Design and Analysis of Experiments
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Lecture – 38
2 k Factorial Design Single Replicate

Welcome, we will continue 2 to the power k factorial design.

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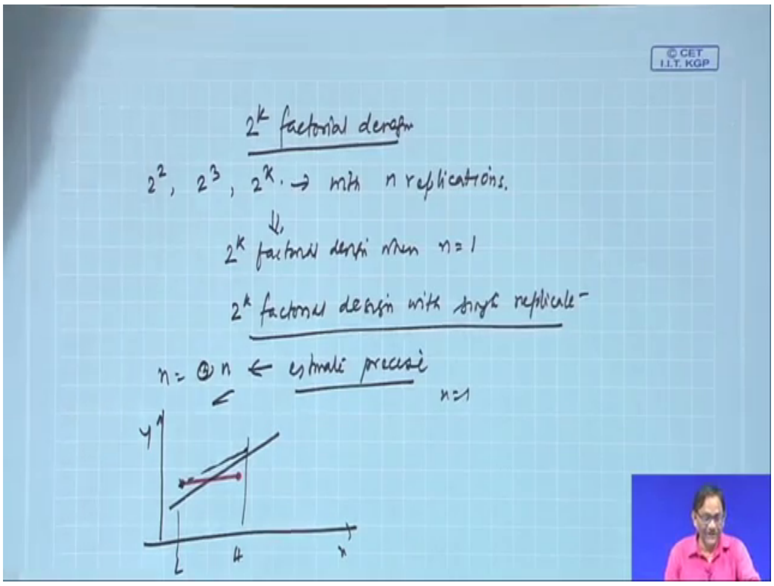

So far we have seen 2 to the power 2 design, 2 to the power 3 design and general 2 to the power k design with n replicates or with n replications. So, in this lecture, we will see when what will happen to 2 to the power k factorial design when n equal to 1, otherwise we say today's lecture is 2 to the power k factorial design with single replicate. So, topic is 2 to the power k factorial design with single replicate.

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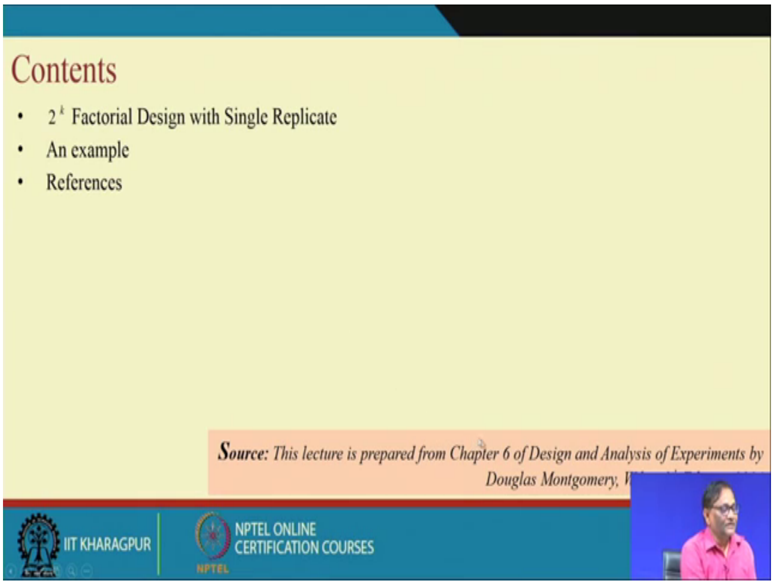

Now, what are the things we will discuss, we will discuss in detail, how do you conduct the analysis when you have single replicate in 2 to the power k factorial design with an example, this lecture also prepared based on chapter 6 of Montgomery design and analysis of experiments.

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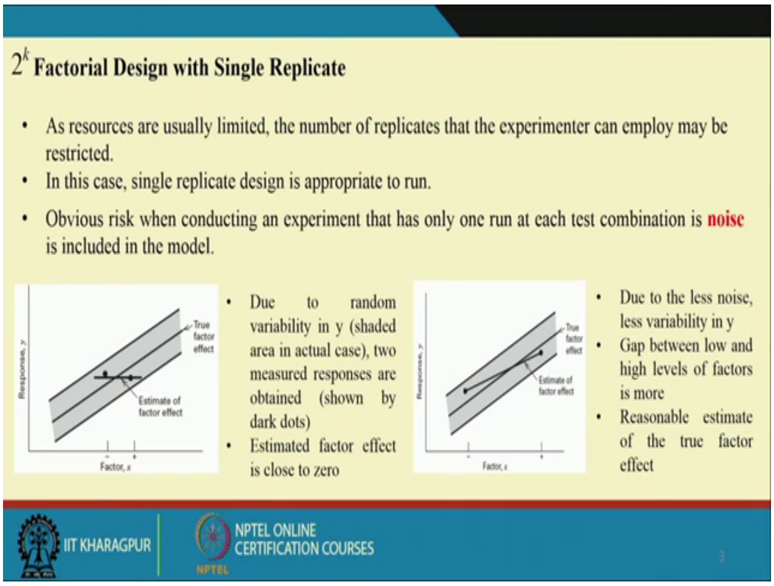

Let us see, some of the issues related to your experimental design and the prediction of response line or response surface, when the number of replication is less. So, we will go for less replication or single replication weighing to the resources or other way in a limited resources. So, that mean if you have limited batches of raw material, in that case what will happen, you cannot conduct full replications, in replications you may require to conduct 1 replications per batch, for a particular combination like this 2 to the power k. So, what I mean to say, you have resource scratch whatever may be the reasons.

So, now if you do that, what will happen that the effect of noise that will be there and that effect cannot be separated out ultimately and the estimate because of the replication n is 1, the error estimation will become incomplete or it is difficult or it may not be able to complete the error you will not be able to do this, that is the true factor effect which is in client slightly this one and this may be the bound 95 percent bound.

So, if you have less number of observations like single replicate case you may get a predicted or fitted line like this. So, in this case what happen? It is debating from the true factor effect like that mean your factor if a factor you keep at minus or low level. So, the true effect is here and when you make it high level the 2 factor difference is there, but because of less number of observations if you get like this where it is showing almost no effect on y.

So, this is due to random variability in y to measured responses are obtained shown by dark dots estimated factor effect is closed to 0. Now, if you have more number of observations, what happen? This estimated effect fact line this will become closer to the true effect. So, here what happen, your due to less noise less variability in y gap between low and high level of factors is more and reasonable estimate of the true factor effect. In a nuts cell what I mean to say, if n equal to n equal to n your estimate will be more precise and it will be suppose, if this is the true effect it may be close to that low to high, this is y and this is x. If n is 1, what will happen, you may find that the same case that may be like this.

So, there is no effect that is not the true sets. So, be careful that the replication is a issue very key important issue. So, it is always advisable that you go for more replications, but many a times you cannot do this. So, here what we are showing you that if this is the situation, how can you analyze the experimental data and find out that what are the effects that are significant and what are the effect that can be excluded from the model.

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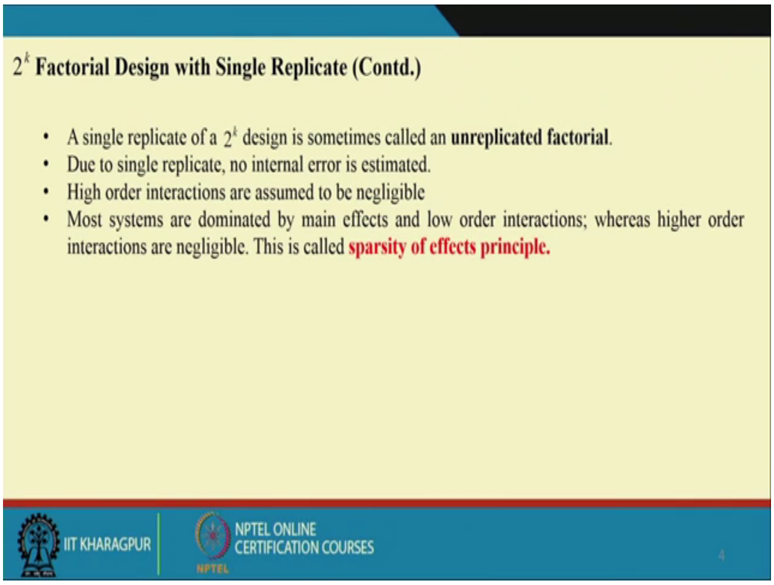

So, a single replicate of a 2 to the power k design is sometime called an unreplicated factorial, due to single replicate no internal error is estimated, high order interactions are assumed to be negligible, most of the systems are dominated by main effects and low order interaction; whereas higher order effect interactions are negligible, this is called the sparsity of effects principle. Now, question is that why single replication will work? This will work because of the sparsity of effect principle.

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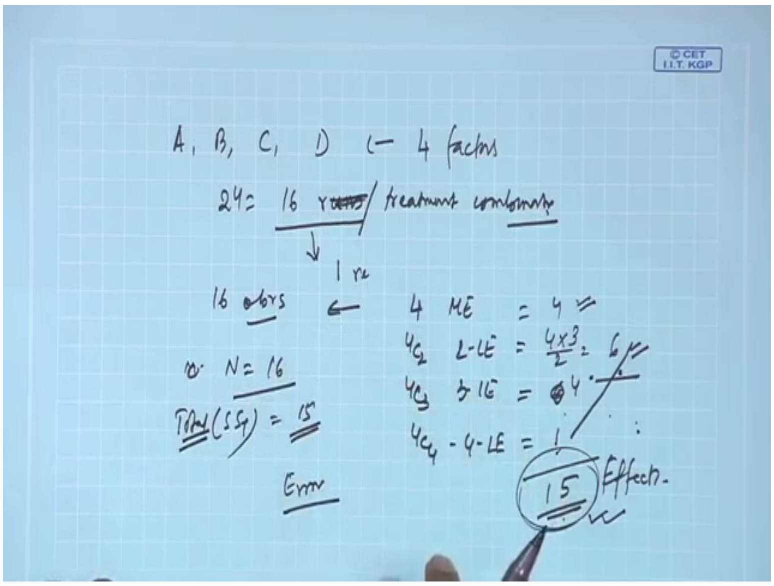

Suppose you have A, B, C and D that 4 factors, each at 2 levels. So, 2 to the power 4 equal to we have 16 runs or 16 run means 16 treatment combination let it be, treatment combination. Now, if you have every treatment combination having 1 replicates, so you have 16 observations and you know that what are the number of parameters to be estimated, 4 main effect you have to estimate, 4 C2 2 a interaction effect, 4 C3 3 a interaction effect, 4 C4 4 a interaction effect, these are the effects you have to estimate. So, what is 4 C2? 4 into 3 by 2 that mean 6; 4 C3 also 6; 4 C4 is 1 and this is 4. So, how many you require 6 plus 6; 4 C3 is 4 factorial 4, this is 4.

So, you require to estimate 15 effects, correct. Now, if your n equals capital n total is 16. So, for total computation of SST, total SST that you have 15 degrees of freedom. Now, you require 1 degree of freedom for each of the effect parameters. So, this 15 degrees of freedom will be lost because 1 for 4 for degree will be lost for main effect 6 and another 6 plus 11 for the interaction effects, for error there is no degrees of freedom available means no free observations available, which help you to estimate the error. So, this is what is the problem you will be facing when you have single replicates, then what is the way out? Way out is that, the sparsity of effect principles.

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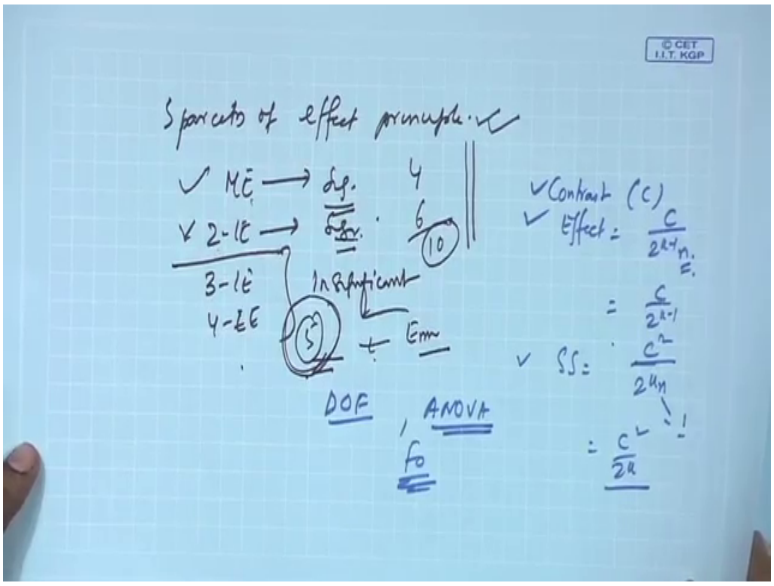

Sparsity of effect principle, what does it mean? Main effect usually becomes significant, may be 2 interaction effect will also may be significant, but 3 interaction effect 4 interaction way interaction effect then that may be insignificant.

So, that means, you do not require to compute all the parameters; effect parameters 4 plus 6 may be you will estimate 10, then remaining this 5 will you do not require to estimate, in that case what happen this 5 degrees of freedom they will go to error estimation.

So, that means, we will be in a position to check whether this effects are significant or not, by saying this am not saying that all the 2 a interaction will be significant or all the main effect will be significant, if it is sparsity principle says that the higher order interactions usually contribute less compared to the lower order interactions.

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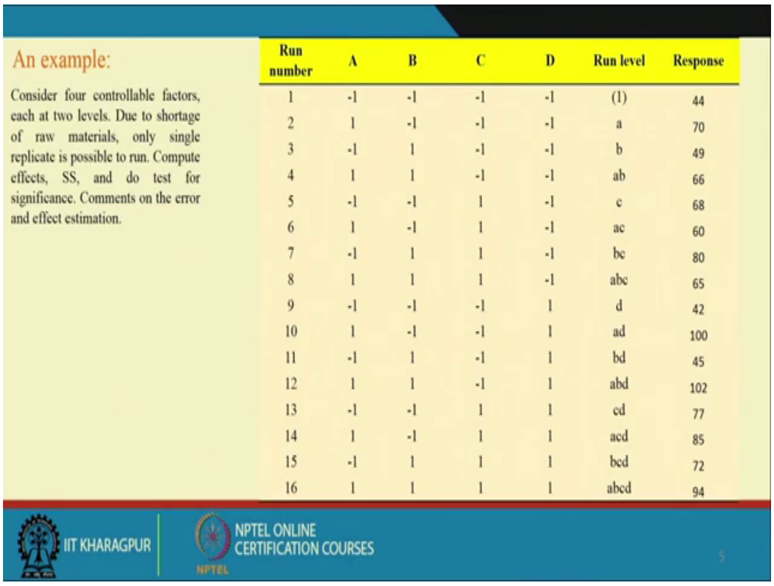

Run number	A	B	C	D	Run level	Response
1	-1	-1	-1	-1	(1)	44
2	1	-1	-1	-1	a	70
3	-1	1	-1	-1	b	49
4	1	1	-1	-1	ab	66
5	-1	-1	1	-1	c	68
6	1	-1	1	-1	ac	60
7	-1	1	1	-1	bc	80
8	1	1	1	-1	abc	65
9	-1	-1	-1	1	d	42
10	1	-1	-1	1	ad	100
11	-1	1	-1	1	bd	45
12	1	1	-1	1	abd	102
13	-1	-1	1	1	cd	77
14	1	-1	1	1	acd	85
15	-1	1	1	1	bcd	72
16	1	1	1	1	abcd	94

We will see a all those things with reference to an example and by this time I am assuming that you all know that what is contrast and you know what are, how to compute the effects equal to contrast by 2 to the power k minus 1 into n, here n is 1, so c by 2 to the power k minus 1, so then, your SS c by c square by 2 to the k into n into n equal to 1. So, that is c square by 2 to the power k like these and also you know degree of freedom, you know what is ANOVA table and how to estimate your F0 all those things that is known to you and we have delivered all those things, it is a what all most all the lectures what is ANOVA, what is degrees of freedom, what is F0.

So, am sure that you will not face any problem related to this test hypothesis testing, please keep in mind in all the lectures so far, start bearing a few in the beginning. So, we are always doing some kind of hypothesis testing, all the classes. So here, our in, all those this later lectures we are talking about main effect and interaction effects; so, there will be hypothesis related to each of the main effects, related to each of the interaction effects and using F statistics you are basically finding out that the effects are significant or not and the basis is that partician the total sum square into the effects sum square and then use mean square, sum square and then do the traditional way of analyzing this things using ANOVA.

So here, what I will show you this is example of that 4 controllable factors there, that means, A, B, C and D, each at 2 levels due to shortage of raw materials only single replicate is possible to run and a compute effect, you require to compute effect of A B C D, AB, BC all those effects. So, we have seen that there will be 15 effects parameter and then compute sum square for each of the effects, 15 effects including the sum square error and do test for significant, comment on the error and effect estimation.

So, I have already given you enough hints on the error calculation and effect estimation, Let the data is like this, so this example is available in Montgomery book. So, so far I am assuming now that, that you know why this is within bracket 1, this is a, this is b, this is ab. So, if you scan through each of the rows for the factors. So, if you find out a row which contains all minus 1, that is within bracket 1.

So, a means, the only the effect that the column of factor A and row 2, you see that this is 1 and all others are minus 1, so a, that means, the a at low another column like this same manner, when abcd you are writing, you see all are positive plus 1, plus 1, plus 1. What is the response here? Response 44 means the y value here, here what we are saying if you conduct an experiment keeping ABCD all at low level, the response value is 44. Suppose, what is 100 here? Here this is ad, that mean you are keeping a at high level and d at high level and both b and c at low level and then if you run the experiment you will be getting the response value 100. Similarly, here ABCD 94 it simply indicates that so in all the factors are at high level the experimental result with reference to the response variable values is 94 that is what you got.

So, now if there would have been more replications you would have more number of response values here, may be 44, may be 50 or something like this, but we have only single replicate. So, that is why these are all single observed values when you have conducted experiments against each of the experimental settings. Now, you that this is what again your design matrix, what is your design matrix here? you have 1, 2 like 16 runs and you have A.

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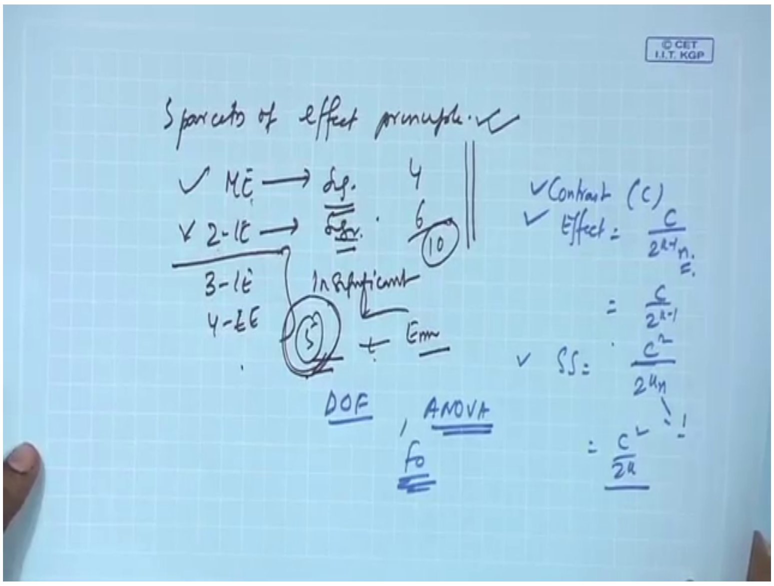

So, it will be minus 1, plus 1, minus 1, plus 1 like this and then B this will be minus, minus, plus, plus like this and C and D, so this is your design matrix. So, from this design matrix you can using the algebraic signs what we have discussed earlier. So, you can find out this design matrix also.

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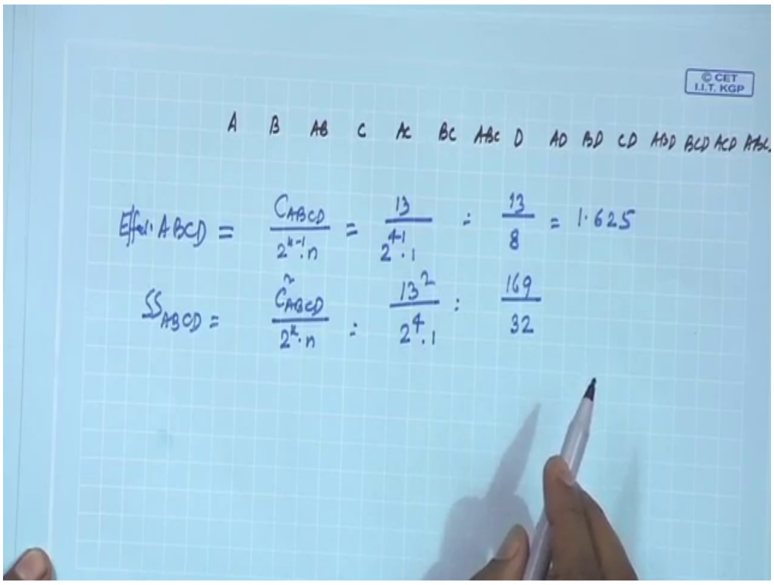

So, you have A, then you have B, then you find out AB, you write C, then you find out AC, you find out BC, then you find out ABC, then you put D, you find out AD, you find out BD, you find out CD, then here ABC, then AB, similarly you will be getting ABD, BCD, ACD, ABCD, so like this. So, that is what is written here.

So, if I miss missing something, you can fill up.

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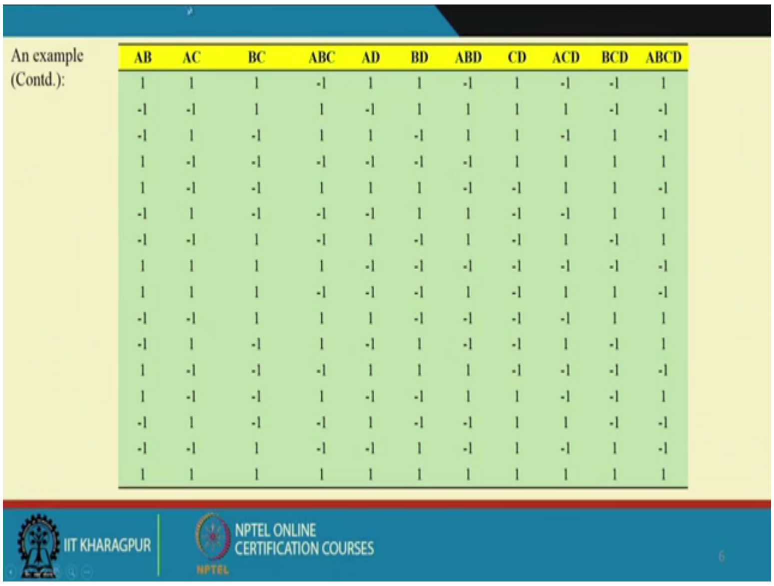

AB	AC	BC	ABC	AD	BD	ABD	CD	ACD	BCD	ABCD
1	1	1	-1	1	1	-1	1	-1	-1	1
-1	-1	1	1	-1	1	1	1	1	-1	-1
-1	1	-1	1	1	-1	1	1	-1	1	-1
1	-1	-1	-1	-1	-1	-1	1	1	1	1
1	-1	-1	1	1	1	-1	-1	1	1	-1
-1	1	-1	-1	-1	1	1	-1	-1	1	1
-1	-1	1	-1	1	-1	1	-1	1	-1	1
1	1	1	1	-1	-1	-1	-1	-1	-1	-1
1	1	1	-1	-1	-1	1	-1	1	1	-1
-1	-1	1	1	1	-1	-1	-1	-1	1	1
-1	1	-1	1	-1	1	-1	-1	1	-1	1
1	-1	-1	-1	1	1	1	-1	-1	-1	-1
1	-1	-1	1	-1	-1	1	1	-1	-1	1
-1	1	-1	-1	1	-1	-1	1	1	-1	-1
-1	-1	1	-1	-1	1	-1	1	-1	1	-1
1	1	1	1	1	1	1	1	1	1	1

What happen? In the previous slide ABCD, the column you got and here AB, AC, BC, ABC. So, what is AB, AC, BC? That multiplying A column, B column you get this multiplying A column, C column you get this like this. So, this interity is known as the basically the what am saying the total that design including the interactions.

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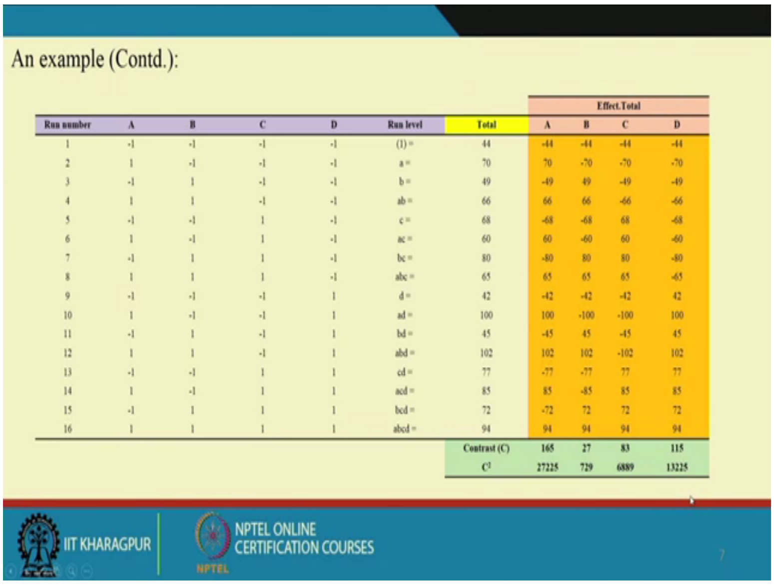

							Effect.Total			
Run number	A	B	C	D	Run level	Total	A	B	C	D
1	-1	-1	-1	-1	(1) =	44	-44	-44	-44	-44
2	1	-1	-1	-1	a =	70	70	-70	-70	-70
3	-1	1	-1	-1	b =	49	-49	49	-49	-49
4	1	1	-1	-1	ab =	66	66	66	-66	-66
5	-1	-1	1	-1	c =	68	-68	-68	68	-68
6	1	-1	1	-1	ac =	60	60	-60	60	-60
7	-1	1	1	-1	bc =	80	-80	80	80	-80
8	1	1	1	-1	abc =	65	65	65	65	-65
9	-1	-1	-1	1	d =	42	-42	-42	-42	42
10	1	-1	-1	1	ad =	100	100	-100	-100	100
11	-1	1	-1	1	bd =	45	-45	45	-45	45
12	1	1	-1	1	abd =	102	102	102	-102	102
13	-1	-1	1	1	cd =	77	-77	-77	77	77
14	1	-1	1	1	acd =	85	85	-85	85	85
15	-1	1	1	1	bcd =	72	-72	72	72	72
16	1	1	1	1	abcd =	94	94	94	94	94
						Contrast (C)	165	27	83	115
						C^2	27225	729	6889	13225

So, this is then what happen, when you have conducted experiment and we said the labels like this, these are the values. Now, what we want basically? We want to compute the contrast and then, we want to compute the contrast square because contrast to will give us the main effect, all effects and contrast square will give us sum square.

So, what we have done? Suppose, you want the contrast for A, then the Ath column will be multiplied by these run level that treatment combination. Here, this 1 will this numerical value will be getting from that this total, this total multiplied by A, then you are getting this column, total multiplied by B column you are getting this column, this column, this column like this.

So, how are you getting this column? You are multiplying this A column with this total column, then when you take the sum you are getting contrast because when you multiply these run column with this A column, factor A column by the run column or treatment column, you will get contrast because this will be just adding or subtracting things. So, now, using the total value, that value of the each of the treatment for y, you will be getting here also numerical values.

Now, when you take sum of all those numerical values you are getting 165, these 165 is the contrast for A, similarly contrast for B, contrast for C, contrast for D and their squares are like this. So, you do by your own, if there is some kind of computational mistakes may be while doing all those thing in excel, if we have committed any mistake somewhere please rectify and, but this is what is the procedure. The more important thing is that, we are giving you the procedure how you will be doing it, if there are some errors in somewhere in calculation, it is always correctable. So, that be ABCD you have computed contrast were computed.

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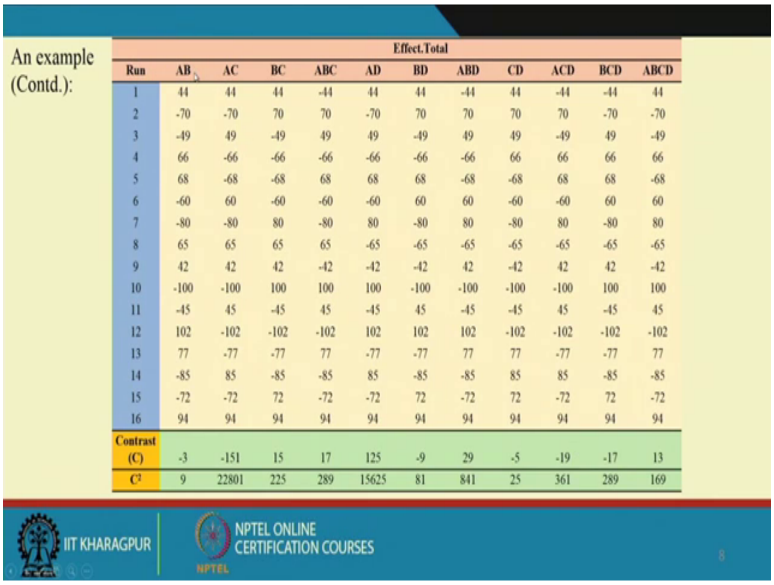

	Effect.Total										
Run	AB	AC	BC	ABC	AD	BD	ABD	CD	ACD	BCD	ABCD
1	44	44	44	-44	44	44	-44	44	-44	-44	44
2	-70	-70	70	70	-70	70	70	70	70	-70	-70
3	-49	49	-49	49	49	-49	49	49	-49	49	-49
4	66	-66	-66	-66	-66	-66	-66	66	66	66	66
5	68	-68	-68	68	68	68	-68	-68	68	68	-68
6	-60	60	-60	-60	-60	60	60	-60	-60	60	60
7	-80	-80	80	-80	80	-80	80	-80	80	-80	80
8	65	65	65	65	-65	-65	-65	-65	-65	-65	-65
9	42	42	42	-42	-42	-42	42	-42	42	42	-42
10	-100	-100	100	100	100	-100	-100	-100	-100	100	100
11	-45	45	-45	45	-45	45	-45	-45	45	-45	45
12	102	-102	-102	-102	102	102	102	-102	-102	-102	-102
13	77	-77	-77	77	-77	-77	77	77	-77	-77	77
14	-85	85	-85	-85	85	-85	-85	85	85	-85	-85
15	-72	-72	72	-72	-72	72	-72	72	-72	72	-72
16	94	94	94	94	94	94	94	94	94	94	94
Contrast (C)	-3	-151	15	17	125	-9	29	-5	-19	-17	13
C^2	9	22801	225	289	15625	81	841	25	361	289	169

Similarly, AB all second order interaction, third order interaction and fourth order interaction you have found out the contrast and contrast square. So, once you have the contrast, you know now the what is the effect and sum square. So, we have found out effect accordingly.

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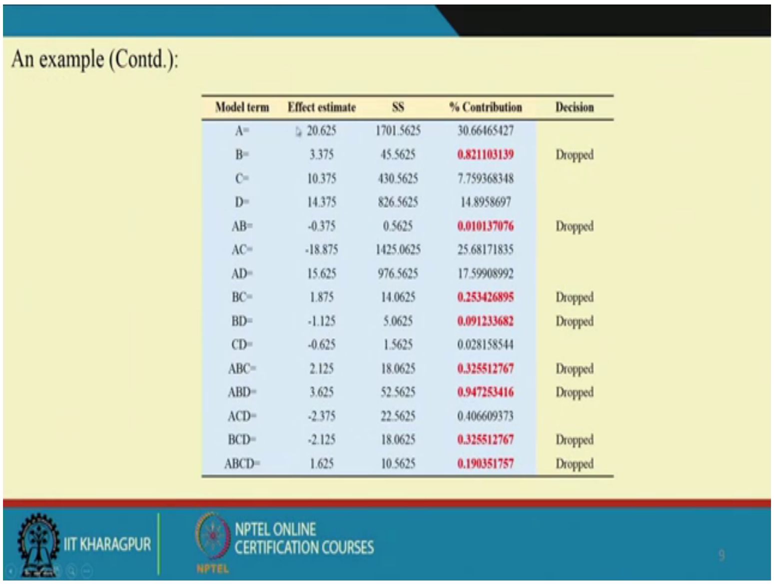

Model term	Effect estimate	SS	% Contribution	Decision
A=	20.625	1701.5625	30.66465427	
B=	3.375	45.5625	**0.821103139**	Dropped
C=	10.375	430.5625	7.759368348	
D=	14.375	826.5625	14.8958697	
AB=	-0.375	0.5625	**0.010137076**	Dropped
AC=	-18.875	1425.0625	25.68171835	
AD=	15.625	976.5625	17.59908992	
BC=	1.875	14.0625	**0.253426895**	Dropped
BD=	-1.125	5.0625	**0.091233682**	Dropped
CD=	-0.625	1.5625	0.028158544	
ABC=	2.125	18.0625	**0.325512767**	Dropped
ABD=	3.625	52.5625	**0.947253416**	Dropped
ACD=	-2.375	22.5625	0.406609373	
BCD=	-2.125	18.0625	**0.325512767**	Dropped
ABCD=	1.625	10.5625	**0.190351757**	Dropped

So, I will show you the last one, suppose ABCD effect, how it will be; if effect ABCD how it will be calculated? ABCD will be contrast ABCD divided by what happen 2 to the power k minus 1 into n. Now, what is contrast ABCD? you see that this is what is the ABCD column. Now, contrast ABCD is 13 sum of all those values. So, that mean this 1 is 13, what is your k? K is here 4, 4 minus 1 that will be 3 and n equal to 1.

So, that mean 13 divided by 2 to the power 3 that is 8. So, your 8 this value is 1.625. So, as a result you see here, what is happening here ABCD is 1.625. Suppose, this is what is the effect calculation; ABCD effect. Suppose, you want to compute sum square ABCD, what you will write down? C square ABCD divided by 2 to the power k into n, so C square is 13 square divided by 2 to the power 4 into 1.

So, this is 169 by 32 is it so, 16, it will be 16; 2 to the power 4 is 16. So, then this will be 10.56. So, see here that SS is 10.56. So, how many parameters you have to estimate here? There are 15 effect parameters, 4 main effects 1, 2, 3, 4, 5, 6, 2 way interaction effects; 1, 2, 3, 4, 4 3 way interaction effect and 1, 4 way interaction effects and their estimation effect is contrast by the 2 to the power k minus 1 into n, that sense you are able to find out all the effects.

You see the effect values are positive and negative also, that mean some of the effect when goes from 1 level to another, low level to high, the why values are increasing, some cases it is decreasing and that is true for interaction effect also and then SS also you are computing in the same manner.

So, what will happen to the total? Total you can compute, but what will happen to error here? You cannot find out error because you do not have free observations for you to compute error. So, as a result that there are 2 way thing that stop here or you find out some innovative way. So, that you cal can compute the errors.

So, the innovative way is that, you just you know the SS already, this SS is basically explanation because we all know from the analysis of variance that sum square total is the sum square A, sum square B, sum square C, plus sum square D, plus sum square AB, plus sum square BC, plus sum square AC, plus sum square AD, plus sum square BD, plus sum square CD, plus sum square ABC, plus sum square your ABD, plus sum square ACD, plus sum square BCD, plus sum square ABCD.

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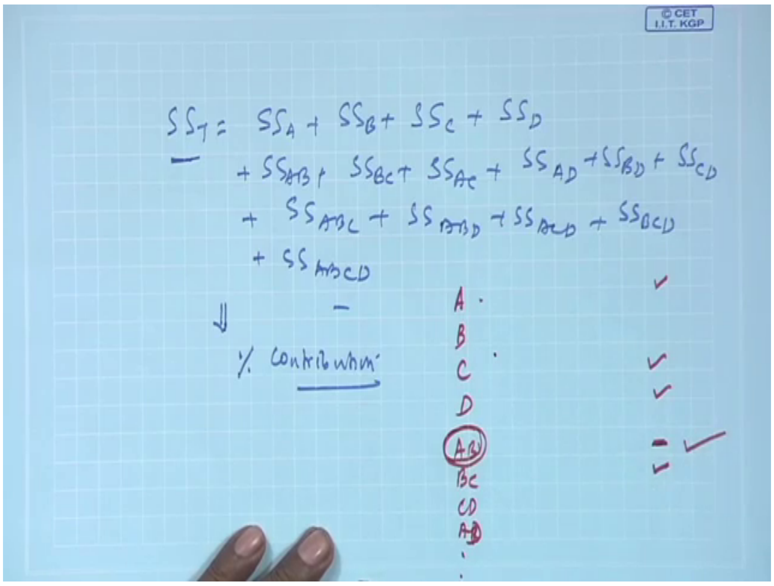

So, this total is sum of all the individual contribution. So, as a result it is advisable that find out the percentage contribution of each of the effects. So, this is nothing but what way you will find out the percentage contribution? That individual contribution divided by total contribution. So, this is what is the contribution of A to SST is 30.66 percent this divided by SST, similarly contribution B is 0.82 percent only, contribution C is 7.75 percent, contribution D is 14.89 percent like this.

So, that means, you are getting each of the contribution. Now, here what happen? You have to drop some of the parameters in order to get the error, as we say that the higher order interactions will be dropped or so in that case what is happening? That is issue that suppose A is a significant, B is significant, then it is are second order interaction AB it is better keep that in the model because if you do not consider that interaction what will happen your result may be erroneous.

So, as a result from this contribution plot you find out which are the effect parameters which contribution is negligible, but whose lower order contributions are not there. Considering these, we found out that B is negligible and we found out that BC, BD, ABC, ABD and your BCD, ABCD all are very, very negligible, but in the same logic why we should not keep this one that is CD because this is also negligible, similarly ACD this is also negligible because percentage contribution is less than even 1, but you will not do these, the reason is here A is significants, C is significantly contributing as well as D is also significantly contributing. So, that is why their interaction AC let it be there, so similarly interaction ACD let it be there.

So, in nuts cell what are we saying? That we are saying that you find out the main effect part first, those are which are contributing keep them, then see the interaction part AB, BC like this and then see that which are contributing in the sense percentage contribution is reasonably high, where it is a subjective 1 at this level. So, then bees AB, BC, CD, AD like these. So, here you find out which are contributing those contributing keep, but it something the contribution is low, but it is both the primary main effects are still significant, then that also you keep.

So, in that sense we have gone, it is always advisable that higher order interaction should be dropped, provided there will be significant evidence that if you dropped it, you will not go for; you will not finally landed in to a wrong model, the approach is first find out the effects; main effects, drop which is not significant and then subsequently find out the second order effect and drop which are not significant or at the same time if you see that your dropping interaction; second order interaction whose primary or main effects are significant advisable that you do not drop this keeping the model. By following this logic what happen? We have dropped 8 effect parameters B, AB, BC, BD, ABC, ABD, BCD and ABCD.

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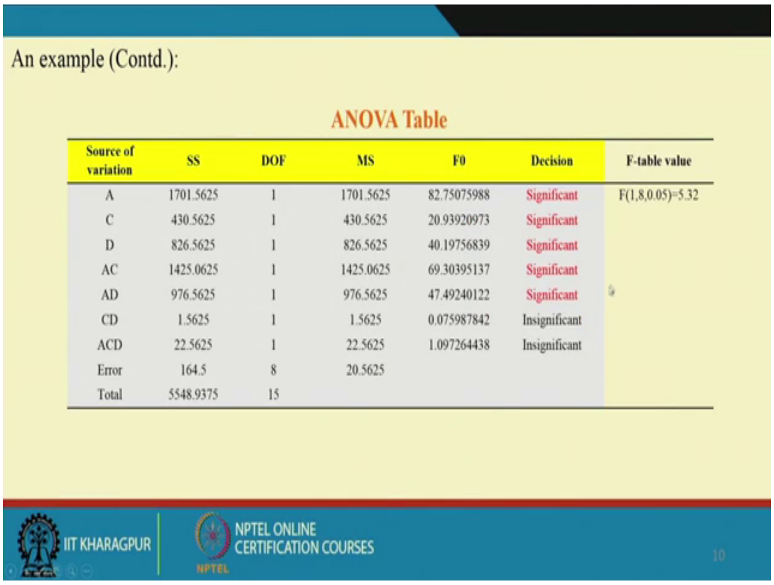

So, then we have a new table, where we are saying the sources of variations are A, C, D, AC, AD, CD and ACD and the rest of the effects the 8 effects what are dropped they are basically comprising the error with 8 degrees of freedom. So, this is our final ANOVA table. Now, you are in a position to find out aminous for everything, also you will be in a position to estimate the a F0 because you have error MSC, MSC is also available with you.

So, as a result if I follow this one F0 value, you find out that A, C, D, AC, AD they are significant and others are not significant. Now, we refer then you can compute the tabulated value and this is what you are getting. So, this is what is the what is logic we are provided to you, but there is no sacrose and it is not sacrosen that you have to follow only this logic they are not be alternative way of thinking, but whatever you do you must keep in mind that the basic objectives of this kind of experiment is to finding out screening out the insignificant factors and then interaction or at their interaction effects. So, that you will get a good model and keep the meaningful factors for further analysis.

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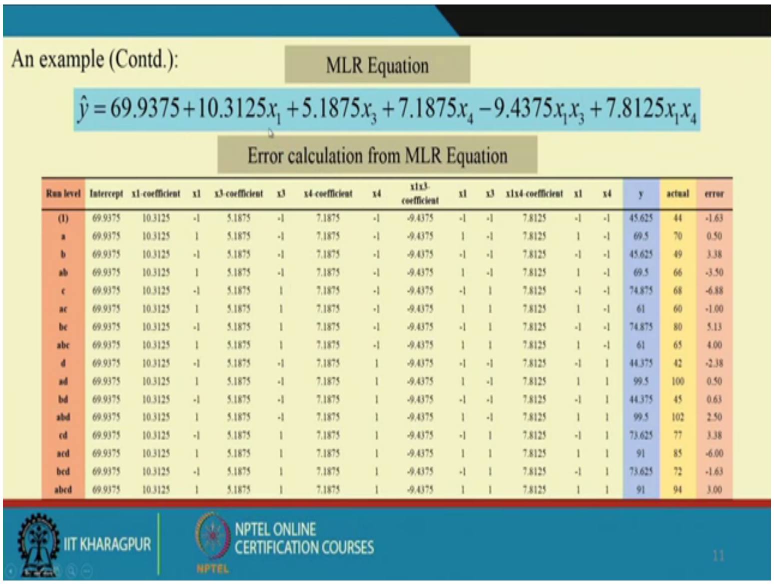

$$\hat{y} = 69.9375 + 10.3125x_1 + 5.1875x_3 + 7.1875x_4 - 9.4375x_1x_3 + 7.8125x_1x_4$$

Run level	Intercept	x1-coefficient	x1	x3-coefficient	x3	x4-coefficient	x4	x1x3-coefficient	x1	x3	x1x4-coefficient	x1	x4	y	actual	error
(1)	69.9375	10.3125	-1	5.1875	-1	7.1875	-1	-9.4375	-1	-1	7.8125	-1	-1	45.625	44	-1.63
a	69.9375	10.3125	1	5.1875	-1	7.1875	-1	-9.4375	1	-1	7.8125	1	-1	69.5	70	0.50
b	69.9375	10.3125	-1	5.1875	-1	7.1875	-1	-9.4375	-1	-1	7.8125	-1	-1	45.625	49	3.38
ab	69.9375	10.3125	1	5.1875	-1	7.1875	-1	-9.4375	1	-1	7.8125	1	-1	69.5	66	-3.50
c	69.9375	10.3125	-1	5.1875	1	7.1875	-1	-9.4375	-1	1	7.8125	-1	-1	74.875	68	-6.88
ac	69.9375	10.3125	1	5.1875	1	7.1875	-1	-9.4375	1	1	7.8125	1	-1	61	60	-1.00
bc	69.9375	10.3125	-1	5.1875	1	7.1875	-1	-9.4375	-1	1	7.8125	-1	-1	74.875	80	5.13
abc	69.9375	10.3125	1	5.1875	1	7.1875	-1	-9.4375	1	1	7.8125	1	-1	61	65	4.00
d	69.9375	10.3125	-1	5.1875	-1	7.1875	1	-9.4375	-1	-1	7.8125	-1	1	44.375	42	-2.38
ad	69.9375	10.3125	1	5.1875	-1	7.1875	1	-9.4375	1	-1	7.8125	1	1	99.5	100	0.50
bd	69.9375	10.3125	-1	5.1875	-1	7.1875	1	-9.4375	-1	-1	7.8125	-1	1	44.375	45	0.63
abd	69.9375	10.3125	1	5.1875	-1	7.1875	1	-9.4375	1	-1	7.8125	1	1	99.5	102	2.50
cd	69.9375	10.3125	-1	5.1875	1	7.1875	1	-9.4375	-1	1	7.8125	-1	1	73.625	77	3.38
acd	69.9375	10.3125	1	5.1875	1	7.1875	1	-9.4375	1	1	7.8125	1	1	91	85	-6.00
bcd	69.9375	10.3125	-1	5.1875	1	7.1875	1	-9.4375	-1	1	7.8125	-1	1	73.625	72	-1.63
abcd	69.9375	10.3125	1	5.1875	1	7.1875	1	-9.4375	1	1	7.8125	1	1	91	94	3.00

So, now if we consider that this are the factors which are basically to be kept in the model, then the you can go for the regression analysis and you will find out x1, x3, x4, x1, x2, x3 and x1, x4. So, A will be denoted by x1, C x3, D x4, x1 x3, x1 x4, this is what we have considered here. So, x1, x3, x4, x1, x3 and x1, x4 considered here.

So, and then these are the different run level intercept and this is using excel sheet we found out the error quantity. Why error is important? because you have to see you have to do residual analysis and that residual analysis will tell you that the model is fit or not, whether the assumptions are violated or not, also from regression point of view you can find out the r square value and other things.

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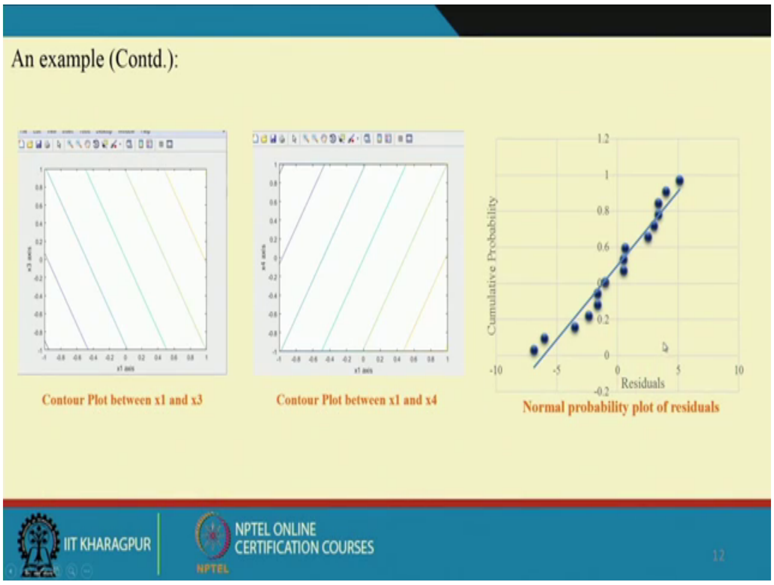

So, this response surface gives us this kind of contour plot. So, I will explain only 1, suppose if you contour develop contour plot for x 1 and x 4 keeping x 3 constant, then what will happen? You will get this kind of plot. So, what is needed here in addition to these, that what is the y value for this line, y value for this line, y value for this line.

So, here what happen they are almost parallel so that means, the interaction effects are not that interaction effects if you see AD, their significant AD AC significant, but any how because this is AC means x3 and AD is x4. So, in this plot we are talking about x1 and x4 keeping other term mean x3 at constant, but anyhow when because of interactions if we precisely find; if we go for other options we may find a curvature; some kind of curvilinear plot also.

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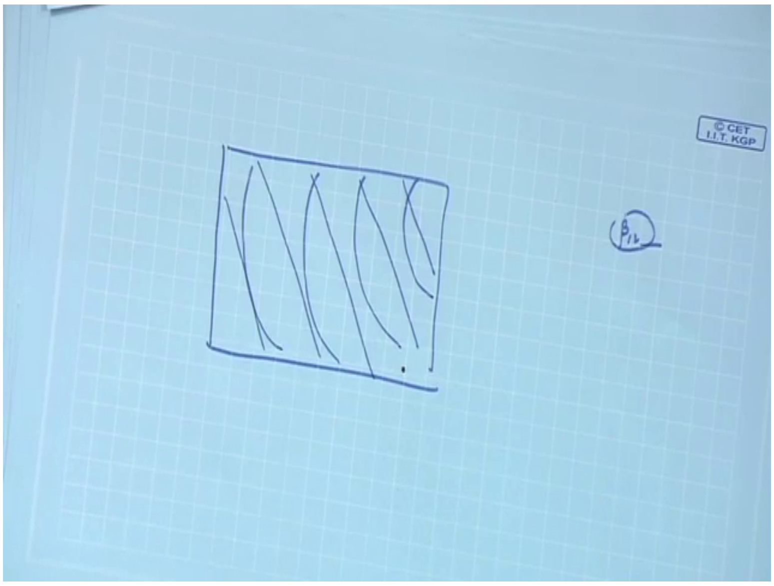

what I mean to say, suppose you are doing this, it is this kind of plots represent that there is no that interaction term like beta 1, 2 these are not there, but in this case from the data we got this kind of things, but many times you may get like this, this kind of plot also, you will get that is what is when the interaction effects are significant.

So, here in the last plot is the normal probability plot. So, the residual B sub is cumulative probability then we plot, we found out that there is a; the, this is basically using a excel we plot, but it is better to use the probability paper. So, in that case what you will get? You will get a straight line, when you join the points you will get a straight line if you get so, then that is what is your that errors are normally and distributed and that is one of the assumptions. So, another major you can do what we have already discussed several times of fitted value verses residual.

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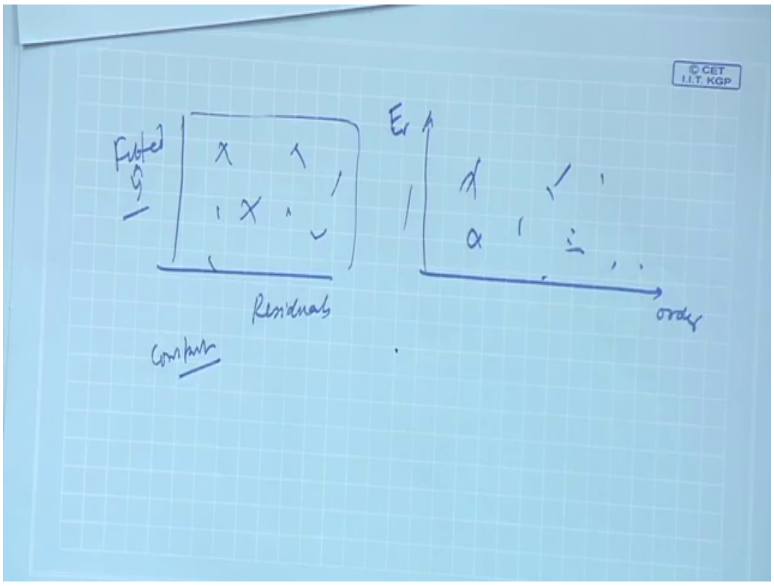

Suppose, this residuals and this side the fitted y; fitted y that is y cap. So, here it is expected that you will get random observations, random plot it is should not be any trained or anything like this. This is one for constant variants this is required, one is normality, another one is may be the order of experiment, the B sub is this side is or this side is the error values. So, that also would be random, it should not show any pattern. So, that mean the auto correlation is not present, that is to be tested also.

All those test, we have discussed earlier in different cases, but please remember this is also valid for this kind of D i. So, here again let me tell you that we have used the content of this book design and analysis of experiment written by D.C Montgomery, published by Willie, thank you very much I hope you have understood the 2 to the power k factorial design, when there will be many replications and when there will be single replication, in case of single replication we have given you a scheme that how to conduct or compute the errors even though you have single replicate and using the sparsity of effect principle, thanks a lot.

Thank you.