Design and Analysis of Experiments
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Lecture – 35
Blocking Factorial Design

Welcome. In this lecture, we will discuss blocking in a factorial design.

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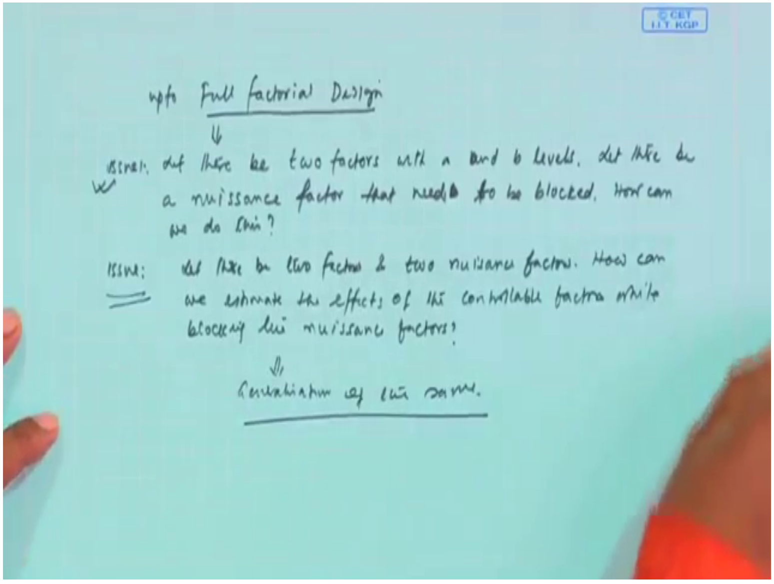

So, so far we have completed up to full factorial design and you know what full factorial design is. Today, we will see that let there be two factors with A and b levels. Let there be a nuisance variable a nuisance factor that needs to be to be blocked. So, how can we do this? So, this is our issue – 1.

Second issue is; let there be two factors which are controllable and two nuisance factors. How can we estimate the effect of the controllable factors while blocking the nuisance factor and definitely issue – 3 will be that this can be generalised generalisation of the same? So, we will discuss first the issue – 1 in details with an example and issue – 2 another example will show you and obviously, this after this you will know that how can it be generalised.

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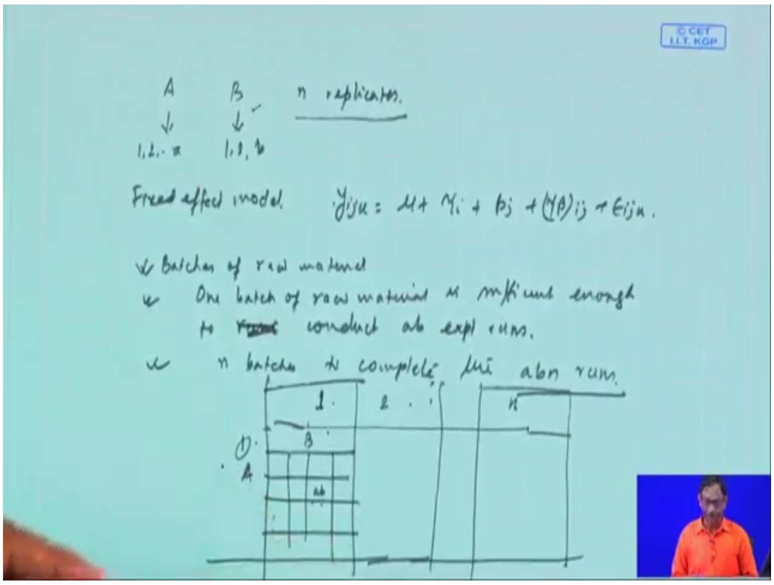

Let us consider first 2 factors A and B and n replicates suppose factor A having a levels factor B having b levels n replicates is there. So, then the fixed effect model is y ijk equal to mu plus tau i plus beta j tau beta ij plus epsilon ijk. So, let when you are running this experiment. So, you are using different batches of raw material, obviously, this batches of raw material may create nuisance and it has to be controlled. Let further that suppose in one batch of raw material is sufficient enough to run or to conduct ab experimental runs.

So, as a result what happened you require n batches n batches to complete the abn runs, experimental runs. So, you are not getting single batch you are getting n batches. So, now, you require to block this n batches. So, how can you block? Every batches of raw material will be used to do ab experimental runs and then next; that means, within a within a full factorial design involving A and B you are having a single replicate. So, n batches will be n blocks and in each suppose 1, 2 like n batch and again I have suppose A and this side B, so, what are you doing? You are doing that B will be having different levels A having different levels. What are you doing here? This batch will be used for ab runs, second batch similarly will bought another ab runs, like n batch will be used for another ab runs. So, within a block there are ab runs. So, then within block A and B will be full factorial with super single replicate. So, that means, raw material represent randomisation restriction or a block and single replicate of a complete full factor factorial experiment is run within a block.

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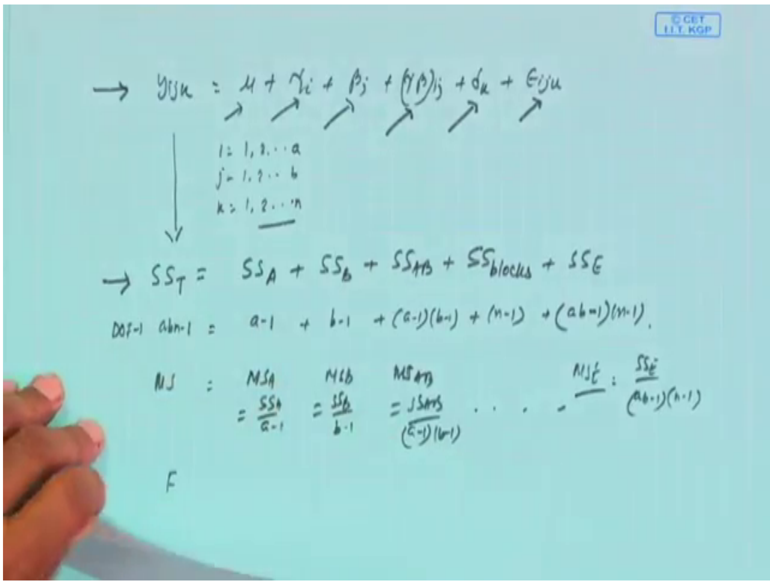

So, if this is the situation then your model fixed effect model will be y ijk. So, it will be your mu plus tau i plus beta j plus tau beta ij plus delta k plus epsilon ijk. So, this is grand mean, this one is factor a effect, factor b effect, interaction effect, block effect, error and I will be from 1, 2, a; j will be from 1, 2, b and k will be from 1, 2, n. If this is the case, now considering our knowledge whatever we have learned so far, what we have done, we partition the observations like these. So, you have to partition the sum square also. So, sum square total this will be your sum square A plus sum square B plus sum square AB plus sum square blocks plus sum square error. This is basically partitioning the observation; this is basically partitioning the sum square total.

Now, what will be the degrees of freedom then DOF partitioning will be like this. DOF here abn minus 1, this is your total. Now, this will be a minus 1 plus b minus 1 plus a minus 1 into b minus 1 plus what will be the how many blocks n blocks n minus 1, then the remaining abn minus 1 minus all those things this will lead to the error degrees of freedom this will be ab minus 1 into n minus 1. Then, what you we actually then if I want the corresponding MS, then what will be the MS; MS A will be SS A by a minus 1, MS B will be SS B by b minus 1, MS AB will be SS AB by a minus 1 b minus 1 and like this and MS E will be SS E by ab minus 1 into n minus 1 and then you compute F. So, this is the general approach and using F you will see that whether the effects are this or not.

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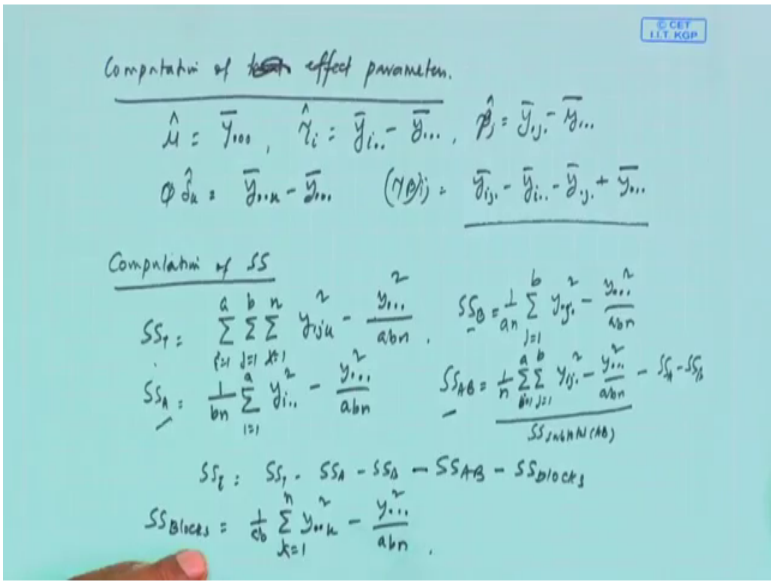

Now, what will be the formula for your computation of treatment effect computation of effect parameter not only it effect parameters, again we know the formula, for example, if I want to estimate the grand mean this will be Y dot dot dot bar. If we interested to estimate tau i this will be your y i dot dot dot bar minus y dot dot dot bar, grand mean. Similarly, if you are interested to estimate beta j this will be y dot j dot bar minus y triple dot bar. If I interested to know delta k, what is the estimate? That will be y dot dot k bar minus y triple dot.

So, then there will be your tau beta ij then this will be y ij dot bar minus y your y i dot dot bar minus y dot j dot bar plus y double dot bar triple dot bar. So, error you will be computing other ways. Now, suppose you are interested to know compute the SS, computation of SS sum square. So, what will be the sum square total? Sum square total here it will be k equal to 1 to n, j equal to 1 to b, i equal to 1 to a, then y ijk square minus y triple dot square by abn. You want to know SS A then, SS A means the row total i equal to 1 to a, y i dot dot square then 1 by b into n minus y triple dot square by abn. If you want to calculate SS B you know this so that b will change mean j is 1 to b, 1 by a n because a n observation will be there then y dot j dot square minus y triple dot square by abn.

So, what will happen to SS AB then SS AB first you find out the subtotal for j equal to 1 to b, i equal to 1 to a then 1 by n yij dot square, this will give you the give you the; that means, in AB cell the square they observe total and they square minus y triple dot square by abn this is your SS subtotal AB and it will be subtracted by SS A minus SS B this will give you SS AB then remaining is SS E which is SS T minus SS A minus SS B minus SS AB, SS E, SS T, SS A, SS B there is another one which is known as minus SS block. So, there is another SS you have to compute that is SS block. How many blocks are there are n blocks. So, in each block ab observations are there. So, k equal to 1 to n, 1 by ab, y dot k square minus y triple dot square by ab into n. So, you got all the computation formulas.

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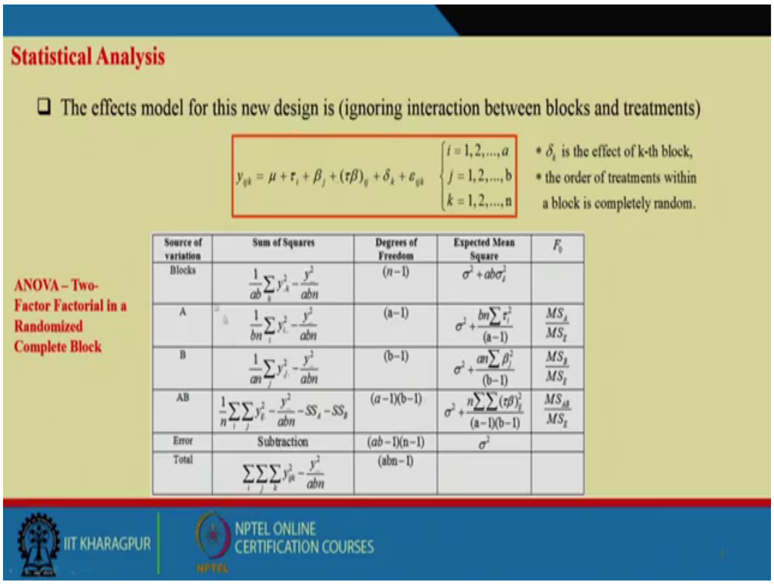

Then, you will be having this ANOVA table you see block this is SS degrees of freedom expected mean square A, B, AB their corresponding everything is given, F 0 is calculated for A, calculated for B, calculated for AB because you want to know whether the factors and their interactions are significant or not using F 0 you will be doing this.

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So, let us see one example here. So, the computation treatment part all those things given here we are considering 2 factors A and B, A with 3 levels B with 2 levels and there are 4 blocks let it be the may be that raw 4 batches of raw materials. The 4 batches of raw materials and each batch of raw material will be able to is enough to conduct 3 into 2, 6 experimental runs. So, what is the condition here, we say that you require 6 experimental runs with single replicate; suppose you want n replicate and it is the situation where that the every batch can conduct only 6 experiment then each of the block will be treated as replicate and each block will be used to do full factorial design with single replicate.

So, then under this situation let us compute what I want to compute on the effect and SS.

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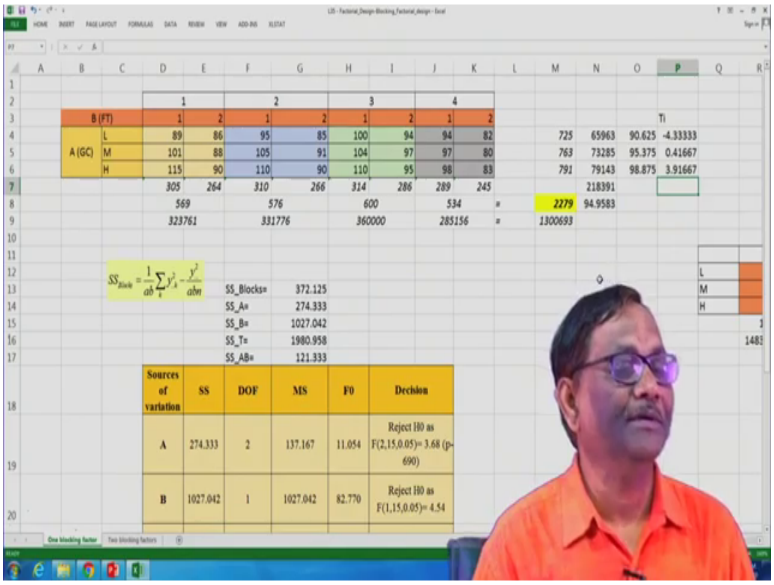

So, this is computation the excel sheet we have I am showing you see the data. This is the upper batches of raw material 1, 2, 3 and 4. So, let me make it little bigger. Now, A having 3 levels, low, medium, high; B having 2 levels that let be 1 and 2. Now, what you want to compute you want to first compute the grand mean, what will be the grand mean? Grand mean will be what is the grand total you see if I click here it is basically the sum of this rows representing 4 blocks. So, and when I click here this is sum of the runs; that means, y sum of the y values against each batch of a raw material. This is the total of all those 3 into 2 into 4, how much? 3 into 2, 6 into 4, 24 observations.

So, if you want a grand mean you can write down these equal to these divided by 24 it will give you 94.95. So, this is your grand mean. Now, suppose what is the total for A at low then this is the row, this row total. This row total is this one, sum total of the rows when A is low this value is 725. Now, if you want the average write down these equal to 725 divided by how many data points are there 1, 2, 3, 4, 5, 6, 7, 8 because 4 into 2, 8 divided by 8 this will give you 90.25.

So, now, if I want for this 3 rows when I click like this, so, 94, 95, 98 suppose you want the treatment effect tau i suppose tau i I am writing little like t i and this will become this will be the row average minus the grand average it is minus 4.33 and in the manner you will calculate the other one what happened sorry. So, 4.33 and the second one will be also these will be second row minus the grand average will be this third one will be the third row average minus the grand average will be this 3.91.

So, these are tau I estimate and similarly, you can get the beta j estimate now if you want to use beta j estimate, you have to see that what is the total row column total against B equal to 1 and B equal to 2. So, B equal to 1, if I say this is 305, here it is 310, here it is 314, here it is 289, so all four. So, 300, 300 plus 600; 6, 3, 900 plus 3 around 1200.

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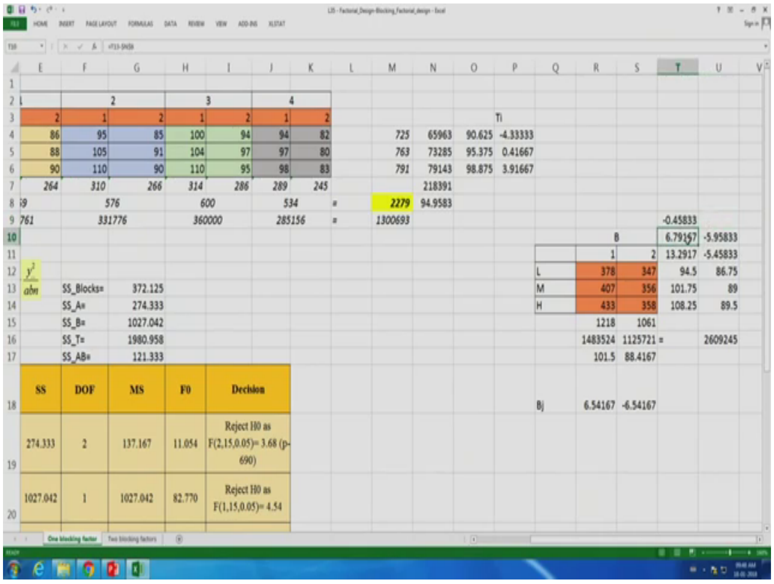

You see here, what we have done L M H. So, this and ultimately if you take sum of these two, this will be nothing, but 725. If you take sum of this 3 this is nothing, but 1218 and this will be 1061. Now, you can find out the average here. So, average of these will be, what will be the average here, we are talking about these average. So, that mean 3 into 4, 12 these divided by 12. So, if I want the average equal to these divided by 12 this one hundred and oh sorry, then what will be the second one? 88. What is the grand average? 94. So, if I want the beta j beta if I write b j here, this will this minus grand average will be 6.5, this will be this minus grand average this will be minus 6.54 because sum of tau i equal to 0, sum of beta j equal to 0 and in this manner.

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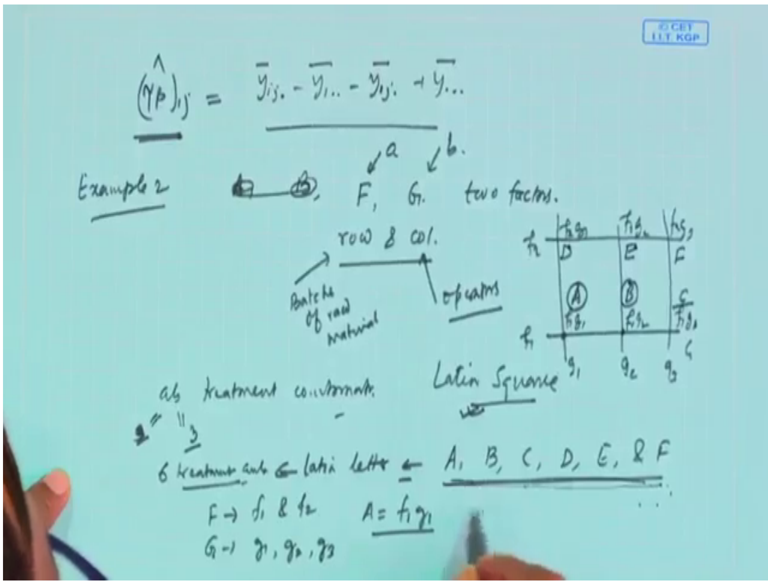

Now, what you required to do if you require to do calculate the tau beta ij the tau beta ij this one if you want to compute this will be basically, here you find out the average this is basically tau beta ij. Here, how many observations are there? Four observations are there because replications four 378. So, these divided by suppose, I want this so that mean these equal to these divided by 4 will give you this and then if I go to this side this is this divided by 4 and if I come down you will get all this one. And, suppose, what happen you want the estimate of tau beta j these equal to this average minus grand average. So, if I fix this one using shift operator then, this is minus 4 point this one.

So, if I change this to this side this will give you with 86 minus 94 is 8.4 and if I make for 3 different, you are getting this, you take sum you. So, probably just one minute, delete it, let me check 378 each whether L 11 this one or not. L 1 L and this one 89 plus 95 plus 100 plus 94, this will be 389, yes or no. So, let main effect check. Yes, you see this one. So, divided by 4 will be that average. So, that is correct.

Now, we are interested to find out this one the average estimate of these. So, formula is y ij bar minus y i dot dot dot bar minus y dot j dot bar plus i dot. So, that means, we have used the wrong formula here.

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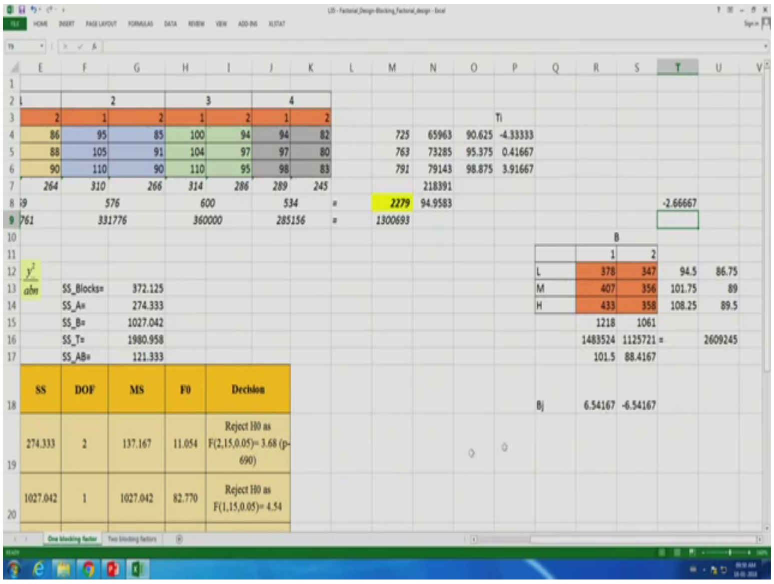

So, what you require to use you require to these. You have to write here suppose equal to these that is basically the cell average y 1 1 dot bar minus row average, row average is these, minus column average these plus grand average this. Now, if you fix (Refer Time: 26:28) you can find out triple. So, this way tau beta will be computed and then delta that our block affects also you can compute, but it is not of interest.

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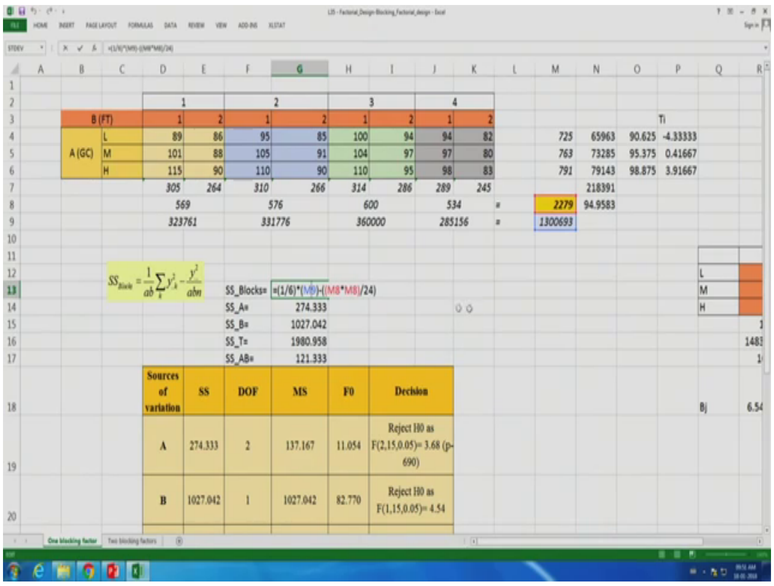

Then find out the SS calculations. So, what is SS T? The SS T is sum total of all you see, that is, we have consider SS T calculation, just a minute, this m 7 minus this now M 7 is this, now how this is computed that we have to check. Sum of these I think I am giving you the SS AB and total. So, what we have you know the formula. Formula is basically you take the square of all those terms minus this because there are in between terms you have computed and that manner it came like this.

Suppose SS treatment, SS block we are writing 1 by 6 because 3 against every block there are 6 observations 1 by 6, M9. M9 mean is this one. So, that means, this is then what is this value? This value is against e s in each block you are take basically the squaring of each of these things and taking sum squaring of second block observation taking sum. So, that sense it is coming. So, that mean, when I am talking about SS block, you are basically first making this one this is your this value and then you are subtracting this value. When you are talking about SS T you are requiring this is the same manner you are. So, in this way you will calculate all the SS parts. Then, after that what will happen you will go for excel sheet ANOVA table.

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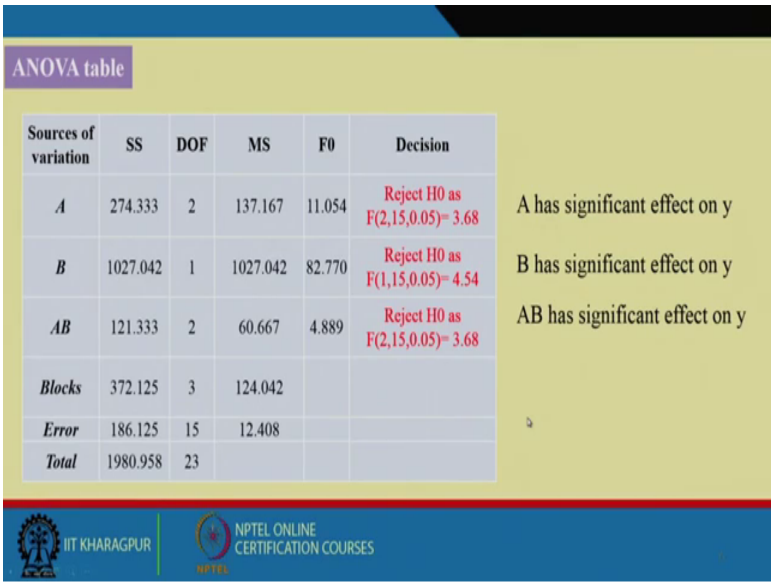

Sources of variation	SS	DOF	MS	F0	Decision
A	274.333	2	137.167	11.054	Reject H0 as F(2,15,0.05)= 3.68
B	1027.042	1	1027.042	82.770	Reject H0 as F(1,15,0.05)= 4.54
AB	121.333	2	60.667	4.889	Reject H0 as F(2,15,0.05)= 3.68
Blocks	372.125	3	124.042		
Error	186.125	15	12.408		
Total	1980.958	23			

Your sources of variation is A, B, AB, blocks, error everything was computed degrees of freedom accordingly computed, MS computed, F 0 computed and we have found out for the first effect that theoretical value is F 2, 15, 0.05 is 3.68, computed value 11.05 more than these significant in this manner it is found that A, B and AB are significant.

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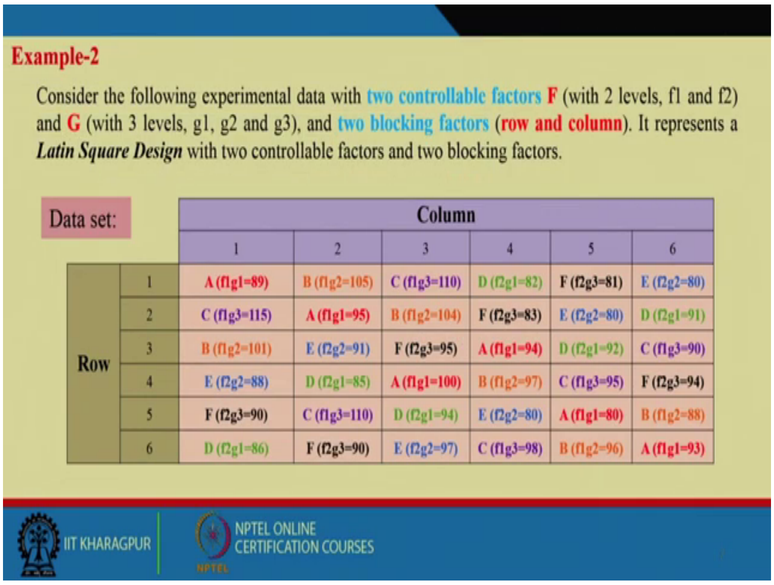

So, now, I will show you another example that example 2, where what happen two factors A and B and two nuisance variables. So, we have considered two factors F and G. These are two factors, we are writing two factors and two nuisance variable in row and column. For example, row may be batches of raw material and column will be column will be operator and controllable two factors may be this may be your temperature this may be pressure or any other controllable factors. So, what will happen here if F has a levels, G has b levels then in a full factorial design, similarly, we have how many independent treatment combination ab treatment combination.

So, in this case we want to use Latin square well this is a very good design. You have seen when there are two blocking factors you can Latin square, if there are 3 blocking factors you can go for Graeco-Latin square here we have 2 blocking factors. So, what we will do here, suppose a equal to 3 or 2 let b equal to 3 then we have 3 plus 6 treatment combination. So, these treatment combinations can be denoted by Latin letters which are basically the treatments or other way I can say that every combination will be process will be treated for every combinations.

So, we can use A, B, C, D, E and F for treatment combinations. For example, just for example, suppose if I say F having 2 levels this is f 1 and f 2 and G having 3 levels g 1, g 2 and g 3. So, A can be that is the both at low level f 1 and g 1 both at low level f 1 and g 1. Now, if I draw like this suppose this side my G and this side F. So, this is g 1 this is g

2, this is g 3 then this one is f 1 this is f 2. So, it is f 1, g 1 this one is f 1 g 2. So, this point is f 1 g 1, f 1 g 2 and f 1 g 3, f 2 g 1, f 2 g 2, f 2 g 3. So, I we are saying these combination is denoted by A, suppose f 1 g 2 denoted by B and f 1 g 3 denoted by C, f 2 g 1 denoted by D, f 2 g 2 denoted by suppose E and this one is F. So, this Latin alphabet these alphabet A, B, C, D these are used for these treatment combination.

So, if you want to use Latin square we require then; that means, 6 rows and 6 columns. So, we require 6 batches of raw material and 6 batches if 6 numbers of operators.

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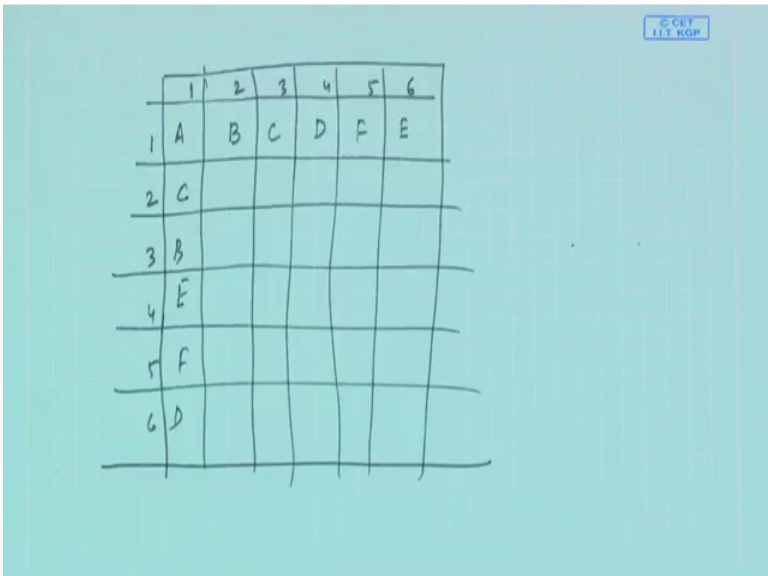

So, yours will be 6 by 6. So, row 1, 2, 3, 4, 5, 6 and here it is 1, 2, 3, 4, 5, 6. So, you have Latin squares 6 by 6 Latin square you have. Now, this all A, B, C, D these will be placed. So, here I let it be A, B, C, D let it be F and E; obviously, you have to follow the Latin square generation rules and accordingly you will be writing down. For example, if would let from that design suppose we got this A then this will be C, let it be B, then E, F, D in this manner for that.

Suppose this is the table. So, A, B, C, D, F, E, C, A, B, F, G all those things using Latin square randomisation scheme it is the treatment combinations are selected like this and given to each of the blocks like row and column blocks. You all know that in Latin square every treatment will occur once against each row, as well as one seconds each column if you compare the column 1 you see A is coming on, again if you compare row though row one also A is once. So, that is the scheme it is done.

So, then what you require to compute, you require to compute treatment effects, you require to compute your row effects, column effects and here we have basically two treatments A and B, so, the treatment A and B and their interactions to be computed.

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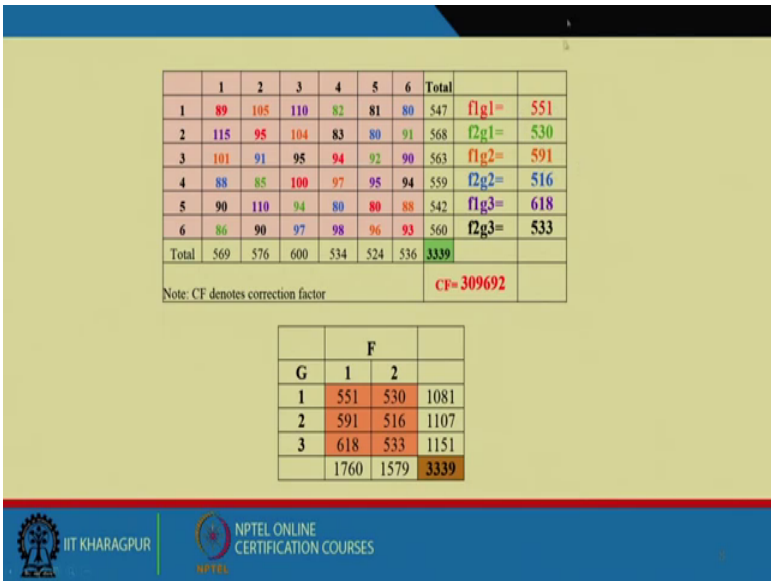

	1	2	3	4	5	6	Total		
1	89	105	110	82	81	80	547	f1g1=	551
2	115	95	104	83	80	91	568	f2g1=	530
3	101	91	95	94	92	90	563	f1g2=	591
4	88	85	100	97	95	94	559	f2g2=	516
5	90	110	94	80	80	88	542	f1g3=	618
6	86	90	97	98	96	93	560	f2g3=	533
Total	569	576	600	534	524	536	3339		
Note: CF denotes correction factor							CF= 309692		

	F		
G	1	2	
1	551	530	1081
2	591	516	1107
3	618	533	1151
	1760	1579	3339

So, this is what the data table what happen when we make row total, we will get row blocks, we will make column total we will get the column blocks. For the A B that mean the factor G and F we will create a separate table from this. So, we know that G at low medium high F at low and medium or F 1 and 2, if we sum the observations, we will be getting this kind of things.

So, that means, from this table you will be basically estimating the blocking factors affect, their sum square from this table you will be estimating the controllable factors effects and their sum squares, but please keep in mind this suppose if you consider 551 this is not because of one observations because of so many observations because 1 and 1, in this case if you see 1 and 1 suppose, we are talking about A that 1, 1 means low levels. So, you have to find out where f 1 g 1 is there means where A is there, here A all those A will be summed up. Similarly all B’s and C’s will be summed up and accordingly there will be 3 into 2, 6 as sum of values.

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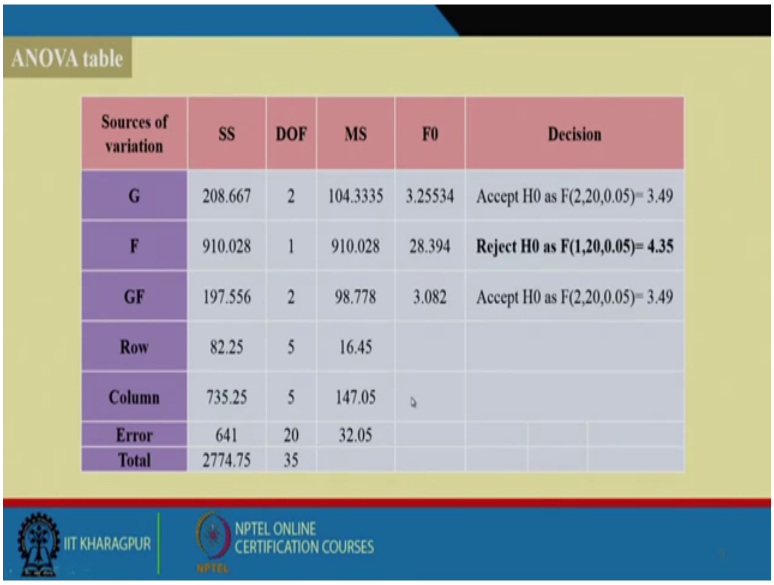

ANOVA table

Sources of variation	SS	DOF	MS	F0	Decision
G	208.667	2	104.3335	3.25534	Accept H0 as F(2,20,0.05)= 3.49
F	910.028	1	910.028	28.394	**Reject H0 as F(1,20,0.05)= 4.35**
GF	197.556	2	98.778	3.082	Accept H0 as F(2,20,0.05)= 3.49
Row	82.25	5	16.45		
Column	735.25	5	147.05		
Error	641	20	32.05		
Total	2774.75	35			

And, then you use the rest formula, the formula for computation all of you know that the formula already we have seen; that means, the SS formula you use and then you find out the G, F, GF row and column. So, SS 208 like this degrees of freedom, there are MS and here what happen, we found that when we block that ultimately only F the second factor F is significant and rest of the factor is insignificant. So, this is what is, our Latin square design for full factorial design and blocking two factors is in Latin square design.

So, what will happen if there is one more factor? So, one more factor in the sense I am taking the one more nuisance factor then you can go for Graeco – Latin square design. So, this is what is blocking in full factorial design and if you go for more factors, more blocks or suppose there are three factors, two blocks and then what will happen you will go for that or more in number ultimately you will go for that is the generalisation of the full factorial with more number of nuisance factors.

So, thank you very much I hope that you have understood this one if you practice it definitely you will be able to solve the problems.