Design and Analysis of Experiments

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Lecture – 33
Full Factorial Single Replicate

Hello, we will discuss full factorial design with single replicate. So, we are continuing with factorial design.

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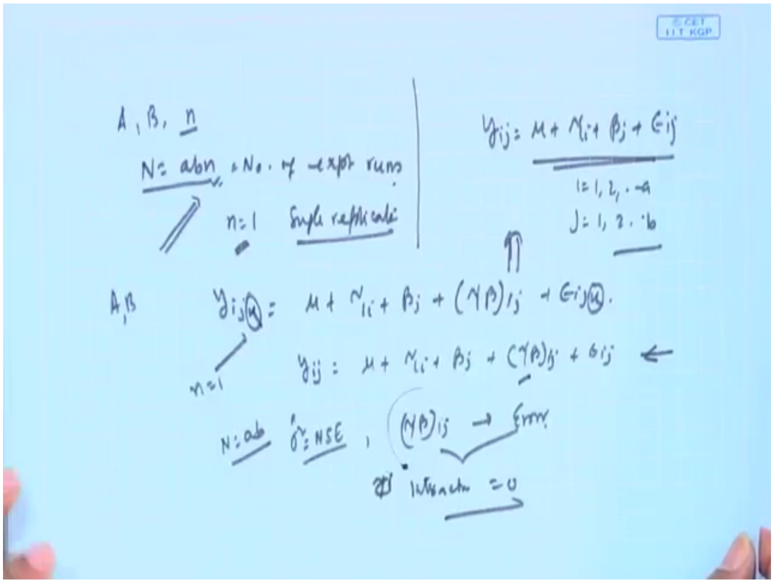

And, you have seen that in case of even two factors with n replicates. So, we require A, B, n this number of experimental runs. Now, it may so happen that there is resource constraint, for example, may be if you are using raw material may be the batch is not sufficient enough to conduct n number of replications for a, b experimental differentiating.

So, or you have time constraint or you have financial constraint. So, all those things the resource constraint will put things differently and may be you will go for n equal to 1, that is known as single replicate or it may so happen that you do not have any other option other than going for single replicate, because more than one is a not available may be somebody else has done replication this experiment or many things.

So, ultimately what I mean to say here that there will be situations when you have to deal with full factorial experiments with single replicate. So, what will happen under such situation and what kind of analysis statistical analysis possible that is what will be discussed in this lecture.

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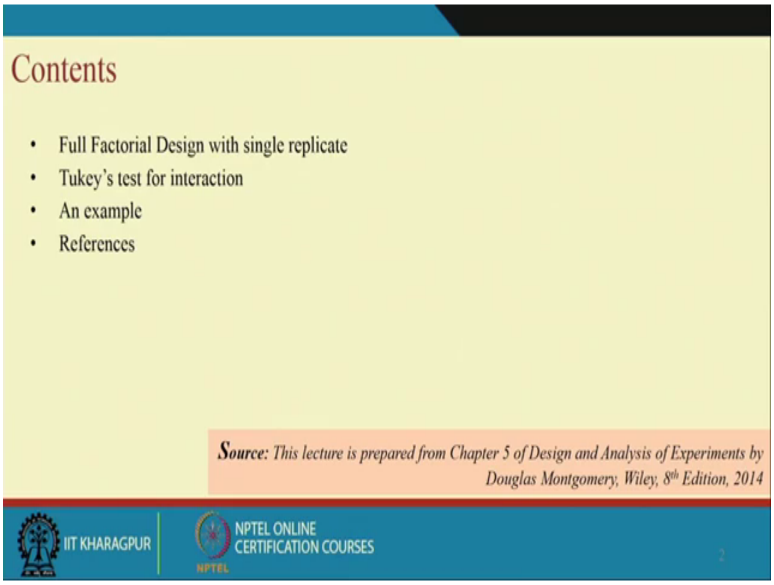

So, full factorial design with single replicate then we will show you a special test known as Tukey test for interactions, this is very important and then I will show you one example and we will explain with example that what how you will analyse the experimental data that is full factorial experimental data with single replications.

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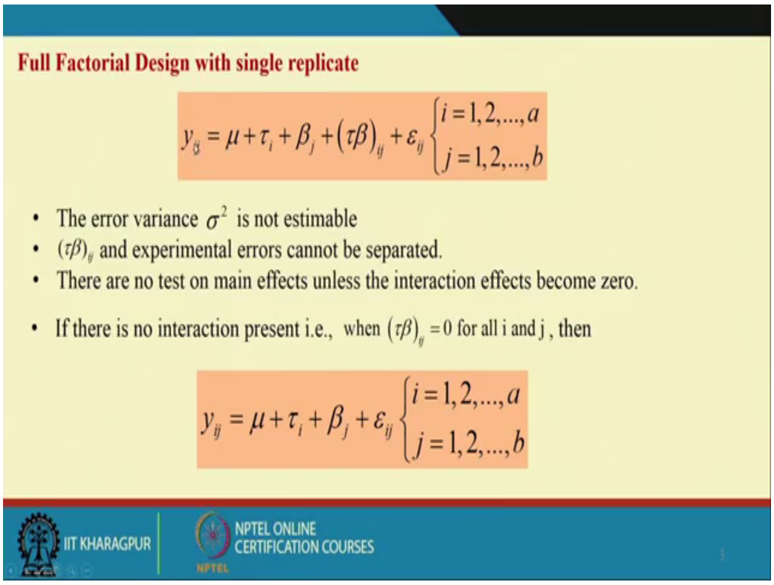

What I am assuming that you all know and, mu y ij is mu plus tau i plus beta j plus tau beta ij plus epsilon ij this is what is the that is what is the fixed effect model and in last lecture we have estimated this parameters with a n replicates. So, also in last 2 last lecture what happened we have seen that ANOVA analysis, analysis of variants and to find out the effect of all those parameters all those and then we have accepted or rejected the null hypothesis for related to factor a, related to factor b, related to their interactions.

Here, when we have we do not have replications or we have only single replicate then the problem is the error variants will not be estimable tau beta ij that is the interaction and the experimental error cannot be separated. So, that mean error these this cannot be estimable because of this thing that error and these are together, confirmed it kind of thing, it cannot be separated. There is no test on main effects unless the interaction effects become 0. So, if there is an no interaction present then what happen we put this equal to 0 and tau beta ij when put equal to 0, the equation become like this.

So, let me repeat this one full factorial design with two factors A and B y ijk we use that is mu plus tau i plus beta j plus tau beta ij plus epsilon ijk. Now, you k stands for replications, if n equal to 1, we do not require this term. So, our observation will be y ij you will be mu plus tau i plus beta j plus tau beta ij plus epsilon ij. So, under such situation if we use this full model what happen because we have only n equal to ab number of observations. This put restriction on estimation of sigma square which is basically NSE error estimate the least NSE, second is that this tau beta ij this one and the error component they will be inseparable.

So, unless the interaction is 0, you cannot go for this kind you have to go for; that means, n equal to more than 1 that is what is this. So, that mean when we can use if tau beta the interaction is 0, then you can go for single replications under the if interaction is 0 then what will happen this y ij equal to mu plus tau i plus beta j plus epsilon ij.

So, this will be our model i equal to 1, 2, a; j equal to 1, 2, b. So, when you do factorial experiments and you are dealing with single replication then please keep in mind that interaction effects will become 0 and that mean you have to test with harsh whether interaction effect is there or not. If you find interaction effect is there then you cannot bill with this you have to go for more number of replications.

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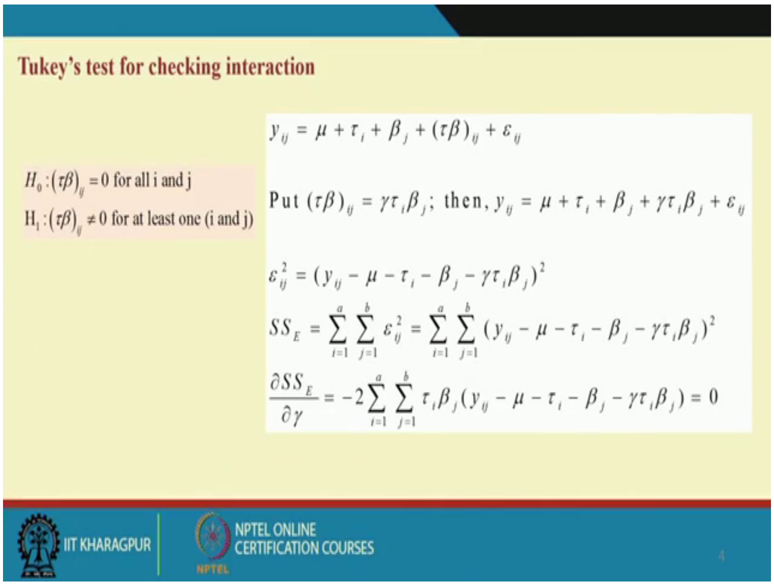

So, if interaction effect is 0, then your model will become simpler and you have to have a test of no interaction, this test is known as Tukeys test.

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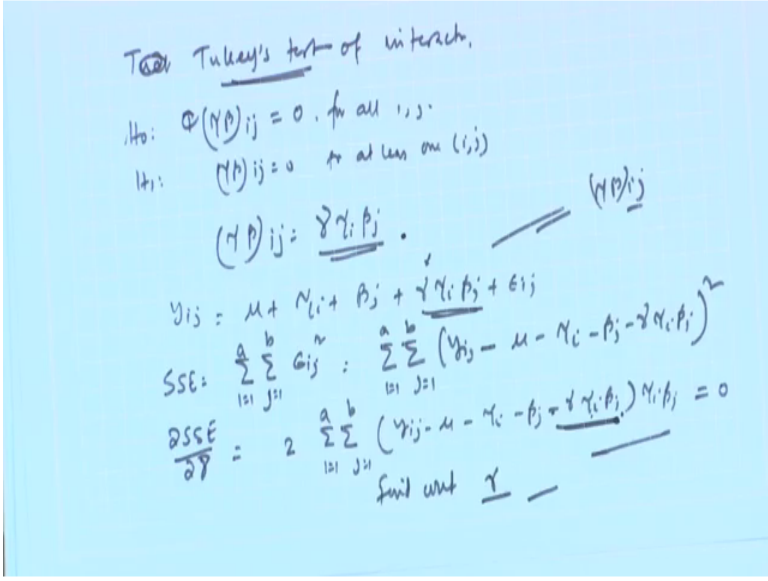

Tukey's test of non additivity or test of interaction. Here, what happen, what is our H 0? H 0 is that tau beta ij this equal to 0 and H 1 is tau beta ij not equal to 0, this is for all ij combination and this is for at least one ij combination. So, here in Tukey's test what happen; you will test this Tukey says that that this tau beta ij this can be taught of gamma a constant into tau i into beta j. Then you put in the equation original equation y ij equal to mu plus tau i plus beta j plus gamma tau i beta j plus epsilon ij because this is what is equal to tau beta ij.

Then you find out the error SS E. SS E will be sum total i equal to j equal to 1, 2, b; i equal to 1, 2, a then epsilon ij square which is nothing, but sum of i equal to 1, 2, a; j equal to 1, 2, b; y ij minus mu minus tau i minus beta j minus gamma tau i beta j this square.

So, what do we want? We want to know the value gamma. So, if you do derivation with respect to gamma then what you will get you will get 2 sum total of i equal to 1, 2, sum total i equal to 1, 2, a j equal to 1, 2, b then all those things will be there y i minus mu minus tau i minus beta j then what happen this minus gamma tau i beta j it will be there plus this one will give you another quantity called that tau i beta j. So, these will give you into tau i beta j put this to 0 and then find out gamma find out gamma.

Now, let us see whether this derivation will take little time. So, I have given you the way you have to do it.

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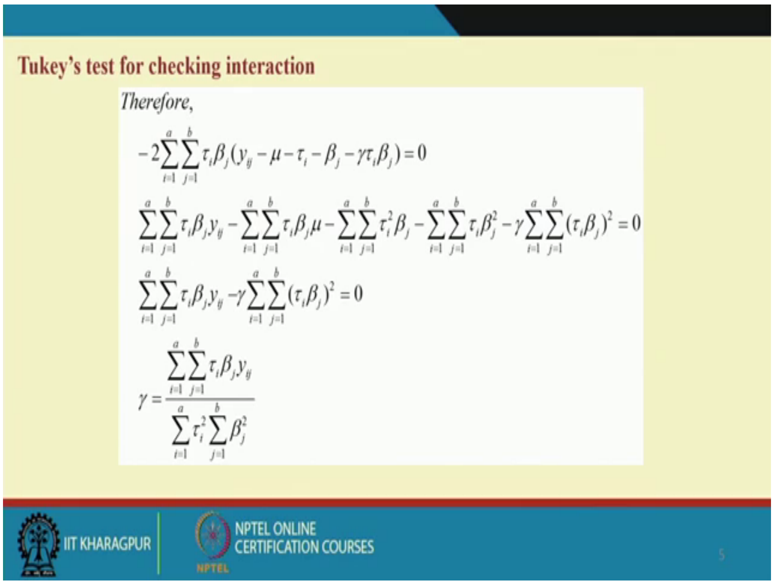

Let us see the second slide here you see this become 0, you multiply all those things. So, the first quantity will be tau i beta j y ij, second one will be tau i beta j mu, third one will be tau i beta j square tau i square beta j, fourth one will be tau i beta j square, fifth one will be gamma tau i square beta j square, what tau i beta j whole square, this will become 0. Now, from these to these what happen these can be the second term we as there is no j a tau i is not involved with j we can bring here, mu we can bring to the left that mean one I just give you one symptom.

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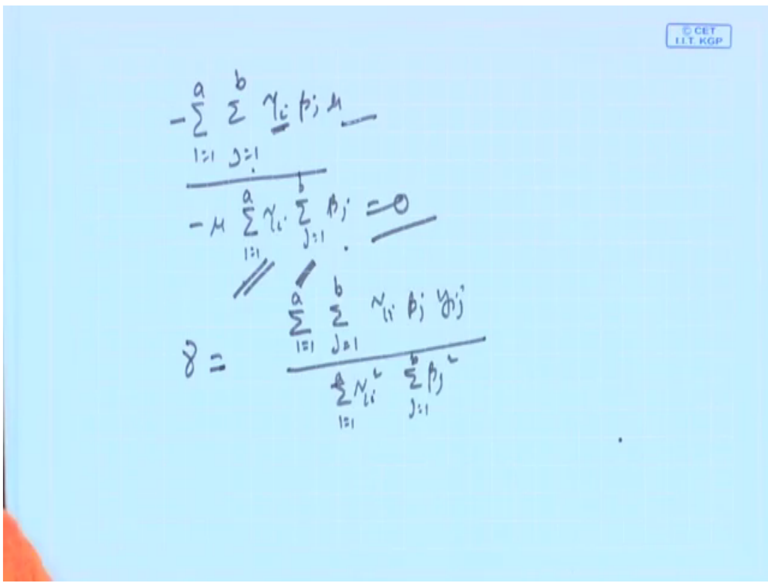

For example, the second term is minus i equal to 1 to a, j equal to 1 to b, tau i beta j and mu. So, these term mu is independent on ij we can write minus mu sum of now tau i is independent of j. So, we can write tau i; i equal to 1 to a, then also we can write j equal to 1 to b beta j this can be write it, but you all know that tau i sum and sum of beta j they are 0. So, this quantity becomes 0 in the same manner the third quantity this will become 0, fourth quantity will become 0. So, we have only two quantity left this one and this one this cannot become 0 because y ij is multiplied here.

So, these minus these, that means, this minus this will be 0. So, what you are getting you are getting gamma equal to sum total i equal to 1 to a, j equal to 1 to b, tau i beta j y ij divided by sum of tau i square sum of beta j square i equal to 1 to a, j equal to 1 to b. So, mean these derivative putting to 0, will give you a gamma value. Now, using this gamma value we will calculate SS N. What is this SS N? That is sum square non additivity (Refer Time: 13:12) this is basically test of interaction. So, that means, it is know the interaction is the tau beta ij is multiplicative with tau i and beta j that sense it is not additive to that.

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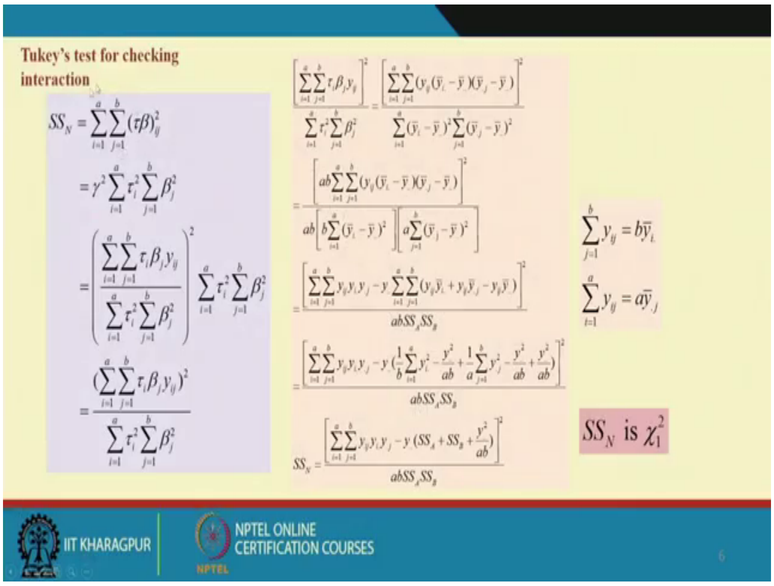

So, SS N is this, we have seen because this is the interaction square sum of this. Now, tau beta ij we have consider at gamma into tau i into beta j. So, that square will be that mean gamma square tau i beta square this for this can be written in this manner and then gamma square value you put here this is the gamma square value you put here. Now, tau i square beta j square this value will be there. So, the lower portion tau i square beta j square this is square term. So, this and this cancel out. So, in the denominator 1 tau i sum of tau i square and into sum of beta j square this will remain and the numerator is the sum of this that i equal to 1 to a, j equal to 1 to b these square.

So, what you will write, these one we know the estimated value of tau i we also know what will be the value of beta j. So, you know y ij it is already given. So, if you put all those values here that estimation part then instead of tau i you are writing y i dot bar minus y dot dot bar, in case of beta j you are writing y dot j minus y dot this bar and y ij as it is you are writing. Then what you do, you manipulate you do algebraic manipulation and bottom hand side this is the estimate for tau i square this is the estimate for beta j square, that you know.

Now, what we have done, we have computed we have multiplied numerator by a square b square and denominator also by a square b square. Then when the a square b square gone in within the bracket to were square is there other is at a b and these a b we keep here and the other two terms; one is attached with b and another one is attached with a because we want to you know that SS a and SS b are nothing, but this is your SS a and this one is SS b, so, ab SS a and SS b it is coming here like this.

Now, here y ij this one now if we know that y j equal to 1 to b y ij is b y i dot bar. Similarly, i equal to 1 to a y ij is a y dot j bar, this is known to us. So, if you put here in the numerator, that mean y ij y i dot and y dot j y dot j minus y i dot and these this quantity you will get. So, this quantity it is just basically the all those summation kind of things.

So, you get from here to here and then with the ab and y dot j this one this can be further a manipulated like these and these finally, result into this. So, what I mean to say here that this these although the equations here looks like a very complicated one, but please there is only algebraic manipulation with here one after I have given all the steps. If you could not understand further you please put your question in the discussion forum. So, if I do the algebraic manipulation it will come like this. So, SS N i equal to 1 to a, j equal to 1 to b, y ij y i dot y dot j minus y i double dot SS a plus SS b plus y dot dot square by ab into square by ab SS a into SS b.

This is the quantity, this is the statistics, which basically test the non additive that is the interaction present or not that sense we will we will use it and this one follows chi square distribution with one degrees of freedom. So, SS N is computed, it is non additivity.

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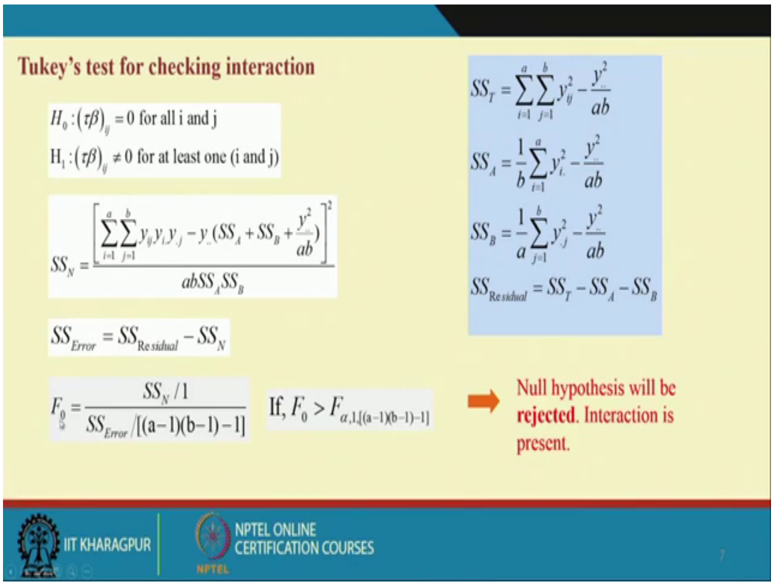

Now, what we have seen that this is what we have seen earlier in the derivation y ij, y i dot this one. So, this now our hypothesis is this then SS error is SS residual minus SS N. You know how to compute SS residual; these are the things we have discussed earlier. So, once you know SS T, A and B SS residual you will be getting from here. Now, SS N is all non additivity part the other part is computed.

So, SS residual minus SS N will give you SS error. So, now, this we will calculate a statistics for F 0 which is SS N by its degrees of freedom by SS error by a degrees of freedom and then this quantity if greater than the threshold F value then we null hypothesis will be rejected then will be interaction will be present our a null hypothesis is no interaction.

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ANOVA for Two-factor model, one observation per cell

Source of variation	Sum of Squares	Degrees of Freedom	Mean Square	Expected Mean Square
Rows (A)	$\sum_{i=1}^{a} \frac{y_{i.}^2}{b} - \frac{y_{..}^2}{ab}$	(a−1)	MS_A	$\sigma^2 + \frac{b\sum \tau_i^2}{(a-1)}$
Columns (B)	$\sum_{j=1}^{b} \frac{y_{.j}^2}{a} - \frac{y_{..}^2}{ab}$	$(b-1)$	MS_B	$\sigma^2 + \frac{a\sum \beta_j^2}{(b-1)}$
Residual or AB	*Subtraction*	$(a-1)(b-1)$	$MS_{Residual}$	$\sigma^2 + \frac{\sum\sum (\tau\beta)_{ij}^2}{(a-1)(b-1)}$
Total	$\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij}^2 - \frac{y_{..}^2}{ab}$	$(ab-1)$		

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After that once you are satisfied with the test that no interaction is there or interaction is absent, in that case what will happen that interaction part will go to the residual part and you will compute the row treatment, column treatment using that additional usual formulas; total, you compute on using another formulas and then residual or AB by subtraction and these are the degrees of freedom.

Similarly, calculate mean squares and then compute the F and you see that whether row treatment effects are there, column treatment effects are there or not, but please remember interaction effects you are not putting into the ANOVA because interaction effect will not be there and that is this will be this is basically only error is there.

And, the expected mean square that from theory; that means, if you say that expected value of MS A will be these expected value will be these expected values will be this and is expected that expected value of MS A and MS B will be more than expected value of MS residual. So, that mean this added quantities this will be these and these will be your will be significant more than this added quantity this one and there will be significant effect.

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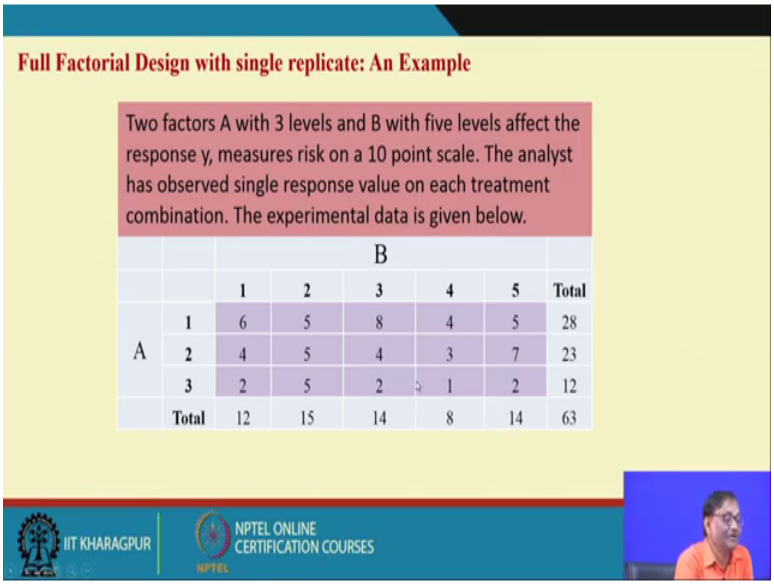

So, one example very quickly; the two factors A with 3 levels and B with 5 levels, suppose effect the response y, we have not are not assumed any physical meaning to AB and response just for the sake of making you understand the computation part we have chosen this example and the analyst has observed single response everywhere single response and we are on the value of each treatment combination the experimental data is given below. So, this is a case of single replicate. Now, what you require to do? You first find out the interaction effect is there or not.

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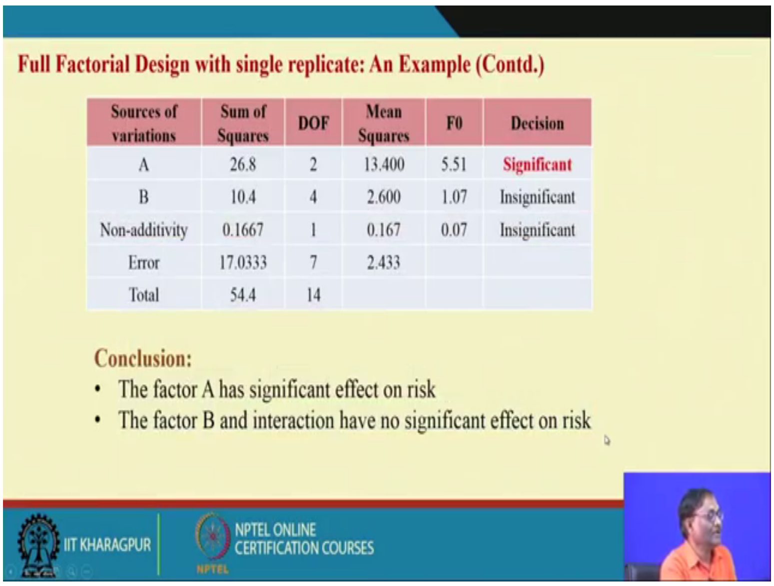

Now, row total, make column total, make overall total and then when you test the non negativity part using this you use these and use F 0 that value is basically 0.1667 that is the sum SS N. SS N is these, SS error is this one and you see that the F 0 value SS N by degree of freedom divided by SS N error by degree of freedom. So, this value is basically and this value is point these value these divided by these this value is 0.07. So, 0.07 it is insignificant value, because if you compare this with the theoretical value like F 17 alpha equal to 0.05 it will be much more than 0.07.

Similarly, if you compute the B effect this value F 0 is 1.07 it is much less than the theoretical one, but A effect is significant, because this 5.51 is more than F2 7, 0.05. So, A effect is significant interaction effect is not there and B effect is not significant. So, what we can conclude here, the factor A has significant effect on risk.

So, if you say that response is a risk response we are saying response is measures on it is a measure of risk on 10 point scale. A and B we have not given any physical meaning, but response measure is a measure of risk on 10 point scale. Then A has no effect A has significant effect on risk factor B and interaction effect no significant effect on risk.

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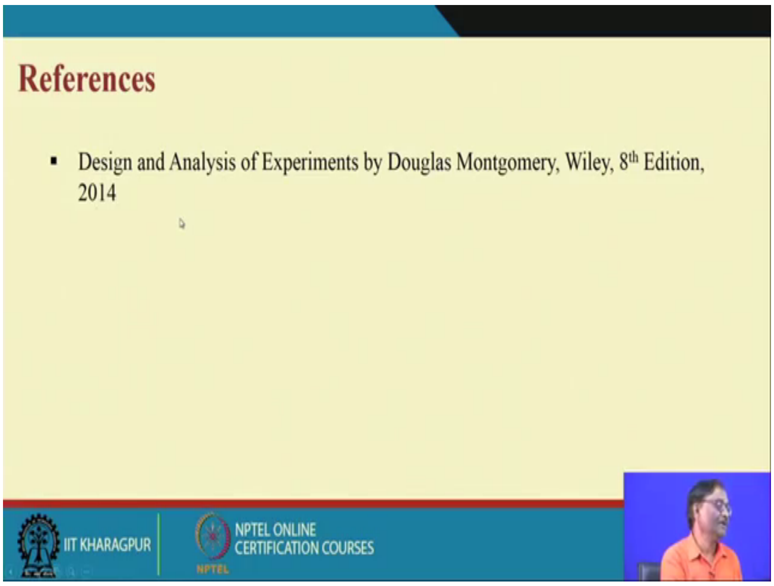

This is what is, that means, factorial design with single replicates. We have used the materials of from the book of Douglas Montgomery, Design Analysis of Experiments. So, I hope that you are in a position to analyse full factorial design with single replicates.

Thank you very much.