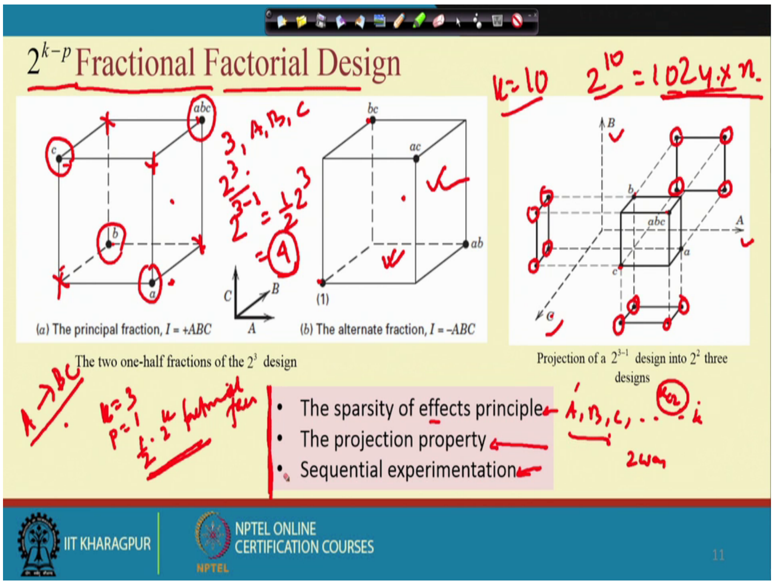

Now you see that even if you go for 2 to the power k factorial design, suppose there are 10 factors k equal to 10 what will happen you require 2 to the power 10 in in distinct treatment combinations for other way get distinct setting which is 1 0 2 4. Now if you choose replication n which is 1 0 2 4 n, even if you considered 2 this will be 2048. So, many runs required. So, even if you have k number of factors each with 2 levels, when k is large number of experimental runs will be very large also. So, that is why what happened it may not be feasible many a times to conduct 2 to the power k factorial design given that k is large.

So, under such situation can you not do experiment with lower number of runs instead of 1 0 2 4 n number experiment runs, will it be possible to make it maybe half of this or maybe 1 fourth of these or maybe 1 8 of these is there any scientific basis can we go for this, all those things will be will be answered in 2 to the power k minus p fractional factorial design.

So, in this class we will not go into details of the 2 to the power k factor effects no factorial design how it is done and also why it is accepted the scientific basis and all those things, but for the time being we want to know what is this 2 to the power k minus p fraction factorial d.

Let us assume that you have 3 3 factors a bc 3 factors A B and C and you are doing 2 to the power 3 factorial 2 to the power 3 factorial 2 to the power k time and 2 to the 3 factorial design you are basically accept accepting this, then you require 8 number of independent runs and if you use that replication it will be 8 l, but you do not have that much money or that much resource materials not available or operators not available you cannot do this.

So, then if I go for 2 to the power 3 design and if I do it; that means, each of the vertice all the settings you are consider considering for this cube, instead of that suppose you want to do 2 to the power 3 minus 1; that means, 1 half of 2 to the power 3 so; that means, 4 number of independence settings you will consider or 4 treatment combination you will considered.

Is it possible it is possible how it is possible we will not be discuss and now we will be discussing in when will be it is analyzing 2 to the power k minus p factorial design. So, in this case my k is 3 p is one this is known as 1 half into 2 to the power k factorial design fractional factorial design. So, then what are those 4 points here 4 settings these, these, these, and these.

So, question is that 4 independent settings or treatment combinations you have not chosen one 2 3 4 this is not chosen this is not chosen this is not chosen. So, whether we lead to work or not. So, it will work because there are certain hypothesis or observations or all experiences already known or some rule rules and laws that are already known who is supporting this type of experiment.

One is that sparsity of effects principles, sparsity of effects principle means suppose you have many fact many factors large number of factors k number of factors. These are the principle says that the mean effect like ABCD all those things and lower order interaction say if it like k c 2 this lower order interaction a (Refer Time: 18:21) has will become significant mostly not the higher order. So, in that case in that case what happen simultaneously if we use the projection property, what is this projection property projection? Property says that a fractional factorial design at the higher level has many full factorial design full means 2 to the power k factorial design at the lower level. What does it mean suppose you do this experiment or this one? Now for example, here we are considering the, which one, this one, may this second one, this one here here here and here.

These 4 experimental settings you have chosen for experiment others you are not chosen. Now you have you have taken 3 variable vectors A B and C, if you project this 2 2 dimensions like either A B or A C or B C, what you are getting you see the projection is such that. In the 2 factor level this is full factor full one all the factorial. Points all the factorial points are having data, you know already conducted experiment there even that higher level it is fractional means some of the factorial points are not considered, but at one lower level here this is full similarly here you see every here point anything here also. So, that is what says that because of this projection property a fractional factorial design at higher level we will have many full factorial design at lower level.

So; that means, if I am interested to know the main effects and interaction effects only that mean 2 O A interaction effects only then this is an half because here you will get a b and ab interaction here ac ac interaction. So, this is possible to get then then another one is sequential experimentation this is also supporting this fractional one. For example, suppose you have done this what we have seen here and you analyze the data, if you analyze the data you will not be using this design you will not be able to estimate all the effects parameters ab c ab ac abc, because sum will be confounded with others because there are not get data points not available in the half of the independent experimental settings. So, there will be a a effect will be confounded with bc b will be confounded with ac c will be confounded with ab.

Confounded in the sense they are the experiments is conducted in such a manner that they are both the effects where, basically I can say take it together what the effects taken together and when you when you analysis and get the effect you will get the confounded effect the total effect of the 2 not individual effect. So, suppose you want the higher order interactions also to be taken to you or some of the confounders also that you have to find out and their effect also separately you want to calculate. In that case suppose you require another experiment to be conducted, then you will not do second fractional experiment second experiment with fractional factorial design at all those (Refer Time: 22:14) you will do the remaining points.

So, if I go for one half fractional factorial design then if I do 2 different experiment time experiment one with that some 4 points into the power 3 design a 3 another another 4 points with 2 to the power 3 that case. So, then if I merge the 2 data set together you are getting full factorial 2 to the power k factorial design for the 3 3 3 variable case here. So, that mean if you do another experiment you have full data set for this. So, these are the these are the reasons while we will go for fractional factorial design and this is the result will be used for screening purpose; that means, what are the factors effecting what are the factors not effecting for example, if I go for fractional factorial and A is compounded with BC.

Suppose while estimating we founded these effect is not significant then none of the effect is significant you can eliminate name.

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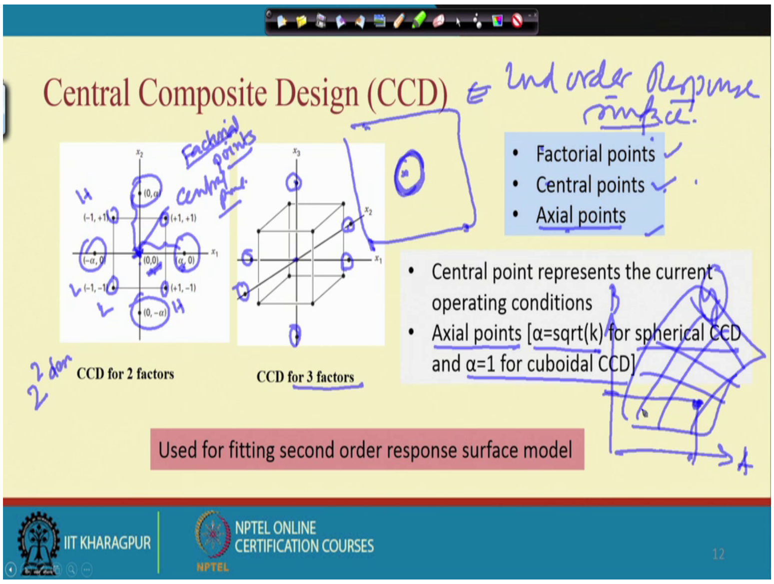

So this is 2 to the power k fractional factorial design. Now another important design is there known as central composite design for example, you consider a 2 to the power a 2 to the power 2 design sorry you consider a 2 to the power 2 design. In this case what you we do you will your settings independent settings these are nothing but known as factorial points factorial points.

So, 4 independent settings, 4 different independent combinations, now it means so happened that suppose if these are in you should if the operator the actual operating situation. So, may not be represented by this because it is obvious because what you will do ultimately you know the process operating zone and you try to find out the low and high value here and here also low and high value. If my that entire operating zone is these. So, low and high value you are considering, but will the operator operates at low and high all the time rather operator will be operating at a point which is middle of this, this is known as most of the time production will be done at this label. So, then this is known as central point.

So, if you have the experiment you do experiment at this central point, then you have more information about the current operating situation and the resulting performance if in terms of response variable. So, in order to capture that we must go for another setting which is or treatment combination if this is known as center, which here in this case we have treated as 00, but why 0 are plus or minus 1 this things we will discuss later on. We basically use some coding scheme here you see that 3 dimensional case 3 variable case 3 factors case. So, this is the center point.

So, you in a design experiment you have a factorial points, you have central points, central point usually talks about the current operating, performance, current operating zone and resulting performance if you will data on it and at this case. And another one is that suppose you may think that that only this low high and centre point is not sufficient to capture the totality. So, you may go for some other points call axial points, what does it mean you may do experiment here experiment here experiment here as well as experiment here or here in this case particular case this one this one this one this one and first second and this is another one this is another one.

So, axial point we mean this there from the centre it is alpha distance apart here on the both the axis x 1 and x 2 x is now what is this alpha there are different-different ways to choose the value of alpha, but sole purpose is you get more experimental settings or more treatment combinations and you get more data. So, that these definitely reach data, if I have only factorial point the data I have and if I go for central point the data I gather will be richer than the first one. Similarly if I have this plus this plus this it will be rich data richer data than the previous one richer data mean give more information.

Now how do you choose the axial point that alpha value maybe square root of k where k is the number of factors, it is known as spherical central composite design or k alpha maybe one for cubical central composite design.

There are there are some other designs also the for optimal the point of be there may be some more issues that that when you go for central composite they may in detail that time you will know. So, that mean here what happened you got something more one is central composite design with central point only central composite design with central and axial points and if these why you require more data, because you require better response surface this is may be used in second order response surface. The sometimes that is why this is known as also response surface design, what is response surface response surface means given the x and suppose factor a and b how the y suppose the y is where behaving how y is behaving.

Now, this is the y y response when I mean in my A is here and B is here my response value will be here. So, you require when you go for second order that mean the quadratic effect will be there curvature effect all effects will be there. So, as you result what happen you require more number of parameters to be estimated? So, you require more data points otherwise you will not be able to estimate many important parameters. So, in order to achieve these central composite design is preferred and you get more data points.

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So, some other design like Latin square design, Graeco Latin square design, mixture design, Plackett Burman design, robust design, nested design, split plot design. So, these are depending on the situation you have to use.

For example Latin square and Graeco Latin square design, in Latin square design is suppose it is special kind of design where you require less number of experimental runs to be conducted and then you will be able to accommodate the blocking variables there also. You require suppose you require a 2 factors one blocking variable go for Latin square you require 2 factors 2 2 number of blocks or 2 number of means this is variable to into 2 into the experiment regulating square, Mister Jain I already told you robust design is very important one because robust design is as I told you that the product you will produce that will be used under different users environment.

Now, if you design is not a robust. So, that it can visit and all another inventions or environmental factors in in in the operation where it will be used. So, then it will no it is not a good products. So, accordingly robust parameter design in the in the area of quality engineering Taguchi has introduced this one, that caused a design another designs are there. So, next a designs need plot design and some more designs are also available.

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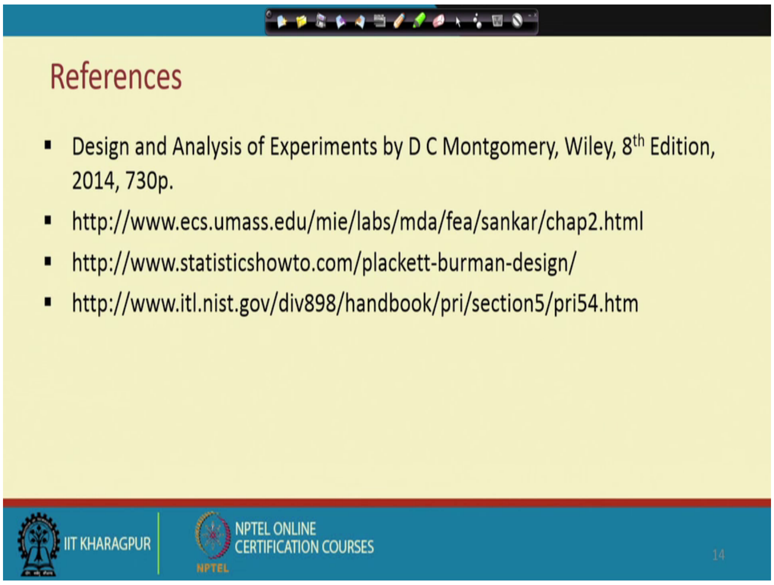

I recommend all of you to go through design and analysis if experiments by D C Montgomery and in different chapters all different types of designs are explained not in any one chapter the way I explain today, but it is has the experimental design is very very important and the types of design is extremely important. So, you must know all those a types and before going for conducting any experiment, because if you know the different types and under what situation all those which experiment to be chosen and then accordingly you will plan for experiment and gets the experimental designs when actually you do in practice.

So thank you very much, this is what the introductory part of experimental design related to the design and it is experiment. And in next class I will start with experimental data and different kind of statistics and how do you compute all those statistics. And finally, goes to probability distribution and different kind of sampling distribution, and then hypothesis testing in maybe next 3-4 hours of time we will be discussing all those things. Do not miss any of those classes, because the continuity means of these particular subjects will be understood from those lectures.